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Seminar in Meson Physics

March 4, 1953

Angular Distribution and Deuteron Polarization in $p + p \rightarrow \pi^+ + d$.

Frank Crawford


In discussing the angular distribution of $p + p \rightarrow \pi^+ + d$, perhaps the first thing to notice is that a priori there can be no odd powers of \( \cos \theta \), where \( \theta \) is the angle in the c.m. system between the direction of meson emission and the direction of the "incident" proton. This is true because the initial state consists of two identical particles either of which may be regarded as the "incident" particle, so that no physical effect can depend on replacing \( \theta \) by \( \pi - \theta \). (This would not be true, for instance, in the reaction $n + p \rightarrow \pi^+ + d$.)

Next, let us write down the initial and final momentum states which will conserve total angular momentum and parity. In this we take the meson as pseudoscalar. That is, the spin is zero, and in an (even/odd) state of orbital angular momentum, the meson's parity with regard to coordinate inversion is opposite to that of two nucleons in an (even/odd) orbital angular momentum state. Also, since the deuteron is in an even parity state \((3S_1 + 3D_1)\) the parity of the $\pi^+D$ system is just that of the meson wave function. On the left hand side we do not write down pp states excluded by the Pauli Principle. The \( \pm \) refer to parity.

The lines, both solid and dotted, represent all transitions conserving both total angular momentum and parity, up through F state mesons. The dotted transitions will be later discarded. We include F state mesons in the diagram merely to call attention to the fact that arguments given later as to expected angular distributions are energy dependent.

For incident protons of lab energy 340 Mev, the meson has only about 20 Mev in the c.m. system, so that meson orbital states of $l = 2$ (D state) or higher should have small probabilities compared to $S$ and $P$ states. (Unless, of course, there are selection rules suppressing lower states in favor of higher states.) We can see this semi-classically by writing:

\[
|l^2| = |\Sigma \xi^2_l|^2 = \xi^2_l \sum_{M_l} (2M_l + 1) E_l, \quad \text{and from quantum mechanics}
\]

\[
|l^2| = 2l(l+1)
\]

or, in units of $\hbar/m_{\pi}$, the meson Compton wave length, and range of nuclear forces,

\[
\xi^2(l) = \frac{m_{\pi} c^2}{E_l} l(l+1) = \frac{\hbar c m_{\pi}}{2E_{\text{lab}}} E_l(l+1) = \frac{1}{2} l(l+1)
\]

\[
\xi^2(l) \equiv \text{classical lever arm for angular momentum } l^2 = \hbar^2 (l+1)
\]
Angular Dist. and Deuteron Polarization in p+p \to \pi^+ + D.

For \( \ell = 2 \),

\[ r_2(2) = \sqrt{7 \cdot 2 \cdot 3} = 6.5 \times \text{range of nuclear force} \]

For \( \ell = 1 \),

\[ r_1(1) = \sqrt{7 \cdot 1 \cdot 2} = 3.7 \times \text{range of nuclear force} \]

to give the correct classical lever arm.

Since the two protons must approach at least within the range of nuclear forces before a real meson can be produced \( r_\pi(DH) \leq \frac{1}{2} \), (giving \( \ell = \frac{1}{2} \) classically.) For a given \( \ell \), the orbital wave functions go as \( r_\pi^\ell \) for \( r_\pi^\ell \geq 1 \)\( r_\pi^\ell \).

Therefore, the D state probability amplitude for meson production is attenuated by about

\[ \left[ \frac{r_2(2)}{r_1(1)} \right]^2 = \left[ \frac{6.5}{3.7} \right]^2 \geq 4.2 \]

and the P state by

\[ \left[ \frac{r_1(1)}{r_2(2)} \right]^2 = \left[ \frac{3.7}{6.5} \right]^2 \geq 3.7 \]

compared to what it would be if production could occur with the classical lever arm. The S state is not affected. These amplitudes should be then squared, to get probabilities. Therefore, to compete with meson S state production, meson D states would have to be interaction-favored over S states by about 1600, and meson P states would have to be favored over S states by about 14.

Theoretical estimates predict no such large favoritism for meson D states. Therefore we will drop them. This means we are dropping any possibility of \( \cos^4 \theta \) terms in the angular distribution, since it takes \( \ell \geq 2 \) to produce \( (\cos^2 \theta)^{\frac{\ell}{2}} \) asymmetry. Present experimental accuracy does not reveal any \( \cos^4 \theta \) contributing up to the highest energies looked at so far, about 50 Mev meson c.m. energy. Of course, we may be also dropping some \( \cos^2 \theta \) and spherically symmetrical terms by neglecting D mesons.

On the other hand, meson P states seem to be so much favored over meson S states as to dominate the production already at 20 Mev meson c.m. energy (340 Mev proton beam), as indicated by the large amount of \( \cos^2 \theta \) in the angular distribution. Our results (Stevenson and Crawford, to be published) on the angular distribution at about 20 Mev meson c.m. energy and excitation at 90° in the c.m., together with the excitation at 0° (of Shultz, thesis, UCRL-1756) indicates that this is still true at about 15 Mev meson c.m. energy. This strong P state favoritism is expected from a weak coupling first order theory where the fundamental meson emission process consists of \( p \to n + \pi^+ \) with a nucleon spin flip, the conservation of angular momentum then demanding a meson P state. At low enough meson energies this is expected to be negligible compared to non spin flip emission into meson S states. Strong coupling theories also favor meson P state emission, but prefer total angular momentum 3/2 for the final nucleon and meson relative momentum. Such an emission by a single nucleon of spin 1/2 obviously violates angular momentum conservation, unless a second nucleon is "strongly coupled" in the emission process, to provide a suitable recoil. (All theories demand momentum conservation in all transitions, real or virtual.)

For the above reasons, we will consider at first only the angular distribution for production into meson orbital P states. In any case it should be noticed that there can be no interference between meson S states, and P states, so that
if S states are present, they can only add to (that is, they can't subtract from) the spherically symmetrical part of the angular distribution. This is because the parity of the system is opposite for these two cases and "parity is a good quantum number."

Our diagram of considered transitions is therefore reduced to:

\[ p \rightarrow \pi + D \]
\[ S_0 \rightarrow P_0 \]
\[ D_2 \rightarrow P_2 \]

There will be interference between these two transitions. That is, total angular momentum, although conserved in every transition, is not a "good quantum number."

(In quantum mechanics, if the operator corresponding to a physical quantity commutes with the Hamiltonian of the system, then the time derivative of the expectation value of the quantity is zero, so that the expectation value is a constant of the motion; that is, it is "conserved" in all transitions permitted by the Hamiltonian. In order that the quantity be in addition a "good quantum number" it must be well defined when the energy is well defined. That is, there must be no degeneracy of the system with regard to this quantity. Obviously, two protons - having a definite relative energy but no definite relative lever arm, i.e., a plane incident wave, are completely degenerate with regard to orbital angular momentum, and hence total angular momentum, so that their wave function consists of a sum of orbital (or total) angular momentum eigenfunctions with appropriate coefficients, and we expect interference between various orbital (or total) angular momenta.)

It is interesting to notice that the spin of the pp system happens to be related uniquely to the parity, through the Pauli Principle, the symmetry properties of the singlet and triplet spin states, and because inversion of coordinates is equivalent to interchange of the two particles. Then the fact that parity is a "good quantum number" makes spin also a good quantum number, rather "accidentally." (For instance, the spin is not well defined for the system n + p.) A little later we will present an argument that, in a way somewhat analogous to this, the total angular momentum may be "dragged along" with the isotopic spin so as to become a good quantum number. This would eliminate the interference terms between the S and D proton contributions. We will carry the interference along, however, in the algebra.

Let A and B be the complex probability amplitude for the transitions

\[ (S_0)_{PP} \rightarrow (P_0)_{\pi D} \]
\[ (D_2)_{PP} \rightarrow (P_2)_{\pi D} \]

and, respectively.

We can express this by writing:

\[ \psi_{PP} = A (S_0)_{PP} + B (D_2)_{PP} \rightarrow \psi_{\pi D} (\phi_2, \phi_2, S_2) = A (P_0)_{\pi D} + B (P_2)_{\pi D} \]

Here \( \psi_{\pi D} (\phi_2, \phi_2, S_2) \) is the angular and spin part of the \( \pi^+ D \) wave function.

\( (P_0)_{\pi D} \) is the \( \pi^+ D \) eigenfunction corresponding to total angular momentum zero, \( \pi \) component \( J_\pi = 0 \), and P state mesons.

\( (P_2)_{\pi D} \) is the \( \pi^+ D \) eigenfunction corresponding to total angular momentum \( J = 2 \), \( \pi \) component \( J_\pi = 0 \), and P state mesons.
Angular Dist. and Deuteron Polarization in p+p→π⁺ + D.

These two eigenfunctions must be made up from proper vector combinations of the meson orbital angular momentum \( l = 1 \), and the deuteron spin \( S = 1 \). (We have \( J_z = 0 \) because we start in a spin zero p-p state and hence there can be no way of telling one azimuth from another. This would not be true, for example, in a triplet p-p state, where we could have an \( \hat{\alpha}_l \hat{\beta}_l e^{-i \phi} \) dependence.)*

Let \( \chi^1, \chi^0 \) and \( \chi^{-1} \) denote the normalized deuteron spin one wave functions for \( \pm \) component \( S_z = 1, 0, -1 \) respectively.

Let

\[
\begin{align*}
\gamma^1 &= \frac{\sin \theta}{\sqrt{2}} \chi^1 \\
\gamma^0 &= -\frac{i \cos \theta}{\sqrt{2}} \chi^1 \\
\gamma^{-1} &= -\frac{\sin \theta}{\sqrt{2}} \chi^{-1}
\end{align*}
\]

be the meson P state wave functions for \( L_z = 1, 0, -1 \) respectively, where \( \theta, \phi \) give the meson emission direction with respect to the proton beam, and where the relative normalization is correct.

Then \( (P_0)_{\pi D} \) and \( (P_2)_{\pi D} \) must be formed from those products \( \gamma^L \chi^S \) that give \( J_z = L_z + S_z = 0 \), that is \( \gamma^1 \chi^{-1}, \gamma^0 \chi^0 \) and \( \gamma^{-1} \chi^1 \), with proper coefficients to correspond to \( J = 0 \) and \( J = 2 \), respectively. The proper choices are (see Condon and Shortley)

\[
(P_0)_{\pi D} = \gamma^1 \chi^{-1} + \gamma^{-1} \chi^1 - \gamma^0 \chi^0
\]

\[
(P_2)_{\pi D} = \frac{\gamma^1 \chi^{-1} + \gamma^{-1} \chi^1 + 2 \gamma^0 \chi^0}{\sqrt{2}}
\]

where the relative normalization is correct. Therefore we can write:

\[
\psi_{\pi D}(\theta, \phi, S_Z) = A (P_0)_{\pi D} + B (P_2)_{\pi D}
\]

or,

\[
\psi_{\pi D}(\theta, \phi, S_Z) = (A + B \frac{\sqrt{2}}{12}) \{ \gamma^1 \chi^{-1} + \gamma^{-1} \chi^1 \} + (\sqrt{2} B - A) \gamma^0 \chi^0
\]

The differential cross section is given by:

\[
\sigma(\theta, S_Z) = |\psi_{\pi D}(\theta, \phi, S_Z) |^2
\]

If the detector is incapable of measuring the deuteron spin orientation then we must "integrate" over deuteron spin. That is, we use the orthogonality of the deuteron wave functions,

\[
\chi^S \chi^S' = \delta_{SS'} = 0 \quad \text{if} \quad S_Z \neq S'_Z
\]

and obtain

\[
\sigma(\theta) = |\psi_{\pi D}(\theta) |^2 = |A + \frac{B}{\sqrt{2}} \{ |\gamma^1 |^2 + |\gamma^{-1} |^2 \} + |\sqrt{2} B - A |^2 |\gamma^0 |^2
\]

\[
= |A|^2 - \frac{1}{2} |B|^2 \left( \frac{1}{3} + \cos^2 \theta \right) + 3 \sqrt{2} RA \frac{5}{3} \left( \frac{1}{3} - \cos^2 \theta \right)
\]

*Of course with an unpolarized beam and target, such effects would give no average \( \phi \) dependence.
Let us look at the angular distributions that result from some special cases of interest.

(1) Pure S wave protons.

\[ B = 0 \]

\[ |\psi_{\pi D}(\theta)|^2 = |A|^2 = \text{spherically symmetric, as expected.} \]

(2) Pure D wave protons.

\[ \lambda = 0 \]

\[ |\psi_{\pi D}(\theta)|^2 = \text{const.} \left( \frac{1}{3} + \cos^2 \theta \right) \]

(3) \( J = \) good quantum number (to be discussed later).

Then the interference between \( P_0 \) and \( P_2 \) states disappears.

\[ \text{Re} \ A \not= B = 0 \]

\[ |\psi_{\pi D}(\theta)|^2 = \text{const.} \left( \alpha + \cos^2 \theta \right) \]

where

\[ \alpha = \frac{1}{3} + \frac{2}{3} \frac{|A|^2}{|B|^2} \geq \frac{1}{3} \]

Lack of uniqueness is evident if we consider the converse.

(4) Spherical symmetry.

Equating the coefficient of \( \cos^2 \theta \) to zero,

\[ \frac{3}{2} |A|^2 - 3 |A||B| \cos \alpha_{AB} = 0 \]

and

\[ \alpha_A - \alpha_B \equiv \alpha_{AB} = \text{phase angle between the two interfering states.} \]

This has the solutions \( B = 0 \) and \( |B| = 2 |A| \) \( \cos \alpha_{AB} \), which can give \( |B| = 0 \) to \( 2 |A| \) with proper phase choice.

(5) Pure \( \cos^2 \theta \):

\[ |\lambda|^2 + \frac{1}{2} |B|^2 + |A||B| \cos \alpha_{AB} = 0 \]

This has the unique solution \( A = -\frac{1}{|B|} B \). This happens to be the only case where the relative values of \( A \) and \( B \) are given uniquely by the angular distribution.

(6) \[ \frac{1}{3} + \cos^2 \theta \]

\[ |\lambda|^2 + \frac{1}{2} |B|^2 + |A||B| \cos \alpha_{AB} = \frac{1}{3} \left( \frac{1}{2} |B|^2 - 3 |A||B| \cos \alpha_{AB} \right) \]

This has the solution \( A = 0 \), and the solution \( |A| = \pm \sqrt{\frac{1}{2} |B| \cos \alpha_{AB}} \) which can go from \( |A| = 0 \) to \( |A| = 2 |B| \) for proper choice of phase.
Angular Dist. and Deuteron Polarization in $p+p \rightarrow \pi^+ + D$.

The argument for "J = a good quantum number" mentioned earlier will now be given.

Write $p-p \rightarrow \pi^+ d$ in the form $p+p \rightarrow (\pi^+ n) + p$.

<table>
<thead>
<tr>
<th>$T_1$</th>
<th>$T_2$</th>
<th>$J$</th>
<th>$J_{1/2}$</th>
<th>$J_{3/2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1/2)</td>
<td>(1/2)</td>
<td>(1/2)</td>
<td>(3/2)</td>
<td>(3/2)</td>
</tr>
</tbody>
</table>

This table summarizes the argument from strong coupling theory. Consider the $n\pi^+$ system alone. The $n\pi^+$ system is regarded as a tightly coupled system having a ground state of isotopic spin 1/2, and a first excited state of isotopic spin 3/2. Isotopic spin is conserved. Since we have non-degeneracy, (i.e., two different energy levels) the isotopic spin is also a good quantum number, and we have no interference between isotopic spin 1/2 and 3/2 states. ($T_{1/2} = 1/2$ and $T_{3/2} = 3/2$)

The meson is in a P orbital state about the neutron so that it can combine vectorially with the neutron spin to give

\[ J_{1/2} \rightarrow 1/2 \quad \text{and} \quad J_{3/2} \rightarrow 3/2 \]

Strong coupling theory links $J_{1/2}$ to $T_{1/2} = 1/2$, and $J_{3/2}$ to $T_{3/2} = 3/2$

Therefore the total angular momentum 1/2 or 3/2 of the $n\pi^+$ system becomes a good quantum number, and $J_{1/2} = 1/2$ and 3/2 do not mix. Now we consider the whole system $n\pi^+ p$. If we neglect relative motion of the proton with regard to the neutron, then the meson P state with regard to the neutron will also be a P state with regard to the center of mass of the whole system. We can therefore combine the spin angular momentum of the proton with the $J_{1/2}$ and 3/2 states, and get $J_{1/2} = 1$ or 2, and $J_{3/2} = 2$ is associated with $T_{1/2} = 3/2$, and there can be no interference between them.

As we have seen, this would give $x = 1/3$ in $a + \cos^2 \theta$. If the isotopic spin 3/2 state of the nucleon meson system predominates over the 1/2 state, we get $x = 1/3$ exactly. Our experimental value is 0.32 (1 ± 0.13), which lends support to a strong isotopic spin 3/2 interaction.

Deuteron Polarization

Qualitatively we should expect deuteron polarization, since the incident D wave protons carry more orienting potentiality than is used up in making P wave mesons.

If we write $S_i = i$ th component of deuteron spin, then:

\[ \langle S_i \rangle = \frac{\sum_{i=1}^{2} \psi_{TD}^*(\theta_i, \phi_i, S_z) S_i \psi_{TD} (\theta, \phi, S_z)}{\sum_{i=1}^{2} |\psi_{TD} (\theta_i, \phi_i, S_z)|^2} = |\psi_{TD} (\theta)|^2 \]
Here we "integrate" over deuteron spins, as before. We use the fact that
\[ S_{±} \chi^{±} = \pm \chi^{±} \quad S_{±} \chi^{0} = 0 \quad S_{±} \chi^{-1} = \pm \chi^{-1} \]
where \( S_{±} = S_{x} \pm \epsilon S_{y} \).

\[ |\psi_{πD}(θ)|^2 \]
is the expression already found for \( σ(θ) \).

\[ \langle S_{±} \rangle : \quad S_{±} \psi_{πD} = (A + \frac{B}{\sqrt{3}}) \left( ± \chi^{0} + \chi^{-1} \chi^{1} \right) \]

This is orthogonal to \( ψ_{πD}^* \), so \( \langle S_{±} \rangle = 0 \)

\[ \langle S_{+} \rangle : \quad S_{+} \psi_{πD} = \frac{1}{\sqrt{3}} \left( A + \frac{B}{\sqrt{3}} \right) \chi^{0} + \frac{1}{\sqrt{3}} (\sqrt{3}B - A) \chi^{-1} \chi^{1} \]
so that
\[ \psi_{πD}^* S_{+} \psi_{πD} = (\sqrt{3}B - A) \chi^{0} + (A + \frac{B}{\sqrt{3}}) \chi^{-1} \chi^{1} \]
\[ = 2 \epsilon \text{Im} \{ ω \} (\sqrt{3}B - A) (A + \frac{B}{\sqrt{3}}) \cos θ \sin θ \sin φ \]

\[ \langle S_{-} \rangle : \quad S_{-} \psi_{πD} = - \frac{2}{\sqrt{3}} \text{Im} \{ ω \} (\sqrt{3}B - A) (A + \frac{B}{\sqrt{3}}) \cos θ \sin θ \sin φ \]

If we now use
\[ S_{x} = \frac{S_{+} + S_{-}}{2}, \quad S_{y} = \frac{S_{+} - S_{-}}{2ε} \]
we see
\[ \langle S_{x} \rangle = S_{D} \cdot (- \sin φ) \]
\[ \langle S_{y} \rangle = S_{D} \cdot (\cos φ) \]

where the amplitude
\[ S_{D} = 3 \sqrt{2} \text{Im} \{ ω \} B^* A \frac{\cos θ \sin θ}{|\psi_{πD}(θ)|^2} \]

We see that the vector \( \langle S_{x}, S_{y}, S_{z} \rangle = S_{D} \hat{e}_{⊥} \)

where \( \hat{e}_{⊥} = (- \sin φ, \cos φ, 0) = \) unit vector \( \perp \) to plane of meson emission and beam axis.

We also see that \( S_{D} \) vanishes unless there is interference between \( J = 0 \) and \( J = 2 \), so that if the strong coupling argument given previously holds, we should expect no polarization.
Angular Dist. and Deuteron Polarization in $p+p \rightarrow \pi^+ + D$

To see what polarizations are in principle possible, let us plot $S$ as a function of $\theta$ and as a function of $A - B$. Let us assume $|W_{\pi D}(\theta)|$ has been measured to be proportional to $\frac{1}{2} + \cos^2 \theta$. Then $\frac{|W|}{|b|}$ can vary from 0 to $2\sqrt{2}$, as we have seen earlier, provided the phase of $A$ and $B$ is chosen correctly. In fact, we can write:

$$S_D = M\left(\frac{|W|}{|b|}\right) f(\theta)$$

where

$$M\left(\frac{|W|}{|b|}\right) = 2\sqrt{2} \frac{|W|}{|b|} \left[ 1 - \frac{1}{8} \frac{|W|^2}{|b|^2} \right]^{1/2}$$

and

$$f(\theta) = \frac{\sin \theta \cos \theta}{\frac{1}{2} + \cos^2 \theta}$$

These are plotted separately, since they are independent factors.

We see that we get maximum polarization for $\theta \approx 65^\circ$ in c.m. with $f(65^\circ) \approx .75$. As a function of $\frac{|W|}{|b|}$ we get maximum polarization for $\frac{|W|}{|b|} \approx 1$, with $M \approx 1.32$.

Therefore if we looked at $65^\circ$ in c.m., and if $\frac{|W|}{|b|} \approx 1$, then the expectation value of $S_D$ is $1.32 \times .75 \approx 1$. Since this is just the value of the deuteron spin, we see that a 100% polarized deuteron beam is in principle possible. As was pointed out, the strong isotopic spin $3/2$ interaction gives no polarization. It is perhaps worth pointing out that if, for example, a very strong deuteron polarization were observed, this would not affect the conclusion that the meson spin is zero, based on the principle of detailed balancing, using the measurements in $p+p \rightarrow \pi^+ + d$ and its inverse.