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NUCLEAR STRUCTURE: RECENT DEVELOPMENTS ON HIGH ANGULAR-MOMENTUM STATES IN NUCLEI

F. S. Stephens

Two developments have occurred recently in the study of high angular-momentum states of nuclei. The first was the discovery of a discontinuity (called backbending) around spin $2\hbar$ in the rotational levels of some rare-earth nuclei. The search for the cause of backbending led to the first general considerations of the response of nuclear matter to the addition of angular momentum, and from these have come not only the probable cause of backbending, but also the prediction that more profound changes in the nuclear structure will occur at still higher angular momentum. This has spurred the experimental attack on the higher spin states and very recently some significant progress has been made there. Information about the moments of inertia for states with spins up to $50\hbar$ has been obtained. This is near the limit of nuclear stability against angular-momentum-induced fission (or particle emission) and these recent experiments demonstrate that this whole range of angular momenta possible for nuclei is now accessible for study.

Nuclear Rotation

A system can rotate provided its orientation in space can be specified. Rotation seems to be possible for systems of all known sizes, from nuclei (and probably also elementary particles) up to at least the
galactic level. However, this does not mean that all nuclei can rotate in this simple classical sense. Nuclei have various shapes depending on the shell-model orbits occupied by the individual nucleons, and in the cases where they are spherically symmetric, a spatial orientation cannot be specified, and there are no collective rotational states. For other nuclei the shell structure leads to deformed shapes, which can be oriented in space and rotational states are observed in these cases. It was A. Bohr in 1952 who first stressed that some, but not all, nuclei can rotate in this way, and he and B. R. Mottelson went on at that time to establish the basic features of nuclear rotational spectra.

The rotational energy levels of an even-even and an odd-mass uranium nucleus are shown in Fig. 1. Several important features can be learned from the general structure of these spectra. First, there is just one sequence of levels in each case (no branching), and this indicates that the nuclei have an axis of symmetry and thus just one value for the moment of inertia (though two equivalent rotational axes). For the even-even nucleus, which has all the intrinsic angular momenta coupled to zero in its ground state (a property of all even-even nuclei), every other spin state is missing. This implies that the two ends of the nucleus are indistinguishable, as is the case
for homonuclear diatomic molecules. From measurements
of the quadrupole moments of nuclei in this region it is
known that the nuclei are shaped approximately like
prolate spheroids, somewhat like a football. The energy
 spacings of the levels in Fig. 1 are given to an
accuracy of ~20% by the simple rotational formula,

\[ E(I) = E_o + \frac{\hbar^2}{2I} I(I + 1) \], \hspace{1cm} (1) \]

where \( I \) is the spin, \( J \) is the moment of inertia of
the nucleus, and \( E_o \) is the bandhead energy. It turns
out that for both these nuclei (and others in this
region) \( J \) is about half of what it would be if the
nucleus rotated like a rigid body. This is an
important point for some of the deviations to be
discussed. A. Bohr and B. R. Mottelson showed in
1955 that if all the nucleons moved independently in
orbits of the average central potential, then the
moment of inertia would be just the rigid-body value.
However, the nucleons do not move entirely independ-
dently; there are correlations among particular
groups of nucleons, one of the most important having
to do with the fact that pairs of nucleons tend to
be coupled to zero spin in time-reversed orbits.

These correlations prevent the nucleons from following
completely the rotation, and thus reduce the moments
of inertia by the observed factor of two. A final
point about Fig. 1 has to do with the odd-mass
nucleus, for which the alternate spin states are not missing. Odd-mass nuclei have a residual internal angular momentum due to the odd nucleon, and in such deformed axially-symmetric nuclei this has a constant projection (7/2 for $^{235}$U) pointed one way or the other along the symmetry axis. Thus the two ends of the nucleus are no longer identical and all spin values are present in the spectrum.

Rotational frames of reference are not Lorentz invariant, which means there are Coriolis and centrifugal forces in rotating systems which are not present in the absence of rotation. For the nucleus, this means that the internal nuclear structure will be affected by these forces more and more as the nuclear rotational frequency increases. This is important because it allows us to apply well-known additional forces to the nuclear system and observe the response. The recent calculations suggest that there will be three types of nuclear response. First, there will be a reduction of the pairing correlations. The nucleon moving in an orbit with the same general direction as the rotational motion of the system is favored by the Coriolis force over the one moving oppositely, and thus the time-reversal degeneracy of the orbits, which is so important for the pairing effects, is removed. Second, there will be obvious centrifugal pressures on the nucleus, both to increase
the deformation and (sometimes) to change to a more favorable type of deformation (shape). Finally, there will occur situations where just one nucleon responds to the Coriolis and centrifugal forces and aligns its angular momentum, \( j \), essentially completely with that of the rotating core. This comes about basically because there are a finite number of nucleons in the nucleus, and some of these have much higher \( j \) values than others, and thus feel the effects of the rotation much more strongly. Essentially this is just a quantized realization of the Coriolis and centrifugal effects, and because it considers the orbits of individual nucleons it is often called a microscopic approach in contrast to the classical or macroscopic approach which considers only behavior averaged over all the individual nucleons. The rotation alignment of nucleons has also been found to play an important role in the one-particle states of odd-mass nuclei at high angular momentum values. These three types of nuclear response can be distinguished experimentally and, in fact, some information about all three is now available.

The energies of the rotational states in \( ^{164}\text{Yb} \) are plotted against spin in Fig. 2. The curve is approximately parabolic as expected from Eq. (1), and is rather smooth although there is a change in slope around \( I = 14 \). This change can be seen more clearly
in the insert, which plots essentially the moment of inertia vs. the square of the rotational frequency for the same levels. Now the change in slope appears as a rather dramatic backbend and it is this discontinuity in the level energies that was first discovered by A. Johnson, H. Ryde, and J. Sztarkier in 1971 and was called backbending. At first all three of the above changes were proposed as possible causes of the backbending: a collapse of the pairing correlations; a change in shape; and alignment with the rotation axis of the angular momentum of two high-j nucleons. The evidence now seems to be accumulating that the last of these mechanisms is probably correct for most cases of backbending. In this region of nuclei, it is two neutrons in $i_{13/2}$ orbits that suddenly align. In other regions alignments have now been observed involving other j values. The suddenness of the alignment comes about because the ground band crosses the aligned band at that point and the lowest levels, which are the ones usually observed, correspond to a shift from one band to the other. Continuations of one or both bands into the regions where they do not lie lowest in energy have now been identified in a few cases.

Other changes are also occurring in $^{164}$Yb as the rotational frequency increases. A perfect rotor (Eq. (1)) would follow the dashed horizontal line in
1. the insert of Fig. 2; whereas, it is apparent that
2. the moment of inertia for $^{164}$Yb increases even at the
3. very lowest rotational frequencies. This kind of
4. increase can arise either from centrifugal stretching
5. or from a slow regular decrease of the pairing
6. correlations. Actually, both effects occur, and can
7. be distinguished from each other by determining whether
8. the deformation (as inferred from the measured qua-
9. drupole moments) increases or not. For most nuclei
10. around $^{164}$Yb there is no measurable increase in
11. deformation so that the observed increase in the moment
12. of inertia is due mostly to a reduction of the pairing
13. correlations, resulting in an approach toward the
14. rigid-body moment of inertia (140 MeV$^{-1}$ on Fig. 2).
15. It is estimated that the pairing will be completely
16. quenched by spins of 20 to 30h. Centrifugal stretching
17. has so far been observed only in nuclei that initially
18. have small deformation ($\beta < 0.3$, where $\beta$ is approxi-
19. mately the difference between the major and minor
20. axes divided by the average value). There are effects
21. in the shell structure that make many medium and heavy
22. nuclei soft toward deformation changes out to $\beta \approx 0.3$;
23. however, these nuclei are not expected to stretch
24. much beyond this deformation until very high spin
25. values. Some Hg and Se nuclei actually change shape
26. suddenly from $\beta \approx 0.1$ to $\beta \approx 0.25$, but this kind of
27. behavior appears to be rare. Thus nuclei change in
several ways in response to the addition of angular momentum, and we are beginning to understand something about these processes.

Very High-Spin States

One of the oldest and best ways to estimate the stability of various nuclear shapes is to compare the nucleus with a liquid drop. The liquid-drop model was first developed by N. Bohr and J. Wheeler in 1939 to explain the phenomenon of nuclear fission. The equilibrium shapes of a charged, rigidly rotating liquid drop have since been calculated and they suggest that a nucleus like $^{164}$Yb will have an oblate shape, with $\beta$ slowly increasing up to 0.3, at spin values around 70h. After that the nucleus will rapidly go through triaxial shapes toward prolate shapes terminating in fission at about 80h. This pattern is similar for most nuclei, although the maximum angular momentum varies, being lower for both the heavier and the lighter nuclei. To this smooth macroscopic behavior one must add the shell effects. These depend on the particular orbits involved, which means on the number of protons and neutrons as well as on the shape of the nuclear potential in which these nucleons are found. Calculations of the nucleon orbits in a modified harmonic-oscillator potential were first made by S. G. Nilsson in 1955, and very recently have been extended by
several groups to include the effects of angular momentum. The results of these calculations for $^{160}$Yb are shown in Fig. 3. Here the points give the calculated equilibrium shape at various spin values in a sector whose coordinates are deformation ($\beta$) and shape asymmetry ($\gamma$). It is assumed in these calculations that all pairing effects will be gone by $I = 30$. Generally, the shell structure prefers prolate shapes at low spin values (which is borne out experimentally), but the calculations show that this should be overcome around spin 50$^h$ in $^{160}$Yb by the classical liquid-drop preference for an oblate shape. It should be very interesting to test these calculations. There is no experimental evidence at present to support the reduction in deformation predicted in Fig. 3 around spins 30-40$^h$, but the present information extends only up to $\sim 20^h$. On the other hand, the aligned high-$j$ nucleons, which are responsible for the observed backbending around $20^h$, have orbits that would represent a triaxial bulge in these prolate nuclei, so that this alignment might be considered as the first quantized step toward the oblate shapes expected at high spin values in Fig. 3. For the oblate nuclei, expected between 50 and 70$^h$, all the angular-momentum should be carried by such individually aligned nucleons. The absence of collective motion (around the symmetry axis) might result in the occurrence of
isomeric states in this spin region, which could provide a rather direct experimental test of these calculations.

Part of the reason the high-spin states are so interesting now is the accessibility of these states to experimental study. The heavy-ions reactions (for example $^{40}\text{Ar} + ^{128}\text{Te} \rightarrow ^{164}\text{Yb} + 4n$) can bring over $100\hbar$ into the compound nucleus, which is even more than these nuclei can hold. (The compound nuclei with too much angular momentum fission and, at still higher values, a true compound nucleus is never formed; the composite system undergoes a new process, called quasi-fission.) The gamma-ray deexcitation of these products can give information on the full range of spin states stable against fission. Below about $20\hbar$ this deexcitation produces resolved gamma-ray lines between heavily populated states, which have been a subject of study for 10-15 years, producing a wealth of data on states in this spin range (including the discovery of backbending). However, above $20\hbar$ the population is apparently spread over many states, each of which is so weakly populated that no individual gamma-ray lines can be resolved. Very recently statistical methods have been applied to this gamma-ray "continuum", and it has been possible to identify features ("bumps") due to collective rotational transitions, even though the individual gamma-ray transitions are not resolved.
Study of the energy and detailed shape of these bumps has enabled R. S. Simon and co-workers this year to determine nuclear moments of inertia up to $50\hbar$ in $^{162}$Yb. The information is still rough in this case, and only shows the general approach to the rigid-body moment of inertia; however, it is likely that these methods will soon produce detailed information on the nuclear moments of inertia up to the fission limit in a wide variety of nuclei. We can then learn whether the present ideas about nuclear structure, which lead to the predictions in Fig. 3, are adequate for the description of nuclei under these extreme conditions.

The study of high angular-momentum states in nuclei is now developing rapidly. Rather extensive calculations have been made of the nuclear response to the addition of angular momentum over the full range of states stable against fission. There is now a considerable amount of experimental information for states with spins up to $\sim 20\hbar$, which shows that some dramatic changes in the nuclear structure are occurring in this spin region, as would be expected due to the presence of strong Coriolis and centrifugal forces. Finally, the experimental techniques are just becoming available to study all states stable against fission (angular momenta up to $70-80\hbar$), where much more profound changes in the nuclear structure are predicted to occur.

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Figure Captions

Fig. 1. Rotational energy levels for $^{238}$U and $^{235}$U. Spin and parity values are given on the left of each level, and energies on the right.

Fig. 2. Energy is plotted vs. angular momentum for the ground state rotational-band members of $^{164}$Yb. The insert shows the same data in the type of plot generally used to show backbending behavior, where: $2\hbar^2 2 = (4I - 2)/E_t$ and $(\hbar\omega)^2 = (E_t/2)^2$, with $E_t = E_I - E_{I-2}$

Fig. 3. The points show the equilibrium shapes calculated for the nucleus $^{160}$Yb at various values of the angular momentum. The points are located on a grid which plots (quadrupole) deformation radially and shape asymmetry as a function of angle.
\[
\begin{array}{ccc}
\pi & E(\text{keV}) & \pi \\
24^+ & 3534 & 25/2^- \\
22^+ & 3067 & 23/2^- \\
20^+ & 2619 & 21/2^- \\
18^+ & 2191 & 19/2^- \\
16^+ & 1788 & 17/2^- \\
14^+ & 1415 & 15/2^- \\
12^+ & 1077 & 13/2^- \\
10^+ & 775.7 & 11/2^- \\
8^+ & 517.8 & 9/2^- \\
6^+ & 307.2 & 7/2^- \\
4^+ & 148.4 & 0 \\
2^+ & 44.9 & 0 \\
0^+ & 238_{\text{U}} & 235_{\text{U}} \\
\end{array}
\]
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