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ON PHASE OSCILLATIONS IN THE BEVATRON

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ABSTRACT

This report presents the theoretical arguments concerning imperfections in tracking and deviations in rf amplitude, with formulas and graphs estimating the magnitude of the various effects.
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INTRODUCTION

In a machine of the synchrotron type the accelerating fields give rise to forces that tend to keep the ions bunched near the proper phase for acceleration. If the parameters of the system -- such as the magnetic field or oscillator frequency or amplitude -- vary slowly compared to the period of oscillation about the stable phase, the ion bunch adjusts its phase and spatial position correspondingly without loss of ions or appreciable increase in amplitude of phase oscillation. This is true for unintentional as well as programmed variations. If the variations are more rapid, the bunch does not follow as well; there is a tendency to stimulate phase oscillations, and some of the ions may fall out of synchronism. For example, if an ion bunch is riding at synchronous phase and the accelerating voltage shifts in phase by $10^\circ$ in a time that is short compared to the period, the ions will thereafter oscillate with a $10^\circ$ amplitude.

In the proton synchrotron, unintentional fluctuations in the machine's parameters are particularly serious. The phase-oscillation frequency is in the range of 1 to 2 kilocycles, and there are many sources of disturbance in this audiofrequency range. The acceleration goes on for a long time, so that random variations can lead to a large cumulative effect. Phase damping accompanying the programmed changes in parameters is negligible, so that there are always ions executing large phase oscillations and ready to be dropped out of synchronism by an unfavorable fluctuation.

It is the purpose of this report to present a quantitative treatment of the effects of fluctuations. The essential results have been worked out by the Brookhaven group; here the derivations are presented somewhat differently and examples of local interest are included.

RANDOM FLUCTUATIONS

The phase motion of the ions for small deviations from synchronous phase is described, if damping effects are neglected, by the differential equation

\[
\frac{d^2}{dt^2} \theta + \omega_p^2 \theta = \frac{dW}{dt} - \frac{\omega_0}{(1-n)(1 + \frac{2L}{\pi R})} \frac{1}{H} \frac{dh}{dt} - \frac{d\omega}{\pi} \tan \phi, \quad (1)
\]
where

\[ \theta = \text{deviation from synchronous phase}, \]
\[ \omega = \text{phase oscillation angular frequency}, \]
\[ W = \text{deviation of oscillator frequency from the prescribed value}, \]
\[ h = \text{deviation of magnetic field from the prescribed value}, \]
\[ \omega_o = \text{rotation angular frequency}, \]
\[ \phi_s = \text{synchronous phase}, \]
\[ a = \text{fractional deviation of rf amplitude from the prescribed value}. \]

This equation is sufficiently good for estimating the effects of fluctuations unless the fluctuations are periodic and of frequency near \( \omega_p \). In that case the nonlinear character of the phase oscillations is important, and a different treatment is required. For the present, it is only necessary to remember that the phase motion becomes unstable for \( \theta \approx 1 \) radian under normal operating conditions, so that a fluctuation should be regarded as very serious if the calculation predicts an induced amplitude of that general magnitude. The solution to Eq. (1) may be written

\[ \theta = \theta_o \sin (\omega_p t + \delta) + \Theta (t), \]  

where

\[ \Theta (t) = \frac{1}{\omega_p} \int_0^t \left[ \frac{dW}{dt'} - \frac{\omega_o}{\pi R} \frac{1}{1-n} \frac{dh}{dt'} \right] \sin \omega_p (t-t') dt'. \]  

The first term represents the oscillation due to the initial phase displacement and velocity, and the second term is the additional phase displacement induced by the fluctuations. The simplest case to consider is that in which the disturbances are sinusoidal with angular frequency \( \mu \). For example, if

\[ a = 0, \]
\[ W = W_o \cos (\mu t), \]
\[ h = 0, \]

then

\[ \Theta (t) = \frac{-W_o \mu}{\omega_p^2 - \mu^2} \left[ \sin \mu t - \sin \frac{\omega_p}{\omega_p^2 - \mu^2} \right]. \]

The frequency modulation \( W \) is of course accompanied by a phase modulation,

\[ \psi = \int_0^t W dt' = \psi_o \sin \mu t = \frac{W_o}{\mu} \sin \mu t. \]
\( \Theta(t) \) is in general small unless \( \mu \approx \omega_p \), in which case the formula does not apply, as mentioned above. However, it is probably indicative for, say,

\[ \mu = \sqrt{2} \omega_p, \]

in which case we have

\begin{align*}
\Theta(t) = & -\frac{2W_0}{\omega_p} \int_0^t a(t') \sin \omega_p (t-t') dt' \\
+ & \int_0^t \left[ \sqrt{W(t')} - \frac{\omega_0}{(1-n)(1+\frac{2L}{\pi R})} \frac{h(t')}{H} \cos \omega_p (t-t') dt' \right. \\
& + \left. \text{boundary terms} \right]
\end{align*}

in the presence of frequency modulation. Thus 20° of phase modulation could be serious even for a frequency 40% off resonance.

Next, consider random fluctuations. For this purpose, it is better to rewrite Eq. (3) by integrating by parts to make frequency and field errors appear explicitly:

\[ \Theta(t) = -\omega_p \tan \phi_s \int_0^t a(t') \sin \omega_p (t-t') dt' \\
+ \int_0^t \left[ \sqrt{W(t')} - \frac{\omega_0}{(1-n)(1+\frac{2L}{\pi R})} \frac{h(t')}{H} \cos \omega_p (t-t') dt' \right. \\
& + \left. \text{boundary terms} \right]
\]

or, if phase errors are of primary interest, to introduce

\[ \psi(t') = \int_0^t W(t'') dt'' \]

and integrate again by parts:

\begin{align*}
\Theta(t) = & \omega_p \tan \phi_s \int_0^t a(t') \sin \omega_p (t-t') dt' \\
& - \frac{\omega_0}{(1-n)(1+\frac{2L}{\pi R})} \int_0^t \frac{h(t')}{H} \cos \omega_p (t-t') dt' \\
& - \omega_p \int_0^t \psi(t') \sin \omega_p (t-t') dt' \\
& + \text{boundary terms.}
\end{align*}

The boundary terms do not lead to cumulative effects and may be dropped.

To illustrate the method of calculation of the cumulative effect of random fluctuations, suppose that the frequency changes by an amount \( w \) for a duration of time \( \tau \) starting at random times \( t_i \). Then, from Eq. (4), we have
\[ \Theta(t) = \sum_{i} w \left( \frac{t_{i} + \tau}{t_{i}} \right) \cos \frac{\omega_{p}}{2} (t - t_{i}) dt \]

\[ = - 2 \sin \frac{\omega_{p}}{2} \sum_{i} \frac{w}{\omega_{p}} \cos \omega_{p} (t - t_{i} - \frac{\tau}{2}) \quad (6) \]

where the sum is for all \( t_{i} < t \).

After \( N \) such disturbances, and before the \((N+1)\), \( \Theta \) will be oscillating freely, as can be seen by rewriting Eq. (6):

\[ \Theta(t) = - 2 \sin \frac{\omega_{p}}{2} \left\{ \cos \omega_{p} t \sum_{i=1}^{N} \frac{w}{\omega_{p}} \cos \omega_{p} (t_{i} + \tau/2) \right. \]

\[ \left. + \sin \omega_{p} t \sum_{i=1}^{N} \frac{w}{\omega_{p}} (t_{i} + \tau/2) \right\} \]

The amplitude is the square root of the sum of the squares of the two parts \(90^\circ\) out of phase:

\[ (\text{ampl})^{2} = 4 \sin^{2} \frac{\omega_{p}}{2} \left\{ \left[ \sum_{i=1}^{N} \frac{w}{\omega_{p}} \cos \omega_{p} (t_{i} + \tau/2) \right]^{2} \right. \]

\[ \left. + \left[ \sum_{i=1}^{N} \frac{w}{\omega_{p}} \sin \omega_{p} (t_{i} + \tau/2) \right]^{2} \right\} \]

\[ = 4 \sin^{2} \frac{\omega_{p}}{2} \left( \frac{w}{\omega_{p}} \right)^{2} \left\{ \sum_{i=1}^{N} \left[ \cos^{2} \omega_{p} (t_{i} + \tau/2) \right. \right. \]

\[ \left. + \sin^{2} \omega_{p} (t_{i} + \tau/2) \right] + \text{cross terms} \right\} . \]

Now if \( N \) is large and phases \( \omega_{p} (t_{i} + \tau/2) \) are random, the cross terms will be positive as often as negative, so that the positive quadratic terms, which add up simply to \( N \), will dominate. Then

\[ \text{ampl} \sim \left| \frac{2w}{\omega_{p}} \sin \frac{\omega_{p}}{2} \right| \sqrt{N}. \quad (7) \]

The actual amplitude induced in a given acceleration period would be larger or smaller depending on the precise succession of \( t_{i} \)'s; but if this mean-square formula indicates an induced amplitude of 1 radian or more, one would expect that successful acceleration pulses would be rare, the more so the larger the indicated amplitude.
Equation (7) demonstrates the sensitivity of the beam to such fluctuations. For example, take
\[
\omega_p = 2 \pi \times 2000 \text{ sec}^{-1},
\]
\[
w = 2 \pi \times 100 \text{ sec}^{-1},
\]
\[
\tau = 200 \mu\text{sec};
\]
then this type of disturbance could only be allowed 100 times; a total duration of 0.02 second with a phase slip each time of ±7°!

Corresponding calculations for a sudden phase shift \(\psi_o\) or jump in rf level lead again to Eq. (7) with \(\frac{w}{\omega_p}\) replaced by \(\psi_o\) or \(a_o\tan \phi_s\) respectively. If the disturbance consists of a positive square pulse followed immediately by a negative one (simulating a differentiated square pulse) the effect is to replace \(\sin \frac{\omega_p \tau}{2}\) by \((1-\cos \frac{\omega_p \tau}{2})\) in Eq. (7). The magnitudes of the effects are thus comparable unless the duration is short compared to a phase-oscillation period, in which case the single square pulse is relatively more serious.

Before random errors are discussed further, it is worth noting that the above development can be applied to the question that occasionally arises concerning the phase motion induced by the use of current markers in tracking. The frequency does not track continuously but in a succession of straight lines, changing slope at each current marker. If the frequency error at each marker is \(w\) and each comes at time \(t_n\), Eq. (4) becomes
\[
\Theta(t) = \sum_n \frac{w_{n+1} - w_n}{\omega_p} \left[ \cos \frac{\omega_p (t-t_{n+1})}{2} - \cos \frac{\omega_p (t-t_n)}{2} \right]
\]
summed over \(t_n < t\).

The successive magnitude and phases are unrelated to the phase-oscillation frequency and may be treated as though random. The mean square amplitude is then
\[
\text{ampl} \sim \sqrt{2N} \left( \frac{\Delta \omega}{\omega_p} \right)_{\text{rms}},
\]
where \(N\) is the number of current markers passed. This might amount to \(10^9\) for a 1% rms error in oscillator frequency.

**Noise**

A type of fluctuation always present is that known as noise. Since it is difficult in this case to distinguish a structure and estimate the number of fluctuations, as demanded by the preceding formulas, a different formulation is indicated. It is shown later that the two approaches lead to the same result.
Consider an ion suddenly subjected to a series of errors in field, rf voltage, frequency, and (or) phase. The disturbances persist for a relatively long time $T$, after which we want to know the magnitude of the induced oscillations. For definiteness we shall think of frequency fluctuations, though the same analysis applies to any of the types. First decompose the fluctuation in a Fourier series of basic period $T$:

$$\frac{W(t)}{\omega_0} = \sum_n \epsilon_n \sin \left( \frac{2\pi nt}{T} + \alpha_n \right). \quad (8)$$

The summation extends from 1 to a value of $n$ corresponding to the highest frequencies passed by the system; that is, to

$$n_{\text{max}} = T \omega_{\text{max}}.$$ 

"Noise" means a type of fluctuation that has a Fourier decomposition in which the phases $\alpha_n$ are random and the amplitudes $\epsilon_n$ are selected from a probability distribution of some sort. That is, on successive acceleration periods the noise is different in detail and the $\epsilon_n$'s quite different if the particular structures are analyzed; but after many periods, the $\epsilon_n$'s fall into a certain probability pattern. "White" noise means that the amplitudes follow a Rayleigh distribution,

$$P(\epsilon) \, d\epsilon = \frac{\epsilon^{2-n}}{2} \exp \left(-\frac{\epsilon^2}{2} \right) \, d\epsilon,$$

where $\alpha$ is the rms value of $\epsilon_n$.

This calculation, however, involves only estimates based on mean square values, so that it is not necessary to distinguish between various distribution laws. Before determining the phase oscillations it is appropriate to relate the rms value of $W$ to the $\epsilon_n$'s:

$$\left( \frac{W}{\omega_0} \right)^2 = \frac{1}{T} \int_0^T dt \sum_{n, m} \epsilon_n \epsilon_m \sin \left( \frac{2\pi nt}{T} + \alpha_n \right) \sin \left( \frac{2\pi mt}{T} + \alpha_m \right)$$

$$= 1/2 \sum_{n=1}^{n_{\text{max}}} \epsilon_n^2 = 1/2 \epsilon_n^2 T \omega_{\text{max}}^2.$$ 

That is,

$$\epsilon_{\text{rms}} = \left[ \frac{2}{T \omega_{\text{max}}} \left( \frac{W}{\omega_0} \right) \right]^{1/2} \quad \text{rms} \quad (9)$$
Inserting Eq. (8) in Eq. (4) and integrating to time $T$, we have

$$
\Theta(T) = \sum_n \frac{\omega_n T}{2\pi} \frac{\sqrt{\cos a_n (1 - \cos \frac{\omega_p T}{2\pi n}) + \frac{\omega_p T}{2\pi n} \sin \omega_p T \sin a_n}}{n^2 - \left(\frac{\omega_p T}{2\pi}\right)^2} \frac{1}{\psi}.
$$

(10)

In this form it is apparent that only the Fourier components in the neighborhood of the phase-oscillation frequency are important.

Let $n = N + \nu$, where $N$ is the largest integer less than $\frac{\omega_p T}{2\pi}$.

Then, if only terms for $\nu << N$ are retained, Eq. (10) becomes

$$
\Theta(T) = \frac{\omega_o T}{2\pi} \sum_n \frac{\epsilon_n}{\psi} \sin \psi/2 \sin (\psi/2 + a_n).
$$

where $\psi = \omega_p T - 2\pi N$.

Then $\Theta^2(T) = 1/2 \left(\frac{\omega_o T}{2\pi}\right)^2 \sum_n \frac{\epsilon_n^2}{\psi^2} \sin^2 \psi/2$,

where the cross terms have been dropped as in Eq. (7), this time because of the randomness of $a_n$, and $\sin^2 (a + \psi/2)$ has been replaced by its average value, $1/2$.

The phase $\psi$ is peculiar to the exact time $T$ of observation and should be averaged out:

$$
\Theta^2(T) = \frac{1}{2} \left(\frac{\omega_o T}{2\pi}\right)^2 \sum_n \epsilon_n^2 \int_0^{2\pi} \frac{\sin^2 \psi/2}{(\nu - \psi/2)^2} d\psi
$$

$$
= \frac{\pi^2}{2} \left(\frac{\omega_o T}{2\pi}\right)^2 \sum_n \epsilon_n^2 \int_0^{2\pi} \frac{\sin x}{2\pi (\nu - 1)} dx.
$$

Finally, to obtain a means square value for $\Theta$, we should replace each $\epsilon_n^2$ by its mean-square value:

$$
\Theta^2(T) = \frac{\pi}{2} \left(\frac{\omega_o T}{2\pi}\right)^2 \sum_n \epsilon_n^2 \int_0^{2\pi} \frac{\sin x}{2\pi (\nu - 1)} dx = \frac{\pi}{2} \left(\frac{\omega_o T}{2\pi}\right)^2 \frac{\epsilon^2}{2}\sum_n \epsilon_n^2.
$$

Then, using Eq. (9), we have

$$
\Theta_{\text{rms}} = 1/2 \omega_{\text{rms}} \left(\frac{T}{\nu_{\text{max}}}\right)^{1/2}.
$$

(11)
Equation (11) imposes a rather strict condition on the rf system. For example, if $T = 1$ sec, $v_{\text{max}} = 2 \times 10^4$ cps and $\theta_{\text{rms}}$ is required to be negligible -- i.e., $5^\circ$ -- the tolerance on noise level is

$$\frac{W_{\text{rms}}}{2\pi} < 4 \text{ cps},$$

which is about $1$ part in $10^5$ of the oscillator frequency. It is noteworthy that the induced amplitude does not depend on phase-oscillation frequency or rotation frequency; in fact, it does not even depend on bandwidth, for $\frac{W_{\text{rms}}}{\sqrt{\nu_{\text{max}}}}$ is strictly the spectral density in the vicinity of the phase-oscillation frequency.

The earlier calculation can be rephrased in noise terminology. For example, Eq. (7), for square impulses of short duration, is

$$(\text{ampl})_{\text{rms}} = w\tau \sqrt{N}.$$  

In this case we have

$$\frac{W^2}{\nu^2} = w^2 \frac{\tau N}{T},$$

for $N$ impulses in total elapsed time $T$. Therefore

$$(\text{ampl})_{\text{rms}} = W_{\text{rms}} \sqrt{T\tau}.$$  

However, a Fourier analysis of a succession of square pulses of duration $\tau$ would contain frequencies up to $v_{\text{max}} \sim 1/\tau$. Therefore the expression for the amplitude agrees in magnitude and functional form with Eq. (11).

Finally, the formulas corresponding to Eq. (11) are:

for rf amplitude noise,

$$\theta_{\text{rms}} = a_{\text{rms}} \frac{\omega}{2} \tan \phi_s \frac{T}{\sqrt{\nu_{\text{max}}}},$$

and

for magnetic field fluctuations,

$$\theta_{\text{rms}} = \frac{1}{2} \frac{\omega_0}{(1-n)(1+\frac{2L}{\pi})} \left(\frac{H_{\text{rms}}}{h}\right) \frac{T}{\sqrt{\nu_{\text{max}}}}.$$  

Influence of Damping

The characteristic effect of damping can be seen by considering a simple case -- that of an oscillator subjected to a succession of impulses, equal in magnitude but applied at random times. First, if damping is neglected, the motion after the first impulse at time $t_1$ will be

$$\theta = \delta \sin \omega_p(t-t_1),$$
where $\delta$ is the amplitude induced by a single impulse applied to the oscillator initially at rest. After the second impulse, the motion will be

$$\theta = \delta \sin \omega_p (t-t_1) + \delta \sin \omega_p (t-t_2),$$

and so on.

Applying the argument following Eq. (6), we have the rms amplitude after $N$ impulses,

$$\text{(ampl)}_{\text{rms}} = \sqrt{N} \delta. \quad (12)$$

If the oscillations are damped, then the motion following $t_1$ will be

$$\theta = \delta D(t, t_1) \sin \omega_p (t-t_1),$$

and so on. Here $D(t, t_1)$ is the amount by which the amplitude has decreased from time $t_1$ to $t$. The argument is now slightly modified, for after averaging out the oscillating terms the damping factors remain,

$$\text{(ampl)}^2 = \delta^2 \sum_{i=1}^{N} D^2(t, t_1) \sim \delta^2 \frac{dN}{dt} \int_{t_1}^{t} D^2(t, t') dt', \quad (13)$$

where $\frac{dN}{dt}$ is the average rate at which the impulses occur.

In most familiar oscillating systems the damping is exponential; i.e.,

$$D(t, t_1) = e^{-a(t-t_1)},$$

where $a$ is a positive number. In this case, Eq. (13) can be integrated, with the result

$$\text{(ampl)}^2 = \delta^2 \frac{dN}{dt} \frac{1 - e^{-2a(t-t_1)}}{2a}.$$

For $t$ sufficiently large,

$$\text{(ampl)}^2 \approx \frac{\delta^2}{2a} \frac{dN}{dt},$$

independent of time. This is why a pendulum does not start swinging by itself under the impulses of air molecules striking it. One should say that it starts to swing but, because of damping, quickly settles down to an unobservably small amplitude.

Damping in an accelerator is qualitatively different in that it follows a power law rather than an exponential. Using

$$D(t, t') = \left( \frac{t'}{t} \right)^n,$$

we get, by integration of Eq. (13),

$$\text{(ampl)}^2 = \delta^2 \frac{dN}{dt} \frac{t}{2n+1} \left[ 1 - \left( \frac{t_1}{t} \right)^{2n+1} \right],$$
which becomes, for \( t \gg t_1 \), \[
(\text{ampl})^2 = 6^2 \frac{N}{2n+1},
\]
identical with Eq. (12) except for the numerical factor in the denominator. For \( n = 1/4 \) or \( 1/2 \), as is common in accelerators, the change is not great. A power-law damping only decreases the rate of build-up somewhat, but does not limit the amplitude. The conclusion would be that damping in an accelerator is not of much help in counteracting random disturbances.

Sinusoidal Perturbation Near Resonance

The differential analyzer has been used to develop the phase motion of a synchronous particle after the initiation of a sinusoidal frequency or phase perturbation. These trajectories are difficult to predict analytically because of the nonlinear nature of the phase oscillations. The nonlinearity is of great value in preventing loss when the perturbing frequency is near the phase-oscillation frequency, so much so that the ions can tolerate a much larger perturbation of pure frequency than of noise.

The results of the analyzer runs are summarized in Table I. Each box represents a run, and shows the amplitude of phase modulation (denoted by A) and the resulting maximum phase deviation of the ion from synchronism or the survival time if the motion is unstable (denoted by B). \( \omega/\omega_p \) is the ratio of driving frequency to the phase-oscillation frequency for small amplitudes. Some of the runs are shown in detail in Fig. 1 as examples; marked *.

The results are also given in the form of graphs (Figs. 2a and 2b). Here an attempt is made to plot the phase- or frequency-modulation amplitude that separates the stable from the unstable region. The division is not at all sharp, for not all ions would be damped by the same perturbation, the effect depends on the amplitude and phase of their phase oscillations. The dividing line does, however, agree reasonably well with experiments in which perturbations are deliberately introduced.

It is worth noting the gain in stability due to the nonlinear effects. From Table I, for example, we see that \( 3^0 \) of phase modulation at resonance produces a \( 60^0 \) excursion of the ion; a \( 60^0 \) amplitude would be induced in a linear oscillator by a modulation of \( 0.01^0 \) acting during a 1-sec acceleration period.

REFERENCES

2. See also N. M. Blachman, Rev. Sci. Instr. 23, p. 250.
Fig. 1 Typical plot of phase motion $\Delta \theta$ as a function of time under the influence of a sinusoidal perturbation.
Fig. 2a Effect of phase-modulation amplitude $\psi$ on phase motion as a function of driving frequency $\omega$. Amplitudes greater than those corresponding to the curve should produce serious loss of beam.
Fig. 2b Effect of frequency-modulation amplitude $\Delta \omega$ on phase motion as a function of driving frequency $\omega$. Amplitudes greater than those corresponding to the curve should produce serious loss of beam.
Table I

Results of analyzer runs for different $\omega/\omega_p$ ratios. A denotes amplitude of phase modulation; B denotes maximum phase deviation from synchronism (or survival time if motion is unstable).

<table>
<thead>
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<th>$\omega/\omega_p$</th>
<th>0.6</th>
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<th>1.0</th>
<th>1.2</th>
<th>1.4</th>
<th>1.8</th>
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A: Amplitude of phase modulation; B: Maximum phase deviation from synchronism (or survival time if motion is unstable).