Injunctions, Hold-Up, and Patent Royalties*

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Abstract

This paper studies royalty negotiations between a patent holder and a downstream firm selling a product which is more valuable if it includes a feature covered by the patent. Royalties are negotiated in the shadow of patent litigation, which will determine whether or not the patent is valid and infringed. If the two firms negotiate after the downstream firm has already designed its product to include the patented feature, the negotiated royalty rate exceeds the natural, normative benchmark level due to the patent holder’s ability, if the patent is found valid and infringed, to obtain a permanent injunction preventing the downstream firm from selling its product until it can introduce a non-infringing version. Royalty over-charges are greatest for weak patents covering minor features of products sold at prices well above marginal cost. The downstream firm’s ability to develop a non-infringing version of its product during the pendency of litigation can reduce but not eliminate these royalty over-charges. Royalty over-charges persist even if negotiations occur before the downstream firm designs its product. Indeed, for weak patents, the downstream firm gets no benefit from the ability to negotiate a license before designing its product. However, royalty over-charges are reduced if the courts stay injunctions to provide time for infringing firms to design non-infringing versions of their products.

Keywords: patents, innovation, licensing, hold-up

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1. Introduction

The U.S. patent system is widely seen as out of balance and in need of reform. The Federal Trade Commission (2003) expressed concern that competition is being harmed because the U.S. Patent and Trademark Office (PTO) is issuing many “questionable” patents, i.e., patents likely to be invalid or contain overly broad claims. The National Academies of Science (2004) expressed concern that many patents are issued for inventions that should in fact be considered “obvious” and called for new administrative procedures allowing third parties to challenge patents. Jaffe and Lerner (2004) argue powerfully that the U.S. patent system is broken and requires fundamental reform. Congress is currently considering major patent reform.

This paper shows how the rules regarding injunctions and damages affect the royalties that will be negotiated between patent holders and licensees who are using the patented technology. Focusing on negotiated royalties is empirically justified since far more patents are licensed than are litigated to judgment.¹ This paper takes as given the set of patents that are being issued by the PTO, while recognizing that patents differ widely in their “strength,” i.e., the probability they will be held valid and infringed if litigated to judgment.

The principal finding in this paper is that the current U.S. patent system systematically over-rewards the owners of patents in the information technology sector who license rather than practice their patents. These over-rewards are greatest for the owners of weak patents that cover minor features of complex products sold at prices well above marginal cost. Holders of such patents are over-rewarded relative to a natural normative benchmark primarily because of their ability to obtain injunctions in the event they prevail in patent litigation. While such permanent

¹ See Lemley and Shapiro (2005) for a general discussion of the empirical evidence regarding patent licensing and litigation.
injunctions are fundamental to the property rights typically associated with patents, some modest reforms in the rules governing permanent injunctions can significantly improve the operation of the patent system.

The issues raised here are starkly illustrated by the widely publicized patent infringement case between NTP, Inc. and Research in Motion (RIM). NTP, a patent holding company, claimed that RIM, the provider of the popular BlackBerry wireless e-mail device, had infringed several of NTP’s patents. After a jury found NTP’s patents valid and infringed by RIM, NTP asked the court to issue an injunction to stop RIM from selling infringing BlackBerry devices. As a result, RIM came under enormous pressure to settle the case to avoid a shutdown of the BlackBerry service, which could have resulted from a court injunction forcing RIM to stop infringing NTP’s patents. In March 2006, RIM paid $612.5 million to NTP to settle the case. To many observers, this payment reflected the strong bargaining position NTP enjoyed by virtue of its threat to shut down BlackBerry service, not the underlying value of NTP’s patented technology.

Even more recently the Supreme Court acknowledged problems with the use of injunctions in patent cases. In eBay v. MercExchange, a unanimous court struck down the approach taken by the Federal Circuit Court of Appeals, under which permanent injunctions were issued “absent exceptional circumstances.” The Supreme Court ruled that the district court has discretion whether to grant or deny injunctive relief based on traditional principles of equity, using a four-factor test. The concurring opinion written by Justice Kennedy, joined by Justices Stephens, Souter, and Breyer, states:

“When the patented invention is but a small component of the product the companies seek to produce and the threat of an injunction is employed simply for undue leverage in negotiations, legal damages may well be sufficient to compensate for the infringement and an injunction may not serve the public interest. In addition injunctive relief may have different consequences for the burgeoning number of patents over business methods, which were not of much economic and legal significance in earlier times. The potential vagueness and suspect validity of some of these patents may affect the calculus under the four-factor test.”

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3 eBay Inc. et. al. v. MercExchange, LLC, Decided May 15, 2006.
Precisely this fear of injunctions has led many leading companies in the information technology sector to complain about so-called “patent trolls” who, while responsible for little or no novel and non-obvious inventions, are able to obtain significant patent royalty payments from companies with revenue streams that can be put at risk in patent infringement cases.4 At least prior to the *eBay* decision, once the defendant in a patent infringement case was found to be infringing a valid patent, the patent holder had a virtually automatic right to obtain a court-ordered injunction preventing the defendant from continuing to sell its infringing product. Such injunctions were routinely granted even if the patent covered only a minor feature of a complex, valuable, and popular product. With this rule, patent owners have been in a strong bargaining position, even the owners of weak patents covering only minor inventions. By obtaining an injunction, the owner of a patent who prevails in patent litigation effectively has the power to stop the defendant from selling even a non-infringing version of the product, at least until the defendant can redesign its product and introduce a non-infringing version. The right to obtain an injunction thus gives the patent holder the power to hold-up the infringing firm. The prospect of such hold-up affects the negotiating strengths of the two parties prior to the onset of litigation.

The problems identified here are especially common in the information technology sector of the economy, including computer software, Internet business methods, semiconductors, and computer hardware and telecommunications products. First, there has been a surge of patenting of software and business methods over the past ten years, as documented by Bessen and Hunt (2004). Second, there have been widespread complaints about patent quality, and about vague and overly broad patents, in this area, as reported by the Federal Trade Commission (2003) and the National Academies of Science (2004). Third, software innovations tend to be incremental, with rapid sequential innovation; see, for example, Cohen and Lemley (2001). Fourth, software and hardware products tend to be complex, so a single product can potentially infringe many patents; see Hall and Ziedonis (2001) and Shapiro (2000). Fifth, software and hardware commonly is sold at prices well above marginal cost. Lastly, it can be costly and time-consuming to redesign these products to avoid infringement claims. Hence, it is no coincidence

that many firms in the information technology sector weighed in strongly in the eBay case and are pushing hard for patent reform.5

The problems of hold-up and opportunism identified here result in large part because patents are probabilistic property rights, as recognized by Gallini (2002) and emphasized by Lemley and Shapiro (2005). Farrell and Shapiro (2005) explore the licensing of a probabilistic patent to a number of competing downstream firms. Assuming that each downstream firm can immediately and costlessly shift to a backstop technology if enjoined from using the patented technology, they focus on the form of licensing agreements and on the competitive interactions among the downstream firms. The current paper takes a complementary approach to the licensing of probabilistic patents. Here, we study licensing negotiations between a patent holder and a single downstream firm, focusing on the cost and disruption imposed on the downstream firm if it is precluded by a court order from using the patented technology.

The model of licensing negotiations developed here is designed to incorporate a number of key features that can to give rise to hold-up and opportunism in patent infringement cases:

- **Probabilistic Patents**: Royalties are negotiated in the shadow of patent litigation. The relationship between patent strength and the level of negotiated royalties is derived.

- **Injunction Threat**: If patent litigation ensues, the downstream firm will continue producing and selling its product during the pendency of the patent litigation. However, if the patent is found valid and infringed, the court will issue an injunction forcing the downstream firm to withdraw its infringing product from the market. This pattern is very common in practice.

- **Patent Surprise**: The analysis includes the all-too-common case in which the downstream firm designs and begins selling its product before it is aware that it may be infringing the patent in question. The model shows how negotiated royalties in this case

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differ from royalties negotiated in situations where the downstream firm is fully aware of the patent holder’s assertions at the time it originally designs its product.

- **Patent on Minor Features**: The fraction of the total value created by the downstream product that is attributable to the patented invention is tracked as a parameter in the analysis. The model predicts that over-rewards to patent holders are greatest for patents covering a minor feature of a high-margin product.\(^6\)

- **Hold-Up**: The cost that the downstream firm would have to bear to redesign its product to avoid using the patented technology, and the time required to conduct this redesign, are tracked as parameters in the analysis. These two elements create the prospect of hold-up.

- **Reasonable Royalties**: The relationship between the level of “reasonable royalties” used to compute the patent damages and the level of negotiated royalties is derived. The level of reasonable royalties in a self-fulfilling equilibrium is calculated.

Section 2, “Royalty Negotiations,” presents the basic modeling elements. Section 3, “Hold-Up,” studies the case in which the downstream firm has already designed its product when it is faced with the patent holder’s infringement claims. This section shows the factors that determine the extent to which holders of weak patents are over-rewarded, assuming that reasonable royalties are set at a benchmark level. Section 4, “Staying Permanent Injunctions to Permit Redesign,” shows that the over-reward to patent holders is reduced if the courts routinely stay injunctions to give infringing firms time to redesign their products and introduce non-infringing versions. Section 5, “Early Negotiations,” shows that the problems identified in the hold-up case do not go away even if the downstream firm is fully aware of the patent infringement claim against it when it initially designs its product. In fact, for weak patents, early knowledge of potential infringement is of no value to the downstream firm. Section 6, “Reasonable Royalties in Self-Fulfilling Equilibrium,” shows that the over-reward to patent holders is even greater if

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\(^6\) As stated by the General Counsel of Intel: “A fundamental invention deserves greater value than a relatively minor tweak to work that went before it. A broad application of the injunction remedy makes all patents “crucial,” whether they are or not.” See “Troll Call,” by Bruce Sewell, *Wall Street Journal*, March 6, 2006.
reasonable royalties are determined endogenously in a self-fulfilling equilibrium. Section 7 concludes by drawing out the implications of this analysis for reform of the patent system.

2. Royalty Negotiations

A patent holder P owns a single patent. A downstream firm D produces a product that can incorporate a feature covered by the patent. The patent holder and the downstream firm are not competitors.⁷

A. Patented Feature

The patented feature increases the value of D’s product by \( v \geq 0 \) to all consumers, in comparison with the best non-infringing alternative.⁸ We call \( v \) the “value of the patented technology.” The downstream firm has a clear incentive to build the patented feature into its product: doing so allows it to charge a price that is higher by \( v \).

Let \( p \) denote the price per unit that D receives with the patented feature, and let \( c \) denote the marginal cost to D, apart from any royalty payments to P. We call D’s per-unit margin \( m = p - c \) for products incorporating the patented feature. D’s per-unit margin for products not incorporating the patented feature equals \( m - v > 0 \). We are interested in situations in which \( m \) is relatively large in comparison with \( v \). We have in mind “complex technologies” that incorporate many features or components, such as a complex piece of hardware like the BlackBerry handheld device or the Intel Pentium microprocessor, or a sophisticated software product such as Microsoft Windows, Microsoft Office, Adobe Acrobat, or Adobe Photoshop.

Let \( X \) denote the number of units produced by D per unit time. We assume that the patented feature comprises a relatively small portion of the overall value of the downstream product, or that the demand facing D is quite inelastic, so we can treat D’s rate of sales per unit time, \( X \), as

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⁷ The analysis presented here would need to be modified to address cases in which the patent holder competes against the alleged infringer and thus can claim patent damages based on lost profits, not just “reasonable royalties.”

⁸ The value of the patented technology is measured net of any extra marginal costs caused by the patented feature. The analysis here is unchanged if the patented technology reduces the unit cost of production by \( v \).
independent of whether or not D incorporates the patented feature into its product.\textsuperscript{9} The relevant patent lifetime is normalized to the period \([0,1]\). For simplicity, we assume no discounting.\textsuperscript{10} So, the total number of units sold during the lifetime of the patent equals \(X\).

**B. Product Design and Redesign**

The downstream firm makes an initial product design decision to either include or exclude the patented feature. We assume that the (fixed) product design costs borne by D are the same, whether or not D chooses to incorporate the patented feature into its product. Hence, we do not need to track these costs in our analysis.

If D initially incorporates the patented feature in its product, it is costly for D to redesign its product later to avoid using this feature, as would be required for D to keep selling its product if P obtains an injunction against D and if the two parties do not sign a licensing agreement. We denote by \(F\) those (fixed) redesign costs. We further assume that the redesign effort takes time. We denote by \(L \geq 0\) the lag from the time that D commits to incurring the redesign costs until the time when the redesigned product is ready for sale.

**C. Patent Strength and Litigation Costs**

If patent litigation occurs, there is a probability \(\theta\) that P’s patent will be held valid and infringed by D’s product, in which case we say that P wins the litigation. We call \(\theta\) the “patent strength.” With complementary probability \(1 - \theta\), the patent is ruled invalid or not infringed by D, in which case we say that D wins the litigation. Patent strength is common knowledge.

If the firms litigate, each must bear litigation costs, which we denote by \(C_p\) and \(C_d\) respectively. Litigation takes time \(T < 1\). Since the patent lifetime is normalized as the period \([0,1]\), \(T\) is the duration of litigation as a fraction of the remaining lifetime of the patent. We define the “end”

\textsuperscript{9} Our analysis could be amended to account for circumstances in which the patented features causes some extra sales to be made. In that case, the analysis here would still apply to the sales that are not caused by the feature in question. Allowing sales to vary with time would not alter the basic analysis, and could be accomplished simply by redefining the time variables in the analysis to reflect sales made as well as time passed.
of litigation to be the time at which the ultimate winner of the litigation is determined. If P wins the patent litigation, we assume that an injunction issues at this same point in time.\textsuperscript{11} For simplicity, we assume that redesigning the product does not take as long at litigation: $L < T$.

\textbf{D. Nash Bargaining Over Royalties}

We assume Nash Bargaining between P and D, so they split any gains from trade available during any negotiations they have. We denote P’s bargaining power by $\beta \in [0,1]$, so P captures its disagreement payoff plus a fraction $\beta$ of the gains from trade. Likewise, D captures its disagreement payoff plus a fraction $1 - \beta$ of the gains from trade.

Since we are studying Nash Bargaining in a model with symmetric information, and since the combined payoffs of the two firms are larger under agreement (initial licensing) than under initial disagreement (no licensing), we know that the model must predict licensing, not litigation. Therefore, this model should not be viewed as offering predictions about the likelihood of litigation. Rather, it informs the terms on which patent settlements, i.e., licensing, will occur.

Even though the parties do not litigate in equilibrium, the rules regarding injunctions and damages do affect the equilibrium royalty rate because they affect the parties’ payoffs from litigation, and the parties negotiate a licensing agreement in the shadow of litigation.\textsuperscript{12}

\textsuperscript{10} Accounting for discounting would be straightforward. Each time variable in the analysis would just be redefined to measure the present discounted value of a constant annuity over that time period as a fraction of the present discounted value of a constant annuity lasting for the entire patent lifetime.

\textsuperscript{11} We use a highly simplified model of the litigation process. We assume that no preliminary injunction issues; in fact, such injunctions are rare. We also are abstracting away from intermediate rulings that cause the parties to update significantly their views on patent strength. We do not believe that our basic results are sensitive to this assumption. The analysis would be quite similar if one were to assume that an intermediate ruling does issue, so long as this ruling is highly accurate in terms of the ultimate disposition of the patent case, in which case one can think of the time from the intermediate ruling to the end of the litigation as a period during which the permanent injunction has been stayed, as advocated below.

\textsuperscript{12} In fact, the vast majority of interactions between patent holders and alleged infringers result in licensing agreements. As reported in Lemley and Shapiro (2005), about 97\% of all filed patent cases settle. Furthermore, we suspect that three to five patent disputes result in a licensing agreement for every one that leads to a patent suit even being filed. If so, there are more than one hundred patent licenses for every patent litigation that results in a final judgment.
E. Benchmark Royalty Rate and Welfare Analysis

For the purposes of performing welfare analysis and making policy recommendations, we compare the negotiated royalty rate, denoted by $r^*$, with a benchmark royalty rate that represents a reasonable reward to the patent holder for its patented technology. The benchmark royalty rate is equal to $v\theta\beta$. This benchmark reflects the value of the patented feature, $v$, the patent strength, $\theta$, and the underlying bargaining power of the patent holder, $\beta$, in this market context, where prices are set through negotiations. For example, if the patented feature is worth $1 per unit, so $v = 1$, if the patent is valid with probability 0.4, so $\theta = 0.4$, and if P and D have equal bargaining power, so $\beta = 0.5$, then the benchmark royalty rate is $0.20 per unit. As shown below, this is the royalty rate that would be negotiated if P had no hold-up power over D. Note that the benchmark royalty rate is proportional to patent strength: the patent holder’s payoff directly reflects the probability that the patent holder in fact innovated to create the feature covered by the patent.

Ultimately, we are interested in learning whether the patent system, as it currently operates, is providing suitable incentives for innovation. In the equilibrium in our model, there is no litigation, no design-around costs are incurred, the quantity produced is not sensitive to the royalty rate, and all units produced incorporate the patented feature. Therefore, within the bounds of the formal model, the equilibrium royalty rate only determines the amount transferred from the downstream firm to the patent holder; it has no direct effect on the allocation of resources. However, the equilibrium royalty rate has profound implications for long-run economic efficiency. In keeping with the basic economic principles underlying the patent system, we postulate that a royalty rate at the benchmark level of $v\theta\beta$ provides an efficient reward to innovators. A royalty rate above this benchmark level acts like a tax on downstream firms and generally encourages the filing of patent applications (or overly broad patent claims) and the assertion of the resulting patents by parties who may not in fact have made novel, non-obvious inventions. These are precisely the concerns emphasized by Jaffe and Lerner (2004) and which have come into bold relief in recent years due to cases like NTP v. RIM and eBay v. MercExchange.
**F. Patent Damages and “Reasonable Royalties”**

If the patent is valid and infringed, D will owe P damages for any past infringement. We denote by $s$ the royalty per unit that the court will require D to pay in this event.\(^{13}\) This variable is measured to include the possibility that D will be judged to have engaged in willful infringement, which can lead to a trebling of damages. To reduce the number of cases, we assume that the expected royalties are less than the value of the patented feature: $\theta s < v$; this inequality is satisfied in the benchmark case (see just below) where $s = \beta v$.

We initially treat $s$ as exogenous. This allows us to see how alternative rules governing the determination of patent damages affect negotiated royalty rates. However, after we calculate the equilibrium royalty rate for a given level of $s$, we perform much of our analysis assuming that $s = \beta v$. This is a natural benchmark level for reasonable royalties under patent law, as explained below. This is precisely the royalty rate that would be negotiated between P and D if P’s patent were proven valid but P did not have any extra leverage based on the threat of holding up D. Put differently, this is the royalty rate that would result if P simply earned its negotiated share of the total value created by its patented technology.\(^{14}\)

**G. Timing of Product Design and Negotiations**

We consider two basic models which are similar in spirit but differ in their timing.

In the first model, “Hold-Up,” the downstream firm is unaware that the relevant feature is patented at the time that it makes its initial product design decision. Naturally, since the feature adds value, D incorporates the patented feature into its product.\(^ {15}\) The analysis begins when P subsequently asserts its patent against D’s product and the two firms negotiate over patent

\(^{13}\) We are not studying the case in which P competes against D and thus asserts damages from D based on lost profits. Our analysis is confined to the case in which P’s damage claim only involves reasonable royalties.

\(^{14}\) In Section 6 we show how $s$ can be determined endogenously in a fulfilled-expectations equilibrium, which results in a higher level of negotiated royalties.

\(^{15}\) This situation, which is very common in the information technology sector, implicitly involves independent invention: P and D both “discover” the patented technology independently. As explained in Shapiro (2006), another, arguably superior way to deal with independent invention is to establish an independent invention defense, at least if D uses the patented invention before the patent application is published or the patent issues.
royalties. If these initial licensing negotiations fail, D decides whether or not to introduce its allegedly infringing product. If D does introduce its product, then P decides whether or not to sue D for patent infringement. If P sues, D can continue selling its product or withdraw it from the market. D also can work on redesigning its product. If P wins the litigation, the parties will have another opportunity to negotiate a patent license subsequent to the court’s ruling.

The second model, “Early Negotiations,” is the same as the first except that D is aware of P’s patent, and the two parties have an opportunity to negotiate over royalties, before D makes its initial product design decision.

3. Hold-Up

In this section we study the situation in which D was unaware of P’s pending or issued patent at the time that D initially designed its product to incorporate the patented feature. This fact pattern occurs frequently, either because D designed its product before P’s patent application was published and before P’s patent issued, or because D was simply unaware of P’s patent application or issued patent when D designed its product, even though D’s design efforts occurred after this information had become public.\(^{16}\)

We are interested in understanding the factors that govern the negotiations between P and D over a patent license once P asserts its patent against D’s product. More specifically, our model is designed to explain the royalty rate likely to emerge from those negotiations. We pay particularly close attention to how injunctions and the rule by which reasonable royalties are determined affect the royalty rate negotiated between P and D.

As usual with Nash Bargaining, to determine the negotiated royalty rate, \(r\), we need to calculate each party’s payoff from agreeing on that royalty rate and each party’s disagreement payoff, which here involves patent litigation. The agreement payoffs are straightforward. The payoff to

\(^{16}\) The second of these possibilities by no means implies that D was derelict or actively ignoring or evading or willfully infringing P’s patent, given the large number of patents, many of which have broad and vague claims.
the downstream firm from accepting a license at rate \( r \) is given by \((m - r)X\). P’s payoff from this license is \( rX \). Their combined payoffs from licensing are simply \( mX \).\(^{17}\)

**A. The Downstream Firm’s Optimal Threat Point**

What happens if P and D do not reach a licensing agreement? In the absence of a licensing agreement, D has several immediate decisions to make: whether to sell its allegedly infringing product, whether to commit resources to redesigning its product to avoid even the chance of infringing the patent, and, if D withdraws its allegedly infringing product from the market, whether to seek a declaratory judgment that P’s patent is invalid.

Since we are interested in permanent injunctions and patent damages, which predominantly arise in situations where D keeps selling its allegedly infringing product during litigation, we study cases in which D’s best course of action is one of the following two strategies:

- **Sell, Do Not Redesign**: Keep selling its product, do not redesign the product.
- **Sell and Redesign**: Keep selling its product, develop a non-infringing version.

Our focus on these strategies fits with our interest in valuable and complex products for which the patent involves only a relatively small component in the overall product. In these settings, downstream firms frequently find it optimal to keep selling their products in the face of patent infringement claims, even though doing so requires bearing litigation costs and runs the risk of incurring liability for infringement.

The Appendix provides sufficient conditions such that D’s optimal strategy in the absence of a license is either “Sell, Do Not Redesign” or “Sell and Redesign.” These conditions are met so long as D’s litigation costs are small relative to the margins earned on the product and the total value of the patented technology.

\(^{17}\) We assume that the parties negotiate a uniform per-unit royalty rate, \( r \), which will apply for the lifetime of the patent. Allowing a fixed licensing fee, or more generally a two-part tariff would not matter at all in our model, given our assumption that D sells a fixed number of units, whether or not D’s product incorporates P’s patented feature. Allowing royalty rates that vary with time also would not add anything to the analysis.
Whether D adopts the “Sell, Do Not Redesign” strategy or the “Sell and Redesign” strategy, P makes the next decision: whether or not to sue D for patent infringement. We assume that P earns positive profits by suing D for patent infringement, so P’s threat to sue D for patent infringement is credible. If reasonable royalties are at their benchmark level, \( s = \beta v \), a sufficient condition for P to earn positive profits from suing D is that \( C_p < \theta \beta v X \). The Appendix proves

**Lemma #1:** The downstream firm’s optimal strategy in the absence of a licensing agreement is “Sell and Redesign” if and only if \( \theta > \frac{1}{\beta} \frac{F}{(m - v) X L + F} \equiv \theta^* \).18

The downstream firm is more likely to redesign its product during litigation (a) the stronger is the patent, (b) the greater is the patent holder’s bargaining power, (c) the larger are the total margins that D could earn without the patented feature which are at risk due to an injunction if P wins the patent litigation, \( (m - v) X L \), and (d) the smaller are the redesign costs. Intuitively, if the patent is weak, D will not find it optimal to incur product redesign costs during the pendency of litigation, since it is relatively unlikely that the redesigned product will ever be needed.

**B. Negotiated Royalties for Relatively Weak Patents**

For \( \theta < \theta^* \), D’s threat point is “Sell, Do Not Redesign.” In this case, the Appendix proves

**Theorem #1:** Suppose that reasonable royalties are set at their benchmark level, \( s = \beta v \). For relatively weak patents, i.e., \( \theta < \theta^* \), the equilibrium royalty rate equals

\[
\begin{align*}
  r^* = \theta \beta v + \theta \beta (m - v) L + \theta \beta \frac{F}{X} + \beta C_D - (1 - \beta) C_p \\
  \frac{1}{X}
\end{align*}
\]

(1)

The first term, \( \theta \beta v \), reflects the expected damages payments during \( [0, T] \), i.e., during the interim period before the patent decision would be rendered, using \( s = \beta v \), plus the expected value of the negotiated royalties during \( [T, 1] \), based on benchmark royalty rate of \( \beta v \).

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18 We assume \( \theta^* < 1 \). If not, D never engages in redesign, regardless of patent strength, and the analysis simplifies to the case in which “Sell, Do Not Redesign” is optimal for D.
The second term, \( \theta \beta (m - v)L \), measures P’s ability to hold up D based on the lag time associated with design-around. Note that this expression reflects patent strength, \( \theta \), P’s bargaining power, \( \beta \), and the value to D of using *non-patented* technology, \( (m - v) \), as well as the redesign lag, \( L \).

The third term, \( \theta \beta \frac{F}{X} \), measures P’s ability to hold up D based on the fixed costs associated with redesigning the product. These fixed costs are divided by \( X \) to convert them into a per-unit royalty. Like the other component of hold-up, this expression reflects patent strength, \( \theta \), and P’s bargaining power, \( \beta \).

The final term, \( \frac{\beta C_D - (1 - \beta)C_P}{X} \), reflects P’s net bargaining power associated with the fact that failure to reach a licensing agreement will impose litigation costs on both parties. The net impact of this threat, which depends upon the two firms’ relative litigation costs and bargaining power, is divided by \( X \) to convert it into per-unit terms. This expression can be positive or negative. This term is larger, the greater is P’s bargaining power, the greater are D’s litigation costs, and the smaller are P’s litigation costs. This term is zero in the neutral case in which \( \beta = 1/2 \) and \( C_P = C_D \).

**C. Negotiated Royalties for Relatively Strong Patents: \( \theta > \theta^* \)**

For \( \theta > \theta^* \), D’s threat point is “Sell and Redesign.” In this case, the Appendix proves

**Theorem #2:** Suppose that reasonable royalties are set at their benchmark level, \( s = \beta v \).

For relatively strong patents, i.e., \( \theta > \theta^* \), the equilibrium royalty rate equals

\[
 r^* = \theta \beta v + \frac{\beta F}{X} + \frac{\beta C_D - (1 - \beta)C_P}{X}.
\]  

(2)

We have already discussed the first and last terms on the right-hand side of this expression. The new term, \( \frac{\beta F}{X} \), is the amount that P can extract per unit because D’s threat point involves redesign costs of \( F \). Note that this term is *not* discounted by patent strength.
D. Negotiated Royalty Rates and Patent Strength

Figure 1 shows how the equilibrium negotiated royalty rate, $r^*$, varies with patent strength. For simplicity, we now introduce the standing assumption that the pure litigation cost term in $r^*$, $\beta C_D - (1 - \beta) C_p$, is zero.

The straight line through the origin depicts equation (1), which applies when $\theta < \theta^*$. The flatter straight line shown in Figure 1, beginning at $r^* = \beta F / X$ when $\theta = 0$, depicts equation (2), which applies when $\theta > \theta^*$. The equilibrium royalty rate, $r^*$, is depicted by the two heavier line segments in Figure 1. Note that the negotiated royalty rate drops discontinuously at $\theta = \theta^*$.

Figure 1 also displays a dotted line representing the benchmark level of royalties, $\theta \beta v$.

The two lines in Figure 1 depicting equations (1) and (2) cross at $\theta^* = \beta \theta^*$. For $\theta \in (\theta^*, \theta^*)$, D would be better off if it could credibly commit to redesigning its product in the event no licensing agreement is reached. A credible threat to redesign the product would help D negotiate with P since redesign would leave P in a weaker negotiating position in the event that P wins the patent litigation.

E. Over-Rewards to Holders of Relatively Weak Patents

We now compare the negotiated royalty rate, $r^*$, with the benchmark level of $\theta \beta v$. For relatively weak patents, i.e., $\theta < \theta^*$, using equation (1), the over-charge received by the patent holder, measured in proportion to the benchmark royalty rate, is given by

$$\frac{r^* - \theta \beta v}{\theta \beta v} = \frac{m - v}{v} L + \frac{F}{v X}$$

The first term on the right-hand side is the ratio of the value of the product without the patented feature to the value of that feature, times the percentage of the patent lifetime that is required for redesign. This term reflects the patent holder’s power based on the threat that the downstream firm will be forced form the market by an injunction while redesigning its product. The second term on the right-hand side is the ratio of the redesign costs to the total value of the patented improvement. The total over-charge is the sum of these two unit-free terms. The royalty over-charge is independent of the patent strength, $\theta$, and of the patent holder’s bargaining power, $\beta$. 
Corollary #1: For relatively weak patents, i.e., $\theta < \theta^*$, the royalty over-charge is larger, the larger is the margin on a non-infringing product relative to the per-unit value of the patented feature, the time required to redesign the product to avoid infringement, and the ratio of the redesign costs to the total value of the patented feature.

We can illustrate the effects of hold-up using a numerical example. Suppose that the product sells for $40 without the patented feature, and the marginal cost of producing the product is $30 per unit, so $m - v = 10$. Suppose the patented feature adds an extra $1 of value, so $v = 1$. This implies that $(m - v) / v = 10$. Suppose that redesign will take one year out of ten years remaining in the patent lifetime, so $L = 0.1$. Then the first term on the right-hand side of equation (3) equals $10 \times 0.1$ or 1.0. There is a 100% royalty over-charge based on the threat of an injunction, so the injunction threat alone causes the per-unit royalty rate to be twice as high as the benchmark level. In addition, redesign is costly. If the redesign costs equal half of the total value of the patented feature, $F / \nu X = 0.5$, causing an additional 50% royalty over-charge.19

F. Over-Rewards to Holders of Relatively Strong Patents

We can perform the same exercise for relatively strong patents, i.e., $\theta > \theta^*$, in which case D’s threat point is “Sell and Redesign.” Using equation (2), the over-charge received by the patent holder, measured in proportion to the benchmark royalty rate, is now given by

$$\frac{r^* - \theta^* \nu}{\theta^* \nu} = \frac{F}{\nu X} \frac{1}{\theta}$$

(4)

In this case, the gap between $r^*$ and the benchmark level $\theta^* \nu$, relative to that benchmark level, is the product of two ratios: the ratio of the redesign costs to the total value of the patented invention to the downstream firm, and the inverse of the patent strength.

Corollary #2: For relatively strong patents, i.e., $\theta > \theta^*$, the royalty over-charge is larger, the larger is the ratio of the redesign costs to the total value of the patented feature, and the weaker is the patent.

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19 The Appendix shows that $\theta^* = 2 / 3$ with these parameter values.
We can illustrate the effects of hold-up by modifying the numerical example given above so the marginal cost of production is $10, which implies \( m - v = 30 \). If the patent strength is \( \theta = 0.5 \), then P can achieve a 100% over-charge based on P’s threat to impose redesign costs on D.²⁰

**G. Royalty Stacking and Standard Setting**

Regardless of the level of patent strength, the percentage royalty over-charge is larger, the smaller is \( v \). One might think that cases involving a small \( v \) are unimportant, precisely because the patented technology is relatively unimportant. However, that would be an error, since there may be many such “small” patents affecting any given product. When multiple patents read on a single product, multiple royalties are stacked on top of each other, so the aggregate royalty over-charge can be highly significant, either in absolute terms or as a percentage of the price of the downstream product. As emphasized by Heller and Eisenberg (1998) and Shapiro (2001), severe problems can arise when hold-up is combined with the patent thicket and royalty stacking. Lemley and Shapiro (2006) give striking examples of the patent thicket involving third-generation mobile phones and Wi-Fi technology (the IEEE 802.11 family of standards).

Patent applicants often are able to obtain patents on minor features that are inadvertently incorporated into complex products. So-called “patent ambush” has been of particular concern in the standard-setting context, where redesign can be especially difficult or time-consuming, as illustrated by the FTC’s complaints against Rambus and Unocal.²¹ Corollaries #1 and #2 warn us that the holders of patents that are essential to a product standard can obtain royalties far out of proportion to their contributions, based on their ability to hold up downstream firms who have inadvertently designed products or adopted a standard incorporating patented features. When this occurs, the patent system is discouraging, not rewarding, innovation.

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²⁰ The Appendix shows that \( \theta^* = 2 / 7 \) with these parameter values, so “Sell and Redesign” is indeed the optimal strategy for D if \( \theta = 1 / 2 \).


H. Pure Hold-Up

We now comment on special case in which the patented technology is no better at all than the next best alternative, so \( v = 0 \). In this case, the downstream firm would simply and costlessly have avoided the patented technology, had it only known about its patent exposure when it designed its product. Therefore, all of P’s payoff results from hold-up. In this case, we cannot talk about the percentage gap between the negotiated royalties and the benchmark level, since the benchmark royalty level of \( \theta \beta v \) equals zero, reflecting the fact that the patent holder has contributed no value to the downstream firm’s product.

For \( \theta < \theta^* \), with \( v = 0 \), Theorem #1 tells us that \( r^* = \theta \beta mL + \theta \beta \frac{F}{X} \). While these over-charges are proportional to patent strength, they can be significant if the product is quite profitable, so \( m \) is large, if the redesign lag, \( L \), is relatively long, and if the redesign costs per unit, \( F/X \), are significant. For \( \theta > \theta^* \), with \( v = 0 \), Theorem #2 tells us that \( r^* = \beta \frac{F}{X} \). In this case, the over-charges are independent of patent strength and determined by the redesign cost per unit.

4. Staying Permanent Injunctions to Permit Redesign

The Supreme Court in eBay has opened up the possibility that a party found to have infringed a valid patent will not be enjoined from selling its infringing product. Injunctions will only be issued if the patent holder passes the Court’s four-factor test, which the Court describes this way:

According to well-established principles of equity, a plaintiff seeking a permanent injunction must satisfy a four-factor test before a court may grant such relief. A plaintiff must demonstrate: (1) that it has suffered an irreparable injury; (2) that remedies available at law, such as monetary damages, are inadequate to compensate for that injury; (3) that, considering the balance of hardships between the plaintiff and defendant, a remedy in equity is warranted; and (4) that the public interest would not be disserved by a permanent injunction.

In the simple model presented in this paper, where the patent holder is purely a licensing entity with no downstream presence, awarding per-unit monetary damages at the benchmark level, \( \beta v \), is a very attractive alternative to injunctive relief. One practical advantage of this approach is that the jury will already have determined the reasonable royalty rate for past infringement, and it
may be reasonable to use this same figure for prospective infringement, at least if market conditions are not changing too much. However, one must also recognize that, in practice, it may be very difficult for juries accurately to estimate $\beta \nu$.

**A. Impact of Stays on Negotiated Royalty Rate**

We now consider a less drastic departure from recent practice. In particular, we show how the initial licensing negotiations are affected if the courts regularly *stay* the permanent injunctions that they grant, and if these stays last long enough to give downstream firms the opportunity to complete their redesign efforts.\(^{22}\) If stays are routinely granted and reasonable royalties equal their benchmark level, $s = \beta \nu$, then D has no incentive to redesign its product prior to the resolution of litigation. D’s optimal strategy for all $\theta$ is “Sell, Do Not Redesign,” and the Appendix shows that negotiated royalties equal

$$r^* = \theta \beta \nu + \theta \beta \frac{F}{X}$$  \hspace{1cm} (5)$$

Figure 2 displays the heavy straight line through the origin which represents the equilibrium royalty rate if injunctions are stayed, as given by equation (5). For comparison purposes, the equilibrium royalty rate without stays, from Figure 1, is also shown on Figure 2. Granting stays allows the downstream firm to delay its redesign efforts until it learns the outcome of the patent litigation. For this reason, stays are of no value for $\theta = 0$ or $\theta = 1$; for these extreme values of patent strength, there is no information to be learned. Stays are especially helpful for patents of intermediate strength, for which learning the outcome of the patent litigation is most informative.

**Theorem #3:** Routinely granting stays to permanent injunctions to provide infringing firms the time to design non-infringing products causes negotiated royalty rates to fall, moving royalty rates closer to their benchmark level. Stays are most valuable to alleged infringers for patents of intermediate strength.

If stays to permanent injunctions are routinely granted, the royalty “over-charge” in proportion to the benchmark royalty level is given by

\[^{22}\text{We assume that redesign is more profitable for D than exiting the market. If not, then stays simply extend the time period in which D can use the patented technology in exchange for damages of reasonable royalties.}\]
\[
\frac{r^* - \theta \beta v}{\theta \beta v} = \frac{F}{v X}.
\]

Even with stays on injunctions, the negotiated royalty still exceeds the benchmark level of \( \theta \beta v \) by an amount depending upon the ratio of the redesign costs to the value of the patented feature.

In our previous numerical examples, we assumed that \( F / v X = 0.5 \), i.e., the redesign costs were half of the total value of the patented feature. In that case, granting stays limits the over-charge to 50%. In contrast, in our numerical example where “Sell, Do Not Redesign” was optimal, the threat of an injunction led to this 50% plus an additional 100% over-charge.

**B. Policy Analysis**

Routinely granting stays provides a clear social benefit: the patent holder who wins the patent litigation has less power based on hold-up. As a result, the royalty rates negotiated in equilibrium fall, moving closer to the benchmark level.

If stays are routinely granted, D will have little incentive to redesign its product prior to the resolution of the patent litigation. While some might see this as a drawback, we regard it as an additional social benefit from a policy of granting stays. If \( \theta < \theta^* \), even in the absence of stays D would not redesign during litigation, so this argument is inapposite. What if \( \theta > \theta^* \) so the expectation by D that it will obtain a stay to the permanent injunction would indeed cause D to delay its redesign efforts if no licensing deal is initially reached? In our model, redesign and litigation do not occur in equilibrium, so no social costs of redesign are actually incurred. The effects of stays arise entirely through the royalty rates that are negotiated. As already argued, long-run welfare is higher if negotiated royalty rates are reduced, bringing them closer to the benchmark level.

Going beyond the model, however, we can ask about the welfare effects of reducing the incentives of allegedly infringing firms to redesign their products during the pendency of litigation. Viewed in this light, stays have additional desirable properties. For patents that are ultimately ruled invalid or not infringed, which occurs in roughly half of patent judgments, unnecessary and wasteful redesign efforts are thereby avoided. For patents that are ultimately ruled valid and infringed, at worst the patent holder suffers a delay in its ability to obtain
injunctive relief. However, for the cases considered here, where the patent holder does not compete against the infringing firm, this harm seems limited, since the infringing firm must pay reasonable royalties on products sold during the period when the permanent injunction was stayed. Any resulting harm to the patent holder is no worse than the harm that arises in the many patent cases where the patent holder is unable to obtain a preliminary injunction and the defendant continues to sell its allegedly infringing product before facing a permanent injunction. Furthermore, in the cases at issue, the court has already determined the reasonable royalty rate for the purpose of past damages, making it relatively easy to use this same rate for infringement that occurs while the permanent injunction is stayed.

In practice, litigating parties update their prospects of victory as litigation proceeds, generating information based on discovery and intermediate rulings. If information is gradually revealed, D, having received unfavorable news, may redesign its product before losing the litigation. In the limiting case, information is revealed very gradually, so before D loses there will arrive a point in time at which the updated patent strength is great enough that D finds it optimal to initiate redesign efforts. If this point in time occurs at least $L$ before the end of litigation, D will have time to redesign its product before facing an injunction, so stays will be far less important.

5. Early Negotiations

We now consider how our analysis changes if the patent holder and the downstream firm negotiate before D initially designs its product. In particular, we now assume that P and D enter into licensing negotiations sufficiently early that D can design its product to include, or exclude, the patented feature, at no extra design cost, and still have sufficient time to introduce its product as planned at time zero.

How are the negotiations between P and D affected if D has not yet designed its product? The only difference from the earlier model is that D has an additional option: D can design its product initially to avoid any chance of infringing P’s patent. We call this the “Design Around” strategy. D’s payoff from “Design Around” is $(m - \nu)X$. The Early Negotiations game differs
from the Hold-Up game if and only if D’s payoff from “Design Around” is higher than D’s payoff from both “Sell, Do Not Redesign” and “Sell and Redesign.”

If the “Design Around” option is valuable to D, then D’s threat point payoff is $(m - v)X$, so the gains from trade equal $vX$. Since P gets a fraction $\beta$ of the gains from trade, the negotiated royalty rate must be $\beta v$, which is above the benchmark level of $\theta \beta v$ for all $\theta < 1$. For sufficiently weak patents, Theorem #1 tells us that D can obtain a lower royalty than $\beta v$ by threatening “Sell, Do Not Redesign.” This proves

**Theorem #4:** If Early Negotiations are of any benefit to the downstream firm, then the negotiated royalty rate equals $\beta v$, which exceeds the benchmark level of $\theta \beta v$. Early Negotiations provide no benefit to the downstream firm for sufficiently weak patents.

In fact, early knowledge of the patent can be positively *harmful* to the downstream firm it increases the chance that the downstream firm will be found to have willfully infringed the patent, which can lead to treble damages. For this reason, many firms in the information technology sector instruct their engineers *not* to read the patents coming out of the PTO.

The finding that, even with early negotiations, P and D will negotiate a royalty rate in excess of the benchmark level may be surprising, since the patent holder would not appear to have any ability to hold up the downstream firm in such early negotiations. However, when early negotiations are valuable to the downstream firm, D’s best threat, designing around the patent, is equivalent to conceding that the patent is valid and infringed without a fight. In this situation, the downstream firm does not get any reduction in royalties to reflect the probabilistic nature of the patent, so the royalty rate, $\beta v$, is not discounted at all to reflect any weakness of the patent.

If the downstream firm does benefit from Early Negotiations, then $r^* = \beta v$, and the proportional gap between the negotiated royalty rate and the benchmark royalty rate, is given by

$$\frac{r^* - \theta \beta v}{\theta \beta v} = \frac{1 - \theta}{\theta}.$$  

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23 D prefers the “Sell and Redesign” strategy to the “Design Around” strategy if the extra redesign costs, $F$, are less than the net benefits of selling a potentially infringing product during the pendency of litigation, $(v - \theta s)XT$. 

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With this threat point, In this case, the over-charge ratio depends on the patent strength but not on any other variables. As an example, if the patent strength is $\theta = 0.5$, the over-charge is 100%. If the patent strength is $\theta = 0.2$, the over-charge is 400%. Even for a rather strong patent, say $\theta = 0.8$, the over-charge is still 25%.

6. Reasonable Royalties in Self-Fulfilling Equilibrium

We have assumed so far that the reasonable royalty rate was set at the benchmark level of $s = \beta v$. We now show that the hold-up problems just identified are magnified when $s$ is determined endogenously.

The law governing patent damages states that “the court shall award a claimant damages adequate to compensate for the infringement, but in no event less than a reasonable royalty for the use made of the invention by the infringer.”24 Under established precedent, the reasonable royalty rate is usually defined to be the royalty rate that would be negotiated initially between the two parties, if the patent were known to be valid and infringed, and if they were willing and able to reach an agreement.25 In terms of the model presented here, the reasonable royalty should therefore equal $\beta v$, which is the benchmark level of $s$ used above.

In practice, however, even if the courts accept the principle that $s$ should equal $\beta v$, they face the problem that $\beta v$ may be very difficult for juries to estimate with accuracy. Therefore, following the Georgia-Pacific case, the courts have developed a series of factors that juries should consider when calculating reasonable royalties. In practice, the courts pay close attention to the royalties actually negotiated by the patent holder with other licensees for the patented technology. They also use the royalty rates negotiated for other “comparable” patents as proxies. In other words, in setting $s$, the courts rely on royalty rates that have actually been negotiated.

25 See, for example, Leonard and Stiroh (2005). The key case articulating this principle is Georgia-Pacific Corp. v. United States Plywood Corp., 446 F. 2d 295 (Second Circuit, 1971).
In terms of the variables in our model, the approach taken by the courts involves using the equilibrium licensing rate for a patent with strength \( \theta = 1 \) as a proxy for \( s \). This approach necessarily introduces some degree of circularity into the definition of reasonable royalties, since \( s \) depends upon the observed royalties that have actually been negotiated, and those royalties in turn depend upon \( s \). We resolve this circularity by looking for a fulfilled-expectations equilibrium. The Appendix proves

**Theorem #5:** Suppose that “reasonable royalties” are based on the actual royalties negotiated between the patent holder and other similarly situated downstream firms. In a fulfilled-expectations equilibrium, the reasonable royalties are elevated above the benchmark level of \( s = \beta v \) by the patent holder’s threat of hold-up.

These higher levels of reasonable royalties further elevate the level of negotiated royalties.

### 7. Conclusions

The model developed here shows that granting patent holders the nearly automatic and immediate right to obtain permanent injunctions after they prevail in patent cases over-rewards the owners of patents who license rather than practice their patents. The over-reward is especially great for the owners of patents on minor features used in high-margin products. These findings hold even after accounting for the ability of downstream firms to redesign their products during the pendency of litigation to protect themselves in the event they face a court injunction to stop selling infringing products. Perhaps most striking, such patent holders are over-rewarded even if the firm using the patented technology is fully aware of the patent and can negotiate a license before it makes its initial product design decision.

The analysis provided here strongly supports the conclusion that patent reform is needed. In particular, the model presented here implies that economic efficiency and innovation will be promoted if permanent injunctions are no longer routinely issued to non-competing patent holders upon a finding of infringement. Granting stays, so that injunctions are not imposed at least until the defendant has a reasonable opportunity to introduce a non-infringing version of its product, would enhance economic efficiency.
The analysis here also shows that significant, adverse effects can arise when the PTO grants weak patents; this same theme can be found in Farrell and Shapiro (2005). This paper therefore gives further support to the growing chorus of voices calling for policy changes that will improve patent quality, e.g., by devoting greater resources to patent examinations. This paper also gives further support for conducting additional post-grant reviews to weed out weak patents before they are licensed or litigated.

The model also addresses a circularity in the manner in which damages based on “reasonable royalties” are calculated in patent infringement cases. The circularity arises because reasonable royalties are often based on the royalties actually negotiated in the shadow of litigation, and these negotiated royalties depend in turn upon the magnitude of damages that courts are expected to award if the parties are unable to sign a licensing deal and instead engage in patent litigation. If this circularity is resolved in a fulfilled-expectations equilibrium, reasonable royalties exceed their benchmark level, further elevating royalty over-charges.

Further research is needed to understand how the effects identified here are altered when the patented feature significantly increases the sales made by the downstream firm, when the patent holder is a direct competitor of the allegedly infringing firm, and when multiple patents are asserted against the same product, simultaneously or sequentially, by one or more patentees.

Finally, a more complex analysis, building on the model presented here, is required in situations where the patent holder negotiates with multiple licensees. In such cases, the patent holder has more at risk in patent litigation. A finding of invalidity will destroy the patent holder’s ability to collect royalties for the patent in question. On the other hand, a finding of validity will bolster the strength of the patent and allow the patent holder to obtain higher royalties in subsequent licensing negotiations. Just how these factors play out, in situations where negotiations take place sequentially and the patent holder behaves strategically, is a topic for future research.
References


Appendix

Sufficient Conditions for the Downstream Firm to Keep Selling its Product

Six strategies are available to D if no licensing agreement with P is reached:

1. **Exit**: Stop selling its product, do not redesign the product, do not seek a declaratory judgment.

2. **Wait and Challenge Patent**: Stop selling its product, do not redesign the product, seek a declaratory judgment that the patent is invalid, if standing rules permit.

3. **Wait, Redesign, Avoid Litigation**: Stop selling its product, redesign the product to avoid any chance of infringing the patent, do not seek a declaratory judgment.

4. **Wait, Redesign, Challenge Patent**: Stop selling its product, redesign the product to avoid any chance of infringing the patent, seek a declaratory judgment.

5. **Sell and Redesign**: Keep selling its product, redesign the product.

6. **Sell, Do Not Redesign**: Keep selling its product, do not redesign the product.

Here we provide conditions under which (6) is better than (2) is better than (1), and conditions under which (5) is better than (4) is better than (3). Together, these conditions imply that either (5) or (6) is optimal for D.

**“Sell, Do Not Redesign” (6) Better than “Wait and Challenge” (2)**

The difference between (6) and (2) is that under (6) D makes sales while litigation is pending. D’s revenues from these sales are $mXT$. However, D incurs possible infringement liability of $\theta sXT$. Since $m > v$ and $v > \theta s$, we know that $m > \theta s$, so D is better off selling, and running the risk of infringement damages, than staying out of the market while litigation is pending.

**“Wait and Challenge” (2) Better than “Exit” (1)**

Strategy (2) is better than (1) if the expected payoff from litigation exceeds the litigation costs. The expected payoff from litigating is $(1 - \theta)mX(1 - T) + \theta(m - w)X(1 - T)$, where the first term here represents D’s profits from selling its product after the litigation ends, free of royalties, if D wins the litigation, and the second term represents D’s profits from selling if it loses and must pay a royalty of $w$ to P. A sufficient (but far from necessary) condition for this to exceed litigation costs is $(1 - \theta)mX(1 - T) > C_D$.
“Sell and Redesign” (5) Better Than “Wait, Redesign, Challenge” (4)

The difference between (5) and (4) is that D makes sales during [0, L] under strategy (5). As just noted, D earns profits of $mXL$ on these sales, from which we must deduct its expected liability to P of $\theta sXL$. Therefore, these sales contribute to D’s profits, since $m > \theta s$, as just noted.

“Wait, Redesign, Challenge” (4) Better Than “Wait, Redesign, Avoid Litigation” (3)

Under strategy (4), D’s gets $(m - \nu)X(T - L) + (1 - \theta)mX(1 - T) + \theta(m - \beta \nu)X(1 - T) - F - C_D$. Note that the negotiated royalty if P wins is equal to $\beta \nu$, since P has no ability to hold up D. Under strategy (3), D’s payoff is $(m - \nu)X(1 - L) - F$. Strategy (4) is better than strategy (3) if and only if $(1 - \beta \theta)\nu X(1 - T) > C_D$. The left-hand side measures the benefits of litigation: instead of bearing the full cost per unit of selling the non-infringing alternative, $\nu$, for sure, during $[T, 1]$, by litigating D only bears an expected cost per unit of $\beta \theta \nu$, reflecting the probability $\theta$ that P will win and the resulting negotiated royalty of $\beta \nu$.

Summary of Sufficient Conditions

Summarizing, so long as $C_D < (1 - \theta)mX(1 - T)$ and $C_D < (1 - \beta \theta)\nu X(1 - T)$, either “Sell and Redesign” or “Sell, Do Not Redesign” is optimal for D. Both of these conditions are met if D’s litigation costs are small. The first condition is most easily met for a high-selling product with a significant price/cost margin, especially if the patent involved is relatively weak. The second condition is met if D’s litigation costs are small relative to the total value to D of using the patented technology, especially if the patent involved is relatively weak.

D’s Payoff from the “Sell, Do Not Redesign” Strategy

We now compute D’s payoff from adopting the “Sell, Do Not Redesign” strategy. This strategy entails litigation, so D incurs its litigation cost $C_D$. While litigation is ongoing, D continues selling its product, so D earns $mX$ per unit time during $[0, T]$, for a total of $mXT$, at which time litigation is completed. If D wins the litigation, which occurs with probability $(1 - \theta)$, its continuation payoff is $mX(1 - T)$.

If P wins the litigation, which occurs with probability $\theta$, then D owes damages to P equal to $sXT$, and P obtains an injunction against D. At that point, P and D can negotiate a license. Again we assume Nash Bargaining between P and D. So again we need to calculate their payoffs from agreement and disagreement. At this point, both firms’ litigation costs have already been incurred, profits have already been earned on products already sold, and damages are already due based on those sales, so we can ignore those parts of the firm’s payoffs when considering the bargaining outcome going forward.
Negotiations if P Wins the Patent Litigation

If P and D sign a licensing agreement, their combined prospective profits are \( mX(1-T) \).

What are the firms’ threat points in the negotiation after P wins the litigation? If P and D do not reach an agreement, the patent holder gets nothing, and the downstream firm is forced to either exit the market or incur the design-around costs and, after a lag, introduce a modified, non-infringing product. While the redesign effort is underway, D must withdraw from the market.

There are two sub-cases, depending upon whether D is better off incurring the redesign costs or just exiting the market. Exiting the market gives D a (prospective) payoff of zero, and redesign gives D a (prospective) payoff of \( (m-v)X(1-T-L)-F \). We focus on the case in which redesign is more profitable. In this case, the downstream firm’s (prospective) profits of \( (m-v)X(1-T-L)-F \) are equal to the combined disagreement profits of P and D. Subtracting this amount from the combined agreement profits of \( mX(1-T) \) gives the gains from trade associated with reaching an agreement, which equal \( vX(1-T)+(m-v)XL+F \). Under Nash Bargaining these gains are split, so the payoff to P is

\[
\beta vX(1-T)+\beta[(m-v)XL+F].
\]

The prospective payoff to D if P wins is the combined prospective payoff, \( mX(1-T) \), minus P’s payoff, as just given in equation (8). Simplifying, D’s prospective payoff equals

\[
(m-\beta v)X(1-T)-\beta[(m-v)XL+F].
\]

To summarize, following a win by P, the two parties will sign a license giving a prospective payoff to P of \( \beta vX(1-T)+(m-v)XL+F \), as shown in equation (8), and a prospective payoff to D of \( (m-\beta v)X(1-T)-\beta[(m-v)XL+F] \), as shown in equation (9).

D’s Overall Payoff From the “Sell, Do Not Redesign” Strategy

D’s expected payoff from the “Sell, Do Not Redesign” strategy is equal to its prospective payoff, as shown in equation (9), if P wins, plus its prospective payoff of \( mX(1-T) \) if D wins, plus D’s expected profits over the period \([0,T]\), which are \( (m-\theta s)XT \), minus D’s litigation costs, \( C_D \). Therefore, D’s overall payoff from the “Sell, Do Not Redesign” strategy equals

\[
(m-\theta s)XT+(1-\theta)mX(1-T)+\theta(m-\beta v)X(1-T)-\theta\beta[(m-v)XL+F]-C_D.
\]

This expression can be written as

---

26 If D’s best option is to exit the market, then P and D split the gains from trade \( mX(1-T) \), which involves a negotiated royalty rate of \( \beta m \). This case is even more favorable to P than the case on which we focus.
\[ mX - \theta X[sT + \beta v(1-T)] - \theta \beta (m-v)XL - \theta \beta F - C_D. \]  

(10)

For future use, we write this as \( mX - E - C_D \), where 
\[ E = \theta X[sT + \beta v(1-T)] + \theta \beta (m-v)XL + \theta \beta F \]  
is the expected payment from D to P. If D adopts this strategy, P’s payoff equals \( E - C_p \). With \( s \geq \beta v \), so long as \( C_p < \theta \beta vX \), \( E > C_p \) so P’s threat to litigate is credible.

**D’s Payoff from the “Sell and Redesign” Strategy**

We now compute D’s payoff from the “Sell and Redesign” strategy. The “Sell and Redesign” strategy entails litigation, so D incurs its litigation cost \( C_D \). This strategy also involves redesign, so D incurs the redesign cost \( F \). The payoff from the “Sell and Redesign” strategy differs only in a few terms from the payoff from the “Sell, Do Not Redesign” strategy just computed.

The only benefit that D enjoys from engaging in redesign immediately rather than waiting for the resolution of the patent litigation is the improved bargaining position D enjoys if in fact P wins the litigation. In that event, which only arises with probability \( \theta \), D saves \( \beta (m-v)XL + \beta F \). The cost to D of improving its bargaining position is the redesign cost, \( F \), which must be incurred before the outcome of the patent litigation is known. Therefore, D’s payoff from “Sell and Redesign” is equal to D’s payoff from “Sell, Do Not Redesign” plus \( \theta \beta (m-v)XL + \theta \beta F - F \). So, the payoff to D from the “Sell and Redesign” strategy equals

\[ mX - \theta X[sT + \beta v(1-T)] - F - C_D. \]  

(11)

For future use, we write this as \( mX - G - F - C_D \), where \( G = \theta X[sT + \beta v(1-T)] \) is the expected payment from D to P. If D adopts this strategy, P’s payoff must equal \( G - C_p \). With \( s \geq \beta v \), so long as \( C_p < \theta \beta vX \), \( G > C_p \) so P’s threat to litigate is credible.

**Proof of Lemma #1**

As just explained, D will find it optimal to redesign its product immediately, rather than waiting for the outcome of the litigation, if and only if \( \theta \beta (m-v)XL + \theta \beta F > F \). Simplifying, D’s optimal strategy in the absence of a licensing agreement is “Sell and Redesign” rather than “Sell, Do Not Redesign” if and only if

\[ \theta > \frac{1}{\beta (m-v)XL + F} \frac{F}{}. \]  

(12)
Proof of Theorem #1

We now complete our analysis for the case in which D’s optimal threat point is to follow the "Sell, Do Not Redesign" strategy, as it will be for sufficiently weak patents. We showed above that the threat points in the initial negotiations when this is D’s optimal strategy are equal to $mX - E - C_D$ for D and $E - C_P$ for P.

Settlement allows the firms to save on litigation costs. Under Nash Bargaining, the two firms split these savings. Therefore, under Nash Bargaining, the initial negotiations give a payoff to P equal to its threat point, $E - C_P$, plus its share, $\beta$, of the gains from reaching agreement, $C_P + C_D$. So, P’s payoff from the initial licensing negotiations must equal $E - C_P + \beta(C_P + C_D)$ or $E + \beta C_D - (1 - \beta) C_P$. Since P receives no revenues other than the payment from D, and incurs no costs, this expression must measure the total negotiated payment from D to P, which equals $rX$. Substituting for $E$, using $E = \theta X[sT + \beta v(1-T)] + \theta \beta(m-v)XL + \theta\beta F$, the equilibrium royalty rate in this case is given by

$$r^* = \theta[sT + \beta v(1-T)] + \theta \beta(m-v)L + \theta\beta \frac{F}{X} + \frac{\beta C_D - (1 - \beta)C_P}{X}.$$  

(13)

Substituting $s = \beta v$ gives the expression for $r^*$ in Theorem #1.

Proof of Theorem #2

We now complete our analysis for the case in which D’s optimal threat point is to follow the “Sell and Redesign” strategy. We showed above that the threat points in the initial negotiations when this is D’s optimal strategy are equal to $mX - F - G - C_D$ for D and $G - C_P$ for P.

Settlement allows the firms to save on litigation and redesign costs; under Nash Bargaining, the two firms split these savings. Therefore, under Nash Bargaining, the initial negotiations give a payoff to P equal to its threat point, $G - C_P$, plus its share, $\beta$, of the gains from reaching agreement, $C_D + C_P + F$. So, P’s payoff from the initial licensing negotiations must equal $G - C_P + \beta(C_P + C_D + F)$ or $G + \beta F + C_D - (1 - \beta) C_P$. As in the previous case, since P receives no revenues other than the payment from D, and incurs no costs, this expression must measure the total negotiated payment from D to P, which equals $rX$. Substituting for G using $G = \theta X[sT + \beta v(1-T)]$, the equilibrium royalty rate in this case equals

$$r^* = \theta[sT + \beta v(1-T)] + \beta \frac{F}{X} + \frac{\beta C_D - (1 - \beta)C_P}{X}.$$  

(14)

Substituting $s = \beta v$ gives the expression for $r^*$ in Theorem #2.
Numerical Examples: “Sell, Do Not Redesign” vs. “Sell and Redesign”

Lemma #1 establishes that D does better adopting the “Sell, Do Not Redesign” strategy rather than the “Sell, Redesign and Litigate” strategy if and only if \( \theta \beta < \frac{F}{(m-v)XL + F} \), which can be written as \( \theta \beta < \frac{F/vX}{m-v/L + F/vX} \). In our numerical example in which “Sell, Do Not Redesign” is optimal for D, we have \( F/vX = 0.5 \), \( (m-v)/v = 10 \), and \( L = 0.1 \), so this inequality becomes \( \theta \beta < \frac{0.5}{1+0.5} = 1/3 \). With equal bargaining power, \( \beta = 1/2 \), this condition in turn becomes \( \theta < 2/3 \). In our numerical example in which “Sell, Litigate and Redesign” is optimal for D, we have \( F/vX = 0.5 \), \( (m-v)/v = 30 \), and \( L = 0.1 \), so this inequality becomes \( \theta \beta < \frac{0.5}{3+0.5} = \frac{1}{7} \). With equal bargaining power, \( \beta = 1/2 \), this condition in turn becomes \( \theta < 2/7 \).

Proof of Theorem #3

Working backward as usual, we ask what payoffs result if P wins but the injunction is stayed while D redesigns its product. In this situation, D’s prospective payoff after P wins, and if no license agreement is then reached is given by \( (m-v)X(1-T-L) - F \). P’s prospective payoff if no license is signed is \( \beta vXL \). The gains from reaching agreement are \( vX(1-T-L) + F \), which represents the use of the patented technology after the injunction would go into force plus D’s redesign costs. D’s prospective payoff under the negotiated license is therefore equal to \( (m-\beta v)X + (m-v)X(1-T-L) - F + (1-\beta)[vX(1-T-L) + F] \) which simplifies to \( (m-\beta v)X(1-T) - \beta F \).

The downstream firm’s overall payoff from the “Sell, Do Not Redesign” strategy thus equals \( (m-\theta \beta v)XT + (1-\theta)mX(1-T) + \theta[(m-\beta v)X(1-T) - \beta F] - C_D \). Simplifying, this expression becomes \( mX - \theta \beta vX - \beta F - C_D \). Under the initial licensing agreement, D gets this payoff plus its share, \( \beta \), of the gains from trade, \( C_D + C_p \), or \( mX - \theta \beta vX - \beta F - [\beta C_D - (1-\beta)C_p] \).

Converting to a per-unit royalty, and assuming \( \beta C_D - (1-\beta)C_p = 0 \), we get \( r^* = \theta \beta vX + \theta F/X \).

Proof of Theorem #5

If \( \theta < \theta^* \) so “Sell, Do Not Redesign” is D’s optimal threat point, the equilibrium royalty rate is given by equation (13). Assuming \( \beta C_D - (1-\beta)C_p = 0 \), and putting \( \theta = 1 \) into that equation gives, \( r^*(1) = sT + \beta vX(1-T) + \beta(m-v)L + \beta F/X \). The condition defining a self-fulfilling
equilibrium is \( s = r^*(1) \). Using this relationship, we can eliminate \( r^*(1) \) from the previous equation to get \( s = sT + \beta v (1-T) + \beta (m-v) L + \beta \frac{F}{X} \). Solving for \( s \) gives

\[
s = \beta v + \frac{\beta}{1-T} [(m-v) L + \frac{F}{X}].
\]

This expression tells us that the reasonable royalty rate will itself be greater than the benchmark level of \( \beta v \) because it is influenced by the threat of hold-up, as reflected by the term in brackets on the right-hand side.

This expression for \( s \) cannot hold if it violates our standing assumption that \( \theta s < v \), in which case D would prefer the strategy of “Wait, Redesign, Litigate” to “Sell and Redesign” and our analysis would need to be modified. In addition, for this equation to apply, if D loses the litigation and is unable to obtain a license, D must prefer redesigning its product to exiting the market. This requires \( (m-v)X(1-T) > F \), which places a lower bound on \( (1-T) \). If, instead, D’s threat point after losing the patent litigation is simply to withdraw from the market, then \( s = \beta m \), which can far exceed the benchmark level of \( \beta v \).

If the expression above for \( s \) does apply, we can use equation (13) to derive a new equation for the equilibrium royalty rate negotiated between P and D:

\[
r^* = \theta \beta v + \frac{\theta \beta}{1-T} [(m-v) L + \frac{F}{X}].
\]

When this equation applies, the hold-up term in the fulfilled expectations equilibrium is magnified by the factor \( 1/(1-T) \), in comparison with the case where \( s = \beta v \).

**“Sell and Redesign” Optimal for D**

If \( \theta > \theta^* \) so “Sell and Redesign” is D’s optimal threat point, the equilibrium royalty rate is given by equation (14). Substituting using \( s = r^*(1) \) and solving for \( s \), we get

\[
s = \beta v + \frac{\beta F}{X(1-T)}.
\]

As in the previous sub-section, this equation cannot apply as \( T \) approaches unity, because it would violate our standing assumption that \( \theta s < v \). So long as \( T \) is not too large, however, this expression is valid and leads to the revised expression for the equilibrium royalty rate:

\[
r^* - \theta \beta v = \theta \beta v \frac{1}{1-T} \frac{F}{vX} \frac{1}{\theta}.
\]

Again, the equilibrium over-charge is magnified by the factor \( 1/(1-T) \).
Figure 1: Negotiated Royalties

Royalty Rate: \( r \)

Negotiated Royalty Rate: \( r^* \)

\[ \beta v + \beta F/X + \beta(m-v)L \]

\[ \beta v + \beta F/X \]

\[ \beta v \]

\[ \text{Benchmark} \]

Patent Strength: \( \theta \)

Do Not Redesign

Redesign

0

\( \theta^{**} \)

\( \theta^* \)

1
Figure 2: Effect of Stays

Royalty Rate: $r$

Patent Strength: $\theta$

$\beta v + \beta F/X + \beta (m-v) L$

$r^* \text{ Without Stays}$

$r^* \text{ With Stays}$