PROPORTIONALITY PROFILES OF WEST EUROPEAN ELECTORAL SYSTEMS

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ABSTRACT

How proportional are the results of electoral systems designed to ensure proportional representation? How large is the deviation from proportionality for relative majority systems? Does a given system favor large, small or medium-sized parties? Are some parties consistently advantaged? Is the system predictable or erratic? Plotting the percentage of seats—percentage of votes ratio versus percentage of votes for all parties, is found to be a simple and yet powerful graphical method to answer such questions. Application of such “proportionality profiles” to West European multi-party systems leads to a typology of electoral systems based on their actual effect (rather than their formal procedure or intended effect). District magnitude, the number of rounds, and nationwide adjustment rules are found to be more important than formal seat distribution rules such as d'Hondt or Sainte-Lagué.

Introduction

Theoretical analysis of electoral laws has cast some light on the effects of the different systems of proportional representation. Inclusion and exclusion thresholds have been determined for various basic proportional representation (PR) formulae (see e.g. Rokkan, 1968; Rae et al., 1971; Loosemore and Hanby, 1972; Grofman, 1975; and Lijphart and Gibberd, 1977). The actual share of votes at which a given party achieves its first seat must be in-between these extremes; the theoretically most likely location has been worked out and partly tested (Laakso, 1978: 85–86). Threshold value theory has been extended to
more general payoff values by Lijphart and Gibberd (1977). For relative majority systems the empirical non-proportional "cube law of elections" can be theoretically justified (Taagepera, 1973). But at the present stage theory can be applied only to the simplest electoral systems. The actual systems in most countries tend to include various complicating features (such as mixtures of several basic electoral formulae, arbitrary votes thresholds, and rules on electoral alliances), and even the very same formula may work out differently for different historical-geographical frameworks. Much empirical work is needed to guide theoretical modeling in the most useful directions.

Comparative empirical study of political effects of electoral laws reached an early landmark with Rae's work (1967) and various rules were ranked for proportionality by Blondel (1969: 191). In view of the large number of data involved (seat and vote shares of all parties for a large number of elections in a large number of countries) emphasis was put on developing indicators that would condense data. Thus the index of fractionalization of votes (Rae, 1967) reduced all votes data into one index per election or even (through averaging) per electoral system. Alternatively, the effective number of parties involved in an election or a series of elections can be defined (Laakso and Taagepera, 1979). Fractionalization and the effective number of parties usually decrease in the process of translating votes into assembly seats, indicating that minor parties tend to be eliminated or under-represented. Plotting seat shares versus vote shares shows that even the supposedly proportional electoral systems tend to favor larger parties at the expense of the smaller ones (Rae, 1967). The differences between the vote and seat shares for individual parties can be summed up to express the deviation from proportionality for a given election (e.g. Loosemore and Hanby, 1972); this sum can be divided by the number of parties involved in order to get an average deviation (Rae, 1967). The total deviation from proportionality can be separated into a systematically party size-dependent part and a residual essentially random part (Laakso and Taagepera, 1978 and 1980). The most recent and thorough tests of the cube law have been carried out by Linehan and Schrodt (1977), and by Schrodt (1980), with mixed results.

However, hidden theoretical assumptions often enter empirical studies. Even the apparently straightforward question of whether a given electoral system favors large or small parties tacitly assumes that middle-size parties cannot be favored over both small and large parties. Once such an implicit assumption is made, the whole statistical analysis is likely to be carried out along lines that hide middle-size party advantage, because the model includes no term where such advantage could
express itself; it would be treated as mere random deviation. This is only one example of how hidden and often unwarranted theoretical assumptions can slip into a presumably empirical study. Purely empirical work hardly exists.

Another problem is that, by the time theoretical or empirical analysis has been carried out, the original information not only is compacted but much of it is lost. A decrease in the effective number of parties from votes to seats level no longer tells us which parties have been boosted at the expense of which others. Also, indices that characterize some aspect of an entire election or electoral system may be so abstract that they may tell very little to the practicing statesmen or even a fellow-scholar who is not part of the same narrow school.

Before we can locate fruitful theoretical avenues and suitable approaches to supposedly empirical analysis we need a way to present all elections results relevant for a given electoral system in a single picture that can be seen at a single glance in all its complexity. Tables of data, though precise in their content, lack this quality. A graphical plot is needed. The variables used should be as directly self-explanatory as possible, and no data should be omitted or condensed. As far as proportionality is concerned, we think we have devised such a simple all-inclusive empirical picture. We will call it a proportionality profile. It is constructed in the following way.

The Procedure

Our basic concern is with the interrelation of a given party’s percent share of votes (%V) and its percent share of seats (%S) in the representative assembly. From the viewpoint of proportional representation, the crucial index is the ratio of these shares — either %V/%S or %S/%V. The difference between shares (%V − %S) also may be considered, but it clearly depends on the party size: a party with only a few percent of the votes is not likely to accumulate as large a difference as could be the case with a major party. Normalizing for the size effect by dividing the difference by %V takes us back to the ratio expression 1 − %S/%V. Of the two possible ratios, %V/%S becomes infinite when a party with some votes happens to get no seats. The ratio %S/%V varies from 0 (for no seats) to values rarely larger than 2; it is thus easier to represent graphically. We will call this ratio the advantage ratio of the given party:

\[ A = \frac{\text{actual representation}}{\text{proportional representation}} = \frac{%S}{%V}. \]
This ratio is analogous to the "index of advantage" that Dahl (1956: 114) used for characterizing the degree of representation of various U.S. states in the U.S. Senate. For ideal proportional representation $A = 1$ by definition. Values below unity indicate a disadvantage while values above unity indicate that a party is getting more than its proportional share of the seats. The upper limit on $A$ at a given $%V$ is $100/(%V)$.

Since it is observed that a party's advantage ratio often depends on its size (in terms of votes), our next step is to plot $A$ versus $%V$. This does not mean that we assume that any electoral system assigns relative advantage according to party size. We are merely plotting a "scattergram". A size-dependent pattern may or may not emerge from the plot. By assigning different symbols to different parties we can visually detect biased cases where one party tends to have a higher advantage ratio than another, for the same share of votes. Our "proportionality profile" (the $A$ vs. $%V$ plot) does not tell us anything about the reasons for such bias (e.g., planned gerrymander, or an inherently favorable geographical distribution of votes). If a party's advantage ratio should fluctuate wildly from one election to another, with little change in votes, our plot will show it, too, again without telling us anything about the reasons for such instability. It is a device for generating informed questions before looking for answers.

The major a priori assumption that does enter the proportionality profile approach is that party size is often a major determinant for its advantage ratio — this is why we plot $A$ against $%V$ rather than against some other parameter. Apart from this assumption, the procedure seems to be fairly empirical, leaving the interpretation of the emerging pattern widely open. We also hope that the variables involved (votes share, and the seat—vote ratio) are sufficiently simple to enable even the non-specialist to understand what is involved.

Figures 1 and 2 offer two disparate examples of the patterns that may emerge. Belgium (Fig. 1) offers a quite regular pattern of size-dependence, over the 58 years (1919–77) during which it has used the basic d'Hondt highest average formula. (For description of various electoral systems see e.g. Rae, 1967; Grofman, 1975; Lijphart and Gibberd, 1977; or Nohlen, 1978.) Belgian parties with more than 18% of the vote always receive a bonus (as reflected by $A$ larger than unity). Parties with less than 18% of the vote are nearly always penalized ($A$ less than unity), and this penalization becomes gradually heavier as party size decreases. The graph also shows at a glance that some geographically concentrated small parties (Flemish, dissident Catholics, and Walloons) have occasionally managed to beat the system. Dates for the most deviant cases are shown in Fig. 1. Cases of parties with less than
2% of the votes and no seats are too numerous to be shown separately; they are lumped together under a single microparties symbol (a grid of dots). The perfect proportionality line \( (A = 1) \) is shown on all our proportionality profile plots. We have also found it useful to show the vertical \( %V = 10\% \) line because this line delimits the region where small-party disadvantage tends to change into large-party advantage, for many electoral systems. In the Belgian case the 10% parties are seen to be still among the disadvantaged.

The German Reichstag elections of 1871–1912 show a completely different proportionality profile (Fig. 2). Almost anything seems possible, at first glance, especially for small parties. At a closer look, different parties have different patterns. The Socialists were most heavily penalized at any level of votes. Seat distribution was by absolute majority in single-seat districts during the first round, followed (if no absolute majority materialized) by a runoff between the two parties with the highest number of votes. The Socialists probably lost most of these runoffs to non-Socialist coalitions. The Zentrum and the Conservative parties, on the other hand, are seen to be advantaged in most, but not all early elections. For National Liberals, advantage ratio rose from 0.65 in 1890 to 1.03 in 1893 although they lost votes. The fluctuations for small parties are often completely erratic. Note that,
for the tiny local Danish Party of Schleswig, going from one seat (in 1878) to two (in 1881) with essentially the same number of votes meant an advantage ratio jumping from 0.8 to 1.7, with intermediate values of $A$ impossible, since seats come in integer values only. In 1907 and 1912, the Danes even reached an advantage ratio of 2.5, and the Elsass-Lothringen party reached 2.0 in 1903 and 1907. For a closer study of German elections one may well want to plot separate proportionality profiles for all parties, label the points with dates, and look for trends in time. For our comparative purpose the contrast between the regular Belgian pattern and the scattered German one is the main point of interest.

In regular cases like the Belgian one, one may want to show the average shape of the size dependence curve. We might visually sketch in the approximate best-fit curve, but this is imprecise. Least square fitting would be precise, but it would require that we first decide on the
general mathematical shape of the curve to be fitted — and this is how unstated theoretical assumptions often slip into presumably empirical studies. (In fact, many political scientists seem to think that all relationships between two variables are linear!) In this study the following approximate and quick empirical fitting method is used. Separate all data points into two percent brackets of %V. Starting from the higher %V values, group these brackets so that they contain at least 4 or (preferably) 5 data points. Within each group, take the median %V and the median $A$. Join these median points. The resulting curve for Belgium (Fig. 1) crosses the perfect proportionality level ($A = 1$) at %V = 18%. This is the vote level below which a party tends to be disadvantaged, and above which it tends to get a bonus. We will call it the break-even percentage. A similar median curve for the scattered German data in Fig. 2 seems to offer little information. (We used 4% brackets of %V in order to smooth out the curve, in this case.)

**West European Profiles**

Rokkan and Meyriat (1969) and Nohlen (1978) have edited excellent collections of electoral rules, statistics and source materials. Rokkan and Meyriat cover 15 European countries where multi-party elections have been held since World War II. We have used all of their seats and votes data which usually range from about 1900 or 1920 to about 1965, with a few exceptions (Germany from 1871 on, Greece, Italy and Iceland from 1926–27 on). For more recent elections in the same 15 countries we have made use of Nohlen's handbook — with its worldwide coverage — which gives data from about 1960 onwards (and often from 1945). Data for periods with different electoral systems were plotted separately. If, upon subsequent inspection, the profiles did not differ, periods were joined in the case of relatively minor changes in the electoral system. In the crowded parts of the plots points have been sometimes slightly shifted in order to avoid superimposition.

It is not always clear what constitutes a party, as distinct from a momentary or long-term alliance. For our purposes we were interested in the smallest units that acted independently during a given campaign to such an extent that their votes and the resulting seats were recorded separately. In practice, we had to depend in this respect on the judgment of the authors in Rokkan and Meyriat (1969) and Nohlen (1978). Whatever they decided to lump together is treated as a single voting bloc in the present study. If data were listed on two levels (e.g., in Germany CDU/CSU as a single bloc, and also CDU and CSU separately),
we used the smaller units. The "Other" or "Diverse" category which
lumps various small (and occasionally even middle-sized) parties pre-
sents a special problem: a large number of tiny parties below any rea-
sonable seats threshold may appear as a rather large bloc of votes with-
out a share of seats. We considered omitting the "Other Parties" cate-
gory from our profiles, but that may also distort the picture. Our solu-
tion has been to plot the "Other" bloc consistently with the same
diamond-shaped symbol, if it involves any seats or more than 2% of
votes. Whenever these "diamond" points tend to disagree with the gen-
eral pattern, the reader may consider discounting them.

Instead of a country-by-country approach, we have tried to group
periods in various countries during which the electoral system remained
unchanged and led to similar profiles in several countries. The profiles
for Denmark, Finland, Norway and Sweden have been published by
Laakso and Taagepera (1978); they are discussed, but not reproduced
here.

During the period covered, several of the 15 countries changed their
electoral system once or repeatedly. The record goes to France which,
from 1919 to 1958, never kept the same rules for more than two or
three successive elections. Fortunately for us these shifts often meant
a return to an electoral system discarded a few decades earlier. Thus the
five French periods with different rules can effectively be grouped into
three basic types. Even with such groupings, the 15 countries present us
with more than 30 clearly distinct periods.

Given this variety, we should try to establish a few basic profile
types. There are several ways to proceed. An electoral system \( E \) can
be presented as a function of its components:

\[
E = f(a, b, c, d, e, \ldots)
\]

where \( a \) = electoral rule, \( b \) = the size of an electoral constituency, \( c \) =
electoral alliances, \( d \) = adjustment seats system, \( e \) = the number of
rounds, etc. . . . The importance of specific components of an electoral
system varies strongly from country to country. Let us first consider an
electoral rule \( (a) \). One can expect that majority systems must differ
strongly from proportionality systems. The classification of proportion-
ality profiles could start from this basic premise. But this distinction
tells us as yet nothing about differences between those countries which
apply proportional electoral rules. Therefore, other components of an
electoral system have to be taken into account. Let us consider the
adjustment seats system \( (d) \). This component can be expected to influ-
ence strongly the proportionality of elections. In the same way, the
proportionality order of different electoral rules among the PR meth-
ods should have an effect on proportionality profiles. Thus, one can base one’s classification on every component of an electoral system \((a, b, c, \ldots)\). However, we have often no a priori ways to tell in which order the components ought to be considered, and which effects of which components are overridden or pre-empted by others. It is not enough to take into account the separate components of the electoral system — one must also consider the complex interactions between those components and the characteristics of a country’s party structure and historical development. Therefore, we have based our profile types not on formal components of an electoral system but on the empirical shapes of the profile. This means that countries are first classified according to the shape of their proportionality profiles during different time periods. The shape of the profile is then explained by the components of an electoral system and the country’s characteristics.

All periods in all 15 countries can be fitted by a typology consisting of only 7 characteristic profiles. The countries and periods belonging to each are tabulated in Table I. These characteristic profiles (which will be discussed in detail later on) are the following:

1. **Middle Valley profile.** Associated with plurality rule in single-seat constituencies, this pattern combines a large bonus (up to \(A = 1.7\)) for the largest party with heavy penalties to runners-up (\(A\) less than 0.5 for parties with 10 to 20% of votes), in basic agreement with the “cube law”. However, small regional parties often have a reduced penalty or even win a bonus not foreseen by the cube law. See examples in Fig. 3.

2. **Flat profile.** Often associated with systems having large electoral districts or nationwide adjustment seats, this pattern is close to ideal proportional representation for any party size, except that for small parties random deviations up or down from proportionality become large due to the integer number of seats. The break-even percentage of votes (beyond which a party’s advantage ratio tends to be above unity) is around 2 to 4%, as shown in Table I. Examples of profiles are shown in Fig. 4.

3. **Early Rise profile.** Often associated with Largest Remainder and large-district d’Hondt rules, this pattern tends to offer a moderate bonus to parties with more than 15% of votes while penalizing heavily parties with less than 5% of the votes. The break-even percentage ranges from 6 to 13%. See examples in Fig. 5.

4. **Late Rise profile.** Often associated with the d’Hondt rule, this pattern is similar to the previous one except that the small party penalty region extends to medium-sized parties with break-even percentage ranging from 17 to 25% of votes. Belgium (Fig. 1) is of that type. See other examples in Fig. 6.
## TABLE I

Countries and Periods Belonging to Seven Characteristic Profiles

<table>
<thead>
<tr>
<th>Profile</th>
<th>Country and period</th>
<th>Electoral rule</th>
<th>Average district magnitude</th>
<th>Break-even percentage b</th>
</tr>
</thead>
<tbody>
<tr>
<td>Middle Valley</td>
<td>United Kingdom 1900–77</td>
<td>Plurality</td>
<td>1</td>
<td>(34)</td>
</tr>
<tr>
<td></td>
<td>Greece 1928, 33–34, 52</td>
<td>Plurality</td>
<td>1</td>
<td>(36)</td>
</tr>
<tr>
<td>Flat</td>
<td>Denmark 1953–77</td>
<td>Modified Ste.-Lagué and adjustment</td>
<td>6 (40) a</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Germany 1920–33</td>
<td>Quota, local and nationwide</td>
<td>~15</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Denmark 1932–53</td>
<td>D'Hondt and adjustment</td>
<td>5 (40) a</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Netherlands 1918–33</td>
<td>Largest Remainder, nationwide</td>
<td>100</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Sweden 1970–76</td>
<td>Modified Ste.-Lagué, adj. 4% threshold</td>
<td>11 (39)</td>
<td>4</td>
</tr>
<tr>
<td>Early Rise</td>
<td>Netherlands 1937–77</td>
<td>D'Hondt, nationwide</td>
<td>100; 150</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>Austria 1971–79</td>
<td>Quota and adjustment by d'Hondt</td>
<td>19</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>Germany 1949–53</td>
<td>Plurality and d'Hondt</td>
<td>?</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>Germany 1957–76</td>
<td>Same, plus 5% threshold</td>
<td>?</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>Greece, 1926, 32, 36, 46, 50</td>
<td>&quot;Proportional representation&quot;</td>
<td>~7</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>Italy 1946–76</td>
<td>Largest Remainder</td>
<td>~20</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>Sweden 1921–49</td>
<td>D'Hondt</td>
<td>10</td>
<td>13</td>
</tr>
<tr>
<td>Late Rise</td>
<td>Country</td>
<td>Period</td>
<td>Method</td>
<td>Seats</td>
</tr>
<tr>
<td>-----------</td>
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<td>--------</td>
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<td>-------</td>
</tr>
<tr>
<td>Norway</td>
<td>1930–49</td>
<td>D'Hondt</td>
<td>8</td>
<td>17</td>
</tr>
<tr>
<td>Finland</td>
<td>1906–75</td>
<td>D'Hondt</td>
<td>15</td>
<td>17</td>
</tr>
<tr>
<td>Belgium</td>
<td>1919–77</td>
<td>D'Hondt</td>
<td>~10</td>
<td>18</td>
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<tr>
<td>Iceland</td>
<td>1959–74</td>
<td>D'Hondt and adjustment</td>
<td>6 (11) (^a)</td>
<td>19</td>
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<tr>
<td>France</td>
<td>1945–46</td>
<td>Largest Remainder, then d'Hondt?</td>
<td>~5</td>
<td>20</td>
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<tr>
<td>Greece</td>
<td>1951, 56–77</td>
<td>“Reinforced PR”</td>
<td>~6</td>
<td>20</td>
</tr>
<tr>
<td>Austria</td>
<td>1923–70</td>
<td>Quota and adjustment by d'Hondt</td>
<td>~6</td>
<td>25</td>
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<tr>
<td>Middle Peak</td>
<td>Norway</td>
<td>1953–77</td>
<td>Modified Ste.-Lagué</td>
<td>8</td>
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<tr>
<td></td>
<td>Sweden</td>
<td>1952–68</td>
<td>Modified Ste.-Lagué</td>
<td>11</td>
</tr>
<tr>
<td>Wide Scatter</td>
<td>Germany</td>
<td>1871–1912</td>
<td>Majority or run-off</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>France</td>
<td>1919, 24, 51, 56</td>
<td>Majority/Quota mix</td>
<td>?</td>
</tr>
<tr>
<td></td>
<td>France</td>
<td>1928, 32, 36, 58–73</td>
<td>Majority or run-off</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Ireland</td>
<td>1923–77</td>
<td>Single transferable vote</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>Italy</td>
<td>1913–29</td>
<td>2 rounds, d'Hondt, etc.</td>
<td>?</td>
</tr>
<tr>
<td></td>
<td>Switzerland</td>
<td>1919–75</td>
<td>Largest Remainder</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>Germany</td>
<td>1919</td>
<td>D'Hondt</td>
<td>?</td>
</tr>
<tr>
<td>Geyser</td>
<td>Iceland</td>
<td>1927–59</td>
<td>Plurality, d'Hondt and adjustment</td>
<td>1 to 2 (11) (^a)</td>
</tr>
</tbody>
</table>


\(^a\) Magnitude in parentheses indicates the number of country-wide adjustment seats.

\(^b\) Percentage of votes above which parties tend to have \% S/\% V ratio \( A > 1.0 \), according to Figs. 1 to 7.
5 Middle Peak profile. Often associated with modified Ste.-Laguë rule, this pattern is similar to the Early Rise, except that middle-sized parties (7 to 15% of votes) tend to get a somewhat higher bonus \(A\) often above 1.05) than do the larger parties for which \(A\) is rarely over 1.05. Examples are shown in Laakso and Taagepera (1978).

6. Wide Scatter profile. Often associated with two rounds (majority, then plurality) rules, this pattern consists of nearly symmetrical scatter of points both above and below the \(A = 1\) line. The widest possible range gradually narrows down for only the very largest parties. Similarly to the Flat profile, the average advantage ratio tends to depend little on party size. Imperial Germany (Fig. 2) is an extreme example; (see Fig. 7 for others).

7. Geyser profile. Unique to Iceland (see Fig. 6), this pattern consistently makes a certain party's advantage surge to unusual heights (\(A\) ranging from 1.1 to 2.1) at the expense of any larger or smaller parties.

These are only broad generalizations. We will now discuss each group of profiles separately.

MIDDLE VALLEY PROFILES

These very distinct profiles arise from single seat districts with plurality (relative majority) rule. For such elections the cube relationship may apply: the seat ratio of two parties (A and B) tends to be close to the cube of their vote ratio:

\[
S_A/S_B = (V_A/V_B)^3
\]

This cube law does not yield a single curve in our proportionality profile plot, since the outcome, for a given party, depends on how splintered the opposition is. Figure 3 shows the cube law curves for two situations. The “Single Opponent” curve shows the situation for a pure two-party system. The party whose votes percentage is shown in the graph in that way faces the worst possible situation: a unified opposition. According to the cube law its advantage ratio should fall off very rapidly below 50% of votes (and also, slower, at very high %V where the opponent’s smallness precludes a large advantage). The other curve (“Two Opponents”) assumes that the opposition to the party plotted is split into two equal-sized parties. This might be a realistic best situation that a party might hope for. (The theoretical optimum situation would, of course, be to face an opposition fragmented into a huge number of one-vote parties.)

Figure 3 shows the UK and Greek simple plurality election profiles for this century. The lowest points tend to locate around the “Single
Fig. 3. Middle Valley profiles: United Kingdom 1900–1977 (Plurality) and Greece 1928, 1933–1935, 1952 (Plurality).

Opponent" curve, as expected. The highest points tend to be not much higher than the "Two Opponents" curve, except for parties with less than 10% of votes for which the cube law visibly fails. The cube law should be expected to apply to nationwide parties only. Under "first-past-the-post" conditions, such parties as the UK Liberals may gather an appreciable percentage of votes while rarely being the winner in any district. Thus the advantage ratio is very low. Many parties receiving less than 10% of votes, however, are regional parties. In their own region they may be the majority, and this is what determines their advantage ratio.

In the UK case, the 1900–18 and 1922–77 periods have been plotted with different symbols because the first period was affected by the peculiar situation of the Irish Nationalist party. Presumably because of limited franchise, the Irish deputies represented (and were elected by) markedly fewer votes than was possible elsewhere, leading to grotesquely high advantage ratios for Irish Nationalists: with 0.6% of total UK votes Irish Nationalists obtained 83 out of 670 M.P. seats in 1906, leading to $A = 21!$ Their other advantage ratios in 1900–10 ranged from 5 to 6, for about 2% of votes.
The most extreme cases of proportionality at any party size, in 20th century Europe, occur for Denmark 1953–73 (curve shown in Laakso and Taagepera, 1978) and Germany 1920–33 (Fig. 4). In the case of Germany all parties with more than 4% of votes had an advantage ratio of $1 \pm 0.05$ in all elections. (Remember that the “Other” category marked by a diamond symbol is likely to consist of several tiny parties.) The only systematically advantaged party was the local Bavarian BVP with 3 to 4% of votes and $A$ ranging from 1.04 to 1.13. The electoral system used was Quota, first in approximately 15-seat districts and then (for remainders) on regional and national levels (Nohlen, 1978: 211).

Denmark 1953–77 and Sweden 1970–76 obtained the same degree of proportionality by modified Ste.-Lagué rule and 6 to 11 seat districts (on the average), with 40 or 39 nationwide adjustment seats. A 4% vote threshold in Sweden makes $A$ drop sharply to zero for parties with less than 4% of votes. The Danish system of 1932–53 with d’Hondt rule (always with 40 adjustment seats) gave an almost as flat a profile.

Netherlands 1918–33 (Fig. 4) achieved fair proportionality using the Largest Remainder rule with the whole country being a single electoral district of 100 seats (Lijphart, 1978). (One could say that it was like the Danish system, with all seats being nationwide adjustment seats.) The advantage ratio tends to be $1 \pm 0.10$ for parties with more than 6% of votes. For smaller parties the range of $A$ widens, with possibly a

Fig. 4. Flat profiles: Germany 1920–1933 (local and nationwide Quota) and Netherlands 1918–1933 (nationwide Largest Remainder).
slight advantage \((A = 1.1)\) for parties with 2 to 6% of votes. The same tendency occurs for Denmark 1932–53.

It would seem that extreme proportionality reflected by a Flat profile is obtained when the last round of seat distribution is carried out on a nationwide basis, with a sufficient number of adjustment seats on hand. The type of rule and the district magnitude used in the main round seem to be rather unimportant: Ste.-Lagüé, d'Hondt, simple Quota, and Largest Remainder may lead to similar profiles, as long as adjustment seats represent at least 10% of all seats. Yet the attention in most descriptions of electoral rules has tended to concentrate on the pre-adjustment phase, with adjustment rules given only passing mention.

**EARLY RISE PROFILES**

Profiles with break-even point around 10% of votes (Fig. 5) seem to arise from a variety of electoral systems. That of Netherlands 1934–77 (nationwide d'Hondt rule for 100 or, after 1956, 150 seats) is rather close to the earlier Flat profile for this country, presumably due to the huge district magnitude. Yet the parties with less than 5% of seats tend to be systematically disadvantaged, presumably due to the d'Hondt rule, much before the legal exclusion threshold of 2/3% is reached.

The small-party pattern is somewhat more scattered for Italy 1946–76. While most parties with less than 10% of votes have suffered, some local parties (Valdois, Sard, South Tyrol) managed to buck the trend until 1963, but not later on. Italian rules are a complex variant of Largest Remainder, with relatively large districts (average around 20 seats) and nationwide adjustment by Largest Remainder.

West Germany also has mixed rules, combining plurality and d'Hondt. The partial 5% threshold introduced in 1957 does not seem to have altered the pattern. The early post-war elections show more scatter because the democratic political patterns had not yet stabilized.

Greece tried "proportional representation" rules several times, from 1926 to 1950. The average outcome is a rather irregular Early Rise profile. In contrast to the Flat profile (where nationwide distribution or adjustment of seats seem to be a common factor), there is no clear single common factor in electoral systems leading to Early Rise profiles.

The profile for Sweden 1921–49 (Laakso and Taagepera, 1978) occupies middle ground between Early Rise and the next category – Late Rise. The cut-off between these two categories is arbitrarily set at a break-even percentage of 15%. Most countries with Sweden's system (d'Hondt and approximately 10-seat districts) have slightly later rises.
Fig. 5. Early Rise profiles: Netherlands 1937-1977 (nationwide d'Hondt), Greece 1926, 1932, 1936, 1946-1950 (PR), Italy 1946-1976 (Largest Remainder), West Germany 1949-1976 (Plurality and d'Hondt).

LATE RISE PROFILES

Profiles with break-even point around 20% of votes seem to be typical of d’Hondt rules with moderate district magnitude (8 to 15 seats): Belgium 1919-77 (Fig. 1), Norway 1930-49, Finland 1906-75 (Laakso and Taagepera, 1978), Austria 1919-20 (Fig. 6). But it also occurs for variants of Largest Remainder rules with somewhat smaller magnitude (5 to 6 seats): Austria 1923-70 (which also had limited regionwide adjustment using d’Hondt) and France 1945-46 (Fig. 6). In 1971-79 elections with increased district magnitude (19 seats), Austria
seems to have shifted to an Early Rise profile (1979 data from *Los Angeles Times*, 5 May, 1979).

Greek "reinforced PR" (Rokkan and Meyriat, 1969) of 1951 and 1956–77 also yielded similar results, but with an unusually steep rise of advantage ratio from 0.6 at 14% of votes to 1.1 at 22%.

**MIDDLE PEAK PROFILES**

Only two cases have been observed: Norway 1953–77 and Sweden 1952–68, both using modified Ste.-Laguë rules. The profiles (shown in

Laakso and Taagepera, 1978) are rather similar to the Early Rise ones, except for a modest peak of advantage ratio at 7 to 15% of votes. This peak came about after a change from d'Hondt to Ste.-Laguë electoral rule, all other factors remaining the same.

Although the Agrarian party accounts for the highest $A$ values in both countries, the middle peak cannot be discarded as a historical-geographical peculiarity of one particular party, since several other parties in the medium votes bracket tend to share it, and this was the way the rules were meant to work, at least in Norway (Rokkan and Hjellum, 1966; Rokkan, 1968). The previously used d'Hondt rule (divisors 1, 2, 3...) worked to the advantage of the large Socialist party and to the disadvantage of the several bourgeois parties who, together, had sufficient weight to obtain a change in rules. Pure Ste.-Laguë (divisors 1, 3, 5...) would have most profited the very smallest parties. Modified Ste.-Laguë (divisors 1.4, 3, 5...) moderates the large party advantage (due to its large later divisors) and penalizes the tiny parties (due to large first divisor), leaving a middle-sized party advantage, both in theory and in practice.

**WIDE SCATTER PROFILES**

These profiles are typical of two-round systems, usually with one seat per district, where the first round requires absolute majority, the absence of which leads to a second round decided by plurality or by a runoff of the two strongest candidates. The party votes reported refer to the first round which is rarely decisive. Seats distribution during the second round depends on coalition formation after the first round. Thus several ideologically close small parties may consistently defeat an isolated large party. In the case of a runoff of the two strongest candidates, neither may be among the first choices of the majority of voters in the case of a highly fragmented constellation. One may expect little correlation between percent votes in the first round and the percent of seats which is largely determined in the second round. This is what we observe, indeed, for Germany 1871–1912 (Fig. 2), France 1928–36 and 1958–73, and also Italy 1913 (Fig. 7). The smaller the party, the more its advantage ratio becomes unpredictable on the basis of size alone, although the average $A$ tends to remain close to 1.0 at all party sizes. In this respect the Wide Scatter profile is similar to the Flat one — compare, e.g., Netherlands in Fig. 4 to Germany in Fig. 2.

The French complex mixed system (see Nohlen, 1978: 183) of 1919, 1924, 1951 and 1956 is seen (Fig. 7) to lead also to wide scatter. To a lesser extent the same is true of the Switzerland 1919–75 Largest
Remainder variant where the wide scatter is limited to parties with less than 20% of votes. The tiny Liberal-Democrats have consistently had a high advantage ratio; without this party-specific anomaly, the profile would be close to an Early Rise one. German 1919 elections (d’Hondt; not plotted) also showed wide scatter, presumably due to lack of stabilization.
The Irish 1918–22 results have been plotted with a distinct symbol because rules varied and the party system had not crystalized, leading to a wide patternless scatter. The Irish 1923–77 "single transferable vote" seems to lead to a wide scatter for parties with less than 20% of votes. However, most points in this range belong to the "Other" and "Independent" categories. The Italian 1919–21 d'Hondt rule also led to appreciable scatter for reasons unclear to us. The 1924 rules were specifically designed to give the largest party 2/3 of the seats; since the Fascists also carried 65% of votes, their advantage ratio actually did not turn out to be high.

GEYSER PROFILE

This profile is unique to Iceland (Fig. 6), where the electoral system has favored the Progressive Party because rural areas have been overrepresented in the Althing — despite the fact that three changes have been made in the system in 1934, 1942 and 1959, always to the disadvantage of the Progressives (Nuechterlein, 1961: 10). Urbanization kept recreating a farm vote advantage. Irrespective of their vote percentage, the Progressives received much more than their proportional share of seats, from 1927 to 1959. In 1956, when they were the fourth largest party in votes (15.6%), they still almost tied with the winner in seats (17 vs. 19), and their advantage ratio was 2.10. In compensation all other parties rarely reached an advantage ratio of unity. The electoral system used was a mix of plurality and d'Hondt with up to 8 members per district, plus up to 11 nationwide supplementary seats which, however, did not suffice to balance the farm vote advantage. Note that plurality rule in single-seat districts is equivalent to d'Hondt with one seat per district. Starting with the second elections of 1959, the geyser was finally brought under control by redistricting into districts of 5 to 12 members per district, plus nationwide adjustment seats (Nohlen, 1978: 285). The new profile has a rather late rise (Fig. 6). A slight Progressive Party advantage remains nonetheless.

Conclusions

The simple method of plotting the "proportionality profiles" for elections has allowed us to distinguish tentatively six types of basic profiles (plus the Icelandic). Several of these types differ only by a matter of degree. Thus Early Rise and Late Rise differ only in the percentage of votes below which parties are penalized, and Flat and Wide Scat-
ter differ only in the extent and range of scatter for small parties. The assignment of borderline cases to one or the other category is somewhat arbitrary; e.g., Ireland and Switzerland, classified as Wide Scatter, also have some Flat and Early Rise features. The Middle Peak profile is close to Early Rise, and it would take further data to ascertain whether the modest peak is real. The Middle Valley profile alone stands quite apart from all others.

Such continuity in profile types, and variety within them, should not surprise us, since electoral systems used by various countries at various times combine the same elements in various complex ways. The basic elements are seat distribution rule, and the district magnitude. The latter can vary quasi-continuously and, even within the same country, the magnitude of individual districts varies. Seat distribution rules (e.g., d'Hondt, Ste.Laguë) may seem to be a discontinuous factor, but quasi-continuity is obtained by mixing the rules (e.g., Austria's Quota in small districts followed by large-district d'Hondt for remainders) or by small modifications in divisors (Ste.Laguë with first divisor 1.4 instead of 1) or in quotas for Largest Remainder systems (1/n, 1/(n + 1) and 1/(n + 2) in Italy, under various conditions). Given the relative uniformity of resulting profiles, much of the electoral rules complexity seems superfluous.

The major determinants of proportionality profile are the district magnitude (cf. Rae et al., 1971: 486; and Lijphart and Gibberd, 1977: 229), the number of rounds and the number of adjustment seats. One-seat districts with one round always seem to lead to the Middle Valley profile (which hurts smaller non-local parties), while with two rounds the unpredictable Wide Scatter profile tends to result. At the other extreme, nationwide seats distribution or adjustment leads to the nearly perfect proportionality of the Flat profile, irrespective of formal distribution rules. In the middle range of district magnitude the effect of seat distribution rules tends to carry some weight, in the following way.

The d'Hondt rule typically leads to Late Rise, but this is partly due to its being used in conjunction with relatively small average district magnitude (5 to 15). Smaller magnitudes lead to later rise of the curve. This is most apparent when one compares the extreme cases of the Dutch nation-wide d'Hondt distribution and the British and Greek one-seat districts. (With one-seat districts, d'Hondt, Ste.Laguë and Largest Remainder rules all become equivalent to simple plurality.) D'Hondt with very large magnitude districts could lead to Flat profile (The Netherlands 1934–77). With fairly large districts it could lead to the border zone between Early Rise and Late Rise (Sweden 1921–49 and Finland 1906–75). With only 2 to 4 seats per district, it should lead to an
extremely Late Rise profile approaching the Middle Valley one.

Quota and Largest Remainder rules tend to lead to Early Rise and more scatter than d'Hondt at comparable district magnitude, but the picture is complex, partly because the quota criterion varies.

Modified Ste.-Laguë tends to lead to a Middle Peak profile with a modest bonus to middle-sized parties. Blondel (1969: 191) ranks electoral systems in the following order of deviation from proportionality: Ste.-Laguë, d'Hondt, Largest Remainder, Plurality. However, our profiles (and related work in Laakso and Taagepera, 1980) suggest that, at the same district magnitude, d'Hondt is clearly less proportional than either Ste.-Laguë or Largest Remainder, in agreement with conclusions by Rae et al. (1971: 486) on thresholds.

These observations regarding the effect of electoral system have been made before, and a long bibliography could be compiled. However, much of it has remained controversial, largely because it has been hard to visualize at once all data available. The proportionality profiles enable us to carry out such visualization, showing not only the main trends, but also the scatter, unexpected peaks, and single-party peculiarities.

While this paper has concentrated on general typology (the range of which we plan to extend further in space and time), national legislators and their academic advisors may want to ponder their own country's profile, and ask: was that what we had in mind when we devised our electoral rules? Comparing the Netherlands Flat profile (1918–33) to the later Early Rise (1934 on), was it intended to do away with occasional small-party advantage without increasing the possible small-party penalty? Did Belgium intend to have penalties for parties below 18% of votes, and a bonus above that level? Did France (from 1958 on) intend to have the observed limited relationship between a party's votes and seats? Did Denmark suspect, when changing electoral rules in 1953, that it already had one of the most proportional systems ever devised, so that further change hardly was worth the hassle? Are the Swiss intentionally giving a non-size-dependent special bonus to Liberal-Democrats? Is the Middle Valley profile the one the British wanted to have? These questions and problems are not new, but the graphical method helps to highlight various odd features.

If the answer to the questions above is "yes", then so be it. Should the answer be negative, however, our analysis may not only highlight problem areas but also offer some rather clear-cut recipes on how to redesign electoral rules in order to obtain an intended effect. One might imitate the rules of a country with a desirable profile. Furthermore, one can use the general typology to adjust for country size and peculiarities,
and to change rules in order to obtain a new profile, with fair confidence of success.

We have been dealing only with the issue of distribution of seats among parties according to their votes. But there is also the issue of apportionment of seats among territorial units according to their population. Mathematically the two problems are identical, but this identity often has gone unnoticed because students of European politics have little concern for apportionment which has, on the other hand, received considerable attention in the case of the United States, in the form of congressional apportionment (see Balinski and Young, 1977, and Still, 1977, for recent discussion). The two groups are largely unaware of each other's work and use a different terminology; e.g. the d'Hondt method is the Jefferson method in the U.S. apportionment tradition. The proportionality profile approach might also have a use in the apportionment studies.

References