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COMPUTER-AIDED EXTRACTOR DESIGN
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Abstract
We have developed a computer program, WOLF, which simulates ion extraction from a plasma and acceleration through an electrode structure and optimizes the performance of the extractor (minimizes the final beam divergence) by varying the potentials, shapes, and positions of the electrodes. WOLF also finds the self-consistent shape and position of the plasma equipotential representing the emitter. The measured performance of our slot extractors agrees well with the computed performance based on our model for the plasma if the ions are assumed to have a temperature of about 1 eV at the emitter. The most important factors affecting the beam divergence are, in decreasing order of importance, 1) gross electrostatic lens effects, including the space-charge of the ions, 2) the initial ion temperature, 3) the shape of the first, or "beam-forming", electrode, and 4) the shape and position of the plasma sheath edge, or emitter.

Introduction
Application of neutral injection for heating and sustaining CTR plasmas requires the production of hydrogen and deuterium ion beams of high total current, high current density, and minimum beam divergence. At modest beam energies (less than about 100 keV), the space-charge of the beam is neutralized immediately as it exits from the extractor by the electrons in a plasma produced by the beam itself in the gas in the adjacent neutralizer. The ions are converted by charge-exchange in the same gas into fast neutral atoms. The production of the required high-quality beams depends then on 1) producing a suitable plasma from which to extract the beam and 2) designing a suitable electrode structure to accelerate the beam. We will concentrate on the latter problem. We require a model for ion extraction from a plasma, a computer program to calculate extractor performance based on this model, and finally, an optimization procedure.

The Model for Ion Extraction from a Plasma
We have made Langmuir probe measurements on one of our plasma sources (the LBL 10-A source), and find that a deuterium plasma capable of providing 0.5 A/cm² of deuterium ions (75% D⁺) at the extractor has \( N_i = 5 \times 10^{17} \text{ cm}^{-3} \) at the center of the plasma and an electron temperature \( kT_e = 16 \text{ eV} \). The plasma is in the shape of a disk 6.6 cm thick and 12 cm in diameter, with the extractor on one of the flat faces. The first electrode of the extractor is also the wall of the chamber, and contains an array of 21 slots, each 0.2 cm wide and 7 cm long, filling a rectangle 7 cm x 7 cm. This wall and all others except for the anode are electrically isolated (the high voltage for beam acceleration is connected to the filaments). All collision mean free paths are comparable to or larger than the plasma dimensions.

If we assume the ion generation rate to be everywhere proportional to the local electron density, and a plane-parallel slab geometry rather than the "pillbox" shape that we have, S. A. Self has solved the problem for us, and while the geometries are not exactly similar, we can use his results to understand the pertinent physics. He gives numerical solutions of Poisson's equation and the equation of motion for the ions in a plane-parallel slab of plasma with a Boltzmann distribution of electrons to determine the potential, densities, and currents as functions of position. Figure 1 shows a typical potential distribution through half of such a slab of plasma. In the steady state, ambipolar fields develop to accelerate the ions to the walls and to repel the electrons from the walls so that the net electrical current to the isolated wall is zero. The walls are negative with respect to the center of the plasma by about 3.9 \( kT_e/e \) volts for a D⁺ plasma; most of this potential drop occurs in a relatively thin sheath at the wall, but a drop of about 0.85 \( kT_e/e \) volts occurs in the body of the plasma out to the classical sheath edge. We say "relatively thin" sheath, because while the Debye length is the appropriate unit for measuring the sheath thickness, the wall...
sheath for this plasma is about 10 Debye lengths thick, or about 0.01 cm, which is not negligible with respect to the thickness of the beam-forming electrode, 0.076 cm.

To extract and continue to accelerate ions, we must make a hole in the wall and put electrodes beyond it. How will this affect the picture? We propose that this basic picture is altered very little if the sheath remains reasonably plane, so that the potential distribution in the plasma is little affected by the presence of the hole. Ions are accelerated through the sheath and are lost whether they hit the wall or the hole in it. Electrons are treated slightly differently in the two cases, though: all electrons are reflected from the hole, but only almost all are reflected by the sheath adjacent to the wall. The electron density adjacent to the wall is \(0.5e^{-0.01} \approx 0.01\) of the density at the center of the plasma, (0.5 because no electrons return from the wall) and is already much less than the density of the streaming ions. We argue, then, that the dominant charged species determines the physics in the hole, and that while the electron density on an equipotential at the wall potential but located in the hole may differ by a factor of 2 from the density at the wall, this density is already low enough to be neglected with respect to the ion density there. This argument is due to Self. We expect, then, that if we provide potentials on other extractor electrodes so that the sheath edge remains reasonably plane, ion trajectories in the plasma, and the potential and electric field distributions along them, will be substantially the same whether the ion ultimately strikes the wall or the hole in the wall.

Now let us assume that we have solved the problem correctly, including also collisions among ions and other mechanisms by which they can gain transverse energy, and that we have a value for the magnitude of the electric field \(E_0\) and an ion velocity distribution function \(f_i(v)\) for any equipotential in the plane plasma. We may now choose any convenient equipotential in the vicinity of the wall (hole), call it the emitter, and continue our calculations of ion trajectories from these initial conditions. Since our numerical calculations will not include collisions between particles or particle generation, the equipotential chosen as the emitter should be near enough to the wall that both collisions and ion generation between the emitter and the wall are negligible. An important point is that there is no unique emitter in ion extraction from a plasma, unlike the case of electron extraction from a solid surface; any convenient equipotential satisfying the above conditions may be considered to be the emitter, provided the self-consistent and correct electric field and ion velocity distribution function for that equipotential are used as initial conditions for subsequent calculation. In the calculations that we will describe later we have chosen the emitter to coincide with the classical sheath edge.

This model is now capable of dealing with both the wall sheath adjacent to the first extractor electrode and with the ion acceleration process through the extractor structure. There is no difference between these regions; indeed, the region of ion flow through the extractor may be considered to be merely a very thick sheath with a very large potential drop across it. This is illustrated in Fig. 1 by the dashed portion of the potential curve, which represents the potential distribution in the hole—the curve is the same whether the wall is present or not.

![Fig. 1. Typical potential distribution through half of a semi-infinite slab of plasma bounded by electrically isolated walls.](image-url)
The program that evaluates the performance of an extractor by finding the self-consistent ion trajectories through a prescribed two-dimensional geometry (including the emitter) with prescribed potentials on the boundary is called FLOW. It does not deal directly with the ion velocity distribution function; instead, $f_i(v)$ is approximated by a finite number of ion beams. We have usually approximated $f_i(v)$ by a drifting Maxwellian distribution, with the drift energy of the order of $kT_e$, corresponding to the potential drop in the body of the plasma. In addition, the ion current density $j$ and an ion temperature $kT_i$ must be specified. We can then divide the distribution into chunks of phase space, integrate through them, and represent each chunk by a beamlet with some current density, direction, and energy at the emitter. The problem is solved on a triangular mesh attached to the boundaries; each triangle on the emitter has a complete set of these beamlets representing $f_i(v)$.

Electrons are included in the region near the emitter as a Boltzmann distribution with temperature $kT_e$. The electron density is assumed to equal the ion density (which is calculated from $j$ and the initial ion energy) at the emitter.

One cycle of beam-tracing proceeds as follows: first, Poisson's equation is solved (the first solution gives the vacuum fields). Negative charges are deposited on the mesh points by integrating the Boltzmann distribution over a mesh triangle, then putting charges on the vertices of the mesh triangle so that the "center of mass" of the charge distribution is unchanged. Denote these two operations symbolically by (EP). This process is repeated $m$ times until it has converged; we can represent this by (EP)$^m$. We now have the solution of Poisson's equation with free electrons (and also with any ion charges from the previous cycle, if this is not the first one).

Next, beams are traced through the mesh. The electric field in any given mesh triangle is assumed to be uniform and constant; the equation of motion can then be solved exactly within the triangle, and energy is rigorously conserved. Since the mesh is flexible, the user can increase the density of mesh points where needed to guarantee that the electric field approximation is valid. The "center of mass" of positive charges due to ions in a mesh triangle is determined by integration along each trajectory through the triangle, and positive charges are deposited on the mesh points so that the position of this "center of mass" of the charges is unchanged.

If we denote the operations of beam tracing and positive charge deposition by $B$, the entire process described so far can be represented by $B(EP)^m$, and represents one complete cycle of beam tracing. It is repeated $n$ times, symbolically $B(EP)^m n$, until the solution has converged, and we have obtained an evaluation of the extractor performance with all potentials, boundaries, and initial conditions specified.

The combination of FLOW and the optimization program PISA$^3$ which controls it is called WOLF. We need optimization, that is, the ability to vary some set of parameters to modify the solution in a desired way, for two reasons: first, to find the self-consistent shape and position of the emitter, and second, to minimize the final beam divergence.

We start the calculation with an assumed shape and position of the emitter—a plane near the position of the classical sheath edge, for instance. Since the ion current density and initial energy are known from the input data, and the potential distribution near the emitter is known after one evaluation (converged beam trace calculation), we can calculate from these data the magnitude of the electric field $E_i$ at each mesh interval across the emitter. We also presume to know, from Self's calculations or the solution of the Fokker-Planck equation or some other means, the magnitude of the electric field $E_0$ that must exist at the emitter if the solution is to be self-consistent. PISA finds the optimum shape and position of the emitter in a least-squares sense by translating and deforming the emitter to minimize the sum $\sum E_i (E_i - E_0)^2$; $w_2$ is a weight. It would also have been possible to find the self-consistent emitter shape and position by calculating a value of $j$ for each mesh interval across the emitter from the $E_i$ and the potential distribution and deforming the surface to obtain the value of $j$ obtained in the self-consistent solution. The two procedures are equivalent; we chose the former because it seemed easier.

Now we are able to evaluate the performance of a given extractor geometry with given potentials on the electrodes and with a given ion current density $j$. It is important to realize that in ion extraction from a plasma, the current density can be considered to be a free variable, again unlike the case of electron extraction from a solid surface. If the ion current density is varied by varying the
properties of the plasma ($N_i$ or $kT_e$), the emitter simply adjusts its shape and position to maintain a self-consistent solution in which the ion flow is simultaneously emission-limited by the plasma and space-charge limited in the extractor.

The ion current density is therefore one of the parameters available to PISA to minimize the beam divergence. Other parameters are the shapes, positions, and potentials of electrodes. PISA varies these parameters to minimize the sum \( \sum w_\theta (\theta_1 - \theta_0)^2 \), where the \( \theta_1 \) are the final angles each beamlet makes with the axis of the problem; \( \theta_0 \) is the desired final angle, normally zero; and \( w_\theta \) is a weight. In fact, PISA can accomplish both optimization tasks simultaneously by minimizing the sum of two terms:

\[
\sum w_s (E_i - E_0)^2 + \sum w_\theta (\theta_1 - \theta_0)^2.
\]

We have verified the accuracy of FLOW calculations and WOLF optimizations by treating problems for which we knew the analytic solution. As an independent check in cases where we have no independent solution, as in the evaluation or optimization of an actual extractor design, we can compare the volume of phase-space occupied by the beam at the emitter with the volume occupied as the beam exits from the extractor; in all cases we have checked, Liouville's theorem is obeyed to within 10% or better.

Our next concern is whether or not the computer model adequately represents ion extraction from a plasma and ion acceleration through the extractor structure. We can check this by comparing calculated and measured extractor performances.

**Comparison of Calculated and Measured Extractor Performance**

In attempting to verify the model by comparing calculated and measured extractor performances, we are hampered by our incomplete knowledge of the plasma properties necessary to provide input data for WOLF. In particular, we do not know \( f_i \) (v) and \( E_0 \) at the sheath edge. We do, however, have Langmuir probe data for the plasma, and from values of \( N_i \) and \( kT_e \) derived from these data, together with Self's calculations (but for a plane-parallel plasma slab, probably a reasonable approximation) and the measured values of potentials at which various elements of the source float, we can calculate the potential difference across the wall sheath at the extractor and the mean ion drift energy there. Self's calculations also give us a value for the magnitude of the electric field at the sheath edge (typically 100-200 V/cm); fortunately, the beam divergence is very insensitive to the value of \( E_o \) used. We also know \( j \) from the measured currents flowing in the system and the extractor area.

We still need to know \( kT_i \) to get the initial ion transverse velocity distribution. The slots in the extractor are so long relative to their width that the transverse velocity distribution at the sheath edge should determine the beam divergence in the direction parallel to the slots. We made an extractor curved so that the slots lay on the surface of a cylinder with a radius of 2.05 m and with its axis normal to the direction of the slot. This extractor produced beams focused in the direction parallel to the slots to a minimum beam width at about 2 m; the beam profile in this direction at 2.05 m was Gaussian, \(^2\) and indicated an ion temperature of 1.25 eV for 20-keV beam and 1.05 eV for 15-keV beams. At worst, this is a good upper limit on \( kT_i \) to best, it is a direct measurement of \( kT_i \).

There is another, empirical, way to determine \( kT_i \). The beam divergence in the direction perpendicular to the slots also depends on \( kT_i \) but in a more complicated way: the beam in these three-element accel-decel extractors is compressed as it is accelerated, and the beam divergence perpendicular to the slots increases as a consequence of Liouville's theorem. We can use WOLF to calculate the beam divergence in this direction based on our model as a function of \( kT_i \) and can find a value that makes the calculated and the measured beam divergences perpendicular to the slots agree. The beam divergence was measured at a calorimeter 3.3 m from the extractor; we have no evidence that the ion trajectories are changed after they enter the downstream plasma adjacent to the extractor. In Fig. 2 we show the calculated value of \( \sqrt{\theta_{rms}} \) for 20-keV deuterium beams, the appropriate quantity to compare with the measured 1/e half-width is 1.98°, \(^5\) which indicates an ion temperature of about 1.4 eV, in good agreement with the above estimate of 1.25 eV for 20-keV beams determined from the focal properties of the beam in the direction parallel to the slots. We believe that the agreement of these two determinations of \( kT_i \) is not fortuitous, and indicates that the ions when they reach the sheath edge really do have a distribution of transverse velocities and a mean energy in that direction slightly in excess of 1 eV.

Figure 2 also shows the calculated and measured beam widths at the exit of the extractor as a function of \( kT_i \). The calculated width is that width that contains 86% of the beamlets, equivalent to the 1/e width if the distribution were Gaussian. The range of measured values comes from examination of
the discoloration or sputtering on the edge of a thin sheet of molybdenum placed across the beams immediately adjacent to the extractor. The apparent full width is 0.05-0.06 cm, in good agreement with the calculated value, and is nearly independent of \(kT_e\). The calculations were done at beam energies of 20 and 10.3 keV, while the measurements were done at 12.5 keV. From the calculations it seems that the beam width scarcely depends on the beam energy, so the agreement is encouraging, even though the calculations and the measurements were done at disparate energies.

Remember that the ion current density is a free variable; we can therefore vary it with all potentials fixed and compare calculated and measured beam divergences as another test of the model. We show this in Figure 3, where the beam divergences—the 1/e Gaussian half-width for the measured beams; and \(\sqrt{2}\theta_{\text{rms}}\) for the calculated beams—are plotted against the current density at the emitter for 20-keV deuterium beams. The measured current density was derived from calorimetric data at 3.3 m and the measured power supply drains; the value for \(j\) for the calculated beam divergences was calculated for pure \(D^+\) beams and was corrected for the known beam composition. The agreement again appears to be reasonable. The measured and calculated values of \(j\) for minimum beam divergence differ by only about 10%. A detailed comparison of the shapes of the curves cannot be made now, since the only plasma parameter varied in the calculations was \(j\). In reality, \(kT_e\) and \(N_i\) also vary with \(j\), but this was not included in the calculation; values appropriate to the optimum plasma were used for all values of \(j\). We were unable to obtain experimental data beyond the minimum of the curve because of electrical breakdown in the extractor. We know that we did reach the minimum, or were at least very close to it, because we could compare this optimum \(j\) with values obtained at lower beam energies, where we were able to go beyond the minimum.

As another test of the model we were also able to vary the electrostatic focal properties of the extractor at constant ion current density and to compare the calculated and measured beam divergences, as is shown in Figure 4. In taking these data, the plasma and the accelerating potential between the first and the second electrodes were kept constant while the final beam energy was varied by varying all the electrode potentials simultaneously. The highest beam energy was 10.3 keV; the lowest, 4 keV. As in Fig. 3,
FIG. 4. Beam divergences, measured (1/e Gaussian half-width) and calculated best-fit 1/e Gaussian half-width for WOLF calculations; half-angle containing 86% of the beam current for BATE calculations), versus beam energy for deuterium beams from 10.3 to 4.0 keV, with the current density and extraction potential kept constant; kTi=1.16 eV in the WOLF calculation.

The BATE program was actually used to design the VIBES extractor used for all the measurements just discussed. The optimum performance of this extractor in producing a 20-keV deuterium beam, evaluated with WOLF, is shown in Fig. 5. Since the problem is symmetric, only half of it has been calculated. Five beamlets per mesh interval were used to approximate the ion velocity distribution function, assumed from the above arguments to be a Maxwellian with kT_i = 1.3 eV, drifting with an energy of 10.2 eV at the sheath edge. The wall sheath behind the first electrode is obvious, as is the only slight effect of the aperture on the shape of the sheath, which remains reasonably plane.

In all these tests made so far, measured and calculated extractor performances are in agreement if the ion temperatures in the source are assumed to be slightly larger than 1 eV, the same value as is required to explain the measured beam divergence in the direction parallel to the slots. Since WOLF also correctly solves problems for which the analytic solution is known and the calculations obey Liouville's theorem, it appears that the most important points of the physics of ion extraction and acceleration through the extractor structure are reasonably represented in the model and correctly calculated by WOLF.
Program Status and Limitations

WOLF is now running on our Control Data 7600 computer, although we are still modifying and improving it. As with any program, there are limitations to its use, some more severe than others. The most severe limitation is that it treats two-dimensional problems only, and cannot deal with cylindrically symmetric ones. We are considering the possibility of modifying it to handle problems with circular symmetry also. Other limitations can easily be eliminated as the need arises. They include the following: 1) ions must be of a single species and have the same initial streaming velocity at the emitter whenever electrons are included in FLOW calculations, 2) there is no magnetic field in the equations of motion, and 3) the downstream plasma in the neutralizer is not treated self-consistently, but is included simply as a specified equipotential surface.

An Example of Extractor Optimization

One disadvantage of the three-element accel-decel extractors such as VIBES that we have made and used is that their focal properties cannot be changed electrically to compensate for mechanical errors in construction or choice of a non-optimum value of \( j \). In fact, even the optimum beam produced with the VIBES extractor is slightly diverging. If we go to a multi-staged design with five elements instead of three, we have enough degrees of freedom to provide this electrostatic focusing. We gain another advantage, too: by arranging the potentials on the electrodes so that ions are alternately accelerated and decelerated, we introduce electrostatic strong-focusing, and can handle higher current densities through the extractor and still produce minimum-divergence beams. This principle has been used for years in the transport of electron beams.

The converged solution is shown in Fig. 7. The beam divergence \((\sqrt{2}\theta_{rms})\) has been reduced to 1.43°, at a \( D^+ \) ion current density of 0.27 A/cm². Other evaluations show that the beam can be electrostatically focused by varying \( V_3 \) with everything else held constant. The advantage of electrostatic strong-focusing is obvious; in this example the beam current density is larger by a factor of 3 than would have been obtained if the potential had been distributed between the first and fourth electrodes in the fashion of a Pierce column. This principle may permit the production of the high-current-density, low-energy (1 to 2 keV) \( H^+ \) or \( D^+ \) beams required by the CTR program for efficient production of intense \( H^- \) and \( D^- \)

![Image of a starting configuration for WOLF optimization of a five-element extractor design for producing 30-keV \( D^+ \) beams; \( kT_i = 1.3 \text{ eV}, kT_e = 46 \text{ eV} \).](image-url)
FIG. 7. Optimized five-element extractor design for producing 30-keV D⁺ beams.

FIG. 8. Evaluation of the optimized five-element extractor design shown in Fig. 7, but with the emitter made flat and displaced halfway to the first electrode.

Some Insights Into Extractor Physics

Provided by WOLF

One advantage of a computer model is that one can examine equipotentials in and ion trajectories through the extractor, quantities that are very difficult to determine experimentally, and can use them as aids in understanding the operation of the extractor. Another advantage of a computer model is that one can turn various facets of the physics on and off at will, and can examine the effect on the solution. For example, Fig. 8 shows the effect of forcing the emitter to be plane and displacing it by half the sheath thickness toward the first electrode, with everything else the same as in the converged solution. Compare this figure with Fig. 7, the converged solution; this change has scarcely any effect on the beam divergence.

On the other hand, if we turn off the beam space-charge by reducing the ion current density by a factor of 10⁶, as is shown in Fig. 9, the result is catastrophic. In this figure, beams appear to be reflected from the top and bottom boundaries because they are symmetry planes; the top is the symmetry plane of the electrodes, and the bottom is the symmetry plane of the slot in the electrodes.

From Fig. 8 and 9 it appears that varying the current density affects the beam divergence in this example not by varying the shape or position of the sheath edge, but by upsetting the delicate balance between the space-charge electric fields and the applied fields that is necessary for optimum performance.

From these and from other calculations it appears that the most important factors...
affecting the design of extractors are, in probable order of decreasing importance:

1) the electrostatic focal properties of the system (suggested also by Thompson and Coupland et al.), but including also the very important effect of the space-charge of the beam,

2) the transverse velocity distribution of the ions at the sheath edge, which limits the density of points in phase-space,

3) the shape of the first, or "beam-forming" electrode, which may have been fortuitously chosen in this example (no attempt has been made yet to optimize its shape), and

4) the shape and position of the plasma sheath edge, or "meniscus".

Conclusion

The extractor is an interface between two plasmas, the source plasma and the target plasma, both of which determine the optimum extractor design. The requirements of the target plasma determine the beam energy, total current, and necessary beam divergence; the source plasma determines the available volume of phase space for acceleration. In a sense the problem of extractor optimization is much like that of optimizing a beam transport system for an accelerator, except for the huge difference in the importance of the space-charge of the beam.

We intend the WOLF program to be a flexible instrument for extractor optimization, to contain as much physics as is necessary. We will build extractors designed with the aid of WOLF, and will compare their calculated and predicted performances; if there is significant disagreement, we will include more relevant physics in the model, such as a self-consistent treatment of the downstream plasma and its accompanying backward-accelerated ion beam, or the additional positive space charge in the extractor due to charge-exchange of and ionization by the beam. We also hope to improve our understanding of the plasma source, to provide better input data for WOLF.

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References


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