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MESON MASSES AND ENERGETICS OF MESON DECAY

Walter H. Barkas

May 15, 1951

Berkeley, California
MESON MASSES AND ENERGETICS OF MESON DECAY*

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I shall review today a number of experiments performed at the University of California Radiation Laboratory which provide information on the mesons masses and on the energy balance in the decay of charged mesons. The work has been a part of a broad program of basic research generously and wisely supported by the Atomic Energy Commission which in time will lead to a more satisfactory understanding of nuclear and subnuclear processes.

It is generally known that the \( \pi \) meson decays into a mu meson and a neutral particle, which is often assumed to be a neutrino. The mu meson in turn decays into an electron and presumably two neutrinos. The measurement of the meson masses has of course a certain intrinsic importance, but the values of the masses also enter into the total energies of the decay schemes, so that the mass measurements serve to confirm or contradict the assumed decay reactions. As of a year ago, the situation was somewhat as follows: A series of experiments at the Radiation Laboratory starting with the work of Gardner and Lattes had been made in which meson momentum and range in nuclear emulsion were the quantities measured, and from which mass estimates were obtained. Each experiment had taught something new about measurement of the range and momentum, the control of background, limitations of the range-energy relation, range straggling, and the technique of using the cyclotron as a precision instrument. Finally, after

making a careful determination of the range energy relation\textsuperscript{1} for protons in Ilford C2 emulsion, new meson measurements were made from which we reported\textsuperscript{2} the mass of the $\pi$-meson to be 276 ± 6 e.m. We also gave the $\mu$-meson mass as 210 ± 4 e.m. based partly on the older measurement of Lattes on the $\pi$ to $\mu$ mass ratio. In the survey of meson behavior, Lattes had also measured the range in emulsion of a number of $\mu$ mesons arising from the decay of $\pi$ mesons at rest. The particular emulsions in which the mesons were found had not been calibrated, however, so that with the uncertainty in the $\mu$ mass, only an approximate value of about 4 Mev was known for the decay energy of the $\mu$ meson. The earlier work is illustrated in Figs. 1 and 2.

The mass values of a year ago were hardly those with which to be satisfied. We particularly regretted that the probable errors which we found it necessary to quote were much larger than the attainable statistical errors. Uncertainties leading to possible systematic errors included the following, as well as a number of minor ones:

(a) Variation of the stopping power of the emulsion from plate to plate and with the ambient humidity.

(b) Uncertainty in the proton range-energy curve. A large part of the curve had to be known for the earlier experiments because a considerable interval of meson momenta had to be accepted to obtain enough good tracks for measurement.

(c) Uncertainties in the absolute value of the cyclotron magnetic field, and in the position of the apparatus in the nonuniform field of the cyclotron. This problem had been partially solved by building a


nuclear induction magnetometer and mapping the field in the vicinity of the apparatus, but a fear still remained that these measurements were inapplicable under the conditions of the experiments. We now know, however, by further experience with these effects that we were unduly pessimistic in estimating the errors.

A new approach has been taken during the past year in which a determined effort to eliminate the objections to the former measurements has been made. Our method of mass measurement depends on the circumstance that if the range ratio of two particles of equal charge is found to be equal to the momentum ratio, their velocities are equal, and the common ratio is equal to their mass ratio. There are at least eight other measurable quantities which could be used in the same way for obtaining the mass of an unknown particle relative to that of a comparison particle, but range and momentum turn out to be the most convenient and accurate in this case.

By dealing only with momentum and range ratios we eliminate from the mass measurement the absolute value of the magnetic field intensity, and also any direct dependence on a range-momentum relation. To provide a source of protons of the correct momentum we introduce a second target into the cyclotron from which protons are scattered so that in the same emulsion we detect simultaneously protons and the various kinds of mesons—all of substantially the same velocity. To design the apparatus, we of course required a prior knowledge of the approximate mass ratios. By improvement of the meson yield relative to the background, we were able to use narrow momentum intervals. Individual determinations of that function of the mass in which the meson range appears linearly are calculated from the measurements, and treated statistically. Deviations from the mean are expected to be normally distributed and the arithmetic average is taken to be the most probable value. For this procedure to be correct, other
statistical errors must be small in comparison with the range straggling error, and the ranges of monoenergetic particles must be symmetrically distributed about the most probable range. Because of these assumptions, we found it important to investigate the phenomenon of range straggling rather closely. Fig. 3 shows the distribution of $\mu^+$ ranges found from $\pi^+$ mesons decaying at rest.

A Chi-squared test indicated that the distribution of the $\mu$ meson ranges is consistent with a hypothesis of normality. Furthermore, the original theory of straggling given by Bohr leads to a standard deviation of 4.0 percent, assuming each electron in the emulsion to be free. The additional straggling we find is probably too much to attribute to observational errors, so that presumably a small effect of electron binding, as estimated by Livingston and Bethe, should be included in the straggling.

To understand more fully the arrangement and philosophy of the meson mass experiments, Figs. 4-9 may be helpful.

The first published results\(^3\) did not permit us to give reliable information on the $\mu$ meson mass because of contamination. This contamination by mesons not coming from the target also affected the $\pi$ meson results, but we are satisfied that the error is small in the final values. We plan, however, to improve the experiment further. In the meantime we have carried through another set of measurements with new apparatus in which we obtained the ratio $\pi/\mu$, and the absolute momentum of the $\mu$ meson, $p_0$, given to it when the $\pi^-$ meson decays from rest. In the later experiment\(^4\) we succeeded in obtaining a $\mu$ meson peak well resolved from the background of $\mu$ mesons coming from $\pi$ mesons decaying.

\(^3\) Walter H. Barkas, Frances M. Smith, and Eugene Gardner, "Meson to Proton Mass Ratios," Phys. Rev. 82, 102 (1951)

at points other than the target. The $\pi/\mu$ mass ratio was obtained in the same way as the meson to proton mass ratio was found.

The experiment is not over, but with our present statistics we find $\pi/\mu = 1.317 \pm 0.004$. The decay momentum of the $\mu$ meson is found as follows: First we find $\mu$ mesons coming from the target having a momentum within a few percent of the full decay momentum. Measuring the momentum and range of these particles, we extrapolate the momentum to match the range of decay mesons found in the emulsion. This requires a knowledge of the slope of the log momentum-log range curve and of the absolute magnetic field intensity. The slope we know rather well from the combination of theory and measurement. The magnetic field was mapped by measuring the Larmor precision frequency of protons in the field. This method yielded a preliminary value of $29.36 \pm 0.06$ Mev/c for the meson momentum.

The experiment gives a number of other quantities of interest if we assume that the neutral particle is a neutrino. Thus we find $\pi-\mu = 66.46 \pm 0.16$ m$_o$, $\pi = 276.1 \pm 2.3$ m$_o$ and $\mu = 209.6 \pm 2.4$ m$_o$ as absolute measurements with no reference to the proton as a comparison particle. These figures imply a value of $4.085 \pm 0.044$ Mev for the kinetic energy of the $\mu$ meson. The relation between the $\pi$ and $\mu$ masses for various values of the neutral particle mass as a function of the momentum of the $\mu$ meson is shown in Fig. 10.

It is appropriate here to pay tribute to Dr. Eugene Gardner and to recall the important part he had in these experiments. This work occupied Dr. Gardner's full attention until his declining strength forced its discontinuance in April, 1950. He died November 26th. His illness resulted from work with beryllium at the Radiation Laboratory. Dr. Gardner's associates will always remember his kindness, his personal integrity, and the soundness of his judgment. He, with Lattes, took the lead in the earlier meson mass experiments, and he was largely
responsible for the present experimental apparatus. He, more than anyone, built up the body of experience upon which our measurement program is based.

In addition to our relatively direct method, two other means of measuring \( \pi^- \) meson masses have now been successfully employed at the Radiation Laboratory. The first of these consists in balancing the equation:

\[
p + p \rightarrow d + \pi^+ + Q
\]

for the production of mesons by the collision of two protons. The line spectrum of mesons was independently observed by V. Peterson et al.\(^5\) and by Cartwright.\(^6\) Each group made a mass estimate for the \( \pi^+ \) meson from the energy of the meson peak. The line spectrum is illustrated in Fig. 11.

This method requires an accurate knowledge of the range-energy relation for charged particles in the material in which the mesons are stopped. This data is available from the work of Bakker and Segre,\(^7\) and the proton energy is known from the work of Mather.\(^8\) Corrections must be made for the range straggling and for the geometrical straggling due to small angle scattering. Their results are as follows:

- **Peterson et al.** 279.0 ± 1.5 e.m.
- **Cartwright et al.** 275.1 ± 2.5 e.m.

Another beautiful experiment was carried out by Panofsky, Aamodt and Hadley\(^9\) who measured the gamma ray energy from the reaction discovered by them:

\[
\pi^- + p \rightarrow n + \gamma
\]

---


6 W. F. Cartwright, "\( \pi^+ \) Meson Mass Determination," Phys. Rev. (to be published)


8 R. L. Mather, Ph.D. dissertation, University of California (1951)

9 W. K. H. Panofsky, R. Lee Aamodt, and James Hadley, "The Gamma Ray Spectrum Resulting from Capture of Negative \( \pi^- \) Mesons in Hydrogen and Deuterium," Phys. Rev. 81, 565 (1951)
This measurement yielded a $\pi^-\,\text{meson mass measurement of } 275.2 \pm 2.5 \text{ e.m.}$

The measurement does not involve a knowledge of a range energy relation, but does require an absolute magnetic field measurement. This was made with a nuclear flux meter. Incidentally, the competing reaction,

$$\pi^- + p \rightarrow n + \pi^0 \rightarrow 2\gamma$$

gives a good estimate of the $\pi^0$ mass. Panofsky et al. found:

$$\pi^- - \pi^0 = 10.6 \pm 2.0 \text{ e.m.}$$

The general agreement found in these precision measurements of $\pi$ meson masses by entirely different methods not only gives us confidence in the correctness of the masses deduced, but also confirms our belief in the reliability of the range energy relations used and the interpretation of the processes assumed in determining the masses.

Finally I shall turn to a remarkable experiment carried out by Sagane, Gardner, and Hubbard.\(^{10}\) For this experiment they utilized the large solid angle and good momentum resolution of the spiral orbit spectrometer to measure the spectrum of decay electrons from the $\mu^+$ meson. Their experimental arrangement and results are shown in Figs. 13 and 14.

This gives another entirely independent measurement of the $\mu^+$ mass; $212 \pm 5 \text{ e.m., assuming the neutral particles are neutrinos.}$

All the results are summarized in Table I.

This work was performed under the auspices of the Atomic Energy Commission.

Information Division
scb/5-15-51

\(^{10}\) Ryokichi Sagane, William L. Gardner, and Harmon W. Hubbard, "Energy Spectrum of the Electrons from $\mu^+$ Meson Decay," Phys. Rev. (to be published)
Table I
Measurements of Meson Masses and Related Quantities

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$\pi^+$</th>
<th>$\pi^-$</th>
<th>$\mu^+$</th>
<th>$\tau\mu$</th>
<th>$\pi^+ - \pi^-$</th>
<th>$\pi^+/\mu^+$</th>
<th>$P_0$</th>
<th>$\pi^+ - \mu^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smith, Barkas, Bradner and Gardner</td>
<td>276±6</td>
<td>276±6</td>
<td>210±4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Barkas, Smith, Gardner</td>
<td>277.4</td>
<td>276.1</td>
<td>±1.1</td>
<td>±1.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cartwright</td>
<td>275.1</td>
<td>±2.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Peterson, Iloff, Sherman</td>
<td>279.0</td>
<td>±1.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Panofsky, Aamodt, Hadley</td>
<td>275.2</td>
<td>±2.5</td>
<td></td>
<td>10.6</td>
<td>±2.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sagane, Gardner, Hubbard</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>212±5</td>
</tr>
<tr>
<td>Birnbaum, Smith, Barkas Preliminary Data</td>
<td>276.1</td>
<td>±2.3</td>
<td>209.6</td>
<td>±2.4</td>
<td>(4.085士0.044)</td>
<td>±0.004</td>
<td>29.86</td>
<td>(66.46士0.16)</td>
</tr>
</tbody>
</table>

All masses in units of the electron mass.
Quantities in parentheses have been calculated assuming neutral particle from decay of $\pi^-$ meson is a neutrino.
Figure Captions

Fig. 1 An assembly used in one of the early meson mass measurements. The mesons leave the thin carbon strip target in the backward direction and enter through the surface of the emulsion. The plate is inclined a few degrees to insure that the mesons will enter through the surface. A channel is provided to limit the momentum interval accepted and to reduce background, but some mesons will scatter from the walls of the channel.

Fig. 2 An illustration prepared by Dr. E. Gardner showing the decrease in the dispersion of meson mass measurements with improved measuring methods. The reference to "Barkas et al. - Current Study" is that reported by Smith et al., Phys. Rev. 78, 86 (1950).

Fig. 3 Histogram of ranges of $\pi^+$ mesons. One meson with a range of 267 microns was found by F. M. Smith. This event clearly does not belong to the main distribution, and no evidence of inelastic scattering was observed in the track. The $\pi$ meson probably decayed in flight.

Fig. 4 A diagram showing the locus of points obtained when individual $\pi^+$ mesons are paired with protons. A point is plotted for each pair. The ordinate is the momentum ratio and the abscissa is the range ratio. Where the locus crosses the 45° line the range ratio is equal to the momentum ratio, and this ratio is the mass ratio. This diagram was prepared from old data in order to obtain a large range of the variables; the later experiments utilize only data in the vicinity of the cross-over. Note the dispersion caused by range straggling. Some of the mesons countless come from points other than the target, so that a least squares straight line fitted to the locus probably does not yield the best mass ratio.

Fig. 5 The curve of log-momentum vs. log-range for protons in emulsion. Note how straight the line is for ranges above 1000 microns.

Fig. 6 Diagram illustrating geometry of experiment to measure meson to proton mass ratio. Scattered protons from the wolfram wire arrive at the plate with momentum greater than that of the mesons from the carbon target in about the ratio of their masses.

Fig. 7 Details of the meson target and plate holder. The target is in the form of a carbon cylinder 0.036 in. in diameter and a quarter inch high. It is mounted on a one mil tungsten wire passing through it axially. Mesons spiral slightly upward from the target which is mounted in the median plane of the cyclotron, and enter the emulsion through the surface, which is facing downward. The large copper block containing the proton channel shields the plate from stray protons.
Fig. 8 Diagram illustrating the trochoidal type of orbit in the median plane followed by a charged particle in the radially decreasing magnetic field of the cyclotron. Careful calculations of the momenta were made from the measurements of the position and direction of the particle as it entered the emulsion.

Fig. 9 Illustration of method of calculating the quantity $M^{1-2}$ for individual mesons. The dispersion in this quantity is assumed to have a Gaussian behavior.

Fig. 10 The relation between the $\pi$ and $\mu$ masses for various values of the neutral particle mass, $\nu$, as a function of the momentum of the $\mu$ meson. It can be seen that while our measurements are consistent with a scheme in which the $\pi$ meson decays into a $\mu$ meson and a neutrino, the mass of the neutral particle can only be determined with an error of several times the electron mass.

Fig. 11 The line spectrum of mesons from which Cartwright measured the $\pi^+$ mass.

Fig. 12 Gamma ray spectrum observed by Panofsky, Aamodt, and Hadley from the absorption of $\pi^-$ mesons in hydrogen. The spectrum is constructed by observing the positron-electron pairs produced by the photons in a thin strip of tantalum. The high energy peak is attributed to the reaction $\pi^- + p \longrightarrow n + \gamma$, and the low energy peak to the reaction $\pi^- + p \longrightarrow n + \pi^0 \longrightarrow n + 2\gamma$.

Fig. 13 Illustration of the spiral orbit spectrometer. The proton beam is directed perpendicular to the paper and traverses a target in the pole gap on the axis of the magnet. Electrons from the decay of $\mu^+$ mesons stopping in the target follow paths as illustrated. Those which traverse all four crystals are counted. An important advantage of this type of spectrometer is the large solid angle of collection.

Fig. 14 The spectrum of positrons from the decay of $\mu^+$ mesons. The points are the measurements, and the curves are the predictions of various theories. The upper limit of the positron energy distribution yields the $\mu$ meson mass, if the meson is assumed to decay into a positron and two neutrinos.
Fig. 1
Fig. 2
RANGE HISTOGRAM - Mu MESONS

RANGE DISTRIBUTION IN C-2 EMULSION OF 163 $\mu^+$ MESON FROM DECAY OF $\pi^+$ MESONS AT REST. DATA FROM THREE PLATES NORMALIZED TO 600 MICRONS MEAN RANGE AND COMBINED.

INDIVIDUAL MEAN RANGES: 591.1 $\pm$ 2.3 $\mu$, 609.8 $\pm$ 2.5 $\mu$, 600.0 $\pm$ 2.3 $\mu$

STANDARD DEVIATION: 26.5 MICRONS $\times$ 4.4 %

Fig. 3
Fig. 4
MOMENTUM IN GAUSS-INCHES VS. RANGE IN MICRONS
FOR PROTONS IN C-2 EMULSION

Fig. 5
Fig. 6
POSITIVE MESON TRAJECTORY
NEGATIVE MESON TRAJECTORY

CIRCULATING PROTON BEAM
C TARGET
Cu BLOCK FOR SHIELDING PLATE
Cu BLOCK WITH PROTON CHANNEL

Fig. 7
MESON TO PROTON MASS RATIO CALCULATION

A. \( \frac{R}{M} = c\left(\frac{P}{N}\right) \)
   (General relation)

B. \( \frac{R}{M} = c\left(\frac{P}{N}\right)^2 \)
   (An approximate relation with \( q = \frac{P}{N} \) and \( c \) a constant of the emulsion - valid for large range of \( N \) and \( P \))

C. \( c = \frac{R_P}{P_P} \) (for average proton)

D. \( N^2 = \left(\frac{P}{P_P}\right)^q \left(\frac{R}{R_P}\right) = c \left(\frac{P}{P_P}\right)^q \)

E. ERROR IN METHOD:
   If momentum ratio, \( \frac{P}{P_P} \), is \( N_0 + \delta \) rather than the true mass ratio, \( N_0 \), and if
   \( q \) is \( q_0 + \delta \) rather than the correct exponent \( q_0 \), then the apparent mass ratio \( N \) will be in error as follows:

   \[ \frac{N - N_0}{N_0} = \frac{\delta}{q_0 - 1} \frac{N_0}{R_0} \left( 1 - \frac{\delta}{q_0 - 1} - \frac{\delta^2}{2!(q_0 - 1)} + \ldots \text{terms of higher order} \right) \]

   For \( \frac{\delta}{q_0} \leq .05, \frac{\delta}{N_0} \leq .05, \left(\frac{\delta}{q_0}\right)^2 \leq .001, \delta \leq .1 \):

   \[ \frac{N - N_0}{N_0} \leq .002 \pm .0001 \pm .00005, \]

   and the average error will be:

   \[ \frac{N - N_0}{N_0} \leq .0004 \pm .00002 \pm .000002 \]

Fig. 9
Fig. 11
Fig. 12
Fig. 13
Fig. 14

--- TENSOR ANTISYMMETRIC THEORY WITH CHARGE EXCHANGE
--- SCALAR CHARGE RETENTION THEORY
--- VECTOR ANTISYMMETRIC THEORY WITH CHARGE EXCHANGE
--- TENSOR SIMPLE CHARGE EXCHANGE