Title
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Permalink
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Publication Date
2015-06-01

DOI
10.1016/j.jhydrol.2015.03.028

Peer reviewed
Objective hydrograph baseflow recession analysis

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\section{Introduction}

A streamflow hydrograph can be separated into a rising limb reflecting increases in discharge resulting from precipitation events, and recession limbs, which represent streamflow maintained at least in part by discharge from watershed aquifer storage. A streamflow hydrograph recession curve exhibits behavior attributed to the relationship between aquifer structure and its associated groundwater outflow to the stream channel. The theory of hydrograph recession analysis emerged from early studies of groundwater flow (Dupuit, 1863; Boussinesq, 1877; Maillet, 1905) and has since led to multiple approaches to characterize the relationship between groundwater and surface water during low flow periods (Tallaksen, 1995; Hall, 1968; Smakhtin, 2001). Increased attention has focused on both the quantity (Famiglietti and Rodell, 2013) and quality (Schirmer et al., 2012) of groundwater discharge to stream channels. This attention is due to groundwater resources being recognized as an important component of the global freshwater budget (Alley et al., 2002; Konikow, 2011) and the identification of global groundwater abstractions and depletion (Famiglietti et al., 2011). In this study, we focus on the relationship between groundwater storage and surface water because understanding the contribution of groundwater to streamflow, termed baseflow or groundwater discharge, is a fundamental focus of engineering, hydrogeologic and ecological studies.

Many investigations have studied streamflow hydrograph recession behavior to further our understanding of watershed processes. Szilagyi et al. (2007) and Shaw et al. (2013) evaluated the influence of watershed evapotranspiration on the behavior of baseflow recessions. Studies have evaluated the influence of watershed geomorphology (Biswal and Marani, 2010; Biswal and Nagesh Kumar, 2013; Biswal and Nagesh Kumar, 2014) and watershed storage on stream network dynamics. Kirchner (2009) characterized catchment behavior by deriving a sensitivity function related to nonlinear storage-discharge relationships. The consequences of an improper characterization of baseflow processes in hydrologic models were addressed by Clark et al. (2011). Lo et al. (2010) developed a parametric model of baseflow behavior to enable estimation of water table depths within a land surface watershed.
model. Carrillo et al. (2011) employed a hydrograph recession plot analysis to illustrate calibration of land surface models using groundwater/surface water behaviors. Staudinger et al. (2011) evaluated hydrologic model structures to simulate seasonal low flow; in that study, results illustrated that data clouds within the recession plot differed between models.

To study river discharge behavior at the watershed scale, Brutsaert and Nieber (1977) introduced an approach to estimate hydrograph recession parameters from a log–log plot of \( -dQ/dt \) versus \( Q \), termed the recession plot, where \( Q \) is streamflow during baseflow conditions. Despite the wide application of the recession plot approach, a theoretical characterization of watershed hydrograph recession behavior remains problematic (Rupp and Selker, 2006; Brutsaert, 2008). Vogel and Kroll (1996) demonstrated the challenge of estimating theoretical baseflow recession constants from individual hydrograph recessions. Stoezlize et al. (2013) illustrated how selection of individual recession hydrographs affects our perception of storage-outflow behavior. Further, Krakauer and Temimi (2011) employed a recession plot analysis with results suggesting that no single power law relationship represented a ‘typical’ recession curve.

The goal of this study is to develop and test a comprehensive scientific approach for the characterization and estimation of theoretical baseflow hydrograph models which should enable improvements in our ability to understand and predict the behavior of watershed hydrograph recessions. There is considerable controversy over the assumption that watersheds exhibit a fixed time constant, known as the baseflow recession constant, in the relation between aquifer storage and baseflow discharge (Zecharias and Brutsaert, 1988; Troch et al., 1993; Vogel and Kroll, 1996; Wittenberg, 1999; Eng and Milly, 2007; Harman and Sivapalan, 2009). Thus, another goal of our study is to improve our ability to construct hypothesis tests which can effectively evaluate whether or not watershed recessions are characterized by a time constant known as the baseflow recession constant, \( k_b \). The following sections provide an introduction to the theoretical derivation of baseflow recession characteristics and the development of our objective methodology for evaluation of the behavior of hydrograph recessions.

1.1. Theoretical background

To characterize relationships between groundwater and surface water systems, we employ the method introduced by Brutsaert and Nieber (1977) which assumes a power law relationship \( (Q = aS^n) \) between watershed aquifer storage \( S \) and baseflow discharge \( Q \) (Hall, 1968; Dooge, 1973) combined with the watershed continuity equation under baseflow conditions which yields \( dS/dt = 1 - Q = -Q \) because there is no inflow \( (I = 0) \) during a hydrograph recession. Combining these two expressions leads to \( dQ/dt = -nQ^{(2n-1)/n} \) which can be further simplified as a power law relationship between \( dQ/dt \) and \( Q \) using \( dQ/dt = -aQ^n \) where the exponent \( b = (2n - 1)/n \) and \( a = nQ^{2n} \). Although the value of \( n \) in the power law model can take on any value in the range \([0, \infty] \), it is often assumed that \( n = b = 1 \) which implies that the aquifer behaves as a linear reservoir with a fixed time constant. Note that Brutsaert and Nieber (1977) also derived solutions to the Boussinesq equation for conditions where the exponent \( b = 1.5 \) and \( b = 3.0 \). Brutsaert and Nieber (1977) graphically illustrate that on a recession plot of \( \ln(dQ/dt) \) versus \( \ln(Q) \), the log of parameter \( a \) is the intercept while \( b \) is the slope of the fitted envelope to the streamflow recession data. Brutsaert and Nieber (1977) recommended fitting a lower envelope to data points created by the recession plot when employing the graphical estimation method. The justification for using the lower envelope arises from the assumption that, for any given streamflow \( Q \), the lowest change in flow per time \( (dQ/dt) \) represents flow originating solely from groundwater storage. In our analysis, we fit a lower envelope as described by Brutsaert and Nieber (1977) to mitigate large \( dQ/dt \) values attributed to surface runoff or small recharge events.

There are numerous challenges to the approach suggested by Brutsaert and Nieber (1977) for estimation of recession parameters. Vogel and Kroll (1992, 1996), Biswal and Marani (2010) and Shaw and Riha (2012) assess baseflow recessions using individual events rather than the cloud of points in the recession plot. Kirchner (2009) assessed watershed behavior using mean values of binned streamflow to estimate recession parameters within a recession plot framework. Other studies (Rupp and Selker, 2006; Wang and Cai, 2009; Thomas et al., 2013) advance numerous different approaches to characterize the relationship between streamflow and \( dQ/dt \). Several studies document seasonal effects on the baseflow response due to changes in evapotranspiration (ET) (Silagyi et al., 2007; Wang and Cai, 2009).

Wang and Cai (2010) and Thomas et al. (2013) derive similar equations to Brutsaert and Nieber (1977) that account for the impact of groundwater withdrawals on hydrograph recessions, which we consider in Section 3.4. Rupp and Selker (2006) address estimation of \( dQ/dt \) given various \( \Delta t \); for this study, we use \( \Delta t = 1 \) day to match the averaging period of the most commonly available U.S. Geological Survey streamflow data. Initially we assume groundwater withdrawals are negligible and ET is either constant or has a negligible impact during baseflow events. These assumptions are consistent with numerous previous studies (Brutsaert, 2008; Silagyi and Parlange, 1988; among many others).

Brutsaert (2008) notes that, at the watershed scale, streamflow measurement error and inconsistent parameter estimation methods result in additional concerns over the interpretation and relevance of estimated baseflow parameters. Given the findings summarized above, combined with concerns raised by Brutsaert (2008), we conjecture that there are numerous issues concerning hydrograph recession analysis which could have an impact on our ability to interpret, model, understand and attribute watershed behavior. It is difficult to decipher the true relationship in the power law model \( (Q = aS^n) \) since one never knows the true value of \( n \). We introduce several innovations in this study with the goal of developing a more objective approach to the analysis of baseflow recessions. Our purpose is to show, by making the analysis more objective, that our analyses lead to a more complete scientific understanding of watershed hydrograph recession behavior. Our overall methodology includes several innovations including: (1) a rigorous quantile regression approach to fit the power law model to the lower envelope of the relationship between \( -dQ/dt \) vs. \( Q \) (Thomas, 2012; Stoezlze et al., 2013), (2) an efficient and reproducible approach for estimation of the numerical derivative \( (dQ/dt) \) and evaluation of the influence of both (3) seasonality and (4) the degree of groundwater withdrawals in the watershed on hydrograph recession characteristics. After introduction of these four innovations, we employ them together to test the hypothesis that groundwater outflow responds as a linear reservoir and is thus characterized by a fixed time constant.

2. Database description

We apply our baseflow estimation schemes to daily streamflow hydrographs for watersheds in New Jersey, USA, because it is currently one of the only regions we are aware of in which time series of monthly groundwater withdrawals are readily available in an electronic format. A total of 45 watersheds were selected for this study because (1) these watersheds are within high density population areas which meets the challenge of Sivapalan et al. (2012) and Vogel (2011) to study anthropogenic impacts on hydrologic
processes and (2) a long-term (19 year) monthly database of groundwater withdrawals is publically available (NJGWS, 2011). USGS stream gage sites were selected based on continuous daily records for the calendar years 1990–2009 to coincide with available groundwater withdrawal data. Spatially distributed databases obtained from the New Jersey Department of Environmental Protection Bureau of Geographic Information Systems (GIS) were used to select watersheds; for the purpose of this study, we selected watersheds with georeferenced surficial aquifers which did not intersect multiple watershed boundaries and which exhibited possible hydraulic connection to streams based on visual intersection of aquifer and hydrography layers (Fig. 1). Vogel and Kroll (1992, 1996), Brutsaert and Lopez (1998) and others have identified different recession hydrograph selection algorithms; Stoelzle et al. (2013) exhaustively compared recession selection methods and identified the approach of Vogel and Kroll (1992, 1996) as one which can accurately characterize hydrograph recession behaviors compared to algorithms used by Brutsaert and Nieber (1977) and Kirchner (2009), though their study raises questions regarding the uniqueness of any baseflow recession characterization. For our analysis, an automated hydrograph selection algorithm was used with daily streamflow records to isolate suitable recession hydrographs for analysis (Fig. 1) described by Vogel and Kroll (1992, 1996). That algorithm identifies the beginning of a streamflow recession when a 3-day moving average begins to decrease and ends when a 3-day moving average begins to increase. Recessions with lengths greater than or equal to 10 days were used for this study. Additionally, the first 3 points of the recession were removed for estimation of the baseflow recession parameters to limit the influence of other runoff processes. Brutsaert and Nieber (1977), Rupp and Selker (2006) and most recently Shaw and Riha (2012) address limitations of discharge estimation from stage-discharge relationships used by the USGS. The precision of stage measurements is approximately 3 mm (0.01 ft); such precision in discharge estimates becomes important during low flow periods as in this study. We estimated the precision of discharge estimates at each gage site and removed streamflow observations which were below the estimated precision based on derived stage-discharge relations reported in Table 1.

Daily precipitation data was obtained from the National Climatic Data Center (NCDC) database for sites with nearly continuous daily precipitation records for the period 1990–2009. Precipitation totals reported as trace (T) are considered to be zero. Precipitation data were used to eliminate any recession hydrographs during which streamflow was observed to decrease while a concurrent precipitation event greater than 0.10 mm was measured at the two precipitation gage sites nearest to the streamflow gage.

![Fig. 1. Locations of 45 watersheds, National Climatic Data Sites (NCDC) sites in New Jersey.](image-url)
Deterministic approaches for the evaluation of the impact of well withdrawals on surface water flows (for example, Hunt, 2012) illustrate a spatial correlation between water withdrawals and streamflow. Naturally, large water withdrawals in close proximity to a stream result in a larger impact on streamflow compared to small withdrawals located further from a stream. Withdrawals from unconfined aquifers with a potential hydraulic connection to the stream generally produce larger impacts on streamflow than withdrawals from confined aquifers where the confining unit is located between the screened material and the stream. Given these concerns, only groundwater withdrawals from unconfined aquifers were used for this analysis. Total watershed withdrawals were calculated by aggregating HUC-14 withdrawals and streamflow measurements within each of the delineated watersheds shown in Fig. 1. Total watershed withdrawals were assumed to be uniformly distributed during each month to obtain daily withdrawal rates.

### 3. Innovations in characterization of hydrograph recessions: experiments

This section describes innovations for characterization of hydrograph recessions and outlines a series of experiments to document the corresponding improvements which are possible when these innovations are employed. One test of our ability to characterize hydrograph recessions relates to our ability to discern whether or not a single watershed time constant exists. Our null hypothesis ($H_0$) is that watershed storage-discharge relations behave as linear reservoirs under baseflow conditions ($H_0$: $b = 1$). In the following sections we examine how evaluation of this hypothesis can be clarified by improvements in our ability to characterize the behavior of hydrograph recessions. No hypothesis tests can ever be accepted with certainty, because one never knows the true behavior of the system. Nevertheless, we illustrate how objective methods for hydrograph recession characterization offer a new window into their underlying behavior.

#### 3.1. Quantile regression to estimate the lower envelope of hydrograph recession plot

Brutsaert and Nieber (1977) recommended fitting a lower envelope to the cloud of data in the plot of $\ln(-dQ/dt)$ versus $\ln(Q)$, termed the hydrograph recession plot. Vogel and Kroll (1992, 1996) used ordinary least squares (OLS) regression to fit the entire data set to estimate recession parameters since they conjectured that the sampling error resulting from the short length of hydrograph recessions combined with streamflow measurement errors result in considerable variability in estimates of $dQ/dt$ and $Q$, as well as estimates of the slope and intercept model parameter.
estimates. Kroll et al. (2004) used 3-day moving averages of streamflow instead of daily flows for estimation of baseflow recession constants from recession plots to minimize the impact of both sampling and measurement error.

Previous methods to fit a lower envelope to clouds of data on the recession plot are generally not reproducible due to the subjective nature of such analyses. For example, Wang (2011) fit recession envelopes “by eye” to allow 5% of data points to lie below the fitted envelope line. Brutsaert (2008) recommended a 5% lower envelope to allow for measurement errors associated with flow estimation during low flow conditions and uncertainty associated with the determination of baseflow conditions. Mendoza et al. (2003) recommended the lower envelope be fit so that 10% of the data is below the regression line. In that study, data were filtered to isolate points around the estimated fit and OLS regression was used to fit the remaining data. As an alternative to an analysis of recession plots based on multiple hydrograph recessions, various studies (Vogel and Kroll, 1992, 1996; Biswal and Marani, 2010; McMillan et al., 2011; Shaw and Riha, 2012) combine estimates of hydrograph recession parameters based on individual recessions. The use of quantile regression provides an objective, rigorous and reproducible method for fitting the lower envelope in the recession plot, which ensures that a fixed percentage of the data falls above and below the fitted relationship (Thomas, 2012; Stoezl et al., 2013).

In general, the method of OLS regression provides an estimate of the conditional mean, or the mean value of the dependent variable as a function of one or more explanatory variables. Interest often is on statistics other than the conditional mean, thus quantile regression was introduced by Koenker and Bassett (1978) to estimate any conditional quantile. Numerous applications of quantile regression following Koenker and Bassett (1978) have appeared in the hydrology literature (Sankarasubramanian and Lall, 2003; Greenwood et al., 2011; Weerts, 2011; Thomas, 2012; Stoezl et al., 2013). Unlike OLS regression, quantile regression minimizes the sum of the errors of the conditional quantile function, typically by linear programming methods to minimize the sum of weighted absolute deviations of a percentile of interest, \( \tau_q \) (Koenker, 2005). A quantile regression approach for recession parameter estimation is attractive since our interest is in the lower envelope, or quantile, of extreme observations (5–10% percentiles, \( \tau_q = 0.05–0.10 \)) within the recession plot.

Quantile regression was implemented to estimate hydrograph recession parameters by fitting a lower envelope assuming a linear relationship between \( \ln(-dQ/dt) \) and \( \ln(Q) \). Quantile regression provides a consistent, well-defined and rigorous procedure to fit a pre-specified relationship to the lower envelope of a set of data. All parameter estimates obtained from quantile regression procedures reported here were significantly different from zero.

3.2. Numerical derivatives provide improved estimators of \( dQ/dt \)

Application of the Brutsaert and Nieber approach requires estimation of numerical derivatives to evaluate the time derivative of streamflow. Numerical differentiation is “ill-posed” in a Hadamard sense (D’Amigo and Ferrigno, 1992) in that small measurement errors in a time series can result in large errors in derivative estimation. Errors associated with numerical estimates of \( dQ/dt \) can result from truncation error from the Taylor series approximations often used in practice or due to streamflow measurement error (Liu et al., 2007; D’Amigo and Ferrigno, 1992).

Brutsaert and Nieber (1977) estimated the time derivative using the backward numerical estimator

\[
\frac{dQ}{dt} \approx \frac{Q_{t-M} - Q_t}{\Delta t} \quad (1)
\]

It is well known that finite different methods to estimate gradients result in error based on step-size approximations (Chapra and Canale, 2005). Although easy to use in practice, James and Conyers (1985) document limitations of finite difference methods which can often result in an inability to perform a meaningful uncertainty analysis. Further, finite difference schemes require estimates of mean discharge \( Q \) using \( (Q_t + Q_{t-1})/2 \) (Brutsaert and Nieber, 1977; Shaw and Riha, 2012).

Various methods have been advocated for fitting relationships and estimating derivatives for noisy time signals. A literature review revealed that smoothing splines perform well for both fitting smooth relationships and for estimating derivatives of noisy data. For example, Craven and Wahba (1979) showed that smoothing splines generate nearly optimal derivatives for noisy data. Ragozin (1983) determined that optimal smoothing of noisy data could result in nearly optimal derivative estimates.

Numerical derivative estimation using cubic splines were used on single recession hydrographs derived from the streamflow time series using an automated algorithm. Numerous methods exist to optimize spline fits to noisy data (Wahba, 1973; Craven and Wahba, 1979). For this analysis, we employed automated kernel estimation procedures for optimal spline fits to hydrograph recessions followed by the derivative estimation schemes for \( dQ/dt \). According to our literature review, such an approach should provide a nearly optimal estimate of \( dQ/dt \) in addition to preserving the observed discharge \( Q \).

Estimates of the numerical derivative using the six finite-difference methods summarized in Table 2 were computed from the entire daily streamflow time series for all sites with an estimated mean discharge \( Q \) following Brutsaert and Nieber (1977) while spline estimation was conducted using individual recession hydrographs. All estimates of \( dQ/dt \) for each method underwent hydrograph separation criteria to select data points for the analysis. Fig. 2 illustrates that the method used to estimate the numerical derivative \( dQ/dt \) can have a tremendous influence on estimates of \( a \) and \( b \) for a particular watershed. Similar results were obtained at other sites documenting that the \( dQ/dt \) estimation scheme could dramatically alter our ability to characterize the behavior of hydrograph recessions.

We compare recession parameters obtained from the six finite difference schemes in Table 2 to recession parameter estimates obtained from splines by reporting percent difference defined as

\[
\% \text{Diff} = 100 \times \left[ \frac{b - b^*}{b^*} \right] \quad (2)
\]

where \( b \) is the hydrograph recession parameter estimate obtained from a finite difference method and \( b^* \) is the estimate obtained from the spline derivative procedure. A graphical assessment of percent

<table>
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<tr>
<th>Table 2</th>
<th>Finite difference estimation schemes.</th>
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<tbody>
<tr>
<td>Finite difference estimation</td>
<td>Equation</td>
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<tr>
<td>Backward</td>
<td>( f'(x_k) = \frac{f(x_k) - f(x_{k+1})}{h} )</td>
</tr>
<tr>
<td></td>
<td>( f'(x_k) = \frac{3f(x_k) - 4f(x_{k+1}) + f(x_{k+2})}{2h} )</td>
</tr>
<tr>
<td>Forward</td>
<td>( f'(x_k) = \frac{f(x_{k+1}) - f(x_k)}{h} )</td>
</tr>
<tr>
<td></td>
<td>( f'(x_k) = \frac{f(x_{k+1}) + 4f(x_k) - 3f(x_{k-1})}{2h} )</td>
</tr>
<tr>
<td>Centered</td>
<td>( f'(x_k) = \frac{f(x_{k+1}) - f(x_{k-1})}{2h} )</td>
</tr>
<tr>
<td></td>
<td>( f'(x_k) = \frac{-f(x_{k+2}) + 8f(x_{k+1}) - 8f(x_{k-1}) + f(x_{k-2})}{12h} )</td>
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</table>
difference for each method is illustrated in Fig. 3. Fig. 3 illustrates that all of the estimators are negatively biased when compared to the optimal spline estimator $b$. The finite difference method which is most commonly employed in recent baseflow research (Brutsaert, 2008; Kirchner, 2009; Wang and Cai, 2009; Wang, 2011) is the 2-point centered difference formula ($C_2$, Table 2). Interestingly, the $C_2$ scheme exhibits the most negative bias relative to spline estimates, among all the estimators considered here.

3.3. The impact of seasonality on the behavior of hydrograph recessions

Seasonal variability plays a dominant role in the hydrologic cycle and thus it plays a large role in determining the interaction between groundwater and surface water systems. Investigations to characterize the effect of seasonality in hydrograph recession parameter estimation were first performed by Knisel (1963) and Federer (1973). Seasonal variability in the baseflow signature of watersheds has generally been attributed to the influence of ET (Wittenberg, 2003; Wang and Cai, 2009; Shaw et al., 2013); however, Zecharias and Brutsaert (1988) noted negligible effects from ET during recession analysis. We conjecture that the objective methods described in the previous sections may enable us to better understand the role of seasonality on hydrograph recessions.

Lins and Slack (2005) report that, for the mid-Atlantic region where our sites are located, low flows tend to occur in September, median flows in January, and high flows in March. In general, groundwater elevations tend to be highest in the spring and lowest in the fall as evidenced in monthly groundwater elevation data obtained from the USGS across New Jersey. Such changes in groundwater elevations may alter active stream networks as described by Biswal and Marani (2010) and Biswal and Nagesh Kumar (2014). To characterize seasonal impacts on hydrograph recessions, we separated recession data from each site into 5 categories: winter (December–February), spring (March–May), summer (June–August), fall (September–November), and annual datasets which include all streamflow data for the period 1990–2009.

Boxplots illustrated in Fig. 4 depict the variations in seasonal estimates of $b$ resulting from the six numerical derivative estimation schemes summarized in Table 2. Fig. 4 illustrates that both season and time derivative estimation scheme influence the estimation of recession parameters. Again, as in Fig. 3, nearly all the numerical schemes led to lower estimates of $b$ than the preferred spline scheme. Interestingly, estimates of $b$ obtained from the $C_2$ estimation approach yielded the lowest variability in the estimates across sites. We conclude that both the season of interest and the choice of derivative scheme can have a significant impact on our ability to discern the true value of $b$. The challenge which remains is to determine, among the various estimators of recession parameters, which estimation scheme leads to improvement in our understanding and our ability to model hydrograph recessions.

3.4. The Impact of water withdrawals on hydrograph recessions

Singh (1968) may have been the first to identify the impact of groundwater withdrawals on baseflow recessions. Wang and Cai (2010) and Thomas et al. (2013) considered the addition of a groundwater withdrawal variable in the traditional formulation of baseflow recessions introduced earlier by Brutsaert and Nieber (1977). We hypothesize that inclusion of water withdrawal terms...
into the analysis of baseflow recessions is needed to improve our understanding of the baseflow response of a river. Thomas et al. (2013) introduced a nondimensional water withdrawal index $\gamma$, defined as the ratio of groundwater withdrawals to overall discharge during the recession:

$$\gamma_i = \frac{\sum_{j=1}^{m_i} w_j}{m_i}$$  \hspace{1cm} (3)

where $w_j$ is the daily average withdrawal for hydrograph recession $j$, $Q_j$ represents the average streamflow over the $j$th hydrograph recession, and $m_i$ is the total number of observed hydrograph recessions available at site $i$. We incorporate the method proposed by Wang and Cai (2010) that includes groundwater withdrawals ($W$) into the analysis, so that now

$$\frac{dQ}{dt} = aQ^{b-1}(Q + W)$$  \hspace{1cm} (4)

An example of the effect of (4) in estimating recession parameters is illustrated in Fig. 5. The parameter $b$ in (4) is estimated by taking the logarithms and rearranging so that the left hand side of (4) becomes $[\ln a - \ln Q] - \ln (Q + W)$; thus, $(b - 1)$ becomes the slope of the resulting plot. Fig. 5 compares the application of the Wang and Cai approach (Eq. (4)) to the Brutsaert approach which amounts to (4) with $W = 0$. Fig. 5(a) illustrates results for a site where withdrawal impacts are small ($\gamma = 0.008$) and, as a result, both methods lead to nearly identical estimates of the baseflow recession slope. Fig. 5(b) compares the hydrograph recession slopes for a basin that experiences large groundwater withdrawals in relation to streamflow ($\gamma = 0.596$). We note that for the basin with significant groundwater withdrawals, use of (4) including groundwater withdrawals leads to a very different recession slope than use of (4) with $W = 0$. Note that all previous studies prior to Wang and Cai (2010) and Thomas et al. (2013) have generally ignored the impact of $W$ when estimating $b$.

The analysis illustrated in Fig. 5 was repeated for all sites and seasons using Eq. (4) including the withdrawal term $W$ (Wang and Cai approach) and without the withdrawal term (Brutsaert approach). Kendall’s tau correlation coefficient (Helsel and Hirsch, 2002) was used to determine if the resulting $b$ values exhibit significant correlation with the withdrawal index $\gamma$. Kendall’s tau is an attractive alternative to Pearson’s correlation coefficient, because it is a nonparametric correlation coefficient which does not assume a linear relationship between the two variables of interest (see Helsel and Hirsch (2002) for further details). The resulting values of Kendall’s tau along with their significance levels are given in Table 3. When one assumes $W = 0$ in (4) there is generally no significant relationship between the estimates of $b$ and $\gamma$ except during the winter, in which case they appear to be inversely related. When groundwater withdrawals are included in (4) the values of $b$ exhibit a positive correlation with $\gamma$ for selected seasons as is expected from previous studies (Ahfeld, 2004; Liu et al., 2007). Table 3 documents that when one properly accounts for withdrawals in (4), resulting estimates of $b$ tend to increase as withdrawals increase, especially during the summer and winter seasons. We conclude that it is necessary to include groundwater withdrawals into the analysis of baseflow recessions if one’s goal is to understand their behavior. These results are further supported in our experiments summarized in the following sections.

4. Does watershed groundwater outflow behave as a linear or nonlinear reservoir?

We have shown that numerous factors involved in estimation of hydrograph recession parameters using the recession plot may impact our ability to understand and model hydrograph recessions. Here, we return to our hypothesis that groundwater outflow responds as a linear reservoir ($b = 1$). We conjecture that the physical relationship between streams and aquifers, at the watershed scale, provides insight to the storage behavior of a watershed. For this analysis, our null hypothesis of linear reservoir behavior ($H_0$: $b = 1$) is tested against the alternative hypothesis of nonlinear reservoir behavior ($H_A$: $b \neq 1$). Although we may never know the true relationship of groundwater discharge to a stream channel, our analysis documents how analysis of hydrograph recessions...

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<th>Table 3</th>
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<td>Summary of Kendall's tau correlation between $\gamma$ and $b$. Bold results highlight significant correlations with significance level of 0.05.</td>
</tr>
<tr>
<td>Test statistic, S</td>
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<tr>
<td>----------</td>
</tr>
<tr>
<td>W = 0 (BN77)</td>
</tr>
<tr>
<td>Annual</td>
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<td>Spring</td>
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Hypothesis test results for those sites which resulted in estimates of bootstrapping methods. The test statistic, $T_{b-2}$, follows a Student’s $t$ distribution with $n-2$ degrees of freedom and can be used to obtain a significance level associated with our null linear reservoir hypothesis which accounts for the differences in sample sizes across watersheds. Hypothesis test results for those sites which resulted in estimates of $b$, which were found to be significantly different from unity, are summarized in Fig. 6 which classifies the test results into categories, depending upon which factors were considered in performing each hypothesis test. Recall that we have shown that the method of numerical differentiation, season, and groundwater withdrawals can all have an impact on the analysis of groundwater recessions. Fig. 6 considers all of these factors and our summary of these findings is discussed in the following sections. Similar figures which summarize hypothesis test results corresponding to use of various numerical derivative schemes summarized earlier in Table 2. Fig. 6 shows that between 9% and 44% of sites led to acceptance of the linear reservoir hypothesis, depending upon which numerical derivative is employed. The derivative procedure used by Brutsaert and Nieber (1977) resulted in acceptance of the linear reservoir hypothesis at 32% of the sites, while the 2-point centered difference formula (C2) employed in recent baseflow studies resulted in similar findings of acceptance of the linear reservoir hypothesis at 31% of the sites. Recall that our literature review recommended the nearly optimal spline estimation of the time derivative for noisy data, such as streamflow. Use of the spline estimation scheme resulted in acceptance of the linear reservoir hypothesis at 49% of the sites. We conclude that the approach used for estimating the derivative $dQ/dt$ can have a significant impact on our ability to discern whether or not groundwater outflow behaves like a linear reservoir.

4.2. The impact of seasonality on the linear reservoir hypothesis

Previous baseflow studies have either selected seasons in which low flow typically occurs (summer and/or fall) or seasons with decreased ET (winter). Here we only consider the numerical derivative estimate obtained from optimal splines to evaluate the impact of season on our hypothesis test results. Hypothesis testing resulted in acceptance of the linear reservoir hypothesis at 61% of the sites during the fall; in the spring, acceptance at 47% of the sites resulted. Several sites (Site 13, 24, 28, 31 and 33, Table 1) exhibited linear reservoir behavior across all seasons while two sites (Sites 1 and 6) exhibited nonlinear behavior. These results suggest high seasonal dependence in explaining the dynamics of the behavior of groundwater outflow based on hydrograph recession plot analyses.

4.3. Impact of degree of withdrawal on the linear reservoir hypothesis

We showed in Section 3.4 that the degree of withdrawal, $\gamma$, affects the nonlinear response of a groundwater/surface water system. This is consistent with other studies which document the impact of withdrawals on groundwater outflow and streamflow (Ahlfeld, 2004; Liu et al. 2007; Wang and Cai, 2010; Thomas et al., 2013). Recall that accounting for withdrawals (Table 3) supports previous studies that showed increasing nonlinear behavior attributed to human impacts (Liu et al., 2007). Fig. 6 illustrates the increased number of sites in which the linear reservoir hypothesis was rejected after introducing withdrawal terms, most notably during summer and fall recessions which increased rejection of the linear reservoir hypothesis from 49% to 56% and 50% to 61%, respectively.

4.4. Physical relation of baseflow recession parameters and streamflow

Brutsaert and Nieber (1977) illustrate that, based on the Boussinesq solutions and Boltzman similarity, combined with a simplified aquifer model, the recession parameter $a$ is a function of physical watershed characteristics including hydraulic conductivity. Assuming groundwater outflow behaves like a linear reservoir, Vogel and Kroll (1992), relate the parameter $a$ to the baseflow recession constant, $k_b$, where $a = -\ln(k_b)$. Because low
streamflow is generally sustained by groundwater discharge, we expect physical watershed properties to be important in explaining low flow behavior. Vogel and Kroll (1996) illustrate improved regression estimates of $Q_{r,10}$ when they included the variable $K_o$ as an explanatory variable. Similarly, Kroll et al. (2004) identified improved regression models for predicting low flow statistics with the inclusion of hydrogeological variables. Thomas et al. (2013) related $K_o$ to physical watershed characteristics and groundwater withdrawals; in that study, a multivariate relationship was developed to evaluate the primary physical and anthropogenic determinants related to $K_o$ following the physically-based model provided by Vogel and Kroll (1992).

Following the work of Vogel and Kroll (1996) and Kroll et al. (2004), we hypothesize that the association between low flow statistics and estimates of the recession parameter ‘$a$’ would be enhanced as the estimates of our recession parameters improved. For this evaluation, we relate low flow statistics ($Q_{r,10}$ as reported by the USGS (Watson et al., 2005)) to our estimates of the recession parameter ‘$a$’ using the rank-based nonparametric correlation coefficient known as Kendall’s tau (Heisel and Hirsch, 2002). The intent here is not to reproduce the multivariate analysis of Kroll et al. (2004) to evaluate the value of various explanatory variables in low-flow statistical regression analysis or the analysis of Thomas et al. (2013) to explore the behavior of $K_o$ in relation to both physical and anthropogenic influences. Instead, we simply wish to highlight the improved association between low flow statistics and recession parameter estimates which result from the various innovations introduced here.

The results in Table 4 illustrate an increased correlation between ‘$a$’ and $Q_{r,10}$ resulting from several innovations introduced here. First, among all the numerical differentiation schemes, the spline estimation scheme led to the highest correlation between ‘$a$’ and $Q_{r,10}$. Secondly, it is apparent that using all available data (the annual dataset) results in estimates of recession parameters that are most correlated to low flow statistics versus estimators obtained from seasonal recession hydrographs. One would expect that recession hydrographs from summer and fall would correlate better to low flow statistics given that flow flows tend to occur between August and October for the study watersheds (Watson et al., 2005); however, it appears given results in Table 4 that using all available hydrographs may provide better estimates of average watershed recession behavior. We attribute this result to the additional information contained within the recession plot when using all recession data available which leads to a better estimate of the intercept. Finally, estimates of ‘$a$’ which include groundwater withdrawals result in higher correlation to low flow statistics as compared to estimates which ignore withdrawals. Such results further corroborate results from Thomas et al. (2013) in illustrating the importance in estimation of recession parameters which include withdrawal information in the estimation methods.

### 5. Conclusions

The primary goals of this research were to evaluate numerous objective innovations for the characterization of hydrograph recessions and to show that the combination of these innovations can lead to an improved scientific understanding of the behavior of streamflow hydrograph recessions. Experiments were also performed to evaluate how the innovations can improve our understanding of watershed processes.

The use of quantile regression is the first of four innovations, previously studied by Thomas (2012) and Stoezl et al. (2013), to fit the lower envelope of a hydrograph recession plot. The use of quantile regression provides a rigorous and reproducible method to estimate recession parameters as compared to traditional subjective approaches, commonly conducted with “best eye” fits. Our second innovation relates to the estimation of the time derivative of streamflow, $dQ/dt$, which has long been recognized as problematic (Brutsaert, 2008). A comparison of six finite-difference estimators (Table 2) with a more efficient spline algorithm to estimate $dQ/dt$ indicated that finite-difference estimates can lead to arbitrary results in estimation of baseflow recession parameters. We attribute this result to the efficiency of spline methods for noisy data such as streamflow. Our third innovation involved an accounting for the seasonality of streamflow data which led to statistically significant differences in recession parameters, across seasons. Our fourth innovation documents that incorporating groundwater withdrawal terms into baseflow recession analysis using the approach introduced by Wang and Cai (2010) and Thomas et al. (2013) led to the expected response of increasing nonlinear behavior (b increasing) as groundwater withdrawals increase. These findings highlight the need for water withdrawal inventories at usable scales, similar to those of Siebert et al. (2010).

Our results in Fig. 6 highlight the inconsistency of linear reservoir hypothesis test results which can be expected to result from differences in the methods used for modeling hydrograph recessions. Our results highlight the need for objective methods for characterizing hydrograph recessions, if our quest is to attempt to relate the behavior of such recessions to the dynamics of watershed groundwater outflow processes. We found few sites to exhibit linear behavior with only a single site exhibiting consistent linearity through all seasonal experiments performed. Hopefully future studies will employ the innovations introduced here resulting in a more objective representation of the relationships between streamflow hydrograph recessions and aquifer dynamics.

### Acknowledgments

Project support was provided by a fellowship awarded to the first author by the Tufts Institute for the Environment (TIE); and by the University of California Office of the President Multicampus Research and Programs Initiative. A portion of the research was carried out at the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration. The authors also express their appreciation to Charles Kroll for helpful comments.

### Table 4

Summary of Kendall’s tau correlation between ‘$a$’ and low flow statistics ($Q_{r,10}$) for watersheds based on estimation scheme, seasonality and incorporation of withdrawal data in Eq. (4). Bold results highlight significant correlations with significance level of 0.05.

<table>
<thead>
<tr>
<th></th>
<th>Test statistic, S</th>
<th>Kendall’s tau (‘$a$’)</th>
<th>p-value</th>
</tr>
</thead>
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<td><strong>Derivative scheme</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>B1</td>
<td>−519</td>
<td>−0.528</td>
<td>3.58E−07</td>
</tr>
<tr>
<td>B2</td>
<td>153</td>
<td>0.171</td>
<td>1.21E−01</td>
</tr>
<tr>
<td>C2</td>
<td>−538</td>
<td>−0.547</td>
<td>1.19E−07</td>
</tr>
<tr>
<td>C4</td>
<td>−239</td>
<td>−0.243</td>
<td>1.98E−02</td>
</tr>
<tr>
<td>F1</td>
<td>−514</td>
<td>−0.322</td>
<td>4.77E−07</td>
</tr>
<tr>
<td>F2</td>
<td>−510</td>
<td>−0.518</td>
<td>5.96E−07</td>
</tr>
<tr>
<td>Spline</td>
<td>−578</td>
<td>−0.586</td>
<td>2.98E−07</td>
</tr>
<tr>
<td><strong>Seasonality</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Annual</strong></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Winter</td>
<td>−290</td>
<td>−0.323</td>
<td>2.47E−03</td>
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<tr>
<td><strong>Spline, W = 0</strong></td>
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<td></td>
<td></td>
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<tr>
<td>Winter</td>
<td>−316</td>
<td>−0.352</td>
<td>9.71E−04</td>
</tr>
</tbody>
</table>
two anonymous reviewers and Editor Peter Kitanidis whose comments contributed to substantial improvements to the original manuscript.

**Appendix A. Supplementary material**

Supplementary data associated with this article can be found, in the online version, at http://dx.doi.org/10.1016/j.jhydrol.2015.03.028.

**References**


