Mechanically tunable plasma frequency

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Schuil, Crystal Joy

2009

Peer reviewed|Thesis/dissertation
MECHANICALLY TUNABLE PLASMA FREQUENCY

A Thesis submitted in partial satisfaction of the requirements for the degree of Master of Science

in

Engineering Sciences (Mechanical Engineering)

by

Crystal Joy Schuil

Committee in charge:

Professor Sia Nemat-Nasser, Chair
Professor Joanna McKittrick
Professor Hidenori Murakami

2009
The Thesis of Crystal Joy Schuil is approved, and it is acceptable in quality and form for publication on microfilm and electronically:

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Chair

University of California, San Diego

2009
In recognition of all their support of my education over the years, this thesis is dedicated to my parents, Marc and Cindy Schuil.
<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Introduction</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>Test Frame Design</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>2.1: Design Constraints</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>2.2: Spring Selection</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>Horn and MUT Setup</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>3.1: Test Setup</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>3.2: S-Parameters</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>Metal Reflector and Rexolite</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>4.1: Metal Reflector</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>4.2: Rexolite</td>
<td>12</td>
</tr>
<tr>
<td>5</td>
<td>Vector Network Analyzer Code</td>
<td>15</td>
</tr>
<tr>
<td>6</td>
<td>Permittivity using Phase Change</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td>6.1: Theory</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td>6.2: Results</td>
<td>20</td>
</tr>
<tr>
<td>7</td>
<td>Calibration</td>
<td>22</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td></td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------</td>
<td></td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Permittivity, Dielectric Constant</td>
<td></td>
</tr>
<tr>
<td>$\varepsilon_r$</td>
<td>Relative Permittivity</td>
<td></td>
</tr>
<tr>
<td>$f$</td>
<td>Frequency</td>
<td></td>
</tr>
<tr>
<td>$f_p$</td>
<td>Plasma Frequency or Turn-on Frequency</td>
<td></td>
</tr>
<tr>
<td>FFT</td>
<td>Fast Fourier Transform</td>
<td></td>
</tr>
<tr>
<td>GRL</td>
<td>Gated, Reflect, Line</td>
<td></td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Propagation Constant</td>
<td></td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>Reflection Parameter</td>
<td></td>
</tr>
<tr>
<td>$k$</td>
<td>Spring Constant</td>
<td></td>
</tr>
<tr>
<td>MUT</td>
<td>Material Under Test</td>
<td></td>
</tr>
<tr>
<td>$n$</td>
<td>Index of Refraction</td>
<td></td>
</tr>
<tr>
<td>S-Parameter</td>
<td>Scattering Parameter</td>
<td></td>
</tr>
<tr>
<td>TRL</td>
<td>Thru, Reflect, Line</td>
<td></td>
</tr>
<tr>
<td>TRM</td>
<td>Thru, Reflect, Match</td>
<td></td>
</tr>
<tr>
<td>$\mu$</td>
<td>Permeability</td>
<td></td>
</tr>
<tr>
<td>$\mu_r$</td>
<td>Relative Permeability</td>
<td></td>
</tr>
<tr>
<td>VNA</td>
<td>Vector Network Analyzer</td>
<td></td>
</tr>
<tr>
<td>$Z$</td>
<td>Impedance</td>
<td></td>
</tr>
</tbody>
</table>
LIST OF FIGURES

Figure 2.1: (a) Test frame and (b) screws holding springs in place on the base. The springs in this picture alternate between left-handed and right-handed. ................................................................. 4

Figure 2.2: (a) Three springs ordered right, left, and right. (b) A single unit cell. ......................... 4

Figure 2.3: Spring constant experimental setup. Spring is attached to a mass on a scale and extended. Displacement and change in load on scale are used to find the spring constant. ........ 6

Figure 2.4: Different spring configurations. .................................................................................... 7

Figure 3.1: VNA and horn antenna configurations. ........................................................................ 8

Figure 3.2: VNA and horn antenna setup for testing the spring material. ..................................... 9

Figure 3.3: Signal flow graph of S-parameters [8]. ........................................................................ 10

Figure 4.1: Three metal plate views: (a) side view, (b) between horns, and (c) holes for connecting to table. ........................................................................................................................ 11

Figure 4.2: Two rexolite views: (a) side view and (b) between horns. ................................... 13

Figure 6.1: Parameters for permittivity equations based on phase change. .............................. 18

Figure 9.1: Schematic of calibrated system under test [15]. .......................................................... 31

Figure A.1: Old VNA user interface .............................................................................................. 47

Figure A.2: New VNA user interface ............................................................................................ 47

Figure B.1: Thru measurement S-parameters ................................................................................ 48

Figure B.2: Line measurement S-parameters ............................................................................... 48

Figure B.3: Reflection measurement S-parameters ..................................................................... 49

Figure C.1: MUT boundary condition problem: m, n, and p are distances in the z direction, and l is the thickness of the material ........................................................................................................ 51
LIST OF TABLES

Table 2.1: A-E springs with h, unit cell height, w, unit cell width, d, spring inner diameter, t, wire thickness, l, minimum spring length, and k, the theoretical spring constant. .................................. 5

Table 2.2: Experimental spring constant results. ............................................................................. 7

Table 10.1: Theoretical results from shifting the test frame, using S22. .......................................... 43
LIST OF GRAPHS

Graph 4.1: Rexolite with (a) a 1mm placement offset and (b) with offset removed. ....................... 14

Graph 4.2: Rexolite offset computed from S-parameter data using Matlab code. (a) S11 and S22 show placement error. (b) No placement error. ................................................................. 14

Graph 6.1: Rexolite and air relative permittivity. Length of air and rexolite is 1.30 cm. Actual values are 1 and 2.53 for air and rexolite, respectively................................................................. 19

Graph 6.2: Spring permittivity, including outlier data points .......................................................... 20

Graph 6.3: Spring permittivity data, raw results (dots) and a data fit (lines). The data fit is to the equation \( \varepsilon_r = c - (fp/f)^2 \).  c and fp are constants. ................................................................. 21

Graph 8.1: Rexolite magnitude data around half-wavelength frequency (a) with a calibrated VNA signal and (b) a calibrated and gated signal. ......................................................... 28

Graph 8.2: Rexolite time domain data (a) from 6-8.5 GHz and (b) from 4.8-6 GHz. The dip in S22 on (a) is due to the half wavelength frequency at 7.4 GHz. ........................................ 28

Graph 9.1: (a) Rexolite permittivity and permeability using both incident and reflected signals and (b) the phases of the data used in calculating (a). ......................................................... 34

Graph 9.2: Calculated n, index of refraction, and \( \Gamma \), the reflection parameter of the springs...... 36

Graph 9.3: Relative permittivity and permeability of the springs.................................................... 36

Graph 9.4: Spring magnitude and phase data, 6-8.2 GHz and 0 cm displacement. (a) Calibrated data and (b) calibrated and gated data................................................................. 37

Graph 9.5: Springs relative permittivity at different extension lengths. "Dots" are calculated from data, "lines" are from the data fit \( \varepsilon_r = c - (fp/f)^2 \). ................................................................. 38

Graph 9.6: Springs relative permittivity at different extensions for calibrated and gated data. "Dots" are calculated from the data, "dashed lines" are the curve \( \varepsilon_r = c - (fp/f)^2 \) fit to the data.... 39

Graph 9.7: Springs relative permittivity at different extensions and the curve \( \varepsilon_r = 1 - (fp/f)^2 \) fit to the data................................................................. 40

Graph 9.8: Plasma frequency vs. spring extension from \( \varepsilon_r = 1 - (fp/f)^2 \) curve fit over frequency range 6-8.2 GHz................................................................. 41

Graph 10.1: Calibrated and gated spring data with springs extended 10 cm.................................. 42

Graph 10.2: Graphical representation permittivity and permeability sensitivity to placement error. .......................................................................................................................... 43

Graph 10.3: Placement offset for calibrated and gated data sets 4.8-6 GHz and 6-8.2 GHz....... 44
Graph 10.4: Plasma frequency vs. spring extension for $S_{11}$, $S_{22}$, and corrected $S_{11}$ and $S_{22}$. The signals used to calculate the permittivity were both calibrated and gated and $f_p$ was calculated through the data fit $\varepsilon_r = 1 - (f_p/f)^2$. 

........................................................................................................................................................................45
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Composite materials with plasma turn-on frequencies in the microwave range can be used as electromagnetic filters. The permittivity of a material changes from negative to positive at the turn-on frequency. Previously, it had been theoretically determined that using non-magnetic metal wire coils could be used to create composites with turn-on frequencies that are dependent on the wire thickness, coil inner diameter, pitch, and coil spacing [1]. This project focuses on creating a composite with a mechanically tunable turn-on frequency. A material is made out of an array of springs is placed within a non-metallic frame and the turn-on frequency of the material is altered through extending the springs and thereby altering the pitch. The springs are arranged with alternating chirality in order to create a non-chiral material. A vector network analyzer and horn antennas are used to send and receive microwave signals through the material.
The measured scattering parameters are then used to calculate the permittivity of the material. The results show an increase in the turn-on frequency with an increase in pitch. Increasing the pitch by about from 3 mm to about 3.9 mm results in a corresponding increase in turn-on frequency of about 1.2 GHz for the non-chiral material setup.
Chapter 1: Introduction

Materials exhibit interesting properties near the plasma frequency. This frequency is also frequency referred to as the turn-on frequency. Above the plasma frequency, the material is transparent. Below the plasma frequency, the material is opaque and highly reflective. The propagation of electromagnetic waves in matter is determined by two material properties: \( \varepsilon \), the permittivity and \( \mu \), the permeability. At the plasma frequency, \( \varepsilon \) becomes negative. This results in the index of refraction, \( n \), and impedance, \( Z \), becoming imaginary. Maxwell’s equations show that when this occurs, the electromagnetic waves decay exponentially in the material and it becomes reflective. Above the plasma frequency, the electromagnetic waves are transmitted harmonically. This frequency is generally in the ultraviolet region [2]. If both \( \varepsilon \) and \( \mu \) are negative, then the material is called either left-handed or a metamaterial. This project focuses on the case when the permeability is positive, and the permittivity goes from positive to negative at the plasma frequency.

Pendry et al. showed that embedding thin, straight wires into a material could create a composite with a low frequency plasma frequency [2]. In order to achieve this at microwave frequencies, the ratio of the wire spacing to that of the wire radius is on the order of 300-1000 [3]. Since the dielectric constant is dependent on the self-inductance of the material, the introduction of wire loops makes it possible to use thicker wires and smaller unit cells [3,4]. This can help to ease the process of manufacturing such materials. Using conductive coils also introduces more parameters that can alter the permittivity, also known as the dielectric constant, of the material. With thin wires, the only two parameters available to change the dielectric constant of the composite are the thickness of the wire and the unit cell thickness [2]. With loops of wire, the inner diameter of the loop and the height between loops (pitch) are also parameters that can be adjusted to modify the dielectric constant of the material. Coils are wound either to the left or the right. Embedding single directional coils in a composite therefore introduces
chirality into the system. This chirality can affect the electromagnetic properties of the material [5]. There are a couple of ways to reduce the chirality of the system. One method is to use a setup that alternates between left-handed and right-handed coils. Another method is to use double coils. A double coil has two wires instead of one and they are wound in opposite directions [5].

Periodic arrays of conducting, non-magnetic elements were shown by Smith et al. to be effective mediums for electromagnetic scattering so long as the wavelength sent through the array is larger than the element dimensions and lattice spacing [6]. Arrays of coils have been numerically studied to determine the different plasma frequency for selections of pitch, inner diameter, wire radius, and unit cell width [1]. Through the selection of these parameters, it is possible to design composites to have specific turn-on frequencies.

These composites may be used to filter electromagnetic radiation at desired frequencies [5]. When the frequency is greater than the plasma frequency, the dielectric constant switches from negative to positive and incident electromagnetic radiation is transmitted through the material. As a result, the plasma frequency is also frequently referred to as the turn-on frequency [5]. It may be desired to change the threshold at which this occurs. One way to do this is to create a new material each time with different parameters. Another alternative is to create a material in which the electromagnetic properties can be altered through mechanical means. This project focuses on the creation and testing of a material to show that it is possible to create a composite with a mechanically tunable turn-on frequency.
Chapter 2: Test Frame Design

2.1: Design Constraints

There were several design constraints in building the test load frame. It has to have the capability to extend the springs the desired pitch change, it needs to not buckle under the force of the stretched springs, it needs to be easily adjustable, and allow for spring replacement. One easy solution to these parameters is to design a frame out of metal. However, metal is highly reflective and would likely interfere with the microwave signals. Therefore an additional design constraint is that the frame needs to be made out of a material other than metal. Wood is a strong, easy to obtain material and was selected for the supporting structure of the frame. Furthermore, the ends are pinned together with wooden joints in order to avoid the use of metal screws or nails. The frame is built to only be loaded under compression. This works well with the testing setup since the springs can only be extended. Due to their long length, they buckle easily when compressed. The height of the frame allows for the springs to be extended 5 inches. This length is the maximum the springs can be stretched and remain in the elastic regime.

The springs have loops on both ends. These loops are placed in holes in plexiglass bases and secured in place by screws. The top base is attached to two threaded rods that extend through the top of the frame. Each rod is held in place by a nut. There are washers in between the nuts and the frames to avoid stress concentrations. A wrench can be used to turn the nuts and adjust the height of the threaded rods. When the rods are raised, the springs apply a downward force. This downward force is then transmitted to the nuts on top of the frame, keeping the springs from recoiling and putting the overall frame under compression.

2.2: Spring Selection

The first step in designing the testing material was to select springs that would behave in the desired manner. There are a couple of important design parameters. The springs need to be
six times longer than the wavelength of the signal sent through them. The turn-on frequency needs to be in the range of 6-12 GHz. In addition, the springs need to be constructed out of a non-magnetic material. The spring constant should be low enough that a number of the springs in parallel can be extended easily and the springs should remain in the elastic regime for the entirety of the tests. This final criterion allows for repeat testing.

Figure 2.1: (a) Test frame and (b) screws holding springs in place on the base. The springs in this picture alternate between left-handed and right-handed.

In order to achieve a plasma frequency in the desired range, it was necessary to scale theoretical calculations from “Numerical Investigation of Effective Electromagnetic

Figure 2.2: (a) Three springs ordered right, left, and right. (b) A single unit cell.
Properties of Periodic Conductive Coil Structures” [1]. Without scaling, the plasma turn-on frequencies are higher than the desired range. In addition, the coils of wire would have smaller dimensions, making it harder to construct an array of coils. The physical dimensions and the corresponding turn-on frequency are inversely related. The dimensions were scaled up by a factor of three, resulting in a reduction in the turn-on frequency by a factor of three from the theoretical model. A single unit cell consists of one coil of one spring and has the dimensions height (spring pitch), inner diameter, cell width, and wire thickness. Unit cell dimensions and spring length under consideration for the springs are in the table below. Several commercially available springs were found to be within the desired measurement scale; however they were constructed out of music wire. Music wire is known for its good material properties, but is also highly magnetic. Therefore, beryllium copper, a popular spring material that is non-magnetic, was selected for the springs. The choice of this spring material made it necessary to custom order the springs. From the chart below, spring A was selected as having the most favorable properties and ordered at a length of 31.8 cm. The actual springs that are found in the test setup do not have the exact same dimensions as spring A. This is largely due to the use of a standard wire size instead of a custom wire size. The dimensions of the actual springs are also found in the table.

There is some variation in the length of the springs. However, due to the number of unit cells, the variation per unit cell is small and the overall effect is an array of springs with the same pitch.

Table 2.1: A-E springs with h, unit cell height, w, unit cell width, d, spring inner diameter, t, wire thickness, l, minimum spring length, and k, the theoretical spring constant.

<table>
<thead>
<tr>
<th>Spring</th>
<th>h (mm)</th>
<th>w (mm)</th>
<th>d (mm)</th>
<th>t (mm)</th>
<th>freq (GHz)</th>
<th>l (cm)</th>
<th>k (N/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>3</td>
<td>6</td>
<td>3</td>
<td>0.5</td>
<td>6.83</td>
<td>26.40</td>
<td>102.61</td>
</tr>
<tr>
<td>B</td>
<td>3</td>
<td>9</td>
<td>3</td>
<td>0.5</td>
<td>4.33</td>
<td>41.70</td>
<td>108.00</td>
</tr>
<tr>
<td>C</td>
<td>3</td>
<td>6</td>
<td>2</td>
<td>0.5</td>
<td>7.33</td>
<td>24.50</td>
<td>302.51</td>
</tr>
<tr>
<td>D</td>
<td>3</td>
<td>9</td>
<td>2</td>
<td>0.5</td>
<td>4.67</td>
<td>38.60</td>
<td>192.74</td>
</tr>
<tr>
<td>E</td>
<td>3</td>
<td>7.5</td>
<td>1.5</td>
<td>0.75</td>
<td>6.00</td>
<td>30.00</td>
<td>1722.65</td>
</tr>
<tr>
<td>Actual</td>
<td>2.93</td>
<td>6</td>
<td>3.07</td>
<td>0.57</td>
<td>7.30</td>
<td>31.80</td>
<td>122.28</td>
</tr>
</tbody>
</table>
The theoretical spring constant was approximated using the widely used formula

\[ K = \frac{Gt^4}{8nD^3} \]  

with \( G \) as the shear modulus, \( t \) is the wire thickness, \( D \) as the mean diameter, and \( n \) as the number of coils. For the test setup, over 100 springs would need to be displaced in parallel. This results in an effective spring constant of

\[ K_{eff} = N \times K \]

where \( N \) is the number of springs in parallel. Before building the test frame, it was important to make sure that the actual spring constant was close to the calculated value. At the estimated value of 122.28 N/m, it would be possible to manually displace the springs. At significantly higher values, it would be necessary to incorporate a motorized load frame in to the design. A spring was attached to a large mass placed on a scale. The scale was then tared. After being displaced a measured amount, the magnitude of the change in mass on the scale was recorded. The spring constant was determined through the following calculation.

\[ K = \frac{\text{mass} \times \text{gravity}}{\text{displacement}} \]

Four tests were performed using this method. The results of the tests are found in the following table. The results are reasonably close to the estimated spring constant of 122 N/m. Since they

![Figure 2.3: Spring constant experimental setup. Spring is attached to a mass on a scale and extended. Displacement and change in load on scale are used to find the spring constant.](image)
are lower than the theoretical value, 122 N/m can be used as the upper bound in determining the force that would be applied on the test frame. When a displacement of 12.5 cm is taken into account, the maximum force on the test frame can be calculated. For 150 springs displaced 12.5 cm, the test frame needs to be able to withstand 15.3 N.

**Table 2.2:** Experimental spring constant results.

<table>
<thead>
<tr>
<th>Change in Mass $\Delta M$ (g)</th>
<th>Mass * Gravity $F$ (N)</th>
<th>Change in Length $\Delta L$ (cm)</th>
<th>Spring Constant $K = F/\Delta L$ (N/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>380</td>
<td>3.7</td>
<td>3.2</td>
<td>116</td>
</tr>
<tr>
<td>620</td>
<td>6.1</td>
<td>5.1</td>
<td>119</td>
</tr>
<tr>
<td>1012</td>
<td>9.9</td>
<td>8.3</td>
<td>119</td>
</tr>
<tr>
<td>1495</td>
<td>14.7</td>
<td>12.7</td>
<td>115</td>
</tr>
</tbody>
</table>

There are several possible spring arrangements. The first one to be tested was with the springs alternating in directionality. This serves to limit the chiral effects of the system [1]. To test a chiral system, the springs could be arranged, either all left-handed or all right-handed springs could be used. Since the directionality of the springs does not affect any of their other physical dimensions, the same load frame design can be used to test the chiral systems. In addition, the spring constant is dependent on the number of coils, not the direction in which the coils are wound. Therefore the load on the test frame can be assumed to be the same for any arrangement of the springs. A picture of the alternating springs used in the test setup can be found in Figure 2.1.

**Figure 2.4:** Different spring configurations.
Chapter 3: Horn and MUT Setup

3.1: Test Setup

There are two main horn setups commonly used for free space measurements. One possibility is to have the horns on the same side of the material under test (MUT). The other is to place the material under test in the middle of the two horns. The first method can be used to perform bi-static reflection measurements [7]. The second measurement is better for transmission measurements and for performing Thru, Reflect, Line calibrations discussed in the calibration chapter. Since most calculations of the permittivity of a material are highly dependent on transmission measurements, the setup with the MUT in the center of the horns was selected.

Figure 3.1: VNA and horn antenna configurations.

The VNA used for testing is an Agilent 8510C Network Analyzer with an 8517B S-Parameter Test Set, and an 83615B Synthesized Sweeper. The VNA is connected to a computer where the signals are converted into Matlab readable data files via a 16-bit general purpose interface bus (GPIB). The horn antennas were manufactured by Rozendal Associates and have a focal length of around 31-46 cm. This means the horns should be placed 62-92 cm apart. Two different sets of horn lenses with similar focal lengths were used. The larger set shown in the following picture is recommended by the manufacturer for use for 6-8.2 GHz and the smaller set is recommended for 8.2-12.4 GHz. When the signals are calibrated, reasonable results have also been measured outside of these frequency ranges with the horns. The horn antennas are placed
on non-metallic mounts to raise their centers close to that of the spring material. There are four holes in the mounts. Threaded rods connect the horns through the mounts to the linear translation stages. Nylon rods are used to ensure that the horn antennas are electrically isolated from the optical table. This becomes important during calibration when a metal sheet is also connected to the optical table. Metal rods were used until conductance was found between one of the horns and the table.

**Figure 3.2:** VNA and horn antenna setup for testing the spring material.
3.2: S-Parameters

Each of the horns sends and receives signals. These values are recorded as scattering parameters (S-parameters). Essentially they are ratios of the amplitude of signals received to signals transmitted. In the signal flow graph, a1 and a2 are the incident waves at port one and port two, respectively. The reflected waves at those ports are b1 and b2. The first index in labeling S-parameters refers to the reflected wave. The second index refers to the incident wave. Therefore S11 is the amplitude of b1 divided by the amplitude of a1. These signals are recorded at the ports. Without calibration, this includes effects from the coaxial cables, horns, and air between the ports and the sample. Calibration is used to move the signals recorded from the ports to the surface of the material under test.
Chapter 4: Metal Reflector and Rexolite

4.1: Metal Reflector

Accurate system calibration is highly dependent on precise material placement with respect to the reference frame. The reference frame is set at the surfaces of the aluminum reflector. As a result, it is important to ensure that the metal sheet is placed at the same place for every test. Initial tests were performed with an aluminum sheet set on the table at a mark half way between the two horns. However, this sheet was not secured to the optical table, and as a result, the necessary precision in placement was difficult to achieve. In order to resolve this problem and achieve testing repeatability, a new reflector was constructed with holes in it so that it could be attached to the optical table. The metal sheet is 0.66 cm thick. The geometry is designed to align the reference planes of the aluminum sheet and the springs on the port two side.

![Figure 4.1: Three metal plate views: (a) side view, (b) between horns, and (c) holes for connecting to table.](image-url)
of the test fixtures. During calibration, horn one has to be moved back 0.66 cm to account for the width of the metal sheet. After calibration is complete and the spring composite is in place, only horn one needs adjustment. Horn one would need to be moved back 1.8 cm from the thru calibration measurement location and then both sides of the spring composite are on the reference planes. There are three sets of holes offset a centimeter from each other. Each set of holes consists of two holes on the left, middle, and right of the base. This allows the option of establishing the reference plane in three different positions without moving the horns. The middle set of holes is used for this project.

4.2: Rexolite

A slab of rexolite is used to verify the results of various methods of testing material properties. Rexolite is a useful material because it has constant, known permittivity and permeability values across a large frequency spectrum. The relative permittivity with respect to free space is 2.53. The relative permeability with respect to free space is unity. A test fixture was designed to place one side of the rexolite directly on the same reference plane as that of the aluminum. The aligning side faces horn two, same as with the aluminum. This allows horn two to remain largely stationary during testing. There are three main design constraints for this fixture. It needs accurate geometry for material placement, the fixture needs to be non-metallic, and the rexolite needs to be aligned with the horns.

Two stanchions made of rexolite are attached to a non-metallic base with the use of two nylon screws per stanchion. There are four holes precisely placed in the base that allow the setup to be attached to the optical table in the correct location. Two clamps with nylon set screws attach the rexolite to the stanchions. This allows the rexolite to be adjusted vertically and align with the horns. In addition, it has the added benefit of not altering the surface of the rexolite piece. Since rexolite is an expensive material, keeping the piece intact makes it possible to use this material in the future for other possible applications. Once the rexolite was in place, a phase
check was performed on the calibrated VNA output to determine if it was accurately placed. When horn one was moved back 1.3 cm and horn two left stationary, the S11 and S22 phases were not symmetric. Some variation in S11 is due to reflections introduced through the movement of the coaxial cable connected to horn one. However, the lack of symmetry between the S11 and S22 signals also seemed to indicate. In order to determine where a placement error might be, and whether or not this is the cause of the phase offset, equations 16-21 were used with the permittivity and permeability considered constant at 2.53 and 1 respectively. The theoretical S12, S11, and S22 were calculated with phase offsets from excess air due to placement error and compared to the measured values. The results showed that the reference planes of the rexolite are approximately 1 mm away from where they theoretically should be located. Careful measurement of the rexolite setup showed that this one millimeter offset does in fact exist. Moving horn one a millimeter closer to the rexolite and horn two a millimeter further from the rexolite results in S11 and S22 having closer phases. This demonstrates that if S11 and S22 are not identical for the spring material, it may be a result of placement error.

**Figure 4.2:** Two rexolite views: (a) side view and (b) between horns.
**Graph 4.1:** Rexolite with (a) a 1mm placement offset and (b) with offset removed.

**Graph 4.2:** Rexolite offset computed from S-parameter data using Matlab code. (a) S11 and S22 show placement error. (b) No placement error.
Chapter 5: Vector Network Analyzer Code

The VNA is connected to a computer in the lab via a serial port. A code, originally written by Cody Wheeland, controls the VNA and extracts the data in a Matlab recognized format. This script was written in python. Several adjustments were made to this code in order to customize it for free space material property calculations. The original code was designed to return the logarithmic magnitude of signals at various x, y, and z coordinates for antenna that could be moved by a control board. However, for this set of tests, x, y, and z are fixed in space, and information on both the phase and the amplitude of the signal are required instead of just the log magnitude. This made it necessary to edit the code in order to extract this additional information.

After examining the code, it became apparent that the python script already extracted the complex data from the VNA and then took the logarithmic magnitude before returning it to the user. This made it possible to make a minor adjustment that returned the complex data instead. The x, y, and z positioning are left as 0, 0, 0, in the user interface, and did not require editing. Since the code is capable of recording measurements at incremental x, y, and z antenna positioning for four different S-parameters, the data is stored in a five dimensional data structure. The first three matrix dimensions store the positioning. Since the antenna position is constant through each test run, these dimensions are all of length one. The fourth dimension has a value ranging from one to four and is as long as the number of data points. Number one corresponds to the S11 parameter, number two to the S12 parameter, number three to the S21 parameter, and number four to the S22 parameter. The fifth dimension of the matrix holds the actual complex number measurement.

While searching for an appropriate calibration sequence, it was necessary to make adjustments in the code to read different types of results. Originally these changes had to be made in the original python code. It was not possible to do it from the user interface. Editing the
code for every situation was both time consuming and occasionally prone to errors. A better solution was to edit the code so that user could control more aspects of the VNA through the program’s graphical user interface. In the current rendition of the code, VNA calibration, smoothing, and gating can all be adjusted via the user interface. In case other potential users preferred the original logarithmic magnitude form of data, an additional option was added to return that form instead. The previous program is set to hold all of the last set of data on the VNA screen. While it is convenient to be able to view what was stored in the data matrices, manual adjustments cannot be made to the VNA while it is in a holding state. A sequence of buttons must be pressed to change the sweep from a single saved sweep to a continual sweep. Since the signal does not change much in time, changing the state to continual at the end of the program makes it faster to adjust settings on the VNA after data has been collected. The last option added to the VNA interface makes it possible to select from the computer whether or not the sweep should be saved through a hold at the end of data collection. As default, it is selected as off and the trace continually updates after test is completed.

Most of the new code was incorporated into two new functions. The first function begins after the code has detected that the user has selected the button “Start Test” on the user interface. This function scans to see which of the new user options have been selected and passes them in a matrix titled “options” to a function in the second part of the program that talks to the VNA. This second part of the program communicates to the VNA all of the options that can be executed at the beginning of the test. For the options that have to be executed within other functions, global variables are assigned to relay those options to them within the program. In particular, smoothing requires a global variable since it needs to be called during the test run execution. Smoothing only works on either real or imaginary Cartesian data. As a result, it is not possible to extract both parts of the complex data from a single test run. When the smoothing function is enabled, it is necessary to scan for the smoothed real part of the data, and then again for the smoothed...
imaginary part of the data. These data points must then be combined to form a complex set of
data points. Since this is a different data collection protocol from that of non-smoothed functions,
it is necessary to set a state variable to 0 or 1 (true or false) depending on whether or not
smoothing has been enabled and then use that variable in the function that collects data to
determine which data collection protocol should be followed. Pictures of the new and old user
interfaces may be found in Appendix A.
Chapter 6: Permittivity using Phase Change

6.1: Theory

Permittivity can be solved for many different ways. Above the plasma frequency of a material, several assumptions can be made. These include that the permittivity is positive, primarily real, and the permeability is approximately equal to that of free space. From these assumptions, it is possible to calculate the permittivity from the transmission measurements, S12 and S21. In the following calculations, S12 is selected. Either one of these measurements may be used to calculate the permittivity, however for an accurate result; the same one must be used for both the thru and material measurement. From the measured phase difference between the two measurements, and the theoretical phase change in space, the permittivity can be determined through the following equations.

\[ \delta_1 - \delta = \frac{2\pi}{\lambda_a} \Delta x_1 + \frac{2\pi}{\lambda_a} \Delta x_2 \]  
\[ \delta_2 - \delta = \frac{2\pi}{\lambda_a} \Delta x_1 + \frac{2\pi}{\lambda_s} T + \frac{2\pi}{\lambda_a} \Delta x_2 \]  
\[ \Delta \phi = \text{phase}(S_2^{12}) - \text{phase}(S_1^{12}) = \delta_2 - \delta_1 \quad [\text{mod } 2\pi] \]  
\[ \lambda_s = \frac{c_s}{f} \quad c_s = \frac{c_a}{n} \quad n = \sqrt{\varepsilon_r \mu_r} \]  
\[ \text{Assume } \mu_r \approx 1 \]  
\[ \text{Solution: } \varepsilon_r = \left( \frac{c_a \Delta \phi}{2\pi f T} \right)^2 \]
In these equations, $\varepsilon_r$ is the relative permittivity, $\mu_r$ is the relative permeability, $c_i$ is the speed of light in the sample, $c_a$ and $c_s$ are the speed of light in air and the sample, respectively, $\lambda_a$ and $\lambda_s$ are the wavelengths of the signal in air and the sample, $f$ is the frequency of the signal, and $T$ is the length of the sample. In $\Delta \phi$, the appropriate change in phase can be selected through an initial guess of the expected permittivity. Since changing this value significantly affects the results, it is easy to determine the appropriate value for any given measurement. These equations were implemented in to an algorithm in Matlab and then tested on rexolite and air for signals that had not been calibrated.

**Graph 6.1:** Rexolite and air relative permittivity. Length of air and rexolite is 1.30 cm. Actual values are 1 and 2.53 for air and rexolite, respectively.

Based on these results, it is clear that although there are some variations in measurements, raw signals could be used to determine the approximate permittivity of a material. The next step was to measure the permittivity of the test platform. The springs were tested at different displacements ranging from 0 to 3 cm at intervals of 0.5 cm. The lower frequency horn antennas were used from the frequency of 5.5 GHz to 8.5 GHz. From 8.5 GHz to 14 GHz, the higher frequency horns were utilized. The tests were performed over four frequency ranges, each with 801 data points. These ranges were 5.5-8.5 GHz, 8.0-10 GHz, 10-12 GHz, and 12-14 GHz. In the case of overlapping frequencies, the frequencies from the lower frequency data set are shown.
in the plot and used for the data fit. There are 163 overlapping data points that are excluded from the analysis. For a material at frequencies just above the turn-on frequency, the Drude-Lorentz Model of permittivity is expected to be followed. This model states that the relative permittivity is equal to one minus the ratio of the turn-on frequency divided by the frequency of the signal squared.

\[ \varepsilon_r = 1 - \frac{\omega_p^2}{\omega^2} \]

6.2: Results

When this model is applied to the data, a trend of increasing turn-on frequencies with increased spring pitch was found. The outlier data points with a relative permittivity that is larger than 1 are a result of phase subtraction not being a valid method for signals with large reflections and permittivity less than 0. For this reason, all frequencies lower than 6 GHz are omitted from the data fit. In order to better show the shifting of the plasma frequency, the y-axis was also given a maximum value of 1.

Graph 6.2: Spring permittivity, including outlier data points.
Graph 6.3: Spring permittivity data, raw results (dots) and a data fit (lines). The data fit is to the equation $\varepsilon_r = c - (fp/f)^2$. $c$ and $fp$ are constants.
Chapter 7: Calibration

7.1: Calibration Methods

Although initial experimental results indicated the existence of a mechanically tunable turn-on frequency, for better results and for results below the turn-on frequency, it is necessary to perform some form of calibration. There are several widely used types of calibrations for VNA free space measurements. Each of them has a distinct set of advantages and disadvantages. The three main methods are TRM (thru, reflect, match), GRL (gate, reflect, line), and TRL (thru, reflect, line) [9]. Each letter in the acronyms represents a measurement that must be taken by the VNA in order to perform the calibration to find the measurement on the reference plane and requires a calibration standard [10]. A thru measurement is made with the two reference planes coincident. In calibration schemes, a reflection measurement can either be an open circuit measurement or a short circuit measurement. In free space calibration, the reflect measurement is made through the use of a metal plate to create a short. The reflection parameter, $\Gamma$, is expected to have a magnitude of one and a phase shift of 180 degrees at the reference plane. A match measurement consists of adding an identical load to each of the two ports and for a line measurement an additional piece of line, or in the case of free space, a length of air, is added. A challenging aspect of TRM measurements is finding an appropriate matched load. If loads are not perfectly equal, there may be errors in the results of the calibration. In a GRL measurement, a two step calibration technique is used to avoid using time domain gating to reduce the number of free space measurements needed from three to two [11]. Gating can also be used in conjunction with other calibration methods to minimize the effects of residual timing mismatch [9]. GRL consists of first performing a TRL calibration on the system without the horns attached. This sets the reference planes at the end of the coaxial cables. Time domain gating is then used to determine the reflection parameter at of the horns using the S11 and S22 signals. The rest of the error parameters are then determined through a reflection and a line measurement [11]. Although
this method requires fewer free space standards, overall it involves more steps and standards than
the TRL or TRM calibration methods.

The main disadvantage to TRL calculations is that the horns must be moved between
measurements and a precision line must be used. The problem of horn movement was addressed
through the use of optical table linear translation stages. For a free space measurement, a high
precision line is simply an addition of a set amount of air. When using the on board VNA
calibration, a line should be chosen such that the phase shift from the thru measurement is
between 20 and 160 degrees [12]. For example, for a measurement at 10 GHz and 1 cm line
measurement, the resultant phase shift is approximately 120 degrees. In the equation for the
\[ \Delta \varphi = \frac{2\pi fL}{c} \]
phase shift, \( f \) represents the frequency, \( L \) the length of line added, and \( c \) the speed of light. Use of
this equation shows that for tests between 5 GHz and 13 GHz, 1 cm of air falls in to the range of
an appropriate line standard. For frequencies greater than 13 GHz, a shorter line should be used.
Since it is possible to precisely move the horns with the selected test set up, TRL was selected as
the best calibration method. For equations behind the TRL sequence, see Appendix B.

7.2: Calibration Sequence
The following are a series of steps that will perform a TRL calibration [12].

1) Return VNA to previously known state. Select **RECALL, MORE, and FACTORY PRESET**. These steps turn off any selections such as smoothing or gating that may not be
desired in the calibration.

2) Choose the start and stop frequencies. Select the desired number of data points and any other
parameters that should be changed from default. The calibration is only valid for the same
values at which the TRL sequence is run. The VNA 8510C can save up to eight different
calibration sets, but only up to four 801 data point calibration sets [13]. This means that the
VNA may ask a user to delete a calibration set even if all eight calibration sets are not full.

3) Select the menu **CAL**. Then press **MORE** and select one of the two calibration sets to modify under **MODIFY CAL “NAME”**.

4) Under **TRL OPTIONS** check and see whether the reference plane is set through the thru standard or the reflect standard. Thru is more commonly used, but if the electrical length is not precisely known, **REFLECT** should be selected underneath the heading **SET REF**: [12]. For the set of experiments performed for this project, the reference plane was determined through the reflect standard since determining it using the thru standard resulted in a reflection measurement that was off by 180 degrees.

5) If desired, change the name of the calibration set and then save. To do this, press **LABEL KIT**, enter the name change and save the name change. To save the changes to the calibration kit, select **KIT DONE**.

6) Return to the **CAL** menu. Select the calibration set from step 5 and then **TRL 2-PORT**. A list of standards should appear on the screen. The measurements from these standards may be taken in any order.

7) For the thru standard, set the horns such that the two reference planes are considered to be coincident. Then press **THRU**. After a measurement is complete, an underline will appear under the title.

8) Move one horn back the distance of the line standard. As previously stated, 1 cm of air is an appropriate line standard for tests between 5 and 13 GHz. For accuracy, it is better to consistently move one horn and leave the other stationary. After the horn has been moved, press **LINE** to record the data.

9) Place the metal plate with the side towards the stationary horn coincident with the reference plane. Move the other horn back a distance equal to the thickness of the metal plate. This is necessary since the total amount of air between the horns needs to be equal to that of the thru
standards and the metal plate displaced a known amount of air when it was placed between the horns. When this is done, press `REFLECT11` followed by `REFLECT22`. Each of these should then be underlined.

10) The TRL calibration has now been completed, however, before saving and exiting the calibration routine, the VNA needs to know that the isolation standard measurement will not be performed. The isolation standard provides information to the VNA on the levels of cross-talk between the two ports. For coaxial measurements, this can help improve the accuracy of the TRL calibration. For free space measurements, there are no available isolation standards. Therefore, select `ISOLATION, OMIT ISOLATION`, and `ISOLATION DONE`. This sequence must be performed before the VNA will allow the calibration sequence to be saved.

11) Select `SAVE TRL CAL` and then the number of the cal set where it will be stored. It is important to note over which frequency range and for how many data points it is valid for future use of the calibration kit.

12) The option to turn the calibration set on and off can be found under the general `CAL` menu. To switch between calibration sets, first turn off the current set, then select `ON`. The numbered list of calibration sets will be shown and a different set may be selected.

13) The calibration set may now be used to run sets. It is important to use the same reference planes as the ones used in the calibration sequence. For material measurements, this means that the horn must be moved back the distance of the thickness of the sample. If the reference plane is not placed directly on the sample, a phase transformation must be applied at the data analysis stage of the test.

Some of the above steps may be performed out of sequence; however it is important to check which settings have been selected before proceeding with the calibration. For example, if in a previous run of `CAL1`, a specialized standard and offset delay were selected for
**REFLECT11**, then these values will carry over in to the next use of **CAL1** unless they have been specifically reset to the defaults. If the calibration yields unexpected results, this can be one of the possible sources of error and the entire TRL calibration must be redone.
Chapter 8: Time Domain

8.1: Time Domain Gating

There are several different domain options on the VNA. The frequency domain is the one in which measurements with respect to frequency are made. In the power domain mode, the measurements can be made with respect to the power level of the signals. In the time domain, the fast Fourier transform of the signal is plotted. The x-axis is usually either electrical length or time; and the y-axis is either the impedance or the voltage of the signal [14]. The time domain results can be calculated in either low-pass or band-pass mode. Low-pass mode has restrictions on which kinds of frequency ranges may be selected. Band-pass works for any frequency range and for this reason was used in conjunction with calibration on some of the spring and rexolite tests. In band-pass, the VNA takes samplings of discrete frequency points between the selected start and stop frequencies and uses them to calculate the FFT [14].

After a calibration has been performed, the time domain can be used to remove remaining small reflections from the system [9]. These reflections can be a result of new errors in the system, such as bends in the coaxial cables, or errors that calibration did not fully negate. TRL cannot fully correct for multiple reflections between the antennas and the sample and this will appear in the results without time domain gating [7]. Viewing the calibrated results in the time domain can also show additional information about the signal. For instance, large reflections such as those produced when the sample length is half the wavelength of the signal can be seen. The following graphs are rexolite examples. There are some known theoretical issues with the VNA time domain that can cause the VNA time domain transform to differ from performing an inverse Fourier transform on the same data. These errors can be the result of discrete data sampling, frequency truncation, windowing, and renormalization [14].
Graph 8.1: Rexolite magnitude data around half-wavelength frequency (a) with a calibrated VNA signal and (b) a calibrated and gated signal.

Graph 8.2: Rexolite time domain data (a) from 6-8.5 GHz and (b) from 4.8-6 GHz. The dip in S22 on (a) is due to the half wavelength frequency at 7.4 GHz.

8.2: Gating Sequence

The following steps can be used to create and activate a time domain gate.

1) Select **DOMAIN** from the Menus option panel. This can be used to switch the domain among several options such as time, power, and frequency.

2) Press **TIME BAND PASS**. This changes the appearance of the signal in the viewing window. The x-axis will now be in seconds instead of Hertz. The scale on the y-axis may need adjusting in order to properly see the signal.

3) The easiest way to adjust the scale is to press **AUTO** found with the Response buttons on the
right side of the VNA. The other Response buttons may be used to manually configure the scale if desired.

4) It may be necessary to also adjust the x-axis in order to see the entire signal. To do this, select START in the Stimulus category of buttons. Enter in the desired start time on the number key pad and then select G/n for ns. The other buttons on the right side of the number pad may be used to select different units. Press STOP next to START and enter a desired stop time followed by G/n.

5) After steps 3 and 4, it may be necessary to repeat steps 1 and 2 in order to return to the TIME BAND PASS menu. Another way to return to this menu is to press PRIOR MENU next to the number key board until this menu is reached.

6) Once back at the menu, select SPECIFY GATE. This will bring up a menu where the time domain gate can be turned on, off, or adjusted.

7) Choose OFF under GATE if not already selected. Having a gate on cuts off part of the signal making it hard to appropriately place a new gate.

8) Further down on the menu, START, STOP, CENTER, and SPAN can all be used to adjust the gate. These values can be changed using either the knob next to the numeric keyboard or through entering values on the keyboard. START and STOP allow the user to change where the gate begins and ends and are represented by flags on the screen. CENTER moves the middle of the gate and SPAN changes the width of the gate. The center of the gate appears on the screen as a T. The gate should be placed around the part of the signal that the user wishes to keep. Usually this should be done with either S11 or S22 selected as the signal.

9) To activate the gate, ON needs to be pressed. ON and OFF may also be toggled using the user interface on the computer.

10) Press either PRIOR MENU or DOMAIN to switch back to the domain menu and then select FREQUENCY. The domain must be returned to frequency in order to view the magnitude
and phase of the gated signal. The user interface code does this automatically. If the user wants to export the time domain data instead of the frequency domain data, the setting should remain **TIME BAND PASS**, and then the command to the VNA “FREQ;” needs to be commented out of the user interface code in the file VNA.py. The start and stop values in the user interface will then be nanoseconds instead of gigahertz and the test may be run as usual.

Time domain gating is typically performed on the S11 or S22 signal. On material measurements with calibrated signals, the gate around the primary response from one should also be around the primary response on the other. If significantly different gates are required, the gate will need to be adjusted between measurements for S11 and S22.
Chapter 9: Permittivity and Permeability using Incident and Reflected Signals

9.1: Equations

The previous method of determining the permittivity of a material makes a couple of assumptions that are not valid for the material under test at all frequencies. It assumes that the material is low-loss and that the permittivity has a real value. At frequencies near and below the turn-on frequency, the metamaterial is a high loss material and has a complex permittivity. In order to determine the complex permittivity at these frequencies, it is necessary to take into account both the transmitted and reflected signals. Since the signals without calibration have high levels of internal reflection due to the coaxial cable connection lines, an accurate system calibration must be performed before making these calculations.

![Schematic of calibrated system under test](image_url)

**Figure 9.1:** Schematic of calibrated system under test [15].

The following equations for permittivity and permeability make the assumption that the reference planes are directly on the surface of the material under test. This can be achieved through calibration to give the configuration shown in the figure above. If the reference planes are not directly on the material, then the S11 and S21 signals can be translated to the surface of the sample through equations 12 and 13. $S_{11}^m$ and $S_{21}^m$ are the measured signals. S11 and S21 are the signals translated to the reference planes. As these measurements are taken in free space, $\gamma$ is the angular frequency divided by the speed of light in air. $l_1$ and $l_2$ are the length of the additional air added on the side of the sample facing antenna one and antenna two, respectively.

\[
(12) \ R_1 = e^{-\gamma l_1} \\
(13) \ R_2 = e^{-\gamma l_2}
\]
Once the signals on the reference planes have been determined, the permittivity and permeability can be readily calculated from a set of equations similar to those in the Nicolson-Ross method and made specific to the application of finding the permittivity of a material in free space determined by Ghodgaonkar et al. [15]. The first step is to relate $S_{11}$ and $S_{21}$ to the reflection coefficient, $\Gamma$, and propagation constant, $\gamma$, through:

\[
S_{11} = \frac{\Gamma(1 - T^2)}{1 - \Gamma^2 T^2}
\]

\[
S_{21} = \frac{\Gamma(1 - T^2)}{1 - \Gamma^2 T^2}
\]

where the propagation constant is related to $T$ by

\[
T = e^{-\gamma d}
\]

and the reflection coefficient is

\[
\Gamma = \frac{Z - 1}{Z + 1}.
\]

The propagation constant of the material is related to that of free space through

\[
\gamma = \gamma_0\sqrt{\varepsilon_r\mu_r}
\]

and the definition of $Z$ is

\[
Z = \sqrt{\frac{\mu_r}{\varepsilon_r}}.
\]

\[
\Gamma = K \pm \sqrt{K^2 - 1}
\]

The root of $\Gamma$ is selected through the constraint that the reflection coefficient must have a magnitude less than or equal to one. $K$ and $T$ in the above equations may be calculated using

\[
K = \frac{S_{11}^2 - S_{21}^2 + 1}{2S_{11}}
\]
The last variable that needs to be calculated to find the relative permittivity and permeability is the propagation constant. Although the rest of the variables are single-valued, for sample sizes greater than signal’s wavelength in the sample, $\gamma$ is multi-valued. This is a result of $T$ being a complex number. The natural logarithm of a complex number has an imaginary component with a modulus of $2n\pi$ where $n = 0, \pm 1, \pm 2$, etc. Using this and the previous relationships, the permittivity and permeability can then be calculated using:

\[
\begin{align*}
\gamma &= \ln \left( \frac{1}{T} \right) / d \\
\varepsilon_r &= \frac{\gamma (1 - \Gamma)}{\gamma_0 (1 + \Gamma)} \\
\mu_r &= \frac{\gamma (1 + \Gamma)}{\gamma_0 (1 - \Gamma)} .
\end{align*}
\]

$\varepsilon_r$ and $\mu_r$ are the complex permittivity and permeability of the sample with relative to that of free space. The S21 and S11 equations are derived from the boundary conditions on the setup. Their validity can be verified through the equations for the electric and magnetic fields at the boundary of the surface of the material and air. An example set of equations for this can be found in Appendix C.

**9.2: Initial Rexolite and Spring Tests**

This algorithm was implemented in to Matlab and then tested on calibrated and gated rexolite data. The results from tests over two different frequency ranges are combined in the graph below. Both tests contained 801 data points and were time domain gated based on the S22 signal to remove residual port timing mismatch. The first set of data spanned the frequencies from 8.2 GHz to 10.4 GHz and the second set of data was from 10.4 GHz to 12.6 GHz. Although the results are not as accurate for permittivity as the phase subtraction was for the rexolite, the
results can be used to show basic trends for both the permittivity and permeability of lossy materials.

**Graph 9.1:** (a) Rexolite permittivity and permeability using both incident and reflected signals and (b) the phases of the data used in calculating (a).

Theoretically, the S11 and S22 signals should be identical for reference planes on the surface of the sample. However, due to inaccuracies in the system, achieving this can be difficult.
For example, in the above graph on calibrated data, the periodic difference is likely attributable to different amounts of coaxial cable bending. However, in this case the slopes of the lines are very similar. The two slopes are not always this coincidental. The rexolite sample was done with the precision mount and with the horns precisely placed. A one centimeter offset from the reference plane may result in a phase shift of up to 180 degrees. Phase differences between S11 and S22 in the following data sets are generally the result of placement errors. These placement errors may be less than a millimeter in magnitude.

This method was next tested on the non-chiral spring material. The springs were not stretched and the same calibration parameters were used as with the rexolite. Instability in the results appears around 10.2 GHz. This is a result of high levels of a low magnitude S11 signal at these frequencies. Results like this are expected at frequencies where the length of the sample material is a multiple of half the wavelength of the signal. These results are a product of the material geometry, not a change in the material properties. The ratio of the propagation constant of the material to that of free space, $n$, is about 0.82 at 10.2 GHz. The wavelength of the signal is then

$$\lambda = \frac{c}{f} = \frac{c_0}{nf} = \frac{2.998 \times 10^8 m/s}{0.82 \times 10.2 GHz} = 0.0358 m$$

and half of the wavelength is 0.0179m, which is close to the material sample length. The rexolite tests did not have a similar response since the frequency test range did not include a frequency at which the length of the rexolite sample was a multiple of half of the wavelength of the signal. Instabilities such as this are well documented for solving the permittivity and permeability using the Nicolson-Ross method which has a similar formulation to this method [16,17]. At low values of S11, equation 23 becomes prone to error. One standard way to avoid these instabilities is the use of a short sample [18]. However, shortening the length of the material under test was not an option in this case.
Graph 9.2: Calculated n, index of refraction, and Γ, the reflection parameter of the springs.

Graph 9.3: Relative permittivity and permeability of the springs.

### 9.3: Experimental Plasma Frequency

After establishing that these algorithms accurately show the permittivity and permeability of a material above the turn-on frequency, the next step was to perform the tests over a frequency range that includes it. The prior experiment without calibration showed the lowest turn-on frequency range to be about 6.25-6.90 GHz. A test frequency range of 6-8.2 GHz was selected to encompass all of the turn-on frequencies. A full TRL calibration was performed on the VNA
over this frequency range. The spring material was placed on the table and horn one was moved back 1.2 cm and horn two forward 0.1 cm in order to place the surfaces of the MUT at the reference planes. Fourteen data sets were recorded, with the springs extended 0.5 cm between each data set for a total displacement of three centimeters from the beginning to the end of the test. Data was recorded for each of the seven spring lengths for calibrated and time domain gated data. In order to perform time domain gating on the data, first the calibration sequence is run. Then the VNA is switched over to time domain mode.

**Graph 9.4:** Spring magnitude and phase data, 6-8.2 GHz and 0 cm displacement. (a) Calibrated data and (b) calibrated and gated data.

It is possible to see the spring material change from low loss to lossy between 6 and 7 GHz. In general, the S22 data results are better behaved. This is true for both the rexolite and the springs used as the MUT. There are several factors that likely lead to this discrepancy. The first is that horn one is moved a larger magnitude than horn two. Calibration removes errors in the
system due to the coaxial cables for the cables set in a specific position. Moving the horns moves the coaxial cables and introduces new reflections into the system that are not accounted for in the calibration sequence. In addition, horn one has a longer coaxial cable attached to it with more bends initially in it. This means that there are more errors in the system initially and that post-calibration the data reflection signal associated with horn one is more likely to have errors than that of horn two. Calculating the complex permittivity and permeability only requires the use of one of the two reflection parameters. S22, as the more reliable reflection parameter, was used in the determination of the permittivity and the permeability of the MUT.

**Graph 9.5:** Springs relative permittivity at different extension lengths. "Dots" are calculated from data, "lines" are from the data fit $\varepsilon_r = c - (fp/f)^2$. 

<table>
<thead>
<tr>
<th>Extension Length</th>
<th>fp</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 cm</td>
<td>6.77</td>
<td>1.11</td>
</tr>
<tr>
<td>2 cm</td>
<td>6.76</td>
<td>1.03</td>
</tr>
<tr>
<td>4 cm</td>
<td>7.04</td>
<td>1.02</td>
</tr>
<tr>
<td>6 cm</td>
<td>7.22</td>
<td>1.00</td>
</tr>
<tr>
<td>8 cm</td>
<td>7.40</td>
<td>0.98</td>
</tr>
<tr>
<td>10 cm</td>
<td>7.54</td>
<td>0.95</td>
</tr>
</tbody>
</table>
Graph 9.6: Springs relative permittivity at different extensions for calibrated and gated data. "Dots" are calculated from the data, "dashed lines" are the curve $\varepsilon_r = c - (fp/f)^2$ fit to the data.

The permittivity calculated using the calibrated and gated data for the springs at different levels of extension can be found in the above plot. A curve fit taking the shape of the Drude-Lorentz model was applied to each of the data sets to find the turn-on frequency. A clear increase in $fp$ with the increase in spring pitch can be seen in both the calculated permittivity and the Drude-Lorentz curves applied to the data sets.
Graph 9.7: Springs relative permittivity at different extensions and the curve $\varepsilon_r = 1 - (fp/f)^2$ fit to the data.
When fp is found for all 21 data sets using this curve fit, a clear upward trend in plasma frequency can be seen for both the calibrated non-gated and calibrated gated data sets. In conclusions, this shows that the spring composite material has a mechanically tunable turn-on frequency.

**Graph 9.8:** Plasma frequency vs. spring extension from $\varepsilon_r = 1 - (fp/f)^2$ curve fit over frequency range 6-8.2 GHz.
Chapter 10: Error Analysis

10.1: S11 and S22 Phase Differences

Further analysis into the S11 and S22 signals showed discrepancies between the phases at some of the extension lengths. For example, at the 10 cm spring extension, there is a clear difference between the S11 and S22 phases. Previous work with rexolite showed that small discrepancies in placement are a likely source of this difference in phases. These placement errors are likely a result of the test frame shifting while the springs are being extended. These placement errors need to be accounted for in the calculations of permittivity and permeability. This is because small displacements have a significant effect on the calculations. For example, if the test frame is shifted a half millimeter closer to horn two, the theoretical change in turn-on frequency can be computed by multiplying it by a phase shift. The phase shift is equal to that in equation 14 using \( d = -0.5 \text{ mm} \) and \( \gamma \) for air. S12 and S21 do not need to be phase shifted if a placement error is assumed. The total amount of air in the transmitted signals is the same. However, shifting the test frame 0.5 mm towards horn one results in less air between horn one and the frame and more air between horn two and the test frame. The plasma frequency was found for both the original signals and the phase shifted signals using both forms of the curve fit.
Calibrated and gated data for the springs at the initial extension length was used. The following table and graph shows the results from theoretically shifting the test frame towards horn two.

Table 10.1: Theoretical results from shifting the test frame, using S22.

<table>
<thead>
<tr>
<th></th>
<th>( y = c - (fp/f)^2 )</th>
<th>( y = 1 - (fp/f)^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( c )</td>
<td>( fp )</td>
</tr>
<tr>
<td>Original:</td>
<td>1.08</td>
<td>6.58</td>
</tr>
<tr>
<td>Shift frame by 0.5 mm towards horn 2:</td>
<td>1.12</td>
<td>6.82</td>
</tr>
<tr>
<td>Percent Change:</td>
<td>4.37</td>
<td>3.57</td>
</tr>
</tbody>
</table>

Graph 10.2: Graphical representation permittivity and permeability sensitivity to placement error.

10.2: Calculating Placement Error

Moving the test frame closer to one horn moves it further away from the other. The total amount of air between the horns remains constant, and the magnitude of the change is the same for between horn one and the sample and horn two and the sample. This means that the phase difference between S11 and S22 can be attributed to equal amounts of air shifting the S11 and S22 signals. Theoretically, the two phases should be equal. This leads to the following equation:

\[
(29) \quad \varphi S11 + 2\gamma L = \varphi S22 - 2\gamma L \mod 2\pi
\]
where $\gamma$ is the propagation constant of free space and $L$ is the placement offset. Since these are reflection signals, they travel through the excess air twice before being recorded. This is why there is a factor of 2 before $\gamma L$. Solving equation 29 in terms of $L$ gives the placement error as

$$
(30) \quad L = \frac{\varphi S_{22} - \varphi S_{11}}{4\gamma}.
$$

This equation was implemented into a Matlab code (Appendix D.4) for each data point. 

**Graph 10.3**: Placement offset for calibrated and gated data sets 4.8-6 GHz and 6-8.2 GHz.

The mean $L$ for data set is then to phase shift $S_{11}$ or $S_{22}$ by the following:

$$
(31) \quad S_{11}' = S_{11}e^{2\gamma L}
$$

$$
(32) \quad S_{22}' = S_{22}e^{-2\gamma L}
$$

The signs in the exponents of equations 31 and 32 are different because a positive $L$ from equation 30 indicates a shift in the test frame in the direction of horn two and away from horn one. When the relative permittivity is found for the corrected $S_{11}$ and $S_{22}$ parameters, much better agreement is found between the two calculations. Additional differences between the two calculations of the relative permittivity may be attributed to different $S_{11}$ and $S_{22}$ magnitudes.
Graph 10.4: Plasma frequency vs. spring extension for S11, S22, and corrected S11 and S22. The signals used to calculate the permittivity were both calibrated and gated and $f_p$ was calculated through the data fit $\varepsilon_r = 1 - \left(\frac{f_p}{f}\right)^2$. 

![Graph showing plasma frequency vs. spring extension for S11, S22, and corrected S11 and S22. The y-axis represents plasma frequency in GHz, and the x-axis represents spring extension in cm. Data points for S11 Corrected, S22 Corrected, S11, and S22 are shown with different markers.]
Chapter 11: Conclusion

The plasma frequency of a composite with coiled wire arrays is dependent on four parameters: unit cell width, wire thickness, coil inner diameter, and pitch. In this project, three of those parameters were held constant. The pitch was mechanically adjusted. Previous theoretical studies on increasing the pitch of the coil showed that the plasma frequency should also increase. Through the use of a vector network analyzer and horn antennas, free space material measurements were made on an array of springs. Since individual springs are wound either left or right-handed, the springs were arranged periodically in order to reduce system chirality. Two different methods were employed to determine the permittivity of the material. The first looked at the phase change between two measurements. This method gave results that showed a Drude-Lorentz form for the permittivity and a turn-on frequency that increased with pitch increase. However, this method assumed a constant, real, and known permeability of the material. It also assumed that the permittivity of the material was equal to one, an assumption not valid for frequencies below the turn-on frequency.

In order to acquire better results, a new system of equations based on the magnetic and electric field boundary conditions of the material was employed. These equations required system calibration in order to achieve reliable reflection and transmission signals. TRL calibration, in conjunction with time domain gating was utilized during data collection. This made it possible to determine the S-parameters of the spring material. After data analysis was performed, it was determined that mechanically adjusting the pitch of the material does in fact increase the plasma frequency of the material. Extending the springs 10 cm results in an increase of the plasma frequency of about 1.4 GHz. This project experimentally demonstrated that it is possible to create composites with mechanically adjustable plasma frequencies.
Appendix A: VNA Computer-User Interface

Figure A.1: Old VNA user interface.

Figure A.2: New VNA user interface.
Appendix B: TRL Equations

Derivation for asymmetric system based off of method for symmetric case found in Microwave Engineering [8].

Thru Measurement

\[
\begin{align*}
T11 &= \frac{b_1}{a_1} = S11 + \frac{S22'(S12)^2}{1 - S22 \times S22'} \\
T22 &= \frac{b_1}{a_1} = S11' + \frac{S22(S12)^2}{1 - S22 \times S22'} \\
T12 &= T21 = \frac{b_1}{a_2} = \frac{b_2}{a_1} = \frac{S12 \times S12'}{1 - S22 \times S22'}
\end{align*}
\]

Figure B.1: Thru measurement S-parameters.

Line Measurement - Add air between horns

\[
\begin{align*}
L11 &= \frac{b_1}{a_1} = S11 + \frac{S22'(S12)^2 e^{-2\gamma l}}{1 - S22 \times S22' e^{-2\gamma l}} \\
L22 &= \frac{b_2}{a_2} = S22 + \frac{S22(S12)^2 e^{-2\gamma l}}{1 - S22 \times S22' e^{-2\gamma l}}
\end{align*}
\]

Figure B.2: Line measurement S-parameters.
\[ L_{12} = L_{21} = \frac{a_2}{b_1} = \frac{b_2}{a_1} = \frac{S_{12} \times S_{12} e^{-r_l}}{1 - S_{22} \times S_{22}' e^{-2r_l}} \]

Reflection- Use metal sheet to reflect signal

Figure B.3: Reflection measurement S-parameters.

\[ R_{11} = \frac{b_1}{a_1} = S_{11} + \frac{(S_{12})^2 \Gamma_L}{1 - \Gamma_L S_{22}} \]

\[ R_{22} = \frac{b_2}{a_2} = S_{22} + \frac{(S_{12})^2 \Gamma_L}{1 - \Gamma_L S_{22}'} \]

8 Equations, 8 Unknowns: \( S_{11}, S_{11}', S_{22}, S_{22}', S_{12}, S_{12}', e^{-r_l}, \Gamma_L \)

Algebra solved:

\[ B = (T_{11} - L_{11})(T_{22} - L_{22}) - L_{12}^2 - T_{12}^2 \]

\[ e^{-r_l} = \frac{-B \pm \sqrt{B^2 - 4(T_{12} \times L_{12})^2}}{2(T_{12} \times L_{12})} \]

Select positive or negative based on proximity of \( l \) to actual length of air added.

\[ S_{11}' = \frac{(L_{22} - T_{22}) \times T_{12}}{T_{12} - L_{12} e^{-r_l}} + T_{22} \]

\[ S_{11} = \frac{(L_{11} - T_{11}) \times T_{12}}{T_{12} - L_{12} e^{-r_l}} + T_{11} \]

\[ S_{22} \times S_{22}' = \frac{L_{12} e^{r_l} - T_{12}}{L_{12} e^{-r_l} - T_{12}} \]

\[ \frac{S_{22}'}{S_{22}} = \frac{S_{22} \times S_{22}'(R_{11} - T_{11}) + T_{11} - S_{11}}{S_{22} \times S_{22}'(R_{22} - T_{22}) + T_{22} - S_{11}'} \]
Sign selected based on the phase of Gamma

\[ \begin{align*}
  (15) \quad S'_{22} &= \pm \sqrt{S_{22} \times S_{22}' \left(\frac{S_{22}'}{S_{22}}\right)} \\
  (16) \quad S_{22} &= \frac{S_{22} \times S_{22}'}{S_{22}'}
\end{align*} \]

Sign selection of \( S_{12} \) does not affect calibrated S values so long as \( S'_{12} \) is evaluated as being dependent on the sign of \( S_{12} \).

\[ \begin{align*}
  (17) \quad S_{12} &= \pm \sqrt{\frac{(T'11 - S11)(1 - S_{22} \times S_{22}')}{S_{22}'}} \\
  (18) \quad S'_{12} &= \frac{T'12(1 - S_{22} \times S_{22}')}{S_{12}} \\
  (19) \quad \Gamma_L &= \frac{R_{22} - S_{11}'}{(R_{22} - S_{11}')S_{22}'+(S_{12})^2}
\end{align*} \]
Appendix C: Material Electromagnetic Boundary Conditions

Boundary condition equations for material under test (g):

\[ (1) \quad E_i e^{-\gamma o m} + E_r e^{\gamma o m} = E_\alpha e^{-\gamma m} + E_\beta e^{\gamma m} \]

\[ (2) \quad \frac{E_i e^{-\gamma o m}}{\eta_o} - \frac{E_r e^{\gamma o m}}{\eta_o} = \frac{E_\alpha e^{-\gamma m}}{\eta} - \frac{E_\beta e^{\gamma m}}{\eta} \]

\[ (3) \quad E_\alpha e^{-\gamma n} + E_\beta e^{\gamma n} = E_t e^{-\gamma o n} \]

\[ (4) \quad \frac{E_\alpha e^{-\gamma n}}{\eta} - \frac{E_\beta e^{\gamma n}}{\eta} = \frac{E_t e^{-\gamma o n}}{\eta_o} \]

\[ (5) \quad \frac{E_t}{E_i} = S12 * e^{\gamma_o (n+p)} \]

\[ (6) \quad S11 = \frac{E_r}{E_t} \]

\[ (7) \quad \eta = \sqrt{\frac{\mu}{\epsilon}} \]

Equations based on the following system simplifications where the reflected and transmitted E-fields are an agglomeration of all of the reflections and transmissions. The distance measurements are labeled m, n, l, and p.

![Figure C.1: MUT boundary condition problem: l, m, n, and p are distances in the z direction, and l is the thickness of the material.](image)
Appendix D: Matlab Code

D.1: Permittivity using Phase Change.

This is Matlab code calculating permittivity through the difference in S12 phases.

function [eps, x] = FindPerm(Thru, Sample, f1, f2, LSample)
% Function FindPerm(Thru,Sample,f1,f2,LSample) takes a Thru measurement and a Sample
% measurement of the same data length and frequency span and calculates the permittivity
% assuming that the permeability = 1. It returns the permittivity (eps) and a frequency matrix
% containing all of the frequency data points.

% Determine number of data points in test sample
n1 = size(Sample);
n = n1(1,5);

% Range is end frequency minus start frequency
range = f2-f1;
step = (range)/(n-1);

% Set up x coordinates for graphing
x = f1:step:f2;

% Speed of light, in m/s
c = 2.998E8;

% The difference in phases is mod 2*pi. This selects that value.
m = 0;

% Initialize results matrix
eps = zeros(n,1);

for i = 1:n
    Sa = angle(Thru(:,:,3,i));
    Sr = angle(Sample(:,:,3,i));

    phi = (Sr - Sa);                        % Phase difference
    if phi > 0
        phi = phi-2*pi;                     % Phase correct
    end
    phi = phi - 2*m*pi;                    % Mod 2pi
    eps(i) = (c*phi/(2*pi*LSample*x(i)*1E9))^2; % Result
end

% Plot results
plot(x,eps,'b');
end
D.2: Complex Permittivity and Permeability

This is a Matlab function calculating the complex permittivity and permeability of a material using both the transmission and reflection measurements.

function eps = ComplexPandP(S,f1,f2,L)
    % This function finds the permeability and the permittivity of a material. The algorithm is based off of Ghodgaonkar, et al., 1990.
    % Either S11 and S21 or S22 and S12 can be used. One reflection and one transmission parameter are necessary. S is the signal, f1 the start frequency, f2 the stop frequency, and L the length of the sample. The function returns the complex permeability values.

    % Phase correct for any additional air that might be in the system. d is the total distance of extra air in the transmission, d2 is the distance of additional air in the reflection measurement. Distances are in meters.
    d = .00;
    d2 = .00;

    % Determine number of data points in test sample
    n1 = size(S);
    n = n1(1,5);

    % Range is end frequency minus start frequency, step is the total distance between data points
    range = f2-f1;
    step = (range)/(n-1);

    % Set up x coordinates for graphing
    x = f1:step:f2;

    % eps is the permittivity, mu the permeability
    eps = zeros(n,1);
    mu = zeros(n,1);

    % Variables useful for testing different parts of the algorithm
    test = zeros(n, 1);
    test2 = zeros(n,1);

    % Speed of light, divided by 10^8 in m/s
    c = 2.998;

    % Code takes the natural log of a complex number. This controls the mod 2*pi associated with that calculation. For samples sizes shorter than the wavelength of the signal, m will equal 0.
    m = 0;

    % Start loop through data points
    for i = 1:n
        % go is the propagation constant of free space
        go = j*x(i)*20*pi/2.998;
% Calculate phase shifts for S21 and S11
shift = exp(-go*d);
shift2 = exp(-go*d2);

% Perform phase shifts on data
S11 = S(:,:,1,4,i)*shift2^2;
S21 = S(:,:,1,2,i)*shift;

K = (S11^2 - S21^2 + 1)/(2*S11);

% Reflection parameter
G = K + sqrt(K^2 - 1);
if (abs(G) > 1)
    G = K - sqrt(K^2 -1);
end

% T = exp(-(propagation constant)*(length of sample))
T = ((S11+S21 - G)/(1-(S11+S21)*G));
phi = angle(T);
B = -j*(phi-m*2*pi)/L;

% g is the propagation constant of the material
g = (log(1/abs(T))/L)+B;

% Calculate the permittivity and permeability
mu(i) = (g/go)*(1+G)/(1-G);
eps(i) = (g/go)*(1-G)/(1+G);

% Calculate Z for result verification purposes
Z = sqrt(mu(i)/eps(i));

% Save Z and phi
test(i) = Z;
test2(i) = phi;
end

% Plots Z and phi, useful for troubleshooting
draw_test(x,test,test2);

% Plots the real and imaginary parts of the permittivity and permeability. It is important to note
% which sign is chosen for the imaginary parts-the convention varies.
figure;
hold on;
D.3: Reference Plane Error

This is a Matlab function calculating reference plane placement error in mm.

function FindL(S, f1, f2, L, eps, mu)
% This function is useful for trouble shooting whether accurate sample placement has been
% achieved. The inputs are S, a calibrated sample signal, f1 and f2, the start and stop
% frequencies, L, the length of the sample, eps, the theoretical permittivity of the sample, and
% mu, the theoretical permeability.

% Determine number of data points in test sample
n1 = size(S);
n2 = n1(1,5);

% Range is end frequency minus start frequency
range = f2-f1;
step = (range)/(n2-1);

% Set up x coordinates for graphing
x = f1:step:f2;
% Calculate Z, G, and n from the known permittivities
Z = sqrt(mu/eps);
G = (Z-1)/(Z+1);
n = sqrt(mu*eps);
c = 2.998;

% Initialize variables to hold the length of air added
Lerr = zeros(n2,1);
Lerr2 = zeros(n2,1);
Lerr3 = zeros(n2,1);

% Check the length of air at every data point
for i = 1:n2
    % g and go are the propagation constant of the sample and free space,
    g = x(i)*20*pi/c*n*j;
go = x(i)*20*pi/c*j;
z = exp(-g*L);

    % Temporary variables hold S-parameter values, S21 is not necessary, it can be switched in
    % for S12
    S11 = S(:,:,1,1,i);
    S12 = S(:,:,1,2,i);
    S21 = S(:,:,1,3,i);
    S22 = S(:,:,1,4,i);
    error = 1000;
    error2 = 1000;
    error3 = 1000;
    for La = -.005:.00001:.005
        % Theoretical phase shift due to excess air
        R = exp(-(go*La));
        % Theoretical thru and reflection values
        CalcS12 = z*(1-G^2)/(1-G^2*z^2)*R;
        CalcS11 = G*(1-z^2)/(1-z^2*G^2)*R^2;
        err = abs(S12-CalcS12);
        if err < error
            Lerr(i) = La*1000;
            error = err;
        end
        err2 = abs(S11-CalcS11);
        if err2 < error2
Lerr2(i) = La*1000;
error2 = err2;
end

err3 = abs(S22-CalcS11);
if err3 < error3
    Lerr3(i) = La*1000;
    error3 = err3;
end
end

end

% Plot results
figure;
hold on;
title('Sample Excess Air', 'FontName', 'Times New Roman', 'FontSize', 12);
xlabel('GHz','FontName', 'Times New Roman', 'FontSize', 12);
ylabel('Length Air (mm)', 'FontName', 'Times New Roman', 'FontSize', 12);
plot(x,Lerr);
plot(x,Lerr2, 'g');
plot(x,Lerr3, 'c');
legend('S12','S11','S22', 'Location','NorthEastOutside');
set(get(gcf,'CurrentAxes'),'FontName','Times New Roman','FontSize',12)
end

D.4: Placement Error

This Matlab code uses the phase difference between S11 and S22 to calculate the placement error of the MUT.

function error2 = PhaseShift(data,f1,f2,title1)
% Returns the placement error, in meters. This placement error can then be used as an argument
% in the permittivity code to calculate the corrected permittivity and permeability.

n1 = size(data);
n = n1(1,5);

% Establish x-axis for plotting.
range = f2-f1;
step = (range)/(n-1);
x = f1:step:f2;

% Variables to store the phases of the S-parameters
y11 = zeros(n,1);
y12 = zeros(n,1);
y21 = zeros(n,1);
y22 = zeros(n,1);

% Placement error
error = zeros(n,1);

% 1/2 the difference between the phases of S11 and S22
phase = zeros(n,1);

for i = 1:n

% Propagation constant, air
g = x(i)*20*pi/2.998;

y11(i) = angle(data(:,:,1,1,i));
y12(i) = angle(data(:,:,1,2,i));
y21(i) = angle(data(:,:,1,3,i));
y22(i) = angle(data(:,:,1,4,i));

% Difference in phases
sub = y22(i)-y11(i);

% Divide by 2
phase(i) = sub(i)/2;

% Stores theoretical error at each data point in mm.
error(i) = phase(i)/g*1000;
end

% Divide by 2: reflection measurement, air travels there and back
error2 = mean(error)/2;

% Phase correct S11 and S22 phases by ½ the mean phase difference between them
y11 = y11 + phase2;
y22 = y22 - phase2;

% Use to plot results, normally commented out.
%plot_all(error, sub, y11, y22, y12, y21, f1, f2, x, title1)
end

function plot_all(error, sub, y11, y22, y12, y21, f1, f2, x, title1)
% This function plots the results

% This plot draws all of the phases, with an adjusted S11 and S22.
figure;
hold on;
xlabel('Frequency (GHz)', 'FontName', 'Times New Roman', 'FontSize', 12);
ylabel('Phase', 'FontName', 'Times New Roman', 'FontSize', 12);
plot(x,y11,'m');
plot(x,y12,'b');
plot(x,y22,'g');
plot(x,y21,'c');
axis([f1,f2,-4,4]);
legend('S11','S12','S22','S21', 'Location','NorthEastOutside');
title(title1, 'FontName', 'Times New Roman', 'FontSize', 12);
set(gcf,'CurrentAxes','FontName','Times New Roman','FontSize',12)

% This figure draws a plot of all the calculated placement error. This error is then averaged to
% calculate the theoretical placement error.
figure;
hold on;
plot(x, error)
title('placement error')
xlabel('Frequency (GHz)', 'FontName', 'Times New Roman', 'FontSize', 12);
ylabel('distance in mm')

% This plots the calculated phase difference between S11 and S22 at each frequency
figure;
hold on;
plot(x, sub);
title('phase subtraction');
xlabel('Frequency (GHz)', 'FontName', 'Times New Roman', 'FontSize', 12);
ylabel('phase difference');
end
References


