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The Demand for Currency Approach and the Size of the Shadow Economy: A Critical Assessment

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ABSTRACT

A commonly used approach to measure the size of the shadow economy, known as “the monetary method”, is based on econometric estimates of the demand for currency. These estimates are used to get the currency held by economic agents in excess of the amount they need to finance registered transactions. This excess of currency multiplied by the income-velocity of circulation (assumed to be equal in the registered and shadow economies) gives a measure of the hidden GDP. This paper shows that the monetary method only produces coherent estimates if the income-elasticity of the demand for currency is one and suggests a way to correct the estimated size of the shadow economy when such elasticity is not one. The correction is applied to existent measures for different countries.

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“…cash leaves no tracks, and makes no demands on anybody else’s integrity.”

Benjamin Friedman, The Economist, July 22, 2000, p.76

1. Introduction

National accounts do not register a whole set of economic transactions. The size and the causes and consequences of the existence of such transactions are studied under different names: hidden, unrecorded, underground, parallel, black or shadow economy. Undeclared, underdeclared, nonmeasured and under-registered transactions made to avoid the burden of taxes or to circumvent regulations, illegal transactions connected with crime and corruption and legal but non-market activities are included in the concept of shadow economy. Not surprisingly, the concepts analyzed are not uniform. Economists have been interested in this topic during the last 25 years. A volume of The Economic Journal (1999) and a survey by Schneider and Enste (2000) thoroughly document such interest. In recent years the issue also attracted the attention of the press. The reason is straightforward: underestimating the GDP implies, for example, an overestimation of the Public Deficit/GDP and Debt/GDP ratios; therefore, any fiscal and monetary policy decision would be based on biased official figures. Additionally, country unions (notably the European Union) usually determine budget targets and member contributions with reference to the GDP and, consequently, any discrepancy in the shadow economy rates across nations is at the center of the problem.

The very nature of the shadow economy makes its measurement a difficult task. Furthermore, different estimation methods target
different concepts. As a result, estimation methods have become an important issue.

The ‘monetary method’ or ‘currency approach’ is essentially based on the computation of discrepancies between declared income and the income implied by the observed currency demand. Although it has been widely used in the literature to measure the size of the shadow economy, it has also been strongly criticized on different grounds.

Recently, Caridi and Passerini (2001) argued against the currency approach. They stress that this method wrongly considers the concepts of unreported and unrecorded activities to be equivalent. The former refers to the fraction of income not declared to the tax authorities while the latter corresponds to the portion of the national product missed by statistical offices. The authors claim that the method is useful to give indications of the amount of tax evasion (or unreported income), rather than the exhaustiveness of statistics (unrecorded income).

Thomas (1999) and Breusch (2005a, 2005b), among others, criticize the method on econometric grounds concerning quantitative accuracy, time series properties, structural breaks and sensitivity to units of measurement.

In this paper we focus on an internal inconsistency of the technique that has appeared repeatedly in applied work and has not been addressed so far in the literature. We point out that in almost every empirical application the steps followed to “measure” the size of the

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1 Authors do not seem to be always aware of this.
2 Besides the monetary method, three others approaches can be identified in the literature: (1) methods based on discrepancies between registered income and income implied by observed expenditure; (2) methods based on direct auditing surveys (mainly fiscal and regulatory) usually known as direct methods; (3) methods based on discrepancies between declared production and observed use of production factors (mainly labor and energy).
shadow economy are inconsistent with the method itself. That is, rather than an extrinsic critique, we stress an intrinsic flaw. In particular the method assumes that the velocity of circulation of cash is the same in both reported and black sectors and we show that this equality is only true when the income elasticity of the demand for cash is one.\(^4\) However, in assuming equal velocities, the literature has made the calculations “as if” the income elasticity were one even when it was not the case.

We do not claim that the adjustment we suggest provides the correct way to estimate the velocities of circulation both in the formal and informal sectors. We just argue that the technique itself is not only based on the idea that the velocity is lower in the shadow economy (in which ‘everything’ is financed with cash) but it also provides two distinct estimates, so that the assumption of equality is wrong. We provide an expression for the velocity of circulation of currency in the shadow sector as a function of the income elasticity as well as a simple way to correct measures that incorrectly assumed the same velocity.

The next section reviews the evolution of the monetary approach. The third section discusses the method based on the econometric estimation of the demand for currency and some conditions that the income-elasticity of the demand for cash should meet to get coherent results. In section 4 a correction for incoherent results is suggested and applied to some existent estimates. Some closing remarks are made in the last part of the paper.

\(^3\) Except in Ahumada et al. (2001).

\(^4\) This is a general statement and it applies to any proxy of income used by different researchers. We refer to “almost every empirical application” because some find unitary income elasticities.
2. A review of the monetary approach evolution

The monetary approach to measure the size of the shadow economy is based on the assumption that cash is used to make transactions that agents want to keep hidden from official records. Transactions made using cash are difficult to trace: they leave no tracks. Other assets are registered in financial institutions and their uses are recorded in such a way that transactions made with them can be easily inspected. If the amount of currency used to make hidden transactions can be estimated, then this amount could be multiplied by the income-velocity of money to get a measure of the size of the shadow economy.

The monetary approach was first presented by Gutman (1977) and Feige (1979) and it has evolved to use econometric tools in estimates made by Tanzi (1982, 1983), which are based on Cagan (1958). The technique was then applied to measure the size of the shadow economy in the US, Italy, Norway, Canada, South Africa, Tanzania, Mexico, India, Australia, Austria, Belgium, Denmark, France, Germany, Great Britain, Ireland, Netherlands, New Zealand, Spain, Sweden, Switzerland Argentina, etc.\(^5\)

Gutman’s method (1977) is based on four key assumptions: (a) high taxes and government regulations are the main causes of the existence of a shadow sector; (b) only cash is used to make transactions in the shadow economy; (c) the ratio of currency to demand deposits, \(C/D\), is only influenced by changes in taxes and regulations, and (d) there was some point in time in the past when no shadow economy existed. As the ratio \(C/D\) of that period should

\(^5\) Tanzi (1999) gives a skeptical view given the wide diversity of the results obtained.
have prevailed except for changes in the level of taxes and regulations, each increase in C/D is directly linked to the extra currency used in the shadow economy. The method assumes that the income-velocity of circulation, \( v \), is equal for the registered and the hidden economies; hence the size of the hidden sector is \( v \) times the extra currency.

Feige’s method (1979) uses the standard version of the quantity theory of money \( Mv = PT \), where \( M \) is money including demand deposits. The value of transactions is \( PT \). Assuming that the ratio of the value of transactions to nominal income remains constant through time and that it is known for a period in which there were no hidden transactions, then total nominal income can be estimated for any period. The difference between estimated total nominal income and observed nominal income is the size of the shadow economy. Feige assumes that hidden transactions are made using either cash or checks.

The work by Tanzi (1982) and all the papers based on his approach use econometric estimates of the demand for currency. This method supposes that the income-velocity depends not only on variables that induce economic agents to make hidden transactions but also on income and the opportunity cost of holding cash. The estimated equation of the demand for currency is used to get the extra cash held by economic agents to finance hidden transactions. Again it is assumed that the income-velocity of circulation for registered and hidden transactions is equal, so the size of the shadow economy is measured by multiplying the extra cash by \( v \).

3. The standard monetary approach
This section is devoted to present a detailed review of the monetary method based on the econometric estimate of the demand for currency and to derive the condition that the income-elasticity should meet to obtain equal velocities in both sectors.

A currency demand function in Cagan’s (1958) tradition can be expressed as:

\[ C_o = A(1 + \Theta)^\alpha Y_o^\beta \exp(\gamma) \]  

(1)

where \( C_o \) denotes observed cash balances, \( \Theta \) is a variable which reflects the incentives agents have to make hidden transactions (for example the ratio of taxes or government expenditure to GDP), \( Y_o \) is a scale variable (for example registered GDP), \( i \) measures the opportunity cost of holding cash (the interest rate or the rate of inflation); \( A, \alpha, \beta \) and \( \gamma \) are positive parameters. Observed currency, \( C_o \), is equal to total currency, \( C_t \), which includes cash used for recorded transactions, \( C_r \), plus cash used for hidden transactions, \( C_h \),

\[ C_o = C_t = C_r + C_h \]  

(2)

Observed GDP, \( Y_o \), is the registered GDP, \( Y_r \), which does not include hidden GDP, \( Y_h \).
\[ Y_T = Y_O + Y_H = Y_R + Y_H \]  \hspace{1cm} (3)

Since observed currency includes \( C_H \) but observed GDP excludes \( Y_H \), the usual econometric regression of \( C_o \) on \( Y_o \) would result in biased and inconsistent estimates.

The empirical applications based on this technique proceeds in the following way: a demand for currency is estimated as in (1).\(^6\) Then, under the assumption that the demands for \( C_R \) and \( C_H \) have the same functional form with equal parameters, \( \Theta \) is set equal to zero to get an estimate of the amount of cash demanded under no incentives to hide transactions, \( \hat{C}_R \),

\[
\hat{C}_R = \hat{A} Y_O^\beta \exp(-\hat{\gamma} i) \]  \hspace{1cm} (4)

Now \( \hat{C}_R \) is known from (4) and \( C_T \) is observed currency, \( C_O \), so \( \hat{C}_H \) can be obtained by difference,\(^7\)

\[
\hat{C}_H = C_T - \hat{C}_R \]  \hspace{1cm} (5)

The velocity of circulation in the registered economy is

\(^6\) To take into account that the time series are integrated, some works consider equation (1) as a long run relation. Other papers estimate first difference equations, partial adjustment models or hybrids.

\(^7\) Tanzi (1982) uses \( \hat{C}_T \) instead of \( C_T \) in (5).
To get the size of the shadow economy it is assumed that the velocity of circulation for both, registered and hidden transactions, is the same, so,

\[ v_R = \frac{Y_R}{C_R} \]  

(6)

and then,

\[ v_R = \frac{Y_R}{C_R} = \frac{Y_H}{C_H} \]  

(7)

\[ \hat{Y}_H = \hat{v}_R \hat{C}_H \]  

(8)

\( \hat{Y}_H \) is ‘the’ estimation of the size of the shadow economy and it is obtained using \( \hat{C}_H \) from (5) and \( \hat{v}_R \) from (6).

So far we have described the procedures followed in the literature. The key assumption made explicit in (7) requires \( \beta = 1 \), which is evident if we recall that the income-velocity defined in (6) is equivalent to

\[ v_R = \frac{Y_R}{C_R} = \frac{Y_R}{A Y_R^\beta \exp(-\gamma i)} = \frac{Y_R^{1-\beta}}{A \exp(-\gamma i)} \]  

(9)
while the velocity for the hidden economy is

\[
v_H = \frac{Y_H}{C_H} = \frac{Y_H}{AY_H^{\beta} \exp(-\gamma i)} = \frac{Y_H^{1-\beta}}{A \exp(-\gamma i)} \tag{10}
\]

The velocity is the same in both sectors if \( \beta = 1 \).\(^8\) Those studies that find \( \hat{\beta} \neq 1 \) but follow the steps described above are therefore incorrect.\(^9\)

4. A correction of the method

The problem addressed in section 3 can be solved by an explicit recognition that \( v \) depends on the value of \( \beta \). In this section we follow the currency approach to compute the size of the hidden economy and we show that there is no need to impose ad hoc restrictions on the velocity of circulation. We also provide a way to “correct” wrong estimates made by imposing \( \beta = 1 \) when it was not the case.

The assumption implicitly made by the traditional technique is that the demands for \( C_R \) and \( C_H \) follow Cagan-type forms with equal parameters. This allows to write the aggregate demand equation \( C_R + C_H \), (2), as

\[^8\) The velocity is also the same if \( Y_R = Y_H \) for any \( \beta \).
\[^9\) It should be stressed that currency is the money aggregate whose demand should have an income elasticity equal to one. While this value may appear reasonable and theory-based in the case of the demand for the aggregate used to finance all transactions (e.g. the demand for M1), it may not necessarily be correct for narrower definitions of money. For instance, in Baumol-Tobin’s model, the value of transactions elasticity is 1/2.
\[ C_T = A Y_R^\beta \exp(-\gamma i) + A Y_H^\beta \exp(-\gamma i) = A Y_R^\beta \exp(-\gamma i) \left( 1 + \left( \frac{Y_H}{Y_R} \right)^\beta \right) \]  \hspace{1cm} \text{(11)}

which can be also written as

\[ C_T = A Y_R^\beta \exp(-\gamma i) \left( 1 + \Theta \right)^\alpha \]  \hspace{1cm} \text{(12)}

Equation (11) is always behind expression (12), which is used in every application. This formulation does not need to be restricted to currency; it is also valid for any wider aggregate (e.g. M1) as long as the interest rate is its opportunity cost and not its yield.\(^\text{10}\) However, \(\beta\) obtained from (12) only matches \(\beta\) in (11) if the ratio \(\frac{Y_H}{Y_R}\) is independent of \(Y_R\).\(^\text{11}\)

Consequently, all papers using the monetary approach make the assumptions, though not explicitly, regarding (i) functional form and aggregation from (11) to (12) and (ii) independency between \(\frac{Y_H}{Y_R}\) and \(Y_R\).

Recalling that \(Y_O = Y_R\), \(C_T = C_O\) and that \(C_T\) and \(Y_R\) are observed variables, (12) can be econometrically estimated as in (1).\(^\text{12}\) Next, as

\(^{10}\) It is enough to add the demand for deposits to (11) after assuming it follows the same Cagan-type form.

\(^{11}\) For this reason it is convenient to measure \(\Theta\) normalized by registered GDP.

\(^{12}\) The variable \((1 + \Theta)^\alpha\) is sensitive to changes in the units in which \(\Theta\) is measured, as pointed out in Breusch (2005). We keep the notation to maintain Tanzi’s original
we already described in the previous section, setting $\Theta=0$ provides an estimate for $C_H$.

The ratio between $C_R$ and $C_H$ is

$$\frac{C_R}{C_H} = \frac{AY_R^\beta \exp(-\gamma i)}{AY_H^\beta \exp(-\gamma i)} = \left(\frac{Y_R}{Y_H}\right)$$  \hspace{1cm} (13)

Equation (13) provides an expression for $Y_H$ given $Y_R$, $C_R$, $C_H$ and $\beta$. Consequently there is no need to make the ad-hoc assumption on the equality of income velocity in both sectors.

If $\beta = 1$, the assumption about equality of $v$ for hidden and registered transactions is accurate. From (13)

$$\frac{Y_R}{C_R} = \left(\frac{Y_R}{Y_H}\right)^{1-\beta} \frac{Y_H}{C_H}$$  \hspace{1cm} (14)

or

$$v_R = \left(\frac{Y_R}{Y_H}\right)^{\beta-1} v_H$$  \hspace{1cm} (15)

Equation (15) shows in another way that it is inaccurate to assume that $v$ is equal for registered and hidden transactions when the functional form. Nevertheless, changing $(1+\Theta)^\alpha$ for $(\exp(\alpha \Theta) - 1)$ could solve this problem.
hypothesis $\beta = 1$ is rejected in the econometric estimation of the demand for currency. Equation (13) allows us to “correct” wrong estimates made by imposing $\beta = 1$ when it was not the case. From (13) it follows that

$$\frac{C_R}{C_H} = \frac{\bar{Y}_R}{\bar{Y}_H}$$  \hspace{1cm} (16)$$

where $\frac{\bar{Y}_R}{\bar{Y}_H}$ is the (faulty) ratio obtained under the restriction $\beta = 1$. From (13) and (16)

$$\frac{Y_H}{Y_R} = \left( \frac{C_H}{C_R} \right)^{\frac{1}{\beta}} = \left( \frac{\bar{Y}_H}{\bar{Y}_R} \right)^{\frac{1}{\beta}}$$  \hspace{1cm} (17)$$

which shows how to correct results obtained under $\beta = 1$ when $\beta \neq 1$.

5. Correcting the size of the shadow economy

In this section we present some results only as an exercise to illustrate the main point of the paper. We do not intend to provide accurate measures of the size of the underground economy. In fact it should be pointed out that the level of econometric analysis in these studies is rather basic and casts doubts on the validity of the
econometric results, particularly after identifying the key role played by the estimated income elasticity.\textsuperscript{13} We only show estimates from other papers to apply the correction described above.\textsuperscript{14} It is straightforward to note that if the method is properly followed then, a smaller estimated income-elasticity of the demand for cash with respect to one implies a smaller recalculated size of the shadow economy. Authors generally present their results as percentages of GDP, implying that they computed the portion of GDP not registered by statistics.

**Argentina 1930-1983**

Guisarri (1986) measures the size of the shadow economy in Argentina for 1930-1983 using annual data. He estimates a demand for currency which could be understood as a long run equation. The share of government expenditure in GDP and the ratio between the official and black exchange rate of the U.S. dollar were the chosen variables to quantify the incentives to hide transactions in \( \Theta \). The econometric estimate for \( \hat{\beta} \) is 0.508. He follows the standard technique described in section 3, that is, the assumption of equal velocities or \( \beta = 1 \). According to his calculations, the size of the shadow economy in 1983 (his last observation) represented 56% of registered GDP. Nevertheless, our correction implies that the magnitude of the hidden sector was 32%.

\textsuperscript{13} As a few examples, Thomas (1989) re-estimated Tanzi’s model for 1930-1980 and found evidence of a structural break in 1945 while the tax variable was statistically not significant after 1946. Tanzi (1982) had found an income elasticity very close to one. Smith (1986) showed that the model of Matthews and Rastogi (1985) was mis-specified.
Australia 1967-2000

Bajada and Schneider (2003) produce a time series estimate of the ‘cash economy’ in Australia between 1967 and 2000. The results are based on an error correction model for money demand in which \( \hat{\beta} = 0.852 \). This is the long run estimate obtained by assuming the static equilibrium value of all variables. They calculate that the size of the shadow economy in Australia between 1990 and 2000 averaged 14.6% of GDP. They also compare this figure with the one resulting from a MIMIC approach: 14.82%. They stress the similarity of estimates. However, if the method is correctly followed, the fraction of the shadow income was 10.4% of the registered economy.

Norway

Isachsen et al. (1982) get an income elasticity of the demand for currency of 0.85 between 1952 and 1978. The correction changes the share of the shadow economy from 8% to 5.1% of the registered economy. Isachsen and Strom (1985) use the currency demand approach to get an estimate of the hidden economy of 6.3% for 1978. This estimate is based on a partial adjustment currency demand equation with a long run income elasticity equal to 0.663. The corrected estimate gives 1.51%.

Austria 1956-1998

Unfortunately, many studies applying the monetary method do not show the results of the estimation of the demand for currency, and they focus on the “one” number: the size of the shadow economy as percentage of the GDP.
Schneider (2000) applies the monetary method to Austria for the period 1956-1998. A partial adjustment currency demand equation is estimated as a function of lagged currency, number of Eurochecks per capita, real interest rate on bonds, a measure of the indirect tax burden, an index of regulations and per capita consumption as a proxy for the income effect. Although the reported short-run income elasticity is 0.734, the implicit long-run value is $\hat{\beta} = 2.04$. The author calculates that the size of the shadow economy was 9.12% of registered GDP in 1998. The correction gives 30.9% using the long run elasticity.

_Tanzania 1968-1990_

Bagachwa and Naho (1995) generate two different time series estimates of the shadow economy, based on different real currency demand functions which differ only in the way the tax and government intervention variables enter the equation. The ratio of hidden to registered economy for 1990 is 33.24% from the first equation and 20.96% from the second. However, the income elasticities they find are 2.323 and 2.569 respectively. For 1990 the corrections give estimates of 62.2% and 54.4%.

6.Final Remarks

The monetary method to measure the size of the shadow economy is mainly based on econometric estimates of the demand for currency. These estimates are used to get the currency held by economic agents in excess of the amount they need to finance registered transactions. The standard currency approach uses the
excess of currency multiplied by the velocity of circulation (assumed to be equal in the registered and the shadow economies) to measure hidden GDP. This paper shows that this procedure is faulty; it is only accurate if the income elasticity is one.

Cash finances all transactions in the underground sector to avoid leaving trails. It plays a much more restricted role in the registered economy, where other aggregates are available. Consequently, the rationale of the method is based on the idea of different income/currency ratios. The assumption usually made in applied works of equal velocities together with income elasticity estimates lower (higher) than one result in figures biased upwards (downwards) for the shadow economy.

References


