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Influence of Interfacial Debonding on Mechanical Responses of Magnetorheological Elastomers

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Influence of Interfacial Debonding on Mechanical Response of Magnetorheological Elastomers

THESIS

submitted in partial satisfaction of the requirements for the degree of MASTER OF SCIENCE in Civil Engineering by Songying Li

Thesis Committee:
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2018
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>List of Figures</td>
<td>v</td>
</tr>
<tr>
<td>Acknowledgements</td>
<td>xi</td>
</tr>
<tr>
<td>Abstract of the Dissertation</td>
<td>xii</td>
</tr>
</tbody>
</table>

**Chapter 1 Introduction**

1.1 Researches of MRE Materials.........................................................1

1.2 Scope........................................................................................................2

**Chapter 2 Micromechanics Theory of Materials**

2.1 Heterogeneous Composite Materials.....................................................4

2.2 Two-scale Method ....................................................................................4

2.3 Overview of Fracture Micromechanics..................................................6

**Chapter 3 Mechanical Response of MREs without Interfacial Debonding**

3.1 Introduction of MREs Simulation.............................................................7

3.2 Material Parameters of Magnetorheological Elastomers..........................8

3.2.1 Material Parameters of Carbonyl Iron Particles....................................8

3.2.2 Material Parameters of Rubber Matrix.................................................10

3.3 Introduction of Representative Volume Element (RVE)..............................12

3.3.1 Representative Volume Element Model...............................................12
Chapter 4 Interfacial Debonding Simulation of Magnetorheological Elastomers

4.1 Constitutive Model of Cohesive Zone Interface

4.2 Continuous Cohesive Zone Model (CCZM)

4.3 Applying CZM in ABAQUS/CAE

4.4 Model Introduction

4.5 Results and Comparison

4.5.1 Uniaxial Tensile Simulation of One-particle RVE Model

4.5.2 Shear Simulation of One-particle RVE Model
4.5.3 Uniaxial Tensile Simulation of Isotropic Multi-particles RVE Model......62
4.5.4 Shear Simulation of Isotropic Multi-particles RVE Model.......................65
4.5.5 Uniaxial Simulation of Anisotropic Multi-particle RVE Model...............68
4.5.6 Shear Simulation of Anisotropic Multi-Particle RVE Model...............70
4.6 Numerical Simulation and Experimental Result of MRE.........................70
4.6.1 Large-strain behavior of MREs with different iron contents...............72
   4.6.1.1 Uniaxial Tension Test Comparison..............................................72
   4.6.1.2 Uniaxial Compression Test Comparison........................................77
   4.6.1.3 Shear Test Comparison..............................................................82
4.6.2 Influence of interfacial adhesion on MREs........................................87
4.7 Conclusions.........................................................................................94

Chapter 5 Parametric Study of Interfacial Debonding in MREs.................95

5.1 Particle Fraction...................................................................................95
   5.1.1 Tensile Simulation of Isotropic Model.............................................95
   5.1.2 Tensile Simulation of Anisotropic Model........................................97
   5.1.3 Shear Simulation of Isotropic Model...............................................99
   5.1.4 Shear Simulation of Anisotropic Model..........................................101
5.2 Interface Properties.............................................................................102
   5.2.1 Tensile Simulation of Isotropic Model...........................................102
   5.2.2 Tensile Simulation of Anisotropic Model.......................................103
   5.2.3 Shear Simulation of Isotropic Model..............................................104
5.2.4 Shear Simulation of Anisotropic Model.............................................105

5.3 Conclusions......................................................................................106

Chapter 6 Conclusions and Future Research........................................107

6.1 Conclusions......................................................................................107

6.2 Suggestions for Future Research......................................................108

References..........................................................................................110
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-1</td>
<td>Conception of Two-Scale Method (from microscale to macroscale).</td>
<td>5</td>
</tr>
<tr>
<td>3-1</td>
<td>Tensile stress-strain curve of carbonyl iron particles.</td>
<td>9</td>
</tr>
<tr>
<td>3-2</td>
<td>Shear stress-strain curve of carbonyl iron particles.</td>
<td>9</td>
</tr>
<tr>
<td>3-3</td>
<td>Tensile and compressive stress-strain curves of matrix model.</td>
<td>11</td>
</tr>
<tr>
<td>3-4</td>
<td>Shear stress-strain curve of matrix model.</td>
<td>11</td>
</tr>
<tr>
<td>3-5</td>
<td>Periodic boundary conditions.</td>
<td>14</td>
</tr>
<tr>
<td>3-6</td>
<td>One-particle isotropic RVE model.</td>
<td>18</td>
</tr>
<tr>
<td>3-7</td>
<td>Isotropic and anisotropic multi-particles RVE model.</td>
<td>19</td>
</tr>
<tr>
<td>3-8</td>
<td>Uniaxial tensile stress contour in simulation.</td>
<td>22</td>
</tr>
<tr>
<td>3-9</td>
<td>Uniaxial tensile test diagram of 3D one-particle RVE model.</td>
<td>22</td>
</tr>
<tr>
<td>3-10</td>
<td>Uniaxial tensile stress contours of one-particle RVE model.</td>
<td>24</td>
</tr>
<tr>
<td>3-11</td>
<td>Uniaxial tensile stress and strain curve of one-particles RVE model with</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>the comparison of Mori-Tanaka model and Double-Inclusion model.</td>
<td></td>
</tr>
<tr>
<td>3-12</td>
<td>Young's modulus-strain curve of one-particles RVE model.</td>
<td>26</td>
</tr>
<tr>
<td>3-13</td>
<td>Shear stress contour of one-particle RVE model.</td>
<td>27</td>
</tr>
<tr>
<td>3-14</td>
<td>Shear test diagram of 3D one-particle RVE model.</td>
<td>27</td>
</tr>
<tr>
<td>3-15</td>
<td>Shear stress-strain curve of one-particle RVE model with</td>
<td>29</td>
</tr>
<tr>
<td></td>
<td>the comparison of Mori-Tanaka model and Double-</td>
<td></td>
</tr>
</tbody>
</table>
Inclusion model.

Figure 3-16 Shear modulus response of one-particles RVE model. 29

Figure 3-17 Uniaxial stress contours of isotropic multi-particles RVE model. 31

Figure 3-18 Uniaxial test diagram of 3D isotropic multi-particle RVE model. 31

Figure 3-19 Uniaxial tensile stress and strain curve of isotropic multi-particles RVE model with the comparison of Mori-Tanaka model and Double-Inclusion model. 33

Figure 3-20 The Young’s modulus-strain curve of isotropic multi-particles RVE Model. 33

Figure 3-21 Shear stress contour of isotropic multi-particle RVE model. 34

Figure 3-22 Shear test diagram of 3D isotropic multi-particle RVE model. 34

Figure 3-23 Shear stress-strain curve of isotropic multi-particle RVE model with the comparison of Mori-Tanaka model and Double-Inclusion model. 36

Figure 3-24 Shear modulus response of isotropic multi-particle RVE model. 36

Figure 3-25 Uniaxial stress contours of anisotropic multi-particles RVE model. 38

Figure 3-26 Uniaxial test diagram of 3D anisotropic multi-particles RVE model. 38
Figure 3-27  Uniaxial tensile stress-strain curves of anisotropic multi-particles RVE models.

Figure 3-28  The Young’s modulus-strain curve of anisotropic multi-particles RVE Model.

Figure 3-29  Shear stress contour of anisotropic multi-particle RVE model.

Figure 3-30  Shear test diagram of 3D anisotropic multi-particle RVE model.

Figure 3-31  Shear stress-strain curves of anisotropic multi-particle RVE models.

Figure 3-32  The shear modulus-strain curve of anisotropic multi-particle RVE model.

Figure 4-1  Some constitutive models of cohesive zone.

Figure 4-2  Evolution law of linear damage

Figure 4-3  Definition of scalar damage variable \( d \).

Figure 4-4  Summary of damage initiation criteria.

Figure 4-5  One-particle RVE model.

Figure 4-6  Constitutive model of cohesive zone.

Figure 4-7  Isotropic and anisotropic muti-particles RVE model.

Figure 4-8  Uniaxial tensile contours of one-particle RVE model.

Figure 4-9  The contact stress response at interface.

Figure 4-10  The uniaxial tensile nominal stress-nominal strain curve.
Figure 4-11 Shear stress contour of one-particle RVE model. 57
Figure 4-12 The shear stress-strain curve at top boundary of model. 58
Figure 4-13 Uniaxial stress contours of isotropic multi-particles RVE model. 59
Figure 4-14 The uniaxial tensile stress-strain curve of isotropic multi-particles RVE model. 61
Figure 4-15 Stiffness comparison of one-particle and isotropic multi-particle RVE model. 62
Figure 4-16 Shear stress contour of isotropic multi-particle RVE model. 63
Figure 4-17 The shear stress-strain curve of isotropic multi-particles RVE model. 64
Figure 4-18 The shear response curves of one-particle and isotropic multi-particles RVE model. 65
Figure 4-19 Uniaxial tensile contours of anisotropic multi-particle RVE Model. 67
Figure 4-20 The uniaxial tensile nominal stress-nominal strain curves of isotropic and anisotropic multi-particle RVE model. 68
Figure 4-21 Shear stress contour of anisotropic multi-particle RVE model 69
Figure 4-22 Shear response curves of isotropic and anisotropic multi-particles RVE model. 70
Figure 4-23 Comparison of experimental tensile stress-strain response 73
of pure silicone (symbols) and the simulation of Mooney-Rivlin model (solid line).

Figure 4-24 The stress-strain curves of all MREs tested under tension test. 75

Figure 4-25 Interfacial debonding in anisotropic RVE model. 77

Figure 4-26 Comparison of experimental compressive and tensile stress-strain response of pure silicone (symbols) and the simulation of Mooney-Rivlin model (solid line). 78

Figure 4-27 The stress-strain curves of all MREs tested under tension test. 82

Figure 4-28 Comparison of experimental shear stress-strain response of pure silicone (symbols) and the simulation of Mooney-Rivlin model (solid line). 83

Figure 4-29 The stress-strain curves of all MREs tested under tension test. 86

Figure 4-30 Comparison of experimental tensile stress-strain response of pure silicone (symbols) and the simulation of Mooney-Rivlin model (solid line). 88

Figure 4-31 Comparison of the SEM micrographs and RVE models. 89

Figure 4-32 Comparison of simulative and experimental results of pure silicone and isotropic MREs. 91

Figure 4-33 Comparison of simulative and experimental results of 93
isotropic MREs with different interface adhesions.

Figure 5-1  Tensile response of isotropic MREs with 3%, 20% and 30% iron contents.
Figure 5-2  Tensile response of anisotropic MREs with 3%, 20% and 30% iron contents.
Figure 5-3  Shear response of isotropic MREs with 3%, 20% and 30% iron contents.
Figure 5-4  Shear response of anisotropic MREs with 3%, 20% and 30% iron contents.
Figure 5-5  Tensile response of isotropic MREs with different interface adhesions.
Figure 5-6  Tensile response of anisotropic MREs with different interface adhesions.
Figure 5-7  Shear response of isotropic MREs with different interface adhesions.
Figure 5-8  Shear response of anisotropic MREs with different interface adhesions.
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In addition, a thank you to Yijia Dong, who worked at the University of California, Irvine as an academic Visitor. His study on Meso-structural of concrete fracture using interface elements rendered me precious enlightenment.
ABSTRACT OF THE THESIS

Influence of Interfacial Debonding on Mechanical Response of Magnetorheological Elastomers

By

Songying Li

Master of Science in Civil Engineering

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Professor Lizhi Sun, Chair

Magneto-Rheological Elastomers (MREs) are smart materials whose mechanical properties can be altered by external magnetic fields rapidly and reversibly. In this thesis, the large-strain behavior of MREs was studied by nonlinear finite element simulations under uniaxial tension and pure shear deformation. A modeling approach has been employed to investigate the mechanical behavior of silicone-rubber based MREs. Mooney-Rivlin model was used to simulate the mechanical response of matrix. Cohesive-zone model depending on the assumption of rubber-iron adhesion energy was used to define interface properties between particles and matrix. All types of models showed strongly non-linear stress-strain behavior. The simulation results agreed well with the experiments carried out by other researchers. Parametric studies were created to analyze the impacts of particle fraction, interface adhesion and microstructures in composite on the mechanical
response of MREs models. Simulations outlined the influence of interface adhesion on the mechanical properties of MREs, which was more obvious in the case of higher particle volume fraction. Anisotropic Representative Volume Element (RVE) models revealed that the increase stiffness response due to the enhancement of chain-like particle alignment structures.
Chapter 1 Introduction

1.1 Research of MRE Materials

Magneto-Rheological Elastomers (MREs) are materials made of a relatively soft elastomer matrix filled with magnetizable particles. Anisotropic MREs which are solidified under magnetic field consisting of a chain or pillar structure and rubber matrix. If curing in the absence of a magnetic field, isotropic magneto-rheological elastomers can be obtained (Jolly, 1996). The magneto-rheological effect was first explored in 1948 (Rabinow, 1948), working on Magneto-rheological Fluids (MRFs). However, this material under magnetic field was not emphasized until 1980. Rigbi and Jilken were the first to conduct tests on MREs and to describe the previously unknown magneto-mechanical effects (Rigbi and Jilken, 1983). Later the dynamic small strain behavior of MREs has become a well-explored property, with different types of matrix and inclusion particles (Ginder and Nichols, 1999). The effect of the magnetic field used during manufacture of anisotropic MREs, the magnetic properties of MREs, were investigated (Bica, 2011).

The properties of MREs can be altered reversibly and instantaneously by the application of external magnetic fields. This behavior is caused by the interaction of micron-sized magnetic particles incorporated in polymer (Carlson and Jolly, 2000). MREs also can undergo very high deformation states (Vidal-Verdu and Hafez, 2007). With these characteristics, MREs gain special interest in a variety
of advanced engineering applications and these materials are widely applied as tunable damping, actuators, artificial muscles or shape control (Hamrock, 2006).

Magneto-rheological elastomers have been applied in material preparation, performance testing and engineering applications in recent years. However, the mechanism of magneto-rheological elastomers is still at an initial stage, and there is a lack of understanding about the evolution of microstructures in magneto-rheological elastomers under the magnetic field. The model based on single-chain magnetic particle structures is too simple, Neglecting its modulus of magneto viscoelasticity. The study of physical, electrical and magnetic properties of the magneto-rheological elastomer is still uncompleted, most stay on the phenomenal description.

1.2 Scope

In this thesis, the numerical simulation and theoretical modeling are combined. The mechanical response of MREs is studied based on the numerical simulation of FEM. The virtual microstructures by the computer program are used to create the Representative Volume Element (RVE) model and simulated magneto-rheological elastomers mechanical experiments. By using the simulations of virtual microstructures, the effects of factors such as particle volume fraction, shape, size and spatial distribution can be analyzed conveniently.
Chapter 2 includes the introduction of multi-scale macroscopic study on composites which is based on the theory of homogenization and overview of Fracture Micromechanics. Finite element method based on RVE is discussed to establish the theoretical basis for the simulations of MREs.

Chapter 3 assumes particle-matrix interface is perfectly bonded in numerical simulations. Several RVE finite element models of isotropic and anisotropic MREs in the absence of magnetic field are established and computed the macroscopic mechanical response of those MREs on uniaxial tensile and shear deformations. Compared the results with the variety of uniform theory solutions based on Eshelby inclusion theory.

Chapter 4 presents the Cohesive Zone Model (CZM) to explain the interfacial fracture evolution. The parameters of cohesive zone model the modeling used depend on the rubber-steel adhesion energy that was measured by peeling test. Then the effect of interfacial debonding is studied based on the numerical simulation of FEM, which is compared with the experimental results to analyze the macroscopic mechanics performance.

Chapter 5 demonstrates the influences of some parameters on the mechanical response of magneto-rheological elastomers. The results directly demonstrate the effect of each parameter on the response of the whole model.

Chapter 6 concludes the dissertation by summarizing the major contributions. In addition, some suggestions for the future work are also discussed.
Chapter 2 Micromechanics Theory of Materials

2.1 Heterogeneous Composite Materials

The micro-structure of the material decides material properties. To have a better understanding of the material properties, not only should we study the macroscopic level of material, but also take the multi-scale characteristic analysis into consideration. So that the macro-structure and micro-structure analysis combined with each other, then push the study of the mechanical properties of the material to a deeper level.

Most of the materials used in engineering have multi-scale characteristic, and when the scale is small to a certain value, all of them will show heterogeneity. Heterogeneous composite materials consist of a matrix material and one or several inclusions in microscope. Magneto-rheological elastomer material is a typical heterogeneous composite material. The purpose of micromechanics of this material is to predict the relationship between microscopic and macroscopic mechanical properties of it.

2.2 Two-scale Method

The basic problem of homogenization is: Under the same boundary conditions, finding an equivalent homogeneous material with the same equivalent
macroscopic modulus as the real multiphase composite material. We used finite element method based on RVE to solve this problem.

To predict the effect of the microstructures of the composite on the macroscope response of the material, it is necessary to distinguish between two scales: microscale and macroscale. For microscale, the material is local non-uniform; For macroscale, material consists of representative volume elements (RVEs). These two scales convert through RVE model. The RVE in microscale should be large enough to represent heterogeneous microstructures, but in macroscale it should be as small as possible and considered as a point in the macrostructure of the composite. This method is widely used and accurate. However, it also has some serious drawbacks: For complex microstructures, a high-quality meshing usually takes a long time, especially for non-linear response of material.

1. **Figure 2-1.** Conception of Two-Scale Method (from microscale to macroscale) (Figure 2-1 is cited from Sun S. I., Research about numerical modeling and constitutive modeling of magnetorheological elastomers under finite deformation. Dissertation of Northwestern Polytechnical University, 2015.)
2.3 Overview of Fracture Micromechanics

The traditional fracture mechanics method can be used in crack propagation process of the linear elastic material and is generally suitable for materials which yield in a small ratio. However, for microscale materials, the plastic zone at crack tip is much larger than the assumptions of linear elastic fracture mechanics, traditional method cannot describe the crack initiation and expansion precisely. D model proposed by Dugdale and Barenblatt (Dugdale, 1960 and Barrenblatt, 1962) was used to describe the elastoplastic fracture of ductile metals. Later the cohesive zone model evolved to solve the problem of infinity stress concentration at crack tip zone.

The constitutive model of cohesive zone is independent of the material and interface, and the cohesive zone model can be applied in the finite element method. Cohesive zone model is a simplified model that characterizes the interaction between atoms or molecules (Needleman, 1987). It assumes that the crack tip is a small cohesive zone with two crack surfaces, and descripts relationship of cohesive force \( \sigma \) and relative displacement \( \delta \) between the two crack surfaces. With appropriate selection of parameters, the mechanical properties can be characterized.
Chapter 3 Mechanical Response of MREs

without Interfacial Debonding

3.1 Introduction of MREs Simulation

Magneto-rheological elastomers are discontinuous particle-reinforced composites, the mechanical properties are extensively studied experimentally. The analytical methods based on Eshelby inclusion theory include: self-consistent model, Mori-Tanaka model, Double-Inclusion model. Those methods have been widely used in this field. But those theoretical analysis method can only get the macroscopic effective modulus, without getting the details of the local field, nor can it contain the impacts of distribution, orientation and shape to macroeconomic influencing factors in researches and analyses.

With the development of the computer, people can simulate the periodic distribution of the composite materials for numerical analysis through the establishment of a representative volume element (RVE) method.

In this paper, we establish several representative volume elements (RVE) for magneto-rheological elastomers under no magnetic field, do finite element calculations with the finite element software ABAQUS, get the stress-strain curves and macroscopic effective moduli of magneto-rheological elastomers under uniaxial tension test and simple shear test, and compared with the result data of two methods: Mori-Tanaka model and Double-Inclusion model,
all are in good agreement. Mori-Tanaka model was proposed by Mori and Tanaka (Mori and Tanaka, 1973), and Double-Inclusion model was proposed by Nemat-Nasser and Hori (Nemat-Nasser and Hori, 1993), both of which are based on Eshelby conception. For linear elastic two-phase composites, these two models usually give an excellent prediction of the effective modulus.

3.2 Material Parameters of Magneto-Rheological Elastomers

3.2.1 Material Parameters of Carbonyl Iron Particles

In this thesis, particle filler material is carbonyl iron powder, which is a typical high permeability, high magnetic saturation rates and low remanence material. Young's modulus $E$ is 210 GPa, Poisson's ratio is 0.33, The shear modulus $G$ is 78.94 GPa.

Figure 3-1 and 3-2 are the tensile stress-strain curve and shear stress-strain curve of carbonyl iron particles. In this paper, we assume the iron particles as linear elastomers, Young's modulus and shear modulus keep the same in simulations.
Figure 3-1. Tensile stress-strain curve of carbonyl iron particles.

Figure 3-2. Shear stress-strain curve of carbonyl iron particles.

We can assign material properties after finishing the model part in ABAQUS software. Linear elasticity is the simplest form of elasticity available in ABAQUS. ABAQUS can define isotropic, orthotropic, or anisotropic material behaviours and compute under small strains.
3.2.2 Material Parameters of Rubber Matrix

The matrix of magneto-rheological elastomer can be regarded as a hyperelasticity material, and its constitutive relation can be derived from the strain energy density function. In this paper, we choose Mooney-Rivlin model as follow:

\[ W = C_{10} \left( t_1 - 3 \right) + C_{01} \left( t_2 - 3 \right) + \frac{\kappa}{2} (J - 1)^2 \]  

(3-1)

In this model, Shear modulus can be calculated by:

\[ G = 2 \left( C_{10} + C_{01} \right) > 0 \]  

(3-2)

To simplify the calculation, assuming the ratio of \( C_{01} \) and \( C_{10} \) is 1:4, and assuming shear modulus \( G \) of matrix is 1.00 MPa, So \( C_{01} \) is 0.40 MPa and \( C_{10} \) is 0.10 MPa, which can be input in ABAQUS software as parameters of Mooney-Rivlin model in material assigning procedure. Figure 3-3 shows the tensile and compressive stress-strain curve and figure 3-4 shear stress-strain curve of matrix. We can see that the material shows nonlinear elasticity under 0.60 strain ratio, and all results of simulations are under that ratio because beyond that ratio software will show convergence problems.
ABAQUS/CAE allows users to evaluate hyperelasticity behavior of one material by automatically creating response curves using selected strain energy potentials, which includes Arruda-Boyce form, Marlow form, Mooney-Rivlin form,
Neo-Hookean form, Ogden form, Polynomial form, Yeoh form. ABAQUS also allows users to input the test data and compute the parameters in those forms.

3.3 Introduction of Representative Volume element (RVE)

In the theory of composite materials, the representative volume element (RVE) can be regarded as the smallest volume over which a measurement can be made that will yield a value representative of the whole (Hill, 1963). In other words, it is the smallest material volume element of the composite for which the usual spatially constant (overall modulus). Macroscopic constitutive representation is a sufficiently accurate model to represent mean constitutive response (Drugan and Willis, 1996). With the development of numerical simulation, the mechanical properties of composites by numerical simulation can be more and more close to the real representative volume elements.

Because we assumed this composite material is homogeneous in simulation, following the Homogenization Theory, we can obtain the macroscopic effective moduli of composite materials by the simulation of RVE and also get the results of stress distributions.

3.3.1 Representative Volume Element Model

To ensure a reasonable meshing size for analysis, the intersection of the particles should be avoided and the spacing between the two particles should be no less than a constant value. If the particle is separated by the edge of RVE, the part of the particle inside the RVE is retained while the part of the particle outside the
RVE is transplanted to the corresponding boundary edg. In this way, the particle distributions at the corresponding boundary of RVE can be continuous, this ensures the continuity of the particles at the boundaries and makes the repeatability of RVE possible.

Generally, periodic quadrilateral representative volume elements can be selected for two-dimensional periodic microstructures. In computer numerical simulation, the quadrilateral representative volume elements are relatively easy to apply periodic boundary conditions. Therefore, in this paper, we chose periodic quadrilateral representative volume elements for numerical analyses.

The size of RVE needs to be optimal. It will increase the steps of computation if RVE is too large, resulting in calculation error, or lose its representativeness if RVE is too small. The general method is to set a moving rectangular frame, and move the rectangular frame continuously, study the relationship between size of the frame and the micro-structure mechanical properties and find an acceptable rectangular frame size, finally make RVE model the smallest size after guaranteeing the variability of micro-structure mechanical properties.

### 3.3.2 Periodic Boundary Condition

Periodic boundary conditions are a set of boundary conditions which are often chosen for approximating a large (infinite) system by using small parts called representative elements, which are the most important thing in RVE model.
When the composite material is homogeneously deformed macroscopically, the simulations of microstructures exhibit periodic microscopic deformation.

Figure 3-5. Periodic boundary conditions.

As shown in figure 3-5, The deformation of the opposite side (Top and bottom, left side and right side) is coordinated:

for each opposite side:

(1) Displacement is continuous. The shape of the adjacent RVE is the same after deformation, cannot be embedded to each other nor have a gap.

(2) The stress is continuous. The stress on the corresponding boundary of RVE is consistent.

This also requires that the meshes of opposite edges are the same, each node has an exact opposite. These two requirements could be managed by generating the
mesh of the unit cell with Netgen (Schöberl, 1997). After applying a periodic boundary condition to RVE, the external load on the RVE boundary can be determined by the concentrated load applied to the vertex, which means the external load of RVE need only be applied to the vertex of the model (Sun, 2015). With this concept, we can just apply concentrated load to the vertex of the RVE model in simulation.

In ABAQUS, periodic boundary conditions can be implemented in the ABAQUS input file using the equation command. Periodic boundary conditions need periodic meshing on the boundary, which ensure that the corresponding boundaries have corresponding node to do FEM calculations.

### 3.3.3 Applying Periodic Boundary Condition in ABAQUS

PBC can be set through linear constraint in ABAQUS (Abaqus 6.12 user manual). Multi-points can be constrained by a general linear combination of nodal variables, the linear combination of nodal variables is equal to zero; that is

\[ A_1 u_1^P + A_2 u_2^Q + \ldots + A_k u_k^R = 0 \]  

(3-3)

where R is node, k is degree of freedom, AN are coefficients that define the relative motion of the nodes.

An example to define PBC in ABAQUS:

If we model a one by one square RVE which is subjected to the following PBCs:

\[ u_1^{left} - u_2^{right} = 0 \]  

(3-4)
\[ u_{1}^{\text{top}} - u_{2}^{\text{bottom}} = 0 \]  

(3-5)

We need to set node sets “Left” to represent nodes which are on the left boundary, node sets “Right” to represent nodes which are on the right boundary and set “Top” and “bottom”, then we get 4 sets of nodes as shown in figure 3-12 and 3-13. Because our models here were 2D models, the degree of freedom 1 means the x direction, and degree of freedom 2 means the y direction. Then add the “*equation” in ABAQUS input data in assemble procedure, which is shown as follow:

*Equation

3

** equation has 3 terms

Left, 1, 1

** left surface node set, dof =1, coeff. =1

Right, 1, -1

** left surface node set, dof =1, coeff. =1

*Equation

3

** equation has 3 terms

Top, 2, 1

** left surface node set, dof =1, coeff. =1

Bottom, 2, -1

** left surface node set, dof =1, coeff. =1
Either node sets or individual nodes can be specified as input. If node sets are used, corresponding set entries will be matched to each other. If sorted node sets are given as input, you must ensure that the nodes are numbered such that they will match up with each other correctly once sorted (Abaqus 6.14 Documentation). The meshing seed along the boundary of model need to be controlled. We can set the meshing seed size to ensure the same amounts of nodes on corresponding edges.

The nodes in an unsorted node set will be used in the order that they are given in defining the set (Abaqus 6.14 Documentation). In this situation, ABAQUS will match the nodes in the order that the node number from small to large, one by one, but after meshing the node numbers on the boundary nodes are disorganized, which need to be renumbered the node numbers. ABAQUS allows to renumber the node numbers in “PART” procedure. After that the periodic boundary condition can be set in simulation.

Weidong Wu demonstrated that using MATLAB to find paired nodes between two opposite surfaces (Wu and Owino, 2014). He presented Algorithm to generate paired nodes, and this method is widely used in finding the matching nodes on two opposite surfaces in 3D cube model. In this paper, I mainly consider 2D model, renumbering node numbers can solve the matching problem sufficiently.
3.4 Introduction of Model

Figure 3-6 is the simplest symmetrical RVE model to simulate, it is 50 micrometers by 50 micrometers size, the diameter of particle is 24 micrometers. Particle volume fraction is 0.20. To compute accurately, the mesh seed we set is very small. The mesh size on the boundary of model is 1 micrometer, which means there are 51 nodes on the edge of model (include vertex), this control is necessary because you must ensure that the nodes will match up between the opposite boundary when you apply the PBCs. The interface between particle and matrix is bonded, mesh size along the interface is 0.50 micrometer, which is smaller than mesh size on boundary. Because the stress concentration will appear at interface during simulation, which, in this paper, we mainly focus on. With smaller mesh size along the interface, the result of stress contour will be closer to the real stress distribution, and on the other hands, the problem of convergence difficulties may be solved if the mesh size is small enough. The mesh involves 4261 elements in the matrix (4145 linear quadrilateral elements of type CPS4R and 116 linear triangular elements of type CPS3) and 1927 elements (1877 linear quadrilateral elements of type CPS4R and 50 linear triangular elements of type CPS3) in the inclusions, leading to a total of 4373 nodes in the matrix and 1973 nodes in the inclusions.
Figure 3-6. One-particle isotropic RVE model.

Figure 3-7 is a periodic geometry of multi-particle isotropic and anisotropic magneto-rheological elastomer with a particle volume fraction of 0.20. Mesh sizes are 1 micrometer on the boundary and 0.50 micrometer along interface, we can see that the particles are separated by the edge of RVE, the part of the particles inside the RVE is retained while the part of the particles outside the RVE is transplanted to the corresponding boundary edge. For isotropic one, the mesh involves 5560 elements in the matrix (5403 linear quadrilateral elements of type CPS4R and 157 linear triangular elements of type CPS3) and 1269 elements (1237 linear quadrilateral elements of type CPS4R and 32 linear triangular elements of type CPS3) in the inclusions, leading to a total of 5655 nodes in the matrix and 1385 nodes in the inclusions. For anisotropic one, the mesh involves 5143 elements in the matrix (4948 linear quadrilateral elements of type CPS4R and 195 linear triangular elements of type CPS3) and 1691 elements (1654 linear quadrilateral elements of type CPS4R and 37 linear triangular elements of type CPS3) in the inclusions, leading to a total of 5230 nodes in the matrix and 1792 nodes in the inclusions.
Figure 3-7. Isotropic and anisotropic multi-particles RVE model.

These three models all satisfy periodic geometry request, after applying PBCs, deformation and stress contour should be continuous.

3.5 Results and Comparison

3.5.1 Mori-Tanaka Model and Double-Inclusion Model

Mori-Tanaka model was proposed by Mori and Tanaka in 1973 (Mori and Tanaka, 1973). This model is very successful in predicting the effective modulus of two-phase composites. In theory, it requires less than 25% inclusion volume ratio, but in fact, good predictions can still be obtained beyond this range. J. W. Ju and T. M. Chen gave the non-interacting approximation of effective moduli for multiphase composites in 1994 (Ju and Chen, 1994). They demonstrated that if particles are spherical and both matrix and particles are isotropic elastic. The effective bulk and shear moduli are:
\[
\bar{\kappa} = \kappa_0 + \frac{3(1-\nu_0)(\kappa_i - \kappa_0)\phi_i}{3(1-\nu_0)\kappa_0 + (1-\phi_i)(1+\nu_0)(\kappa_i - \kappa_0)} \\
\bar{\mu} = \mu_0 + \frac{15(1-\nu_0)(\mu_i - \mu_0)\phi_i}{15(1-\nu_0)\mu_0 + 2(1-\phi_i)(4-5\nu_0)(\mu_i - \mu_0)}
\]

(3-6)

Where \( \kappa_0, \kappa_i \) are bulk moduli of matrix and particles, \( \mu_0, \mu_i \) are shear moduli of matrix and particles, \( \nu_0 \) is Poisson’s ratio of matrix, \( \phi_i \) is particles fraction.

Double-Inclusion model was proposed by Nemat-Nasser and Horiin in 1993 (Nemat-Nasser and Horiin, 1993). The composite RVE is composed of a reference material whose stiffness is \( \bar{C} \), consisting of an inclusion phase \( I_i \) surround with phase \( I_0 \) with elasticity tensors \( C_i \) and \( C_0 \). In fact, the D-I model is a series of MFH models that can be derived as different models by choosing different matrix stiffness:

1. \( \bar{C} = \bar{C} \): Self-consistent model

2. \( \bar{C} = C_0(C_i) \): Mori-Tanaka model (two-phase composites)

For linearly elastic two-phase composites, the D-I model usually gives an excellent prediction of the effective modulus. In Sun’s paper (Sun, 2015), He gave the effective bulk and shear moduli as:

\[
\bar{\kappa} = \kappa_0 + \frac{\kappa_i - \kappa_0}{1 + \frac{9(1-\phi_i)(\kappa_i - \kappa_0)}{3(4\mu_0 + 3\kappa_0)}\phi_i} \\
\bar{\mu} = \mu_0 + \frac{\mu_i - \mu_0}{1 + \frac{12(1-\phi_i)(2\mu_0 + \kappa_0)(\mu_i - \mu_0)}{10\mu_0(4\mu_0 + 3\kappa_0)}\phi_i}
\]

(3-7)

where \( \kappa_0, \kappa_i \) are bulk moduli of matrix and particles, \( \mu_0, \mu_i \) are shear moduli of matrix and particles, \( \nu_0 \) is Poisson’s ratio of matrix, \( \phi_i \) is particles fraction.
The result of Mori-Tanaka model and Double-Inclusion model are very similar in this case. We can compute the initial Young's modulus of MRE is 4.6 MPa, the initial shear modulus is 1.6 MPa. Both moduli will change during material deformation because non-linear elasticity of matrix, bulk modulus and Poisson's ratio will not keep the same.

3.5.2 Uniaxial Tensile Simulation of One-particle RVE Model

Figure 3-8 shows uniaxial tensile stress contour of one-particle RVE model. From the diagram, we can know that PBC equations are working, the deformation and stress contour of model is continuous. Because this model is perfect symmetrical on both X and Y direction, the edges are not distorted. Figure 3-9 is the result of uniaxial tensile test diagram which Sun got in his paper (Sun, 2015), compare these two results, the deformation laws of one-particle RVE model are the same.

Figure 3-8. Uniaxial tensile stress contour in simulation.
**Figure 3-9.** Uniaxial tensile test diagram of 3D one-particle RVE model.

(Figure 3-9 is cited from Sun S. I., Research about numerical modeling and constitutive modeling of magnetorheological elastomers under finite deformation. Dissertation of Northwestern Polytechnical University, 2015.)

Figure 3-10 shows the stress contours on both X(S11) and Y(S22) direction. In S11 stress contour, we can find that the S11 compressive stress is concentrated on the left and right sides of particle (blue area) in matrix, that is because the whole model is stretched along the Y direction, but the particle inside is relatively undeformed (Young's modulus is much bigger than matrix's), these zones are being compressed during the simulation. S11 tensile stress is concentrated on the two poles of particle, that is due to periodic boundary conditions, if we suppress that setting, the model will be deformed as a spindle shape. To satisfy the request of continuous displacement and stress distribution on the boundary conditions, the pole area is being stretched on the X direction during simulation, and the tensile stress is concentrated on two poles zones.
The S22 stress is nearly ten times higher than S11 in most zones, there is no element being compressed on the Y direction, and the S22 tensile stress is concentrated on the two poles, because the interface between particle and matrix is assumed to be bonded, and the particle inside is relatively undeformed, the deformation of these zones relatively large, we can infer that interface will debond here if we assign the interface properties. On the other hand, the tensile stress is relatively small on the equator of particles, with the result if S11 stress contour, we can infer that interfacial debonding is most unlikely to happen at these zones.

![S11 stress contour](image1) ![S22 stress contour](image2)

**Figure 3-10.** Uniaxial tensile stress contours of one-particle RVE model.

Figure 3-11 shows the uniaxial tensile stress-strain curve. The stiffness of model increase compared with the rubber material, because iron particles are assumed rigid and interface is bonded. In this assumption, the addition of iron particles does enhance the stiffness of matrix by 40%. Compared to the curves of
Mori-Tanaka model and Double-Inclusion model results cited from Sun’s paper (Sun, 2015), as shown in figure 3-11, the simulation result is robust, all curves are almost in good agreement (value are not exactly matched, the reason may be the different dimension of model). Figure 3-12 is the Young’s modulus-strain curve of model under the finite deformation obtained from Figure 3-11. From the figure, the initial Young’s modulus is 3.10 MPa. Young’s modulus increased nearly 40% related to rubber when the model strain is 0.50 in simulation. It can also be seen from Figure 3-17 that in the case of finite deformation, the macroscope Young’s modulus reduces with the increase of strain. Combined with the Young’s modulus of the matrix and the particles under large deformation, the particle and the matrix present a complex relationship, not a simple series or parallel relationship.

**Figure 3-11.** Uniaxial tensile stress and strain curve of one-particles RVE model with the comparison of Mori-Tanaka model and Double-Inclusion model.
Figure 3-12. Young's modulus-strain curve of one-particles RVE model.

3.5.3 Shear Simulation of One-particle RVE Model

Figure 3-13 is a shear stress contour of shear test deformation. It can be observed that after applying the periodic boundary conditions, the deformations of the opposite boundary surfaces are coordinated and match the requirements of periodic boundary conditions. Figure 3-14 is the result of shear test diagram which Sun got in his paper (Sun, 2015), compare these two results, the deformation rules of one-particle RVE model are the same. As shown in Figure 3-13, we can observe that no obvious stress concentration in shear stress contour, the shear stress in the model vertex zones is relatively large.
Figure 3-13. Shear stress contour of one-particle RVE model.

Figure 3-14. Shear test diagram of 3D one-particle RVE model.

(Figure 3-14 is cited from Sun S. l., Research about numerical modeling and constitutive modeling of magnetorheological elastomers under finite deformation. Dissertation of Northwestern Polytechnical University, 2015.)
Figure 3-15 shows the shear stress-strain curve for both one-particle RVE model and rubber matrix. Because the particle is assumed as a rigid body in simulation, and interface is assumed as bonded, compare to isotropic rubber matrix, the addition of iron particles does enhance the shear strength of the matrix material by 40%. Compared to the curves of Mori-Tanaka model and Double-Inclusion model, as shown in figure 3-15, the simulation result underestimates the response compared to those two methods. Shear modulus of all models do not change much during the simulation, although Mori-Tanaka model and Double-Inclusion model show a slightly increase of shear moduli. Simulations result of shear modulus is shown in figure 3-16. For one-particle RVE model, the initial shear modulus is 1.46 MPa, which increase by 46% compared to rubber matrix. But with the increase of shear strain, Shear modulus of the model decreases slightly. This may be due to the interaction of the matrix material and particles.
**Figure 3-15.** Shear stress-strain curve of one-particle RVE model with the comparison of Mori-Tanaka model and Double-Inclusion model.

**Figure 3-16.** Shear modulus response of one-particles RVE model.
3.5.4 Uniaxial Tensile Simulation of Isotropic Multi-particle RVE Model

Figure 3-17 is a uniaxial tensile test contour of isotropic multi-particles RVE model. From the diagram, we can know that PBC equations are working, the deformation and stress contour of model are continuous. Figure 3-18 is the result of uniaxial test diagram which Sun got in his paper (Sun, 2015), compare these two results, the deformation rules of multi-particles RVE model are the same.

Figure 3-17 also shows the stress contour on both X(S11) and Y(S22) direction of multi-particles RVE model. In S11 stress contour, the S11 compressive stress is higher between the particles (light blue area), that is because the whole model is stretched along the Y direction, but the particle inside is relatively undeformed, these zones are being compressed during the simulation. S11 tensile stress is concentrated on the two poles of particle, Those S11 stress distributions are very similar to one-particle RVE model, stress distributions are continuous at opposite boundaries.

The magnitude of S22 stress is much bigger than S11 in most regions, there is no element being compressed on the Y direction, and the S22 tensile stress distributions are very similar to one-particle RVE model, the difference in stress distributions are due to the asymmetry locations of particles, tensile stress in the region between two particles increases if the distance of that two particles decreases , but we can still infer that interface will debond at polar of particles if
we assign the interface properties, and more likely to debond if two particles are closer. On the other hand, the tensile stress is relatively small on the equator of particles, with the result if S11 stress contour, we can infer that interfacial debonding is most unlikely to happen at these zones.

![S11 and S22 stress contours](image)

**Figure 3-17.** Uniaxial stress contours of isotropic multi-particles RVE model.

![Uniaxial test diagram](image)

**Figure 3-18.** Uniaxial test diagram of 3D isotropic multi-particle RVE model.
(Figure 3-18 is cited from Sun S. l., Research about numerical modeling and constitutive modeling of magnetorheological elastomers under finite deformation. Dissertation of Northwestern Polytechnical University, 2015.)

Figure 3-19 describes the uniaxial tensile nominal stress-nominal strain curve. As shown in figure 3-19, the stiffness of model increases comparing to the rubber material. In this simulation, the addition of iron particles increases the stiffness of matrix by 48%. Compared to the curves of Mori-Tanaka model and Double-Inclusion model, as shown in figure 3-19, the simulation is robust and all curves are almost in good agreement. Figure 3-20 is the Young's modulus and strain curve under the finite deformation obtained from Figure 3-19. From the figure, the initial Young's modulus is 3.05 MPa. Young's modulus increases nearly 40% related to rubber when the model strain is 0.50 in simulation. It can also be seen from Figure 3-25 that in the case of finite deformation, the macroscopic Young's modulus reduces with the increase of strain. This relation is very similar to the result of one-particle RVE model.
Figure 3-19. Uniaxial tensile stress and strain curve of isotropic multi-particles RVE model with the comparison of Mori-Tanaka model and Double-Inclusion model.

![Graph of Young's modulus-strain curve](image)

Figure 3-20. The Young’s modulus-strain curve of isotropic multi-particles RVE Model.

3.5.5 Shear Simulation of Isotropic Multi-Particle RVE Model

Figure 3-21 shows a shear stress diagram of isotropic multi-particles RVE model shear test. It can be observed that after applying the periodic boundary conditions, the deformations of the opposite boundary surfaces are coordinated and match the requirements of periodic boundary conditions. Figure 3-22 is the result of shear test diagram which Sun got in his paper (Sun, 2015), compare
these two diagrams, the deformation rules are the same. As shown in Figure 3-22, shear stress is concentrated between adjacent particles, and shear stress in particles is relatively large.

**Figure 3-21.** Shear stress contour of isotropic multi-particle RVE model.

**Figure 3-22.** Shear test diagram of 3D isotropic multi-particle RVE model.

(Figure 3-22 is cited from Sun S. l., Research about numerical modeling and constitutive modeling of magnetorheological elastomers under finite deformation. Dissertation of Northwestern Polytechnical University, 2015.)
Figure 3-23 shows the shear stress-strain curve for both isotropic multi-particle RVE model and rubber matrix. The addition of iron particles increases the shear strength of the rubber matrix by 48%. Compared to the curves of Mori-Tanaka model and Double-Inclusion model, as shown in figure 3-23, all curves are almost in good agreement. For multi-particle RVE model, the initial shear modulus is 1.47 MPa, which increase by 47% compared to rubber matrix. Shear modulus of multi-particle RVE model increases slightly when the strain is lower than 0.50, which is shown in figure 3-24.

**Figure 3-23.** Shear stress-strain curve of isotropic multi-particle RVE model with the comparison of Mori-Tanaka model and Double-Inclusion model.
3.5.6 Uniaxial Tensile Simulation of Anisotropic Multi-particle RVE Model

Figure 3-25 shows a uniaxial tensile stress contour of anisotropic multi-particles RVE model. From the diagram, we can know that PBC equations are working, the deformation and stress contour of model is continuous. Figure 3-26 is the result of uniaxial test diagram which Sun got in his paper (Sun, 2015), compare these two diagrams, the deformation rules of multi-particle RVE model are the same.

In S11 stress contour, we can find that the S11 compressive stress is concentrated on the four zones of particle (blue area) in matrix, most of matrix zones are being compressed during the simulation. S11 tensile stress is concentrated on the two poles of particle, which is very similar to one-particle
model. Distributions of stress both in particles and matrix are almost the same from each particle region to another, which show the effect of periodic boundary conditions.

There is no element being compressed on the Y direction, and the S22 tensile stress is concentrated on the two poles, the deformation of these zones relatively large. On the other hand, the tensile stress is relatively small on the equator of particles. It shows the same response laws as one-particle model simulation.

**Figure 3-25.** Uniaxial stress contours of anisotropic multi-particles RVE model.
**Figure 3-26.** Uniaxial test diagram of 3D anisotropic multi-particles RVE model. (Figure 3-26 is cited from Sun S. l., Research about numerical modeling and constitutive modeling of magnetorheological elastomers under finite deformation. Dissertation of Northwestern Polytechnical University, 2015.)

Figure 3-27 shows uniaxial tensile stress-strain curves of both isotropic and anisotropic multi-particles RVE models and rubber material. As shown in figure 3-27, the chain-lie structures of iron particles increase the stiffness of rubber matrix by 65%. Compared to the stress-strain curve of isotropic multi-particles RVE model, with the same particle volume fraction, the chain-like structures increase the stiffness by 30% along the direction of chain formation. Figure 3-28 is the Young's modulus and strain under the finite deformation obtained from Figure 3-27. From the figure, the initial Young's modulus is 4.80 MPa. Young's modulus increased nearly 100% compared to rubber and 35% compared to isotropic multi-particles RVE Model when the model strain is 0.50. The macro Young's modulus decreases with the increase of strain. The particle and the matrix present a complex relationship, not a simple series or parallel relationship.
Figure 3-27. Uniaxial tensile stress-strain curves of isotropic and anisotropic multi-particles RVE models.

Figure 3-28. The Young's modulus-strain curve of anisotropic multi-particles RVE Model.

3.5.7 Shear Simulation of Anisotropic Multi-Particle RVE Model
Figure 3-29 shows shear stress contour of anisotropic multi-particle RVE model. The deformations of the opposite boundary surfaces are coordinated and match the requirements of periodic boundary conditions. Figure 3-30 is the result of shear test diagram which Sun got in his paper (Sun, 2015), compare these two diagrams, the deformation rules of multi-particle RVE model are the same. The difference compared to isotropic RVE model is that each boundary deformation is straight, not wavy, which is very interesting. Different model structures show different response to the same shear load condition.

Figure 3-29. Shear stress contour of anisotropic multi-particle RVE model.
**Figure 3-30.** Shear test diagram of 3D anisotropic multi-particle RVE model.

(Figure 3-30 is cited from Sun S. l., Research about numerical modeling and constitutive modeling of magnetorheological elastomers under finite deformation. Dissertation of Northwestern Polytechnical University, 2015.)

Figure 3-31 shows the shear stress-strain curves of two types of multi-particle RVE models and rubber matrix. For anisotropic multi-particle RVE model, the initial shear modulus is 1.41 MPa, which increase by 41% compared to rubber matrix. Compared to isotropic multi-particle RVE model, the chain-like structures of iron particles in MRE reduce the shear stiffness about 10%. Shear modulus of anisotropic multi-particle RVE model do not change much in simulation, which is shown in figure 3-32.

![Shear stress-strain curves of anisotropic multi-particle RVE models](image)

**Figure 3-31.** Shear stress-strain curves of anisotropic multi-particle RVE models.
3.6 Conclusions

1) The RVE model numerical simulation methods and periodic boundary conditions are briefly introduced in this chapter.

2) Comparing the macroscopic mechanical properties of the MRE without magnetic field by numerical simulation method and the Mori-Tanaka model and the Double-Inclusion model, the three are in good agreement.

3) Particle addition has an influence on the mechanical properties of rubber matrix. The comparison between the finite element method and the theoretical solution of the magneto-rheological elastomers RVE model show that the initial
Young's modulus and the initial shear modulus of isotropic MRE increase over rubber material.

4) The chain-like structure of iron particles increases the stiffness of MRE along the chain-like structure direction but reduce the shear strength which perpendicular to the chain-like structure.

5) The stress-strain curves of the isotropic MRE model show that the mechanical properties of the materials are non-linear, not a simple series or parallel relationship.
Chapter 4 Interfacial Debonding Simulation of Magneto-Rheological Elastomers

4.1 Constitutive Model of Cohesive Zone Interface

People use the Cohesive Zone Model (CZM) to explain the fracture evolution. CZM method introduces a degradation mechanism (material softening or weakening) at the crack zone to form a cohesive damage zone. Assuming a relationship between interface stresses and crack interface displacements (separations), which is called constitutive model of cohesive zone.

CZM prediction of crack evolution requiring two parameters (strength and ductility or energy): (1) the cohesive strength obtained at the crack tip zone; (2) the fracture energy resulting from the release of new crack. These two parameters, combined with the constitutive model of cohesive zone, describe the crack evolution in the cohesion zone.

Figure 4-1 shows the constitutive models of cohesive zone widely used in study. For different materials, different constitutive models can be chosen to establish the relationship between cohesion \( \sigma \) and relative displacement \( \delta \) of two crack surfaces (Bazant, Z.P, 1984). To support the experiment better, the critical value in the analysis can be simulated by the cohesive zone model.
4.2 Continuous Cohesive Zone Model (CCZM)

Many researchers consider CZM as continuous series of elements, as shown in Figure 4-2. In this assumption, CZM are a series of continuous elements, and constitutive model of cohesive zone can be compute as the stress-strain curve at the integration point. Therefore, it is also called continuous cohesion zone model (CCZM). There are two main approaches of applying CCZM: smeared cohesive elements and interface cohesive elements (Xie and Waas, 2006).

smeared cohesive elements method likes the traditional continuous element method, can be shown as stress-strain relationship. These methods have been combined with the user-defined material subroutine which is called UMAT in ABAQUS software. smeared cohesive elements method is not suitable for predicting interface crack propagation. First, inserting a very thin layer of elements as smeared cohesive elements between two crack surfaces with zero initial separation will probably result in a large aspect ratio of the element, so that finite element analysis maybe difficult to proceed, which also lead to difficulties in mesh generation. On the other hand, you may experience serious
difficulties in simulation. References describe different smeared cohesive elements and their finite element theory (Reedy, Mello and Guess, 1997).

To overcome the limitations of smeared cohesive elements method, a cohesive zone model about zero thickness has been proposed. This cohesive zone model is expressed in terms of stress-displacement relationship, and its procedure is simpler than that of smeared cohesive elements method. It includes Newton-Cotes integration method instead of the Gauss integration method to eliminate the oscillation generation when the traction gradient is large on the element. This element method was implemented in ABAQUS and valuable results for a broader range in the field were obtained (Camanho, Davila and de Moura, 2003). In 2003, De Borst introduced the overall development of CZM (De Borst, 2003).

4.3 Applying CZM in ABAQUS/CAE

Cohesive behavior is useful in modeling adhesives, bonded interfaces, and gaskets. The cohesive behavior setting in ABAQUS can be 2 types:

1. Element-based cohesive behavior. Model another series of cohesive elements along the interface. Cohesive elements in ABAQUS allow very detailed modeling of adhesive connections, and primarily address Adhesive joints with finite thickness layer and delamination of zero thickness layer.
2. Surface-based cohesive behavior. Model contact surface pairs in ABAQUS/Standard and general contact in ABAQUS/Explicit (which can compute viscoelasticity problems), which is a simplified and easy way to model cohesive connections, using the traction-separation interface behavior. It offers capabilities that are very similar to cohesive elements modeled with the traction-separation constitutive response.

Cohesive surface does not require element definitions, it is defined as a surface interaction property, so damage for cohesive surfaces is an interaction property, not a material property. It is primarily intended for situations in which interface thickness is negligibly small. For cohesive surfaces, refining the slave surface as compared to the master surface will likely lead to improved constraint satisfaction and more accurate results.

The laws that govern surface-based cohesive behavior in ABAQUS are: linear elastic traction-separation curve, damage initiation criteria and damage evolution laws. We can input those parameters in contact setting function.

When the situation of interface satisfies a given initial damage criterion, cracks will subsequently expand in accordance of a given evolution law. We can set damage evolution with two parts. As can be observed in Figure 4-2, the first part is the separation displacement $\delta_n^f$ at initial damage and the model's effective separation displacement $\delta_n^f$ at complete failure, or the energy $G$ consumed during the failure, which is the area under the constitutive model curve.
Figure 4-2. Evolution law of linear damage.

The second part is the scalar damage variable $d$ of damage evolution from initial damage to eventual failure. The initial value of $d$ is 0 in no damage situation, and the value of $d$ is 1 when the crack is fully expanded. The value of $d$ can be calculated by linear or exponential type of softening rules, as shown in Figure 4-3.

Figure 4-3. Definition of scalar damage variable $d$.

Damage initiation refers to the beginning of degradation. The process of degradation begins when the stresses or strains satisfy certain damage initiation
criteria. Four damage initiation criteria are available and are discussed in ABAQUS (Figure 4-4). Each damage initiation criterion also has an output variable associated with it to indicate whether the criterion is met (ABAQUS 6.14 documentation). In this paper, I choose quadratic nominal strain criterion for damage evolution, Damage will initiate when the quadratic interaction function reaches the value of one.

\[
\text{Maximum nominal stress criterion:} \quad \text{MAX} \left( \frac{\sigma_n}{N_{\text{max}}} \frac{\sigma_s}{S_{\text{max}}} \frac{\sigma_t}{T_{\text{max}}} \right) = 1
\]

\[
\text{Maximum nominal strain criterion:} \quad \text{MAX} \left( \frac{\varepsilon_n}{\varepsilon_{n,\text{max}}} \frac{\varepsilon_s}{\varepsilon_{s,\text{max}}} \frac{\varepsilon_t}{\varepsilon_{t,\text{max}}} \right) = 1
\]

\[
\text{Quadratic nominal stress criterion:} \quad \left( \frac{\sigma_n}{N_{\text{max}}} \right)^2 + \left( \frac{\sigma_s}{S_{\text{max}}} \right)^2 + \left( \frac{\sigma_t}{T_{\text{max}}} \right)^2 = 1
\]

\[
\text{Quadratic nominal strain criterion:} \quad \left( \frac{\varepsilon_n}{\varepsilon_{n,\text{max}}} \right)^2 + \left( \frac{\varepsilon_s}{\varepsilon_{s,\text{max}}} \right)^2 + \left( \frac{\varepsilon_t}{\varepsilon_{t,\text{max}}} \right)^2 = 1
\]

**Figure 4-4.** Summary of damage initiation criteria.

(Figure 4-2,4-3,4-4 are cited from DASSAULT SYSTEMES Surface-based Cohesive Behavior Lecture presentation)

4.4 **Introduction of model**

Figure 4-5 is the one-particle symmetrical RVE model, it is 50 micrometers by 50 micrometers size, the diameter of particle is 28 micrometers. Particle volume fraction is 0.30 (It is one case in simulations, Chapter 5 shows parametric study for all cases). Meshing step is the same as Chapter 3 did and apply periodic boundary conditions.
For constitutive model of cohesive zone at interface, we used peel test data as assumption base of the constitutive model of cohesive zone. The peel separation work per unit area $\phi$ can be related to the measured steady peel force $F$ and to the width $b$ of the inextensible peeled strip as $F/b$ (Rivlin, 1944; Lindley, 1971). We follow the assumption of the peel separation work per unit area $\phi$ equal the CZM adhesion energy. We set the $\phi$ as $10\text{J/m}^2$ in simulation, so the peak value of the contact stress $t$ is 2 MPa by computation.

Figure 4-6 shows the constitutive model of cohesive zone in traction-separation behavior. normal and tangential stiffness components $K_{nn}$, $K_{tt}$, $K_{ss}$ are all 1.00MPa/µm, peak values of the contact separation are all 2 µm, separations at failure are all 10 µm. The peak value of the contact stress $t$ is 2 MPa, which is showed directly in Figure 4-6.

**Figure 4-5.** One-particle RVE model.
Figure 4-6. Constitutive model of cohesive zone.

Figure 4-7 is a periodic geometry of an isotropic magneto-rheological elastomer with a particle volume fraction of 0.30, but it is not centrosymmetric. Mesh sizes on the boundary are 1 micrometer and 0.50 micrometer along interface. In this model, we apply CZM to four pairs of interface contact and periodic boundary conditions, keep the parameters of CZM the same as one-particle RVE model.

Figure 4-7. Isotropic and anisotropic muti-particles RVE model.
4.5 Results and comparison

4.5.1 Uniaxial Tensile Simulation of One-particle RVE Model

Figure 4-8 is a uniaxial test diagram of one-particle RVE model. From the diagram, we can know that PBC equations are working, the deformation and stress contour of model are continuous. Because this model is perfect symmetrical on both X and Y direction, the edges are not distorted.
Figure 4-8. Uniaxial tensile contours of one-particle RVE model.

Figure 4-8 shows the stress contour on both X(S11) and Y(S22) direction. In S11 stress contour, compressive stress is continuous at the left and right zones of particle and matrix through the interface, the matrix here is compressed on X direction (blue zone), which is similar with the bonded sample in Chapter 3. But what different is that the compressive stress concentration on two poles of particle, and the concentration starts from the region where interface damage evolves (surface separation bigger than 2 µm). In my opinion, particle is being compressed on X direction by the force from matrix, the poles of particle loss the tensile stress which comes from the contact interface after initial damage, remain a concentration zone of compressive stress on X direction. S11 Tensile
stress of matrix is concentrated on two poles which is due to periodic boundary conditions, which is similar with bonded sample in chapter 3.

In S22 stress contour, tensile stress is continuous from matrix to particle and back to matrix on Y direction, which means CZM at interface do work (contact stress is not continuous if no adhesion between surfaces of particle and matrix). What else, tensile stress is concentrated at the region where surface separation is reaching to 2 µm, in constitutive model of cohesive zone, the contact stress reaches the peak. That is the reason why contact stress concentrate on here both in particle and matrix. This stress contour directly shows the influences of CZM contact in stress distribution of the model.

Figure 4-9 shows the contact stress response at interface during simulation. Normal stiffness components K is 1.00Mpa, until 0.50 of model strain, contact stress continues to increase but still not reach the peak value. In Chapter 5 we set relatively weak interface stiffness. In that case, interface contact stress reaches the peak and start to drop, and the Young’s modulus of the whole model drop because of the initial damage of interface. The contact stress response is perfectly match the previous settings about constitutive model of cohesive zone.
Figure 4-9. The contact stress response at interface.

The uniaxial tensile nominal stress-nominal strain curve up to 50% strain is shown in Figure 4-10, we can observe that when the model strain is under 0.50, no damage evolves at interface yet, addition of particles and adhesion of interface enhance the stiffness of the whole model, Young's modulus (the slope of curve) increases by 33% compared to isotropic rubber matrix. In this simulation, the initial damage does not generate under the strain ratio of 0.50. We can infer that once the damage generates and the interface keeps losing adhesion, influence of particle enforcement to the material reduces and the stiffness of elastomer will be less than the isotropic matrix eventually. In this assumption, the relatively weak interface counteracts the effect of the particles enhancement on the material.
Figure 4-10. The uniaxial tensile nominal stress-nominal strain curve of one-particle RVE model.

But this curve cannot represent the real tensile response of MRE, it assumes that interfacial debonding on all particles happens at same time because the concept of RVE model. In experiment, this material is not isotropic, interfacial debonding happens gradually, the curve will be smoother. Besides, this is idealized because in experimental result, interfacial debonding occurs at very low strain ratio.

4.5.2 Shear Simulation of One-particle RVE Model

Figure 4-11 shows shear stress contour of one-particle RVE model. from the diagram, we can know that PBC equations are working, the deformation and stress contour of model are continuous. The distribution of stress is symmetrical diagonally. As we can see, the separation of two material is very small in this simulation, which is due to the relatively hard interface stiffness. What else,
matrix shear stress is concentrated at short diagonal line where attached to particle, where the material is mainly compressed.

Figure 4-11. Shear stress contour of one-particle RVE model.

The shear stress-strain curve of model is shown in Figure 4-12, it is detected by extracting S12 stress (shear stress) of the nodes on top boundary. From chapter 3 we can conclude that the addition of iron particles does enhance the shear strength of the material (if interface is bonded). But compare to one-particle RVE model with bonded interface, the model with CZM interface weaken the shear strength of the material. As shown in Figure 4-12, shear modulus (the slope of curve) does not change rapidly but rise slightly during the simulation.

This curve is not able to represent the real shear response of MRE, it assumes that interfacial debonding on all particles happens at same time. In experiments,
interfacial debonding happens gradually, the response will be affected by many other conditions.

![Shear stress-strain curve at top boundary of model](image)

**Figure 4-12.** The shear stress-strain curve at top boundary of model.

### 4.5.3 Uniaxial Tensile Simulation of Isotropic Multi-particles RVE Model

Figure 4-13 shows uniaxial tensile stress contour of isotropic multi-particles RVE model. From the diagram, we can know that PBC equations are working, boundary deformation and stress distribution of model are continuous.
Figure 4-13. Uniaxial stress contours of isotropic multi-particles RVE model.
In S11 stress contour, the S11 compressive stress is relatively large on equators of particles, that is because the whole model is stretched along the Y direction, but the particle inside is relatively undeformed, these zones are being compressed during the simulation, which is very similar to result of one-particle model. Stress distribution and interface debonding shape, affected by particles location, are no longer particle symmetrical. The stress concentration in matrix starts from the region where interface damage evolves. Interface failure is more likely to generate where matrix deformation becomes larger.

There is no element being compressed on the Y direction, and the S22 tensile stress is concentrated at where the matrix deformation is relatively large, especially where the separation between particles and matrix is reaching initial damage separation. Combine with the result of S11 stress contour, we can infer that these zones are more likely to crack if simulation continue. On the other hand, the tensile stress is relatively small on the equator of particles, so does the stress in matrix those used to adhere to.

The uniaxial tensile nominal stress-nominal strain curve is shown in Figure 4-14, we can observe no damage evolves at interface yet, Young’s modulus (the slope of curve) increases by 32% compared to isotropic rubber matrix. We can infer that after most of interface separation reaching initial damage criteria, interface contact drop, so the enhancement of particle addition drop, leave the rest of matrix to resist the traction. Eventually, stiffness of MRE model will be less than rubber.
**Figure 4-14.** The uniaxial tensile stress-strain curve of isotropic multi-particles RVE model.

Compared to the result of mechanical response of one-particle RVE model, as shown in Figure 4-15, One-particle RVE model and isotropic multi-particle RVE model show almost the same mechanical response of tensile test simulation. Two curves almost overlap because there is still no initial damage yet. If we assume the interface damage occurs, the response of one-particle model would experience a modulus drop since the interfacial debonding, but the response curve of multi-particle would be smoother because interfacial debonding of two particles do not occurs at same time, we can infer that the result of simulation will be more authentic if RVE model consist of more particles.
Figure 4-15. Stiffness comparison of one-particle and isotropic multi-particle RVE model.

4.5.4 Shear Simulation of Isotropic Multi-particles RVE Model

Figure 4-16 shows shear stress contour of isotropic multi-particle RVE model. From the diagram, we can know that PBC equations are working, the deformation and stress contour of model are continuous. As we can see, shear stress is concentrated at model short diagonal line direction between particles, which is very similar to the result of one-particle RVE model. If two particles are close, the shear stress is more likely to concentrate between these two particles.
The shear stress-strain curve of model is shown in Figure 4-17, it is detected by extracting S12 stress (shear stress) of the nodes on top boundary. Compared to one-particle RVE model with same interface properties, the multi-particle model stiffness does not change much. As shown in Figure 4-17 and 4-18, we cannot see a drop of shear modulus of model in simulation because most of interface does not reach initial damage criteria.
Figure 4-17. The shear stress-strain curve of isotropic multi-particles RVE model.

Compare two shear response curves of one-particle and isotropic multi-particles RVE model in Figure 4-18, we can conclude that these two models show the same shear response law. An exception is that the shear stiffness of isotropic multi-particle, which shows about 15% larger than shear stiffness of one-particle model. There is some uncertainty and the reasons are not yet clear.

It must be mentioned that changing the size of particles will not affect the result of idealized RVE numerical simulation. Yanceng Fan reported that shear storage modulus of MRE becomes larger with smaller size iron particles (Fan and Gong, 2011). In his research he concludes that the increases of MRE shear storage modulus with the decrease particles size is mainly because of the increase of agglomeration ratio. when the iron particles agglomerate in the matrix, some rubber segments will be entrapped among the particles, the rubber which is
located inside the agglomerate is defined as restrained rubber. And Li et al reported that the restrained rubber has made the zero-field shear storage modulus increased in MREs. The more the agglomeration is, the more restrained rubber there will be, and the more shear resistance the model will response. In Yanceng Fan’s result, when particle volume fraction is low in material, the iron particles agglomerates are not obvious, shear storage modulus of composite do not change much with different size of iron particles. In this way, Yanceng Fan’s experimental data and numerical simulation of FEM are in a good agreement.

Figure 4-18. The shear response curves of one-particle and isotropic multi-particles RVE model.

4.5.5 Uniaxial Simulation of Anisotropic Multi-particle RVE Model

Figure 4-19 shows uniaxial tensile contour of anisotropic multi-particle RVE Model. The deformation and stress distribution of model are continuous.
Because this model is perfect symmetrical on Y direction, the deformations of edges are straight.

From the stress contour on X (S11) and Y (S22) direction, the stress distributions are similar with one-particle RVE model. Stress concentrates on where the interface reaches to initial damage separation. The interface stiffness is relatively large than matrix, deformation of matrix is relatively large at the central axis of the model, S22 stress is relatively large in these zones. This stress contour directly shows the influences of CZM contact in stress distribution of the anisotropic multi-particle RVE model.
**Figure 4-19.** Uniaxial tensile contours of anisotropic multi-particle RVE Model.

The uniaxial tensile nominal stress-nominal strain curve is shown in Figure 4-20, we can observe that the tensile response of anisotropic multi-particle RVE Model increases by 20% compared to isotropic multi-particle RVE model, when the model strain is under 0.50, both the curves are smooth because the absence of initial damage. This is idealized because in experimental result, interfacial debonding occurs at very low strain ratio.

Compared to the result of mechanical response of isotropic multi-particle RVE model in Figure 4-20, we can conclude chain-like structures of iron particles increase the stiffness of total model under tensile loading.
**Figure 4-20.** The uniaxial tensile nominal stress-nominal strain curves of isotropic and anisotropic multi-particle RVE model.

### 4.5.6 Shear Simulation of Anisotropic Multi-Particle RVE Model

Figure 4-21 shows shear stress contour of anisotropic multi-particle RVE model. Shear stress is relatively large at model short diagonal line direction of the model, which is very similar to the result of one-particle RVE model. The distribution of stress is centrosymmetric, which is similar with the response of one-particle RVE model. An exception to this is stress concentration at several elements in particles. It seems there are some errors in the periodic boundary conditions computations, the reasons are not clear yet. There is still some uncertainty about how to avoid it.
Figure 4-21. Shear stress contour of anisotropic multi-particle RVE model.

Compare two shear response curves of anisotropic multi-particle and isotropic multi-particles RVE model in Figure 4-22, we can observe that the shear modulus of anisotropic multi-particle model is less than isotropic multi-particle model. In this case, the chain-like structures of inclusions increase the shear modulus compared to the same material with uniform particle distribution. This effect is small in this case because both RVE models only include several particles. For anisotropic model with 30% particle volume fraction, anisotropic property is not obvious: there is no big difference between the tensile stiffness on X and Y directions because the structure is very similar on both directions. In Chapter 4.6, a more complex anisotropic model with a chain-like structure of ten particles shows a significantly reinforcement in model stiffness and anisotropic property.
Figure 4-22. Shear response curves of isotropic and anisotropic multi-particles RVE model.

4.6 Numerical Simulation and Experimental Result of MRE

4.6.1 Large-strain behavior of MREs with different iron contents

Gerlind Schubert and Philip Harrison gave Large-strain behavior of magneto-rheological elastomers under uniaxial tension, compression and shear test (Schubert and Harrison, 2015). Silicone rubber MM 240TV mixed with 30% silicone oil ACC 34 were used to create the elastomeric matrix material, with volume particle concentrations of 10%, 20% and 30%. Their study samples include material together with both isotropic and anisotropic MREs and demonstrated the stress-strain response of all MREs tested under compression, tension and shear test in the absence of a magnetic field.
This part is organized as follows. First, the material characterizations are described. Then, the cohesive-zone model is considered on interface between particles and matrix in model structure. The finite element model is detailed. Finally, the stress-strain response by the simulations are compared with the experimental results.

The parameters $C_{10}$ and $C_{01}$ of Mooney-Rivlin model would be obtained by fitting silicone rubber uniaxial tensile test data. To Determine the parameters of pure silicone matrix by Mooney-rivlin model. Guiyong Ma and Dazhi Jiang gave (Ma and Jiang, 2013):

$$\frac{\sigma_1}{2\left(\lambda_1^2-1\right)} = C_{01} + \frac{1}{\lambda_1} C_{10}$$  \hspace{1cm} (4.1)

Where $\sigma_1$ is stress response in uniaxial tensile test and $\lambda_1$ is elongation of material.

From the tensile test data of silicone matrix, $C_{01}$ and $C_{10}$ can be computed. We pick $\frac{1}{\lambda_1}$ as X-axis and $\frac{\sigma_1}{2\left(\lambda_1^2-1\right)}$ as Y-axis from Gerlind Schubert’s experimental data, then fit experimental data into a straight line in MATLAB and get the parameters $C_{10}$ and $C_{01}$ from function of line.

In our simulation, $C_{01}$ is 0.32 MPa and $C_{10}$ is 0.80 MPa. To test the simulation of pure silicone matrix, the simulative result need to be matched with experimental data.
Particle filler material is carbonyl iron powder, Young’s modulus E is 210 GPa, Poisson’s ratio is 0.33, The shear modulus G is 78.94 GPa. The constitutive model of cohesive zone is defined in traction-separation behavior. normal and tangential stiffness components Knn, Ktt, Kss are all 1.00MPa/µm, peak values of the contact separation are all 2 µm, separations at failure are all 10 µm. The peak value of the contact stress t is 2 MPa.

It must be mentioned that all the experimental stress-strain response is showed as symbols and the simulations results are showed as solid line in each comparison. The models in software simulation are 2D and we ignore viscoelasticity and many other parameters which effect macroscopic behavior of the MRE composite, some simulation results are not matched with experimental results quiet well, but simulation results show good prediction about the mechanical response of MREs at large applying strain.

4.6.1.1 Uniaxial Tension Test Comparison

Figure 4-23 compares the tensile stress-strain curve in simulation with tensile test experimental data, we can observe the simulation of pure matrix is matched quite well up to strain of 0.50. Numerical simulation of pure silicone matrix by Mooney-Rivlin method is robust, and base on that the cohesive-zone model can be applied on interface between particles and matrix for the further simulations.
**Figure 4-23.** Comparison of experimental tensile stress-strain response of pure silicone (symbols) and the simulation of Mooney-Rivlin model (solid line).

The stress-strain curves of all MREs tested under tension test are illustrated in Figure 4-24:
a: The simulative and experimental stress-strain response of pure silicone and isotropic MREs.
b: The simulative and experimental stress-strain results of isotropic and anisotropic MREs.

**Figure 4-24.** The stress-strain curves of all MREs tested under tension test.

Figure 4-24a shows the stress-strain results of pure rubber and isotropic MREs with 10% and 20% iron contents in the absence of a magnetic field. The data of MRE with 30% iron content tested under uniaxial tension was found questionable in Gerlind Schubert’s paper, so here we do not compare the data of MRE with 30% iron content in this paper.

As we can see, simulations are quite robust to an applied strain of 0.50. The experimental results start with a steep slope that flattens from approximately 10% strain and increases again above 30% strain, which may be due to the viscoelasticity of MREs or reinforcement of aggregates we do not consider in simulations. The simulative data shows an obvious response trend about the reinforcement of iron particles to MREs. The stiffness of MRE with 10% iron
content increases by 50% compared to pure silicone and stiffness of MRE with 20% iron content increases by 105%. The stress softening is more pronounced in anisotropic MREs and in MREs with high iron contents.

Figure 4-24b shows the stress-strain results of isotropic and anisotropic MREs with 10% to 20% iron contents in the absence of a magnetic field. The model slightly underestimates the experimental tensile stress response of anisotropic MREs on both vertical and horizontal direction, the reason is that we ignored many parameters like viscoelasticity and complex micro-structure in real material. Both simulative data the experimental data demonstrate that the response of anisotropic MREs on horizontal direction are very similar compared to isotropic MREs, the chain-like structures of particles only enhances the stiffness along the chain direction, which is vertical direction in test data, stiffness increases by 25% to 35% compared to isotropic MREs.

What is more, the simulation of anisotropic MRE model with 20% iron fraction which is symmetrical experiences initial damage at interface, as shown in Figure 4-25, and interfacial debonding happens at the same time on all particles, the response curves of simulations show a stiffness drop at strain of 0.45 for MRE with 20% particle volume fraction, but MRE with lower particle volume fraction does not show such response.
4.6.1.2 Uniaxial Compression Test Comparison

Figure 4-26 shows the comparison of the pure silicone compressive and tensile stress-strain curves results in simulation with compression test experimental data, we can observe that the simulation of pure matrix fits quite well up to strain of 0.50. The macroscope stiffness of pure silicone rises with the increase of strain ratio, and MREs are stiffer under compressive loading than tensile loading with the same strain ratio.
Figure 4-26. Comparison of experimental compressive and tensile stress-strain response of pure silicone (symbols) and the simulation of Mooney-Rivlin model (solid line).

Figure 4-27a shows the stress-strain compressive response of pure rubber and isotropic MREs with 10% to 30% iron contents. Simulations are matched quite well up to an applied strain of 0.50. The simulative data shows obvious response trends about the reinforcement of iron particles to MREs in compression loading. The compressive stiffness of MRE with 10% iron content increases slightly compared to pure silicone, stiffness of MRE with 20% iron content increases by 42% and stiffness of MRE with 30% iron content increases by 132%. Both experimental and simulative data demonstrate that the compression moduli grow up with the increase of strain ratio, this increase is more pronounced in MREs with high iron contents.
Figure 4-27b shows the compressive stress-strain response of isotropic and anisotropic MREs with 10% to 30% iron contents, strain is up to 0.25 since the limitation of convergence problem. The model slightly underestimates the experimental compressive stress response of anisotropic MRE on horizontal direction, but both results demonstrate the stiffness of anisotropic MRE on horizontal increases slightly compared to isotropic one. Both simulative data the experimental data demonstrate that Anisotropic samples with vertical particle alignment exhibit the highest stresses and moduli, the vertical chain-like structures of particles also increase the vertical compressive stiffness, stiffness increases by nearly 100% compared to isotropic MREs.

Unfortunately, the experimental results of anisotropic MRE on vertical direction starts with a steep slope that flattens, which means the stiffness continue to reduce up to strain of 0.25 in the test. The simulations not successfully shows the real anisotropic response here, which still needs further research of FEM simulation.
a: The simulative and experimental stress-strain response of pure silicone and isotropic MREs.
b: The simulative and experimental stress-strain results of isotropic and anisotropic MREs.

**Figure 4-27.** The stress-strain curves of all MREs tested under tension test.

### 4.6.1.3 Shear Test Comparison

Figure 4-28 shows the comparison of the pure silicone shear stress-strain curve results in simulation with shear test experimental data, we can observe that the simulation of pure matrix fits quite well up to strain of 0.30, although the simulation result shows an approximately linear stress-strain response, experimental result shows a non-linear stress-strain behaviour.
Figure 4-28. Comparison of experimental shear stress-strain response of pure silicone (symbols) and the simulation of Mooney-Rivlin model (solid line).

Figure 4-29a shows the stress-strain shear response of pure rubber and isotropic MREs with 10% to 20% iron contents, data of sample 30% was questioned and eliminated in this paper. The simulative data shows obvious response trends about the reinforcement of iron particles to MREs in shear condition, although not quite match exactly with experimental data. All types of MREs show strongly non-linear stress-strain behaviour in shear test, the simulative result gives the linear response which cannot demonstrate real behaviour for some reasons. The shear stiffness of MRE with 10% iron content increases slightly compared to pure silicone, stiffness of MRE with 20% iron content increases by 73%. Simulative data demonstrate the particle enhancement to isotropic MREs, this improvement is more pronounced in MREs with higher iron contents.
Figure 4-29b shows the shear stress-strain response of isotropic and anisotropic MREs with 10% to 20% iron contents, strain is up to 0.25 since the limitation of convergence problem. The model response is not quite matched with the experimental shear stress response of anisotropic MRE on horizontal direction: In simulations, anisotropic MREs model with horizontal particle alignment shows a very similar stress-strain curve shape with isotropic MREs, although the anisotropic MREs exhibit slightly lower stresses response. This is different in shear test, which demonstrated that anisotropic one is slightly higher. Both simulative data the experimental data demonstrate that Anisotropic samples with vertical particle alignment exhibit the highest stresses and moduli, the vertical chain-like structures of particles also increase the vertical shear stiffness, stiffness increases by nearly 80% compared to isotropic MREs.

The model underestimates the experimental shear stress response of anisotropic MRE on both vertical and horizontal direction. The simulative results do not show non-linear shear behaviors of MREs either.
a: The simulative and experimental stress-strain response of pure silicone and isotropic MREs.
b: The simulative and experimental stress-strain results of isotropic and anisotropic MREs.

**Figure 4-29.** The stress-strain curves of all MREs tested under tension test.
4.6.2 Influence of interfacial adhesion on MREs

T. Pössinger investigated the interfacial adhesion between the iron fillers and the silicone matrix in MREs at high strain and gave the experimental data of mechanical response of MREs with different interfacial adhesion.

In his experiment, carbonyl iron powder was mixed in a soft silicone matrix with volume fraction of 3.5 and 30 %, tensile tests of these samples were conducted under scanning electron multi-scale. The silane primer was applied to the iron particles to change interfacial adhesion between two phases. He demonstrated that the effect of the primer on the debonding of the particles from the silicone matrix has an impact on the macroscopic behaviour of the MRE composite and gave the mechanical response of MREs with different applications of the primer.

The matrix material is a kind of very soft and stretchable silicone elastomer. In simulation, $C_{01}$ we picked is 0.01 MPa and $C_{10}$ is 0.0025 MPa, which were calculated from experimental tensile response of pure silicone sample. As shown in Figure 4-30, the simulative tensile stress-strain response matches quite well with experimental data.
He set 3 different level of interfacial adhesion: 1. Untreated MRE: a series of MRE samples without silane particle treatment, the two materials were mixed, degassed and cured as the pure silicone samples with iron particle volume fraction of 3.5% and 30%; 2. Spray-coated MRE: a series of MRE samples was prepared by first dispersing the particles in a plastic container and spraying them with the primer while shaking the container; 3. Primer stirred MRE: a series of MRE samples was prepared by directly stirring the iron particles in the silane primer dilution. For all samples, the magnetic field was applied and caused the particles to align in chain-like structures.

Figure 4-31a shows The SEM micrographs of the 3.5 % and 30% particle volume fraction MREs with untreated particles in experiment, and Figure 4-33b shows the model we built for these two cases. The microstructure of MREs is
heterogeneous and complex. The models in our simulations are relatively simple, although show good predictions mechanical behaviour in different cases.

![SEM micrographs](image1)

**a:** The SEM micrographs of the 3.5 % and 30% particle volume fraction MREs (Cited from Pössinger, T.; Bolzmacher, C.; Bodelot, L.; Triantafyllidis, N., Influence of interfacial adhesion on the mechanical response of magnetorheological elastomers at high strain. *Microsyst Technol* 2013, 20, 803–814)

![RVE models](image2)

**b:** Models of the 3.5 % and 30% particle volume fraction MREs.

*Figure 4-31.* Comparison of the SEM micrographs and RVE models.
Figure 4-32a shows the stress-strain response of pure rubber and isotropic MREs with 3.5% and 30% iron contents in tensile test. The simulative data shows obvious response trends about the reinforcement of iron particles to MREs as the same conclusion of former experimental data, although not quite match exactly with experimental data. The model response is not quite matched with the experimental response of MREs and it underestimates a bit. Both simulative and experimental results show strongly non-linear stress-strain behavior. With a higher volume fraction, the stiffness of the material increases. The tensile stiffness of MRE with 3.5% iron particle volume fraction increases by nearly 50% compared to pure silicone, stiffness of MRE with 30% iron content increases by 200%. Simulative data demonstrate the particle enhancement to isotropic MREs, in MREs with higher iron contents this enhancement is more pronounced. MREs with 3.5% iron particle volume fraction show a significant improvement of stress response because of the chain-like microstructure, MREs with very low inclusion fraction produced in magnetic field can still show significant stiffness increase in the direction of the field.

The experimental data symbols in figure 4-32a are selected from upper response curves of hysteresis curves in figure 4-32b test results, these hysteresis curves show the average response after applying the loading/unloading cycles in experiment.
a: The simulative and experimental stress-strain response of pure silicone and isotropic MREs.


**Figure 4-32.** Comparison of simulative and experimental results of pure silicone and isotropic MREs.
The application of the primer also influences the stress–strain response. The stabilized upload curves showed in test result have been studied and compared to simulation model response. For untreated MREs, we assume the same energy loss as peel test data and build the CZM model interface. For spray-coated MREs, the operation increases the interface adhesion between two phases, in absence of test data support, we assume the stiffness components doubled to the original interface. For primer stirred MREs, the operation changes the interface adhesion and even the composition of the matrix material, so here we assume the stiffness components doubled as spray-coated MREs, then rose $C_{01}$ and $C_{10}$ parameters to increase the stiffness of silicone matrix.

Figure 4-33a shows the tensile stress-strain response of MREs with 30% iron volume fraction, we did not compare the result of MREs with 3.5% iron volume fraction since the benefits of both spray-coat and primer stirring are too small. We can observe that the models give good predictions, although not perfectly matched especially with MREs with higher particle volume fraction. In the case of the 30 % volume fraction MREs, both simulative and experimental results highlight that the addition of the spray-coat primer has very little benefit, although simulative response underestimates slightly. However, the 30 % volume fraction primer stirred MRE stiffness increases by 43% throughout the loading curve compared to pure silicone. Simulative results are not matched with experimental result since the particle agglomerates existences further increase the stiffness and effective filler volume, which we did not take into consideration. The experimental data symbols in figure 4-35a are selected from test data figure 4-33b from T. Pössinger’s results.
a: Tensile stress-strain response of MREs with 30% iron volume fraction.


**Figure 4-33.** Comparison of simulative and experimental results of isotropic MREs with different interface adhesions.
4.7 Conclusions

1) Continuous cohesive zone model and the constitutive model of it are briefly introduced in this chapter.

2) We defined surface-based cohesive behaviour parameters in ABAQUS with the assumption of the peel strength equal the CZM adhesion energy. Results showed that the stiffness of interface is relatively large than average stiffness of rubber-like matrix.

3) The tensile and shear response of isotropic and anisotropic MRE models with CZM interface is studied, Particle addition benefit on the mechanical properties of rubber matrix since the relatively strong interface.

4) The chain-like structures of iron particles increase the stiffness of MRE along the chain-like structure direction but reduce the shear strength which perpendicular to the chain-like structure.

5) The simulation response results were compared with several experimental results, the simulations showed good prediction on uniaxial compression, uniaxial tension and shear test, although some values are not matched well. The model also described the enhancement of interfacial adhesion, this matched well with experimental results.
Chapter 5 Parametric Study of Interfacial Debonding in MREs

In this chapter, we analysed the influences of some parameters, such as particle volume fraction, interface properties and model structure, on the mechanical response of magneto-rheological elastomers. We can change some value of properties and control others to get the response results, which is more convenience compared to experimental operations. The results directly demonstrate the effect of each parameter on response of the whole model. To control variable and simplify the calculation, all isotropic and anisotropic models only include 2 particles, the results of isotropic and anisotropic with the same conditions are very close but still demonstrate the differences.

5.1 Particle Volume Fraction

5.1.1 Tensile Simulation of Isotropic Model

Figure 5-1 shows the stress-strain response of isotropic MREs with 3%, 20% and 30% iron contents in tensile test. We set five levels of interface stiffness with different stiffness components $K$ in cohesive zone model, from no adhesion (preventing interpenetration between the matrix and the particles, and no friction at interface) case to perfect bonded case (setting bonded interface contact). The simulative data shows obvious response trends about the reinforcement of different iron particles fractions to isotropic MREs. Simulative results show strongly non-linear stress-strain behavior.
For no adhesion interface simulation, the addition of particles cannot benefit the tensile stiffness of MREs, Young's modulus reduces with the increase of particle volume fraction. This may be due to the occurrence of void, tensile resistance mainly comes from the rubber matrix, the higher particle volume fraction, the lower matrix fraction, the lower tensile stiffness.

For weak interface case ($k=2e5$ Pa/µm), stiffness increases compared to no adhesion case for each sample. It must be mentioned that the damage evolution occurs in sample of 20% and 30% particle volume fraction, interface contact stress of interface drops, surfaces of particle and matrix delaminate, Young’s modulus of the model keep decreasing, the stiffness of model become less than rubber matrix when model strain reach to certain strain ratio.

For the rest samples with relatively strong interface, stiffness of interface is higher than matrix, the enhancement of particle addition become obvious. With higher volume fraction, the stiffness of the material increases: stiffness of MRE with 3.5% iron content increases slightly compared to pure matrix stiffness; Stiffness of MRE with 20% iron content increases by 20%; Stiffness of MRE with 30% iron content increases by 45%.
Figure 5-1. Tensile response of isotropic MREs with 3%, 20% and 30% iron contents (unit of k is Pa/μm).

5.1.2 Tensile Simulation of Anisotropic Model

Figure 5-2 shows the stress-strain response of anisotropic MREs with 3%, 20% and 30% iron contents in tensile test. To study the benefit of Chain-like particle structures to macroscope response, tensile loading is parallel to the direction of chain-like particle structures.
Results of no adhesion case are quite similar with isotropic MREs, Young's modulus reduces with the increase of particle volume fraction. For weak interface sample, models show an obvious drop in Young's modulus at certain strain ratios except MRE with 3.5% iron content, which is relatively sharp than isotropic results since interfacial debonding occurs at same time on each particle. The drop of MRE with 20% iron content starts from strain of 0.27 and MRE with 30% iron content starts from strain of 0.22. Both of two models become softer than pure matrix at approximately 0.27 strain. For the rest samples with stronger interface, the higher volume fraction, the higher stiffness of the material. From comparison of isotropic and anisotropic MREs, Chain-like particle structures benefit the enhancement of particle addition. Compared to pure matrix, Stiffness of MRE with 20% iron content increases by 30%; Stiffness of MRE with 30% iron content increases by 75%. Compare the result of anisotropic models with isotropic models, with the same particle volume fraction, stiffness of former is larger than latter, Chain-like microstructures of materials benefit the macroscopic mechanical response.
5.1.3 Shear Simulation of Isotropic Model

Figure 5-3 shows the stress-strain response of isotropic MREs with 3%, 20% and 30% iron contents in shear test. Response of MREs with 3% particle volume fraction does not show obvious change in shear stiffness. For no adhesion interface simulation, the addition of particles cannot benefit the shear stiffness of MREs, shear modulus drops slightly with the increase of particle volume fraction. For weak interface case (k=2e5 Pa/µm), shear modulus drops at certain strain since the interfacial debonding. Before that, with higher volume fraction, the shear stiffness of the material increases. We cannot predict the response over strain of 0.50 since the convergence problem occurs. For the rest samples with
relatively strong interface, stiffness of interface is higher than matrix, the enhancement of particle addition become obvious, which is same as tensile response: Stiffness of MRE with 20% iron content increases by 20% to 30%; Stiffness of MRE with 30% iron content increases by 45% to 55%. This enhancement is more pronounced when interface contact become stronger.

Figure 5-3. Shear response of isotropic MREs with 3%, 20% and 30% iron contents (unit of k is Pa/µm).
5.1.4 Shear Simulation of Anisotropic Model

Figure 5-4 shows the shear stress-strain response of anisotropic MREs with 3%, 20% and 30% iron contents in shear test. Response laws are very similar with isotropic MREs, although the weak interface samples do not show any stiffness changes during the simulations. For the samples with relatively strong interface, with higher volume fraction, the stiffness of the material increases: Stiffness of MRE with 20% iron content increases by 20% to 45%; Stiffness of MRE with 30% iron content increases by 25% to 50%. Chain-like particle structures do not have obviously effect compared to the isotropic material with the same particle volume fraction.
**Figure 5-4.** Shear response of anisotropic MREs with 3%, 20% and 30% iron contents (unit of k is Pa/µm).

### 5.2 Interface Properties

#### 5.2.1 Tensile Simulation of Isotropic Model

The results in Chapter 5.1 can also be used in studying the influences of interface properties on material mechanical response. Figure 5-4 shows the tensile stress-strain response of isotropic MREs with different interface properties in tensile test. We can observe that the tensile stiffness of model increase if the interface contact is stronger for all the samples, and this effect is more obvious with higher particle volume fraction.
Figure 5-5. Tensile response of isotropic MREs with different interface adhesions.

5.2.2 Tensile Simulation of Anisotropic Model

Figure 5-5 shows the tensile stress-strain response of anisotropic MREs with different interface properties in tensile test. The tensile stiffness of model increases if the interface contact is stronger and this effect is more obvious with higher particle volume fraction. This law is similar with isotropic MREs, the difference is that the chain-like particle structures improve the effect of interface adhesions.
Figure 5-6. Tensile response of anisotropic MREs with different interface adhesions.

5.2.3 Shear Simulation of Isotropic Model

Figure 5-7 shows the shear stress-strain response of isotropic MREs with different interface properties in tensile test. The shear stiffness of model increases if the interface contact is stronger and this effect is more obvious with higher particle volume fraction, and this effect is more obvious with higher particle volume fraction.
Figure 5-7. S55hear response of isotropic MREs with different interface adhesions.

5.2.4 Shear Simulation of Anisotropic Model

Figure 5-8 shows the shear stress-strain response of anisotropic MREs with different interface properties in tensile test. Compared with isotropic MREs, results are very similar, the chain-like particle structures do not have significant effect on the enhancement of interface adhesions.
Figure 5-8. Shear response of anisotropic MREs with different interface adhesions.

5.3 Conclusions

We analysed the influences of some parameters, such as particle volume fraction, interface properties and model structure, on the mechanical response of magneto-rheological elastomers. The results showed that both tensile and shear stiffness of model increase with the interface property increase, and this effect is more pronounced with higher iron particle volume fraction. From the comparison of isotropic and anisotropic model, we can observe that the chain-like particle structures increase the tensile stiffness along the chain direction, but do not have significant effect on perpendicular shear stiffness.
Chapter 6 Conclusions and Future Research

6.1 Conclusions

This thesis is devoted to study the behavior of rubber based MREs with carbonyl iron under large strain, basically under tension and pure shear deformations and analyzed the influence of interfacial debonding on the response of MREs. Particle volume fraction, interface properties and microstructure of MREs, which impact the interfacial debonding in MREs, are highly investigated.

Mooney-Rivlin model was used to simulate the mechanical response of pure matrix, compared the macroscopic mechanical properties of the MREs by numerical simulation method with the Mori-Tanaka model and the Double-Inclusion model, those three are in good agreement.

Interface properties between particles and matrix were defined by CZM method, with the assumption of the peel strength equal the CZM adhesion energy. The simulation response results were compared with several experimental results, both are in good agreement.

All types of MREs show strongly non-linear stress-strain behavior, not a simple series or parallel relationship. Both isotropic and anisotropic model described the benefit of interface adhesion on the mechanical properties of MREs. The macroscope tensile and shear stiffness of composite rise with the increase of
interface properties. The stiffness would drop even below the pure soft rubber-like matrix if the interface adhesion reduces to a certain value. This effect of interface adhesion is more obvious with higher particle volume fraction.

The chain-like structures of iron particles in vertically aligned MREs increase both the compressive and tensile stiffness of MRE on vertical direction but slightly reduce the horizontal shear strength, this effect on shear strength is not matched with the experimental data which showed a not significant improvement on shear response.

In summary, the RVE simulations with cohesive interface based on FEM explain the reasonable experimental findings on MREs by other researchers, may be further developed and applied to various types of polymer composites.

**6.2 Suggestions for Future Research**

Comparing with the experimental data in chapter 5, we summarized the limitation of simulations since RVE model we used was too small, it cannot describe the real interfacial debonding in heterogeneous MREs precisely.

To further challenge the model, additional types of loadings might be applied to the MRE models, such as biaxial tension or shearing. It would also be interesting to explore the impact of changing in the particles size in MREs.
Many other factors such as viscoelasticity of MREs, particle aggregates during the solidification and cracks evolution in rubber-like matrix which have significant impacts on mechanical response of MREs are still not considered.


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