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Author
Nussinov, Shmuel N.

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University of California

Ernest O. Lawrence Radiation Laboratory

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Shmuel N. Nussinov

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Shmuel N. Nussinov

Lawrence Radiation Laboratory
University of California
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Using Sugawara theory and a universal current x current N.L. Hamiltonian an upper bound on non-leptonic transition is obtained.

There has been recently considerable interest in "dynamical current theories". Such theories not only utilize postulated current algebras but are completely based on currents as the basic variables.\(^1\)

The following expression for the energy momentum tensor in terms of axial and vector SU(3) x SU(3) currents

\[ \theta_{\mu \nu}^S = \frac{1}{2C} \sum_{a=1}^{8} \left( [V^a_\mu(x), V^a_\nu(x)] - g_{\mu \nu} [V^a_\mu(x) \gamma^\mu(x)] \right) + (V \leftrightarrow A), \]

(1)

was shown by Sugawara\(^2\) to be consistent with, and almost uniquely determined by, covariance requirements and the algebra of fields commutation relations.\(^3\)

The parameter C appears in these commutation relations as the coefficient of the Schwinger term:
\[ [V_0^a(x), V_k^b(y)] \delta(x^0 - y^0) = i f_{abc} V_k^c(x) \delta^b(x - y) \]
\[ + i C \delta_{a,b} \partial_k \delta^h(x - y) \]  \hspace{1cm} (2)

and therefore satisfies the spectral function sum rule:
\[ \int \frac{d(m^2)}{m^2} = C \]  \hspace{1cm} (3)

In the following we show that if the non-leptonic Hamiltonian has the universal (Cabibbo current) x (Cabibbo current) form:
\[ H_{PC}^{\Delta S = 1} = \frac{G \sin \theta \cos \theta}{2 \sqrt{2}} [V_{\mu}^{\pi^+}(x), V_{\mu}^{K^-}(x)]_{+} + (V \leftrightarrow A) \]  \hspace{1cm} (4)
then Eq. (1) implies the "universal bound" on all non-leptonic matrix elements:
\[ C \geq \frac{|\langle \alpha | H_{PC}^W |\beta \rangle|}{G \sqrt{2 \sin \theta \cos \theta} \bar{m}} \]  \hspace{1cm} (5)

where \(|\alpha\rangle, |\beta\rangle\) belong to an SU(3) multiplet with a common mass \(m\), \(G = 10^{-5}/m^2_p\), and \(\sin \theta \sim 0, 24\). Since the Suzuki-Sugawara analysis relates the matrix elements of the parity conserving weak Hamiltonian to the \(S\) wave part of the non-leptonic decays and \(C\) is given by Eq. (3) the inequality (5) can in principle be confronted with experiment.

In order to derive the inequality we use the fact that
\[ m = \langle \alpha | \delta_{\alpha 0} | \alpha \rangle - \langle \phi \rangle_0 \]

\[ = \frac{1}{\mathcal{N}_0} \langle \alpha | [V_\mu^k, V_\mu^k]_+ + [V_\mu^\pi^+, V_\mu^\pi^-]_+ + [V_\mu^k_0, V_\mu^k_0]_+ \]

\[ + 2 V_\mu^\eta_0 V_\mu^\eta_0 + 2 V_\mu^\pi_0 V_\mu^\pi_0 \langle \alpha \rangle + (V \leftrightarrow A) - \langle \phi \rangle_0, \]

(6)

where \( |\alpha\rangle \) is a rest state of the multiplet considered.

For simplicity neglect first the subtraction of the vacuum expectation value \( \langle \phi \rangle_0 \) required in (6). The diagonal matrix elements \( N_{\alpha \alpha} \) of:

\[ N = (V_\mu^k_+ + V_\mu^\pi^+) \left( V_\mu^k_+ \pm V_\mu^\pi_+ \right)^* + (V \leftrightarrow A) \]

are proportional to the contribution of the charged currents \([k^+ k^-] + [\pi^+ \pi^-]\) to the mass \( m \) in Eq. 6. Nondiagonal elements \( N_{\alpha \beta} \) between states of equal charge and one unit difference in strangeness are proportional to \( \langle \alpha | H_{\text{PC}} | \beta \rangle \). Since \( N \) is positive definite \( N_{\alpha \alpha} N_{\beta \beta} \geq |N_{\alpha \beta}|^2 \) and we obtain the inequality (5).

Returning to the vacuum expectation value \( \langle \phi \rangle_0 \) in Eq. (6), we realize that its subtraction amounts to the omission of all disconnected contributions in which one current creates an intermediate state from the vacuum and the other current annihilates it with the initial hadron \( \alpha \) passing uneffected:

\[ \langle \alpha | J J | \beta \rangle_{\text{disconnected}} = \delta_{\alpha \beta} \langle 0 | J J | 0 \rangle. \]
If we omit from the completion sum
\[ \langle \alpha | J | \beta \rangle = \sum_n \langle \alpha | J | n \rangle \langle n | J | \beta \rangle , \]
the corresponding "disconnected" intermediate states \(|n\rangle\) which contain a hadron \(|\alpha\rangle \ (= \beta\rangle\) at rest, we achieve the required subtraction of \(\langle \cdot \rangle_0\) while keeping the \(J\) product positive definite within the subspace spanned by the rest states \(|\alpha\rangle\), and the derivation of Eq. (5) still goes through.

Our result (5) is not restricted to theories in which \(\vartheta_{\mu \nu}\) has the form (1) but holds whenever
\[ \vartheta_{\mu \nu} = \vartheta_{\mu \nu}^S + R_{\mu \nu} , \]
where \(\vartheta^S\) is given in Eq. (1) and \(R_{00}\) is positive definite. In such a case there will be an additional contribution \(\langle \alpha | R_{00} | \alpha \rangle\) to the masses which could make the inequality stronger. Only when we appeal to the spectral function sum rule (3) in order to evaluate \(C\) do we make use of the detailed features of Sugawara's model. For the purpose of checking the inequality (5) let us assume "vector dominance" namely that the sum rule (3) is saturated by the \(\rho^0\) contribution:
\[ C = \frac{g_{\rho^0}^2}{m_{\rho^2}} = 0.025 \text{ or } 0.018 \text{ BeV}^2 . \]  

We consider here only hyperon decays where the Suzuki-Sugawara analysis is most successful and ambiguities due to mass differences are smallest. The fit yields the following value of the non-leptonic matrix elements:
\[ \langle \alpha | H_W | \beta \rangle = f_{60}^\alpha F + d_{60}^\beta D \]

\[ F = 3.6 \times 10^{-8} \text{ BeV} \quad D = -1.6 \times 10^{-8} \text{ BeV}. \]

It is most advantageous to choose \( |\alpha\rangle = \Sigma^+ \quad |\beta\rangle = p \) in which case we obtain the large matrix element \( |\langle \Sigma^+ | H_W^{PC} | p \rangle| = |D - F| = 5.2 \times 10^{-8} \text{ BeV}. \)

Using this value and \( m = 1.1 \text{ BeV} \) in (5) we find \( C \geq 0.0115 \text{ (BeV)}^2 \) consistent with (8).

This result may be interpreted as indicative of the possible consistency of the various ingredients used in deriving Eq. (6) - including the vector dominance assumption. In view of the fact that the contribution of the four neutral currents in Eq. (6)
\[ n^+ n^- + n^- n^+ + k^0 + k^0 \]
was ignored in deriving (5) and only the four charged currents retained we find the margin by which (5) is satisfied rather slim.

More direct tests of Sugawara's model involving a consistency sum rule or inequalities for integrated electroproduction cross sections were recently derived by Gross\textsuperscript{10} and by Gross and Callan\textsuperscript{11}. These tests do not involve \( C \) or any assumptions on the non-leptonic Hamiltonian. The practical evaluation of those sum rules is rather difficult, whereas our inequality which [neglecting the saturation problem in (3)] involves directly complete matrix elements.

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FOOTNOTES AND REFERENCES

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1. For a general discussion of such theories see papers by Sharp and Dashen and Sharp, Phys. Rev. 165, 1857-1881 (1968).


6. In particular the extension of Sugawara's theory to include PCAC suggested by K. Bardakci, Y. Frishman, and M. B. Halpern, Phys. Rev. 170, 1353 (1968) results in such modifications of $\Theta_{\mu\nu}$.

7. Such vector dominance assumptions are often used in hard pion calculations and in analysis of various photoproduction experiments so that the assumption could be independently checked.

8. The two values of $g_p$ correspond to the experimental values quoted in S. G. Asbury et al., Phys. Rev. Letters 20, 227 (1968) and to the value obtained by using the K.S.F.R. relation.

9. Since the "m" appearing in the denominator of (5) is actually $(m_Q m_B)^{\frac{1}{3}}$ one is tempted to consider $\langle k | H_w | \pi \rangle$. The symmetry breaking effects in this case are large and difficult to assess.


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