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Publication Date
1984
Submitted for publication

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January 1984

Prepared for the U.S. Department of Energy under Contract DE-AC03-76SF00098
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MASSIVE NEUTRINOS AND COSMOLOGY*

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Abstract

We show that in a neutrino dominated universe the cosmological bound on neutrino masses cannot be evaded by heavy and unstable $\mu$ and $\tau$ neutrinos whose decay is mediated by large flavor changing neutral current. It is argued that the unique possibility to have a neutrino dominated universe is the existence of a light ($m_\nu \sim 0$ (1keV)) and stable right-handed neutrino.

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January 1984

In recent years there has been a large attention to the possibilities and implications of massive neutrinos. Experimental, theoretical as well as cosmological motivations suggest that the neutrino mass is a key for the physics beyond the standard model.

The laboratory upper limits on neutrino masses are: $m_{\nu_e} < 46$ eV, $m_{\nu_\mu} < 0.5$ MeV and $m_{\nu_\tau} < 250$ MeV, and studies of left-right symmetric models and galaxy formation in a neutrino dominated universe highly welcome $\nu_\mu$ and $\nu_\tau$ masses in the keV and MeV ranges. The first for aesthetics of model building, and the second to explain the formation of structures of galactic sizes in the early universe.

The compelling desire for heavy neutrino ($\nu_H$) masses forces the existence of a mechanism to speed up the $\nu_H$ decay into light ones ($\nu_L$), to evade the astrophysical constraint stating that the $\nu_H$ contribution to the energy density of the universe cannot surpass the presently estimated total energy density.

Despite very clever mechanisms that have been proposed to produce a fast $\nu_H$ decay, they have found obstacles in one or another way. Particularly, as shown recently by Pal, mechanisms using light scalars to accelerate the $\nu_H$ decay at tree level are unlikely to survive a confrontation with experiment. As far as we know there is one remaining proposition discussed by Hosotani and others, consisting of the decay $\nu_H \rightarrow 3\nu_L$ mediated by large flavor changing neutral (Z) current (FCNC).

Our intent in this letter is to show that this possibility is also ruled out.

We are going to consider models with the most general neutrino mass matrix...
where $D$ is a Dirac mass matrix, and $A$ and $B$ are Majorana ones (for left and right-handed neutrinos respectively). As a necessary condition to have large FCNC (i.e. large nondiagonal mixing angles) the Majorana masses in (1) must be approximately of the same order of the Dirac masses, what can be easily verified by means of perturbation theory. Mass matrices following this condition can be achieved in the Glashow-Weinberg-Salam model and in left-right symmetric models.\textsuperscript{(2,4,10,11)}

Our approach to study the $Z$ mediated heavy neutrino decay consist in the simple analysis of experimental and cosmological constraints on neutrino mixing angles.\textsuperscript{(2)} To start with we notice that the $\nu^H_1$ lifetime is given by:

$$\tau_{\nu^H_1} \sim [10^{18}/m_{\nu_{\text{kev}}}] [1/(\Sigma U_{ij}U_{\alpha j}^\dagger)^2] \text{ s},$$

(2)

where $m_{\nu_{\text{kev}}}$ is $m_{\nu^H_1}$ in kev units and $U_{ij}$ are mixing angles.

We follow Kobzarev et al.\textsuperscript{(15)} defining the matrix $K$, which diagonalize the mass matrix, formed by submatrices $U$ and $V$ ($K = [U|V]$), and Majorana fields $\chi_a$ (the mass eigenstates) in such a way that

$$\chi_a = \phi_{aL} + \eta_a \phi_{aR},$$

where $\eta_a$ can take values $\pm 1$, $a$ goes from 1 to 6 for 3 generations, and the initial fields $\nu_i$ are related to $\phi_a$ by

$$\nu_{iL} = U_{ia}^T \phi_{aL},$$

$$\nu_{iR} = V_{ia} \phi_{aR}$$

(recall that only $K$ is unitary).

In the presence of some right-handed current characterized by a "Fermi coupling" $G$ we obtain

$$\tau_{\nu^H_1} \sim [10^{18}/m_{\nu_{\text{kev}}}] [1/(\Sigma U_{ij}U_{\alpha j}^\dagger)^2] \exp(-G^2/\Sigma V_{ij}V_{\alpha j}^\dagger/V_{\alpha j}^\dagger V_{ij} + 2)^{1/2} \text{ s},$$

(3)

The sum over mixing angles in Eq. (2) and (3) goes over the light neutrinos, and we can approximate $(\Sigma U_{ij}U_{\alpha j})^2 \sim U_{iH}^2$ and $(\Sigma V_{ij}V_{\alpha j})^2 \sim V_{iH}^2$, where $U_{iH}$ and $V_{iH}$ stand for the mixing between the heavy and light neutrinos in one of the vertices, with full mixing in the other vertex.

To obtain the astrophysical limit on neutrino lifetime that we referred to in the beginning of the paper we proceed as Dicus et al.,\textsuperscript{(16)} first determining the $\chi$ decoupling temperature $T_D$ (and decoupling time $t_D$), and secondly computing the heavy neutrino density as a function of $T_D$ and the age of the universe $t_U$ - $T_D$ is obtained equalling the interaction time ($t_I$) of a given neutrino with the other relativistic particles to the age of the universe.

The energy density of decaying neutrinos after the decoupling time is obtained from

$$\rho_{XH} = m_{XH} n_{XH} (T_D) [(1.9)^2K/T_D]^3 \int_{t_D}^{t_U} [(U_{ij})^2/(t/t_D) \exp(-t/t_D)] dt,$$

(4)

where $\tau$ is given by Eq. (2), the factor $n_{XH}(T_D)\exp[-(t/t_D)^2]$ accounts for the volume expansion from the decoupling to the present era, and the factor $(t/t_U)^4$ times the neutrino mass gives the redshifted value of the initial energy ($m_{XH}$). For $t_U >> \tau >> t_D$ we can approximate (4) by

$$\rho_{XH} = \rho_{XH} n_{XH}(T_D) [(1.9)^2K/T_D]^3 (t/t_U)^4.$$

(5)
Assuming that the contribution of light neutrinos is negligible we impose that \( \rho_{\chi_{H}} < \rho_c \), where \( \rho_c \) is the critical density (\( \rho_c \sim 11.2 \text{ keV cm}^{-3} \)). In Fig. 1 we plot the lower bound in \( U_{\nu_{H}} \) for masses up to 500 keV obtained from the inequality described above together with experimental upper bounds for the same mixing angles. We can see that if the Hosotani mechanism is the only one able to get rid of heavy neutrinos the existence of a heavy muon neutrino is ruled out.

A stronger cosmological constraint can be determined for higher masses. For \( m_{\chi} > 1 \text{ MeV} \) new decay channels are open, as \( \chi_{H} \rightarrow e^+ e^- \chi_L \). The \( e^+ e^- \) pairs will annihilate into photons, contributing to an increase of the entropy, which, if it occurs at the nucleosynthesis epoch, can modify considerably the baryon abundances. In the sequence we make a quantitative analysis showing how the mixing angles are constrained when we restrict the increase of entropy at the nucleosynthesis era.

Supposing that the photons coming from \( e^+ e^- \) annihilation thermalize very quickly, we expect that at time \( t_E \) when the \( \chi_{H} \) decays the photon temperature increases from \( T_{E_b} \) to \( T_{E_a} \). From energy conservation we have

\[
\begin{align*}
\frac{T_{E_a}^4}{T_{E_b}^4} &= 1 + \frac{r}{2} \frac{m_{\chi_{H}} \eta_H(T_1)}{(g_*(T) T_{E_b})^3}. \\
\end{align*}
\]

(6)

where \( a \) is equal to \( 4.72 \times 10^9 \text{ MeV} \text{K}^{-4} \text{cm}^3 \) and \( r \) is the proportion of energy carried by the electrons in the heavy neutrino decay (\( r = 2/9 (2/3) \) for neutral (charged) current events).

Equation (6) can be written as a function of the time \( t_1 \) (and temperature \( T_1 \)) when the universe becomes matter (neutrino) dominated, which can be related to the masses and mixing angles. We can determine the time \( t_1 \) equalling the energy density of heavy neutrinos to the energy density of relativistic particles

\[
\begin{align*}
m_{\chi_{H}} \eta_H(T_1) &= \frac{1}{2} g(T) a T_1^4. \\
\end{align*}
\]

(7)

where \( g(T) \) is the effective number of degrees of freedom of bosons and fermions, and \( \eta_H(T_1) \) is the neutrino equilibrium number density at temperature \( T_1 \) related to \( \eta_H(T_D) \) by

\[
\eta_H(T_1) = \frac{4}{11} \eta_H(T_D) (T_1/T_D)^3.
\]

(8)

Equation 8 shows the modification of the previous density (\( \eta_H(T_D) \)) by the universe expansion, since no new heavy neutrino has been created since decoupling.

Knowing that the age of the universe (during the radiation dominated epoch) is \( t \approx (10^{20}/T^4) \text{s} \), using Eqs. (7) and (8), and assuming \( t_E = \tau_H \) we transform Eq. (6) into

\[
\begin{align*}
\frac{T_{E_a}^4}{T_{E_b}^4} &= 1 + \left( \frac{r}{2} g(T) (T_{E_b}/T_1) \right)^3. \\
\end{align*}
\]

(9)

As remarked by Dicus et al., nucleosynthesis predicts a baryon density (at \( T = 2.7 \text{ K} \)) equal to

\[
\rho_B = 7.15 \times 10^{-11} \left( T_{E_b}/T_{E_a} \right)^3 h_0
\]

(10)

where the factor \( (T_{E_b}/T_{E_a}) \) accounts for density deviations in presence of entropy generation, and \( h_0 \) is a parameter uniquely determined matching a given baryon density with different elements abundance, being the deuterium abundance the one that shows the more accentuated dependence on \( h_0 \).

We will assume as standard values \( \rho_B = 3.4 \times 10^{-31} \text{ g cm}^{-3} \) and \( h_0 = 3 \times 10^{-4} \text{ g cm}^{-3} \) in Eq. (10), doing so we obtain a specific value for \( (T_{E_b}/T_{E_a}) \), and consequently the following (lower) bound on the mixing angle comes out from Eq. (9):

\[
U_{\nu_{H}}^2 \geq 2 \times 10^{-5} (m_{\nu_{ev}}^2)^2.
\]

(11)

Note that our lower bound (Eq. (11)) corresponds to the upper bound in the \( \nu_{H} \) lifetime of Dicus et al., with the difference that we adopted a certain \( h_0 \) which is consistent with a medium value of the \( 2H \) abundance (given in Ref. (17)) for the masses that we are dealing with.
In Fig. (2) we compare the experimental constraints with Eq. (11), and we see that only $m_{XH} \geq 20$ MeV is allowed. Notice that if $m_{XH} < 50$ eV and $m_{XT} > 20$ MeV we would have to explain a hierarchy of masses between them of $O(10^6)$ that is not expected when we compare this one to the other mass ratios within the fermion spectrum, which casts some doubt about this possibility (i.e. $m_{XT} > 20$ MeV).

Our next step will consist in a discussion of a further cosmological constraint, which is based on the study of large scale structure formation in the early universe, and will definitively rule out the mechanism that we have been analyzing.

The neutrinos, as the best candidates for the missing mass of the universe, would be responsible for forming large scale structures as superclusters ($M_C \sim 10^{15}$ $M_\odot$) and galaxies ($M_C \sim 10^{12}$ $M_\odot$ or smaller). These structures are a result of primeval density fluctuations which can grow or be damped as they are larger or smaller than the Jeans mass, which can be related to the neutrino mass (in the free-streaming case) by

$$M_{\nu,V,\text{MAX}} \sim 4 \times 10^{15} m_{30}^{-2} M_\odot,$$

where $m_{30}$ is the mass of the dominant neutrino species in units of 30 eV. For $m_{30} \sim O(1)$ Eq. (12) gives a characteristic mass scale of superclusters, and clearly a more massive neutrino is necessary to form galactic scales, which is corroborated by further studies of the non-linear and linear evolution of fluctuations. Evidently this heavier neutrino must decay fast enough to avoid the problems we discussed before, though not so fast since it would not be able to generate structures of galactic sizes. Hut and White determined from the analysis of the non-linear evolution of fluctuations a minimum lifetime for a heavy neutrino generator of galactic scales. We plot their result in Fig. (2), and not surprisingly this limit on the mixing angles is incompatible with the one determined previously.

It seems that we are left without this mechanism to speed up $\nu_H$ decay in a neutrino dominated universe, and since the explanation of galaxies formation asks for a long lived neutrino, we can presume that our attention should be drawn in other directions rather than the discovery of mechanisms for fast heavy neutrino decay. A radical change in this scenario would be to abandon the possibility of a neutrino dominated universe in favor of more exotic particles, however the same requirements of large scale structure formation probably exclude this possibility. We would like to point out that there is one remaining hope in the neutrino dominated universe scenario whose feasibility had been proposed some time ago and appeals to the presence of light right-handed neutrinos.

We notice that one universe whose dark matter is constituted by stable "predominantly left-handed" neutrinos with masses $m_{X\leq 3} \leq 50$ eV and stable "predominantly right-handed" neutrinos with $m_{X\geq 4} \sim O(1$ keV) is in agreement with experimental and cosmological constraints, including the Tremaine and Gunn's phase-space limit for neutrino dominated galactic halos. The evolution of fluctuations in this case would be a mixing between the universe evolution with a light ($\sim 30$ eV) neutrino, and the 1 keV gravitino of Blumenthal, Pagels and Primack, where the gravitino is substituted by $\chi_i$ ($i = 4, 5$ or 6).

Acknowledgments

The comments of P. Binetruy and B. Machet as well as correspondence with S. T. Petcov are acknowledged. The author also thanks Prof. M. K. Gaillard for the reading of the manuscript and the hospitality of the Lawrence Berkeley Laboratory.
This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098.

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Footnotes

F1. The condition $B - D$ is not so evident when left-right symmetric models are embedded in a grand unified theory like the SO(10) model. Actually Witten\(^2\) has shown that in the SO(10) model the right-handed neutrinos acquire a very
heavy Majorana mass at the two loop level. We consider this large mass generation as a peculiarity of the gauge hierarchy problem transferred to the fermionic sector, due to the particular neutrino behavior under charge conjugation. Indeed, Ibáñez\textsuperscript{13} has shown that in the case of supersymmetric theories there is an inhibition of the large mass feeddown for the $v_R$'s, which turn out to be light. We can also add that in these theories a large Majorana mass must be put by hand,\textsuperscript{13} therefore there are no reasons to believe that right-handed neutrinos should be extremely heavy. Obviously we shall admit small Yukawa couplings for these masses.

F2. We will be assuming throughout the paper that any Higgs structure is sufficiently heavy, not interfering in the processes under consideration, and also the existence of only 3 lepton generations. Therefore, we do not need to be concerned about constraints coming from decays like $v_H \to v_L Y$, since they are suppressed by a GIM leptonic mechanism\textsuperscript{14} (and within the limits imposed by astrophysics).

F3. In the range 0.5 to 1 MeV there are few experimental limits in the mixing angles, and the cosmological lower limit can be fitted by the straight line

$$U_{HL}^2 = 4.1 \times 10^{-2} (1 - 7.8 \times 10^{-4} m_{\nu_{eV}}),$$

thereby the comparison with experiment would show a very small window of allowed masses near 1 MeV, which shall also be discarded if we extrapolate to that region the experimental limits shown in Fig. 1 and 2.

F4. This result is approximately the same obtained by Krauss,\textsuperscript{18} originated from the constraint $\rho_X \leq 10 \rho_{\text{radiation}}$ at the time of $X$ decay.

F5. Freese and Schramm call attention to the fact that one universe whose dark matter consists of a mixing of neutrinos and exotic particles although could solve the problems of large scale structure formation looks rather artificial.\textsuperscript{24}

F6. The denomination "predominantly left-handed (right-handed)" comes from the fact that if the mixing of helicities exist, it is small, and, for example, in the presence of a right-handed current with coupling $G$ the neutrino eigenstates $\chi_{4-6}$ will interact through their right component as long as $G^2 > G_F^2 (U_{HH})^2$.

F7. The $\chi_{4-6}$ decouple much earlier, at temperatures (in the presence of right-handed currents) roughly given by\textsuperscript{25} $T_D(X_{4-6}) \sim (G_F/G)^3$ MeV. Olive and Turner\textsuperscript{25} have shown that masses $O(1 \text{ keV})$ are allowed cosmologically if $G_F/G \geq 3 \times 10^3$. This value agrees to the condition of supremacy of right-handed over left-handed interactions for these neutrinos ($G^2 > G_F^2 (U_{HH})^2$), and is what can be expected from the most recent discussions about the right-handed current mass scale.\textsuperscript{26} Similar argument could be followed in the absence of these currents, even considering that right-handed neutrinos do not look natural in those theories.

Figure Captions

Fig. 1 Bounds on mixing angles. 1 to 4-are upper bounds obtained from the $\beta$ spectral shape analysis and neutrino oscillations given respectively in Ref. (28) (a to d). 5 - is the lower bound imposed by cosmology.

Fig. 2 Bounds on mixing angles (MeV region). 1 and 2 - upper bounds, from peak search and proton beam dump experiment (Ref. (29)a and b) 3 - lower bound, nucleosynthesis (Eq. (11)). 4 - upper bound, formation of large structures (Ref. (22)).
\[ U_{eH}^2 \]

FIGURE 1
Figure 2
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