A BURNOUT SAFETY CONDITION FOR SUPERCONDUCTING MAGNETS AND SOME OF ITS APPLICATIONS

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ABSTRACT

A burnout safety condition for superconducting magnets and some of its applications.

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ABSTRACT

From the time evolution of the current in a superconducting coil during a quench, an upper limit can be computed for the temperature reached anywhere in the coil. A condition under which the danger of burnout is eliminated is derived here. It is used to show how the tests of superconducting magnets can be made safe against burnout and it provides constraints for the design of some type of magnets.
I. INTRODUCTION

A safe condition against burnout has been established for superconducting magnets during quenches.\textsuperscript{1} In this paper, a similar condition is derived. It is based on an inequality binding the temperature in the coil to known characteristics of the wire and the time integral of the square of the current. Using that inequality, it is possible to determine an upper limit for the temperature reached anywhere in the coil knowing the time evolution of the current. Given a maximum value for the temperature that can be tolerated in the coil, a condition (the burnout safety condition) can be established for the current evolution which makes certain that the temperature does not exceed the maximum value. Using a testing method that respects that condition at all times, magnets can be tested without any risk of burnout. Some constraints have to be imposed on their design to insure that a required current can be reached ultimately.

The quench is a phenomenon in which a small piece of superconducting wire turns normal accidentally and gets heated by the current faster than can be handled by the cooling power and the heat conduction.\textsuperscript{2,3} The peak temperature rises in the region where the wire is normal and, at the same time, the normal region spreads because of thermal conduction. The danger of burnout comes from leaving the current on long enough so that the energy deposited at the hottest point in the coil raises its temperature to a destructive value.\textsuperscript{1,4} In such a case, the insulation gets damaged, or a solder connection melts, or the superconductor properties are modified, etc. (see Table 1, next page).

A sure technique to avoid burnout is to turn the current off quickly after a quench has been detected.\textsuperscript{1,5,6} However, in general, this requires
Table 1. Destructive temperatures.

<table>
<thead>
<tr>
<th>Destructive Condition</th>
<th>Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Melting of mylar and epoxy</td>
<td>400-500K</td>
</tr>
<tr>
<td>Melting of solder</td>
<td>500K</td>
</tr>
<tr>
<td>Melting of aluminum</td>
<td>900K</td>
</tr>
<tr>
<td>Deterioration of superconductor</td>
<td>1000-1100K</td>
</tr>
<tr>
<td>Melting of copper</td>
<td>1350K</td>
</tr>
</tbody>
</table>

A fast change of flux and, therefore, produces high voltages. A compromise has to be found in the speed of the current decrease to avoid both dangers. The burnout condition of this paper gives an expression indicating how fast the current should be turned off. Some safety factor, real, but difficult to estimate, is included. Suggestions of ways to avoid burnout will follow. The rules are summarized in Section VII.

II. THE BASIC INEQUALITY

The basic inequality is derived from elementary principles only. Once the temperature of an element of volume \( dV \) is above the critical temperature of the superconductor, there is a non-zero resistivity \( \rho \) in the material, an electric field \( \vec{E} \) to maintain the current running, and an energy \( d\epsilon_J \) dissipated by Joule effect in \( dV \) during an instant \( dt \).

\[
 d\epsilon_J = \frac{2}{\rho} \left( \frac{1}{I} \right) \frac{dV}{1 + r_w} \ dt
\]

where the bar over \( 1/\rho \) represents an average over the two constituents of the wire (superconductor and matrix) above critical temperature, and \( r_w \) is the ratio in the volume \( dV \) of the volume \( V_{\text{ins}} \) of the material that does not carry any current to the volume \( V_{\text{wire}} \) of the wire.
material. If \( r_{sc} \) is the volume ratio of matrix to superconducting material

\[
\left( \frac{1}{\rho} \right) \approx \frac{1}{1 + r_{sc}} \left( \frac{1}{\rho_{sc}} + \frac{r_{sc}}{\rho_{m}} \right) \gtrsim \frac{r_{sc}}{1 + r_{sc}} \frac{1}{\rho_{m}} \tag{2.2}
\]

where \( \rho_{sc} \) and \( \rho_{m} \) are the resistivities of the superconductor turned normal and of the matrix respectively. The symbol \( \gtrsim \) means "larger but almost equal to." This is true in Eq. (2.2) because \( \rho_{sc} \) is substantially larger than \( \rho_{m} \) in general. As in (2.1), the average \( (1/\rho) \) is extended only to the fraction \( dV/1 + r_{w} \) of the volume \( dV \) that consists of wire material, since there is no Joule effect in the rest of \( dV \). If \( A_{\text{wire}} \) is the cross-sectional area of the wire including superconductor and matrix, and if \( i \) is the current in the wire

\[
i = A_{\text{wire}} \frac{1}{\rho} |\dot{E}| \tag{2.3}
\]

Using (2.3), \( \dot{E} \) can be replaced in (2.1).

\[
d\varepsilon_{J} = \frac{i^{2}}{A_{\text{wire}}} \frac{dVdt}{(1 + r_{w})(1/\rho)} \approx \frac{i^{2}}{A_{\text{wire}}} \frac{1 + r_{sc}}{r_{sc}(1 + r_{w})} \rho_{m} dV dt \tag{2.4}
\]

During the same time interval \( dt \), the temperature \( T \) of the element of volume \( dV \) is raised by \( dT \). The wire material itself absorbs an energy \( d\varepsilon_{H,\text{wire}} \) proportional to its volume \( dV/1 + r_{w} \), to its average specific heat \( \bar{C}_{p} \) per unit of volume and to \( dT \).

\[
d\varepsilon_{H,\text{wire}} = \bar{C}_{p} \frac{dV}{1 + r_{w}} dT \tag{2.5}
\]

Per unit of volume, the specific heat of NbTi is about the same as the specific heat of copper, and therefore, is also larger than the specific heat of copper.
heat of aluminum. Let $C_{p,m}$ be the specific heat per unit of volume of the matrix material, whether it is copper or aluminum,

$$\frac{C_p}{C_{p,m}} > C_{p,m} \quad (2.6)$$

Heat conduction effects are hard to estimate accurately. However, our definition of $dV$ takes into account the possibility that some material is so close to the wire that it is practically at the same temperature as the wire at all times. Such material may be insulation or strengthening structure, etc. The volume of this material is $r_w/1+r_w$ $dV$ and $C_{p,ins}$ is its specific heat per unit of volume. A temperature rise $dT$ requires that this material absorb energy.

$$de_{H,ins} = C_{p,ins} dT \frac{r_w}{1+r_w} dV = r_c C_{p,m} dT \frac{dV}{1+r_w} \quad (2.7)$$

where

$$r_c = r_w \frac{C_{p,ins}}{C_{p,m}} = \frac{V_{ins} C_{p,ins}}{V_{wire} C_{p,m}} \quad (2.8)$$

i.e., where $r_c$ would be the ratio of the heat capacity of the volume of material that does not carry any current to the one of wire material if (2.6) were an equality.

The total heat absorbed by the temperature rise $dT$ in $dV$ is

$$de_H = de_{H,wire} + de_{H,ins} > \frac{1+r_c}{1+r_w} C_{p,m} dV dT \quad (2.9)$$

The energies $de_J$ and $de_H$ are not the same because of heat conduction effects. At the hottest point of the coil, heat conduction takes heat away from $dV$. Therefore, at the hottest point,
Therefore, from Eqs. (2.10), (2.9), and (2.4), we get

\[
\frac{C_{p,m}}{\rho_m} \frac{dT}{dt} < \left( \frac{i}{A_{\text{wire}}} \right)^2 \frac{1 + r_{sc}}{r_{sc}(1 + r_c)}
\]

(2.11)

The left-hand side of inequality (2.11) is the time derivative of a function of temperature and of the matrix material only, that we define as

\[
F_J(T) = \int_{T_{\text{crit}}}^{T} \frac{C_p}{\rho_m} dT
\]

(2.12)

where \( T_{\text{crit}} \) is the critical temperature of the superconductor. The function \( F_J(T) \) has been computed for standard materials, neglecting the difference between \( T_{\text{crit}} \) and 0°K. It is plotted for copper of different resistance ratios \( r_R \) on Fig. 1 and for aluminum on Fig. 2, where

\[
r_R = \frac{\rho_m \text{ at } 273°K}{\rho_m \text{ at } 4°K}
\]

(2.13)

\( F_J(T) \) is a monotonic function of \( T \). It can be used instead of \( T \) to define a temperature of the matrix material. Inequality (2.11) implies at the hottest point

\[
\frac{dF_J}{dt} < \left( \frac{i}{A_{\text{wire}}} \right)^2 \frac{1 + r_{sc}}{r_{sc}(1 + r_c)}
\]

(2.14)

and

\[
F_J(T) < \frac{\int_0^t i^2 dt}{A_{\text{wire}}} \frac{1 + r_{sc}}{r_{sc}(1 + r_c)}
\]

(2.15)
where the time integral has to be carried out from the time $t = 0$ at which the first element of the coil turns normal, i.e., the beginning of the quench. Since the specific heats $C_{p,\text{ins}}$ and $C_{p,\text{m}}$ vary with temperature, the ratio $C_{p,\text{ins}} / C_{p,\text{m}}$ of Eq. (2.8) is not constant. To make inequality (2.15) valid, $r_c$ has to be computed with the minimum value of the ratio $r_c$ as a function of temperature. In case of doubt, $r_c$ can always be chosen equal to zero, then inequality (2.15) will always be valid.

Inequality (2.15) has been derived for the hottest point of the wire. Since $F_J(T)$ is monotonic in $T$, $F_J(T)$ at any point of the wire is smaller than $F_J(T)$ at the hottest point. Therefore, inequality (2.15) applies to the temperature $T$ of all points of the wire. Inequality (2.15) is our basic inequality. It is valid for any time $t$ during the quench.

III. THE BURNOUT SAFETY CONDITION

The basic inequality (2.15) involves many parameters, some of which depend on the physical construction of the magnet and others on the characteristics of the external electrical circuit. The inequality can be expressed using coefficients that summarize the effect of several of the parameters in the same area of the magnet design.

Among the quantities that are determined by the magnet construction, there is a maximum temperature $T_{\max}$ that can be tolerated in the magnet, corresponding to a value $F_J(T_{\max})$ of the function $F_J(T)$. $A_{\text{wire}}$ is the cross-sectional area, $r_{\text{sc}}$ the volume ratio of matrix to superconducting material, and $r_c$ the heat capacity ratio of the material that does not carry current but gets heated with the wire to the wire material itself. The ratio $r_c$ defined in Eq. (2.8) is taken at its minimum value.
as a function of temperature. For maximum safety, it can be made equal to zero. The coefficients $f_j$ (dimensionless and of the order of one) and $C_j$ are defined as

$$f_j = \frac{(1+r_s)c}{1+r_s}$$

and

$$C_j = f_j A_{wire}$$

The matrix material determines the function $F_j(T)$ defined by Eq. (2.9) and plotted on Fig. 1 or 2.

The current in the magnet before quench, $i_0$, is a variable parameter which corresponds to different values of the magnetic field in a given magnet. The effective turn off time $\tau_j$ is defined by an integral involving the current $i(t)$

$$\tau_j = \int_0^{\tau_z} \left( \frac{i}{i_0} \right)^2 dt$$

where $t = 0$ is the beginning of the quench and $t = \tau_z$ is the time at which the current $i$ is essentially reduced to zero. The quantity $\tau_j$ can be changed by adding a quench protection circuit or modifying it in the electrical circuit.

Using the quantities defined above, the basic inequality (2.15) can be rewritten for the temperature $T$ anywhere in the coil

$$C_j F_j(T) < i_0 \tau_j$$

It is not easy to measure or predict $T$ everywhere in the coil but $C_j$, $i_0$, and the matrix material determine $F_j(T)$. Therefore, it is sufficient to measure and predict $F_j(T)$ along the axis of the coil.
and $\tau_J$ are easily accessible. Inequality (3.4) shows that, for a given current $i_o$, the temperatures produced in the coil during a quench will be smaller than a limit $T_{\text{lim}}$ that depends on $\tau_J$, i.e., on how fast the current is being turned off.

$$\frac{i_0 \tau_J}{C_J} = J$$  \hspace{1cm} (3.6)

The quantity $J$ is easier to measure than the temperatures in the coil and can be used to derive an upper limit for them.

An efficient technique to avoid burnout is to make sure that in case of quench the external electrical circuit turns the current off quickly enough, so that

$$T_{\text{lim}} < T_{\text{max}}$$  \hspace{1cm} (3.7)

This condition, together with inequality (3.5), shows that the temperature in the coil is always less than $T_{\text{max}}$. This condition (3.7) is our burnout safety condition and can be written as

$$i_o^2 \tau_J < q_{J,\text{max}}$$  \hspace{1cm} (3.8)

where the coefficient $q_{J,\text{max}}$ depends on the magnet construction only

$$q_{J,\text{max}} = C_J F_J(T_{\text{max}})$$  \hspace{1cm} (3.9)

The burnout safety condition determines a range of values for $\tau_J$ defined by (3.3) such that the danger of burnout is eliminated. This condition can be used for the design of the magnet or of the quench protection circuit as it is used in Section V. It can be used when testing
the magnet as shown in Section IV. Moreover, whenever a quench occurs, it is possible to check the burnout safety condition if an ad hoc circuit detects the quench quickly and if the time evolution of the current $i$ is recorded. The effective time $\tau_J$ of (3.3) can be computed from the time of the detection and, knowing the values of $i_0$ and $q_{J,\text{max}}$ of (3.9), the burnout condition can be checked. If it is violated, it may be advisable to improve the quench protection device or to introduce one if none was used before.

If (3.8) is satisfied, burnout does not occur but, if (3.8) is violated, the peak temperature may still be much smaller than $T_{\text{max}}$. The temperature $T$ would be equal to $T_{\text{lim}}$ only if, in place of (2.10), we could write $d\xi_J = d\xi_H$, i.e., if the heat conduction effects could be neglected. Thanks to heat conduction, $T$ may in fact be much smaller than $T_{\text{lim}}$. However, if deterioration is noticed in the coil after a quench, inequality (3.5) can still be used. $T_{\text{lim}}$ can be compared to some crucial temperatures given by Table 1 (page 3) to gain some insight as to the possible damage. The materials affected by burnout can only be those which are sensitive to temperatures less than $T_{\text{lim}}$.

The choice of $r_c$ to be used in (3.1) is not obvious. The safest choice is $r_c = 0$ that corresponds to the case where the insulator does not help to cool the wire at all. We have conducted tests with a coil without any quench protection and monitored $T_{\text{lim}}$ in all the quenches. The largest value of $T_{\text{lim}}$ was 900°K if we used $r_c = 0$ and 400°K if $r_c$ corresponded more or less to the packing fraction of the wire. No damage was found in the coil; therefore, the temperature probably never reached 900°K. This indicates that the value of $T_{\text{lim}}$ with $r_c = 0$ (i.e., ignoring the heat taken
by the insulator) represents a substantial over-estimation of the real peak temperature.

IV. A TESTING METHOD THAT IS SAFE AGAINST BURNOUT

This method consists of inducing quenches at increasing values of the current and analyzing the quench each time. This way we can find out if it is safe to proceed or if we risk burnout. It requires that the magnet be equipped with adequate hardware, capable of inducing quenches within milliseconds. Such hardware may consist of small coils (1 cm in diameter, 3 mm high, 200 turns, 0.5 mH), such as the ones we have used, installed within millimeters of the superconducting wire. The quench can be induced by $B$ effect, by discharging a capacitor of the order of 1000 µF charged up to several tens of volts in one of these small coils. In addition, the magnet should be connected to some electronic device that records the time evolution of the current.

The maximum allowable temperature, $T_{\text{max}}$, is determined by the detail design and the characteristics of the materials used (see Table 1). The constants $c_3$, $F_3(T_{\text{max}})$, and $q_3$ are determined as described in Section III, using (3.2) and (3.9). To avoid burnout, we make sure that the burnout safety condition (3.8) is always satisfied.

First of all, a current is run though the magnet at such a low value that we are sure the quenches are not dangerous. A quench is then induced and the current recorded versus time starting when the quench was initiated. From this record, $\tau_J$ of Eq. (3.3) can be computed. Since we deal with artificially induced quenches, the time at which the quench was initiated is known; therefore, the integral of Eq. (3.3) can be computed from the
actual start of the quench. Hopefully, the burnout condition of Eq. (3.8) is fulfilled. We can then try a larger value of the current, induce a quench again and check the burnout condition. If the burnout safety condition is still fulfilled, even larger currents can be tried until the danger of burnout prevents further current increases.

This stepping procedure can be made absolutely safe against burnout if, at each step, the new value of the current is made no larger than the value

$$i_{\text{max}} = \sqrt{\frac{q_{J, \text{max}}}{\tau_{J, \text{last}}}}$$  \hspace{1cm} (4.1)

where $\tau_{J, \text{last}}$ is the effective time $\tau_J$ of (3.3) computed from the evolution of the current during the last quench. As long as the last quench satisfies (3.8), $i_{\text{max}}$ is larger than the last value of the current, $i_{\text{last}}$. For the magnets we have tested so far, it is an experimental fact that $\tau_J$ decreases with $i$. Then, the next quench will be at a value $i_0$ such that

$$i_{\text{last}} < i_0 < i_{\text{max}}$$  \hspace{1cm} (4.2)

$$\tau_J < \tau_{J, \text{last}}$$  \hspace{1cm} (4.3)

$$i_0^2 \tau_J < i_{\text{max}}^2 \tau_{J, \text{last}} = q_{J, \text{max}}$$  \hspace{1cm} (4.4)

The burnout safety condition (3.8) will be satisfied for sure. This stepping procedure stops when $i_{\text{max}}$ and $i_{\text{last}}$ converge toward one another. Then, (4.1) shows that condition (3.8) will no longer be satisfied. Everything is fine if the design value for $i_0$ is reached. If not, it seems advisable to improve the quench protection.
It may be desirable to try the magnet without a quench protection circuit first. If the design current is not reached using the above-testing method, one should then introduce a quench protection device that reduces the effective time \( \tau_J \) in which the current is turned off. If the design value is still not reached with the protection device, that device should be improved so as to turn the current off more quickly.

In order to illustrate the testing method, Fig. 3 shows the results of a test we made of a 1-m-diameter coil with a conducting bore tube and without a protection circuit. The wire was 1 mm in diameter, \( r_{sc} \) was 1.8, \( r_R = 73 \); the coil self-inductance was 0.79 H. The coefficient \( r_c \) of (2.8) is estimated at 0.3. The steps were taken according to the method described here so as never to exceed 250\(^\circ\)K for \( r_c = 0.3 \), i.e., also 500\(^\circ\)K for \( r_c = 0 \). However, they were rounded down to an even 100 A. The coil never showed any sign of damage by burnout.

If no quench-inducing hardware has been built in the magnet, the monitoring of the burnout condition can only be done when spontaneous quenches occur, measuring \( \tau_J \) of (3.3) from the time the quench is detected. If quenches from training are not frequent enough, the monitoring of the burnout safety condition may be insufficient for complete safety against burnout. However, it is safer than a test of the magnet without any monitoring of that condition at all.

V. CONSTRAINTS ON THE MAGNET DESIGN

As pointed out in Ref. 1, quenches produce voltages and these voltages depend on the quench protection. Some consideration should be given to quench protection at the time of the design of the magnet to determine
the amount of insulation required. If the coil resistance $R_C$ grows fast enough without external intervention, no quench protection circuit is necessary. This fortunate circumstance may be revealed at the time of the test, if the method of Section IV is used for testing. However, it may be risky to rely on it before the test is made. Therefore, at the time of the design, the possible requirement of a quench protection should be considered. If the magnet is not insulated well enough, the safe testing technique of Section IV may require a quench circuit that would induce too much voltage in the magnet at the design current.

The most pessimistic case is the one where the resistance $R_C$ grows very little spontaneously during the required time and contributes very little to the turning off of the current. This most pessimistic case is considered in this Section. The resistance $R_C$ is set to zero in all equations. If quench protection works with $R_C = 0$, it will work even better if $R_C > 0$. This design of the quench protection circuit could even be the final design if, for one reason or another, the testing method of Section IV cannot be used. The magnet will still be safe against burnout, but the quench protection circuit will probably be overdesigned.

Moreover, this Section V describes magnets with a single coil, without conducting bore tubes, protected by a circuit as shown in Fig. 4. During normal operation, the switch $S_o$ is closed, and the current $i = i_0$ flows through the magnet. Suppose, at time $t = 0$, a quench starts. After it has been detected, the switch $S_o$ is opened at time $t_{S_o}$. The subsequent decay of the current is shown in Fig. 5

$$i = i_0 e^{-\frac{R_{ext}(t-t_{S_o})}{L}}$$  (5.1)
with $R_{\text{ext}}$ being the shunt resistance and $L$ the coil inductance. The effective time $\tau_J$ is given by (3.3)

$$\tau_J = t_{S_0} + \frac{L}{2R_{\text{ext}}}$$  \hspace{1cm} (5.2)

The burnout safety condition (3.8) can be written

$$R_{\text{ext}} > \frac{L}{2(\tau_J - t_{S_0})} > \frac{L}{2\left(\frac{q_{J,max}}{i^2_{o}} - t_{S_0}\right)} \approx \frac{LI^2_{o}}{2q_{J,max}}$$  \hspace{1cm} (5.3)

because, in general, for this sort of magnet it is possible to build a quench detector device that reacts so fast that $t_{S_0}$ is negligible.

$$R_{\text{ext}} > \frac{E_{o}}{q_{J,max}}$$  \hspace{1cm} (5.4)

where $E_{o}$ is the stored energy of the magnet

$$E_{o} = \frac{1}{2}L i^2_{o}$$  \hspace{1cm} (5.5)

When the switch $S_{o}$ opens, the current is still at its maximum value $i_{o}$ and is suddenly driven into the resistor $R_{\text{ext}}$. At that time the voltage is maximum (see Fig. 5).

$$v_{\text{max}} = R_{\text{ext}} i_{o}$$  \hspace{1cm} (5.6)

To satisfy the burnout condition (5.4), $v_{\text{max}}$ should satisfy

$$v_{\text{max}} > \frac{E_{o} i_{o}}{q_{J,max}}$$  \hspace{1cm} (5.7)
For a magnet with a conducting bore tube or with secondary windings, Eq. (5.1) is not valid anymore. It is possible to turn off the current very quickly without inducing large voltages because the induced current in the bore tube keeps the flux for a long time. It can then be shown that the condition (5.7) can be relaxed considerably. In addition, there are techniques to force the coil to become normal as soon as a quench is detected. Then $R_C$ can be proved to be substantial. These possibilities will be the subject of future papers.

VI. CURRENT DENSITY IN LARGE MAGNETS

Replacing $q_{J, \text{max}}$ by its expression (3.9) and using (3.2), we get another expression of the burnout safety condition for the type of magnets considered in Section V:

$$A_{\text{wire}}^2 > \frac{E_0 i_0}{f_J F_J(T_{\text{max}})}$$

(6.1)

For the most common magnets, $f_J$ given by (3.1) will be of the order of 1, $T_{\text{max}}$ will be less than 300°K, and the matrix material will be copper with a resistance ratio of the order of 100. It follows that

$$F_J(T_{\text{max}}) \approx 10^{17}$$

(6.2)

$$A_{\text{wire}} > \frac{E_0 i_0}{10^{17} v_{\text{max}}}$$

(6.3)

Inequality (6.3) gives a condition for the wire dimensions. It can be modified to give a constraint on the effective current density commonly defined as
\[ j_{\text{wire}} = \frac{i_o}{A_{\text{wire}}} \]  

\[ j_{\text{wire}} < \sqrt{\frac{10^{17} i_o v_{\text{max}}}{E_0}} \]  

(6.4)  

(6.5)

For practical reasons, \( i_o \) will not be much more than the order of one kiloampere, and \( v_{\text{max}} \) not much more than the order of one kilovolt. It follows that

\[ j_{\text{wire}} \approx \sqrt{\frac{10^{23}}{E_0}} \]  

(6.6)

In order to illustrate the principles discussed in Section V, we show a scatter diagram on Fig. 6, on which each dot represents a magnet that works or that is being built. The abscissa is the magnet stored energy \( E_0 \) and the ordinate is the current \( j_{\text{wire}} \) of (6.4). The solid line represents the expression (6.6) with an equal sign in it. It is significant that almost all dots fall below the solid line, as predicted by (6.6) and our analysis of Section V.

The dots, very much above the line, correspond to magnets of a recent design. These magnets have a bore tube made out of conducting material. As indicated above, these magnets escape the conditions expressed in Section V. Therefore, (6.6) does not apply. The use of a conducting bore tube permits higher current densities for the same energy \( E_0 \).

Note that for magnets with a conducting bore tube, the properties of the safe testing method (Section IV) still apply. Only the constraints on the design exposed in Section V can be revised.
VII. CONCLUSIONS

From the above developments, it seems advisable to

a) Design magnets so they are able to withstand voltages as large as

\[ v = \frac{E_i}{q_{J,max}} \]  

(7.1)

where \( E_i \) is the magnet-stored energy, \( i_0 \) the design current and

\[ q_{J,max} = \frac{r_{sc}(1+r_c)}{1+r_{sc}} A_{wire}^2 F_j(T_{max}) \]  

(7.2)

where \( r_{sc} \) is the matrix material to superconductor volume ratio, and \( r_c \) is the volume of the material that does not carry any current but gets heated with the wire, multiplied by its specific heat per unit volume, divided by the volume of the wire times the specific heat per unit of volume of the matrix material (Eq. 2.8). \( A_{wire} \) is the wire cross-sectional area, \( T_{max} \) is the maximum allowable peak temperature in the coil, and \( F_j(T) \) is the curve on Figs. 1 and 2 that corresponds to the proper matrix material. (See Section III for more detailed explanation.)

b) Use one of the special features such as conducting bore tube and/or induced normal transitions if item a) is difficult to fulfill. Future papers will discuss the consequences of the burnout condition for magnets equipped with such features.

c) Introduce devices in the magnet for inducing quenches and for recording the time evolution of the current during quench.

d) Use the testing method of Section IV, where quenches are induced in the magnet at different values of the current. The first value is low,
the following are in increasing order. The size of each step is limited by the value given by the algorithm (4.1), where \( \tau_{J,\text{last}} \) is the effective time defined by Eq. (3.3) applied to the evolution of the current during the last quench, and \( i_{\text{max}} \) is the maximum value of the current to be used at the next step.

e) Improve the quench protection circuit every time the burnout condition (3.8) becomes marginal.

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REFERENCES


11. M.A. Green, "The Large Superconducting Solenoid for the MINIMAG Experiment," Advances in Cryogen. Engineer. 21 (1975); LBL-3677.


15. We disagree with the statement made in Ref. 1 that secondary windings represent an inefficient use of material. That topic will be the subject of future papers.
Fig. 1. $F_j(T)$ versus temperature $T$ and enthalpy per unit of volume for copper-based superconducting wires of different resistance ratios $r_R$. $F_j(T)$ is defined by Eq. (2.12) and $r_R$ by (2.13).

Fig. 2. $F_j(T)$ versus temperature $T$ and enthalpy per unit of volume for aluminum-based superconducting wires of different resistance ratios $r_R$. $F_j(T)$ is defined by (2.12) and $r_R$ by (2.13).

Fig. 3. Each dot represents an induced quench with the quench number as a label, the value of the current $i_o$ at which the quench was induced in abscissa and the value of $T_J$ calculated from Eq. (3.3) in ordinate. The solid sloped line indicates the value of $T_J$ that corresponds to an equal sign in Eq. (3.8) for $c = 0.3$ and $T_{max} = 250^\circ K$ or $c = 0$ and $T_{max} = 500^\circ K$. When the dot corresponding to a quench is plotted on the figure, the horizontal line drawn through that dot crosses the sloped line at a abscissa equal to the $i_{next}$ of the quench for the corresponding $T_{max}$. The dots' numbers show a sequence in time order from left to right.

Fig. 4. Electrical circuit representing a single coil magnet equipped with its current supply and the simple quench protection circuit discussed in Section V.

Fig. 5. Time evolution of the current and of the voltage for a single coil magnet protected by the circuit of Fig. 4 when the coil resistance $R_c$ can be neglected. The quench starts at $t = 0$ and the switch $S_o$ opens at time $t_{S_o}$.

Fig. 6. Scatter plot of current densities $j_{wire}$ versus energy $E_o$ for some superconducting magnets. The solid line represents the theoretical limit (6.5) for single coil magnet protected by a circuit of the type shown on Fig. 4, for $i_o \cdot v_{max} = 1000 A \cdot 1000 V$.  

FIGURE CAPTIONS
Fig. 1
Fig. 2

ALUMINUM MATRIX

$F_j(T)$ (amp$^3$ sec/m$^4$)

Temperature (K)

Enthalpy (MJ/m$^3$)

$r_R = 1000, 300, 100, 30, 14, 10, 5$
Fig. 3
Switch $S_0$

Magnet coil

$R_{ext}$

Current supply

Fig. 4
Current $i$ vs Time

Voltage $v$

$t_{S_0}$

Fig. 5
Fig. 6

Superconductor matrix current density, $j_{\text{wire}}$ (A m$^{-2}$) vs. Magnet stored energy $E_0$ (J)

- A Magnet
- B Magnet
- C Magnet
- TPC Magnet
- CELLO Magnet
- PLUTO Magnet
- ISR Magnet

- Solenoids built
- Other types built
- Storage ring solenoids built
- Storage ring solenoids proposed
- LBL test coils

$\mathbf{i_0 V_{\text{max}} = 1000 \text{ A} \cdot 1000 \text{ V}}$

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