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REMOTE-REFERENCE MAGNETOTELLURICS: EQUIPMENT AND PROCEDURES

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ABSTRACT

During the past three years, major advances in the magnetotelluric technique have improved the quality of magnetotelluric data to the point where random errors in the data are generally smaller than the uncertainty in the interpretation of the data. The major factor in this improvement has been the introduction of the remote reference technique, although the use of ultrasensitive magnetometers and minicomputers for in-field data processing have also been important. After reviewing the remote reference technique, this paper describes the equipment and procedures used for remote-reference magnetotellurics by a group at the University of California, Berkeley. Magnetometers using ac Superconducting QUantum Interference Devices typically have a

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sensitivity of $10^{-14} \text{ T Hz}^{-1/2}$, a dynamic range of $10^7$ in a 1 Hz bandwidth, and a slewing rate of $3 \times 10^{-5} \text{ T s}^{-1}$ at 10 kHz. The electric field measurements use conventional Cu - CuSO₄ electrodes. The remote magnetic reference signals are transmitted to the base station using FM analog telemetry. The data are collected and processed by a minicomputer based on a LSI-11 microprocessor; the essential results, for example the apparent resistivities and the tipper components, with their probable errors, are available in the field.
1. INTRODUCTION

Major advances in the magnetotelluric (MT) technique* over the past three years have improved the quality of MT data to a level where random errors are now typically smaller than the uncertainty in the interpretation of the results. The single biggest contributor to this improvement has probably been the remote reference technique, although technological advances in magnetic field sensors and in minicomputers for in-field processing of the data have also played vital roles. Our own work on magnetotellurics began in 1975 when we started using very sensitive cryogenic magnetometers incorporating SQUIDs (Superconducting QUantum Interference Devices).** In using these magnetometers, which have a sensitivity $10^{-14}$ T Hz$^{-1/2}$ ($10$ nT Hz$^{-1/2}$), we hoped we could significantly reduce the bias errors that often plagued magnetotelluric measurements. After several field experiments using the conventional single-station scheme in which one measures the two orthogonal horizontal components of the electric and magnetic fields, we concluded that the greatly reduced instrument noise offered by our magnetometers had not significantly improved the quality of our MT data, and that the noise was inherent in the measurement rather than in the measuring devices. This unequivocal conclusion ultimately led Gamble et al. (1979a) to introduce a second remote magnetometer to enable one to "lock-in detect" the naturally

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*For a review of the magnetotelluric technique, see Vozoff 1972.
**For a recent review and references to the literature, see Clarke 1980.
occurring plane wave magnetic and electric signals at the MT site. This remote reference technique substantially reduces bias errors in estimates of the various quantities measured in MT, and furthermore enables one to place reliable confidence limits on them (Gamble et al. 1979b). Originally our data were processed in the laboratory, but, more recently, we have added a minicomputer to collect and process data in the field.

The purpose of this paper is to describe our equipment and field procedures and to give details of our processing techniques. Primary considerations in the design were that the equipment be inexpensive enough for a rather restricted budget, yet flexible enough to enable us to make a variety of different measurements other than just magnetotelluric measurements. Since we have limited manpower, we designed a system that can be operated in the field by a two-man crew and transported in a single vehicle; however, we require a second vehicle to install and service the remote magnetometer. After our initial field experience, we also decided to power the system entirely with rechargeable batteries, thereby eliminating the inconvenience and possible dangers of a generator and avoiding its electrical and acoustical noise.

The outline of the paper is as follows: In Sec. 2 we review the remote reference method. In Secs. 3, 4, 5, and 6, we describe the H-field measurement, the E-field measurement, the telemetry, and the data acquisition system. In Sec. 7 we comment on data collecting techniques, while Section 8 contains some concluding remarks.
2. REMOTE REFERENCE METHOD

The goal of magnetotellurics is to determine the impedance tensor of the ground, \( Z(\omega) \), from simultaneous measurements of the horizontal components of the fluctuating magnetic and electric fields \( H_x(t), H_y(t), E_x(t), \) and \( E_y(t) \). The Fourier transforms of these fields are related by the equations

\[
E_x(\omega) = Z_{xx}(\omega)H_x(\omega) + Z_{xy}(\omega)H_y(\omega), \tag{1}
\]

and

\[
E_y(\omega) = Z_{yx}(\omega)H_x(\omega) + Z_{yy}(\omega)H_y(\omega). \tag{2}
\]

To obtain \( Z(\omega) \) from Eqs. (1) and (2) it was usual, prior to the remote reference technique, to multiply them by the complex conjugates of appropriate pairs of components of \( E(\omega) \) or \( H(\omega) \), and to average the resulting cross powers at each frequency over all sets of data. If the local fields are used for this purpose, the solution for each element of \( Z(\omega) \) inevitably contains at least one autopower (for example, \( |H_x(\omega)|^2 \)). The presence of noise increases the value of any autopower, however large the data set over which it is averaged, thereby introducing a bias (Sims et al. 1971; Kao et al. 1977; Goubau et al. 1978) into the estimate of \( Z(\omega) \).

To overcome this well-known difficulty, we developed the remote reference technique, which requires that two components of the magnetotelluric signals be measured additionally at a remote site. The remote fields are at least partially correlated with the MT signals, and
are used to "lock-in" detect them. In principle, the electric or magnetic field can serve as a reference, but since one usually expects the horizontal components of $H$ to be less contaminated by noise, and less highly polarized than the electric fields, we use $H_{x\text{r}}$ and $H_{y\text{r}}$ as the reference fields. The complex conjugates of the Fourier transforms of the quantities are used to multiply Eqs. (1) and (2) in turn, thus generating four equations that can be solved for $Z(\omega)$:

$$Z_{xx} = \frac{(E_{x\text{r}}^* H_{y\text{r}}^* - E_{x\text{r}} H_{y\text{r}}^*)}{D},$$

$$Z_{xy} = \frac{(E_{x\text{r}}^* H_{y\text{r}}^* - E_{x\text{r}} H_{y\text{r}}^*)}{D},$$

$$Z_{yx} = \frac{(E_{y\text{r}}^* H_{x\text{r}}^* - E_{y\text{r}} H_{x\text{r}}^*)}{D},$$

and

$$Z_{yy} = \frac{(E_{y\text{r}}^* H_{x\text{r}}^* - E_{y\text{r}} H_{x\text{r}}^*)}{D},$$

where

$$D = \frac{H_{x\text{r}}^* H_{y\text{r}}^* - H_{x\text{r}} H_{y\text{r}}^*}{H_{x\text{r}}^* H_{y\text{r}} - H_{x\text{r}} H_{y\text{r}}^*}.$$
shortest time. Inspection of Eqs. (3) to (6) shows that estimates of the impedance elements are independent of any time-independent gain or phase factor in the reference channel; it is necessary only that the MT and reference fields show at least a reasonable degree of correlation.

As a supplement to information gained from $Z(\omega)$, it is extremely useful to obtain an estimate of the tipper, $\tilde{T}(\omega)$, defined as

$$H_z(\omega) = T_x(\omega)H_x(\omega) + T_y(\omega)H_y(\omega), \quad (7)$$

where $H_z(t)$ is the vertical component of the fluctuating magnetic field. A nonvanishing tipper indicates horizontal contrasts in the resistivity of the ground. We estimate $\tilde{T}(\omega)$ by multiplying Eq. (7) in turn by $H_{x_r}^*(\omega)$ and $H_{y_r}^*(\omega)$, and averaging to obtain

$$T_x = (H_z H_{x_r}^* H_y H_{y_r}^* - H_z H_{y_r}^* H_y H_{x_r}^*)/D, \quad (8)$$

and

$$T_y = (H_z H_{y_r}^* H_x H_{x_r}^* - H_z H_{x_r}^* H_x H_{y_r}^*)/D. \quad (9)$$

Thus, a complete set of measurements involves the local fields $H_x, H_y, H_z, E_x, E_y$, and the remote fields $H_{x_r}$ and $H_{y_r}$. At present, the minimum separation required between the local and remote stations is unknown; we have used distances as short as 3 km and have found no evident bias errors in our results.

The following minimum equipment is needed to perform magneto-tellurics with a remote magnetic reference: a two-axis and a three-axis
magnetometer, means for measuring the electric fields, telemetry to transmit data from the remote reference site to the local site, a monitor for all seven channels, and a system to record simultaneously all seven channels. (In fact, we use a three-axis magnetometer for the remote reference, although we transmit only the two horizontal channels; obviously, it is logistically much more convenient to have two inter-changeable three-axis magnetometers.) Figure 1 is a schematic of our system showing the interconnection of these various components.

3. MAGNETIC FIELD MEASUREMENTS

There are two basic types of superconducting magnetometers based on the Josephson effect; namely, the rf SQUID and the dc SQUID (Clarke 1980). The rf SQUID is commercially available and much more widely used than the dc SQUID. However, the dc SQUID offers advantages in both sensitivity and ease of operation, and we have preferred to use a version of this device developed and fabricated by ourselves (Clarke et al. 1976). For this reason, we give a brief account of the fabrication and operation of this device, and of its use in the field.

The ac SQUID consists of two resistively shunted Josephson tunnel junctions connected in series on a superconducting ring (Fig. 2a). The practical device (Fig. 3) is constructed in a series of thin-film depositions through shadow masks onto a 3-mm diameter quartz tube. First, a 300-nm thick band of Pb (10 wt.% In) 10.7 mm long is evaporated around the tube, followed by the evaporation of a 250-μm wide 75-nm thick Au strip. Then, two 150-μm wide 300-nm thick Nb strips are dc-sputtered as
shown (at low temperatures the Nb makes a superconducting connection to the Pbln band). The niobium is thermally oxidized in air for 12 min at 130°C (typically), and immediately afterwards a 300-nm thick Pbln "tee" is deposited to form two Josephson tunnel junctions with areas of about $10^4 \mu m^2$. Next, the Pbln band is scribed midway between the Nb strips. Electrical connection is made to the SQUID by pressing on two pellets of In, as shown in Fig. 3. The entire sensor is coated with a thin insulating layer of Duco cement by submerging it twice in a solution of five parts acetone to one part Duco, and a 300-nm thick Pbln ground plane (not shown in Fig. 3) is evaporated over the front surface of the SQUID. This superconducting sheet reduces the stray inductance of the various metal strips, and minimizes magnetic flux leakage through the slit in the Pbln band. The SQUID is completed by attaching 80 µm-diameter copper wires to the In pellets.

The SQUID is operated immersed in liquid helium at 4.2 K. The current-voltage (I-V) characteristic of a typical device is shown in Fig. 2b. As the current is increased from zero, no voltage appears until it exceeds the critical current, a few µA. At higher currents, the resistance is typically 0.5 Ω. The critical current of the SQUID and the shape of the curve at low voltages oscillate as a function of the magnetic flux, $\phi$, applied along the axis of the SQUID, the period being exactly one flux quantum, $\phi_0 = h/2e = 2 \times 10^{-15}$ Wb (h is Planck's constant and $e$ is the electronic charge). One flux quantum applied to the SQUID corresponds to approximately 0.3 nT. The dependence of voltage on $\phi$ at fixed bias current is shown in Fig. 2c. Thus, the SQUID is, in essence, $\varepsilon$
flux-to-voltage transducer. One measures changes in magnetic flux of much less than $\phi_0$ by measuring the corresponding change in voltage; typically $dV/d\phi = 1 \mu V/\phi_0$, or about $5x10^8$ Vwb$^{-1}$.

To measure small changes in voltage across the SQUID, a 100-kHz magnetic flux with a peak-to-peak amplitude of $\phi_0/2$ is applied by means of an alternating current in a 10-turn coil mounted inside the quartz tube, and the resultant ac voltage is amplified by a superconducting transformer (Fig. 4) that optimally matches the low-output impedance of the SQUID to a low-noise FET preamplifier at room temperature. The primary coil, which is connected across the SQUID, is in series with a resistance of 0.2 $\Omega$ to avoid short-circuiting the SQUID. The secondary coil is coupled to a low-noise FET preamplifier at room temperature via a shielded coaxial cable. The transformer is resonated at 100 kHz by adjusting a trimmer capacitor at the input of the amplifier to give a Q of about 5, that is, a bandwidth of about 20 kHz. The flux modulation principle is shown in Fig. 5. When the quasistatic applied flux is $n\phi_0$ ($n$ is an integer), the 100-kHz ac flux generates a voltage at 200 kHz across the SQUID. However, when the applied flux differs from $n\phi_0$, a 100-kHz component appears across the SQUID, reaching a maximum at $(2n+1)\phi_0/4$. The phase of the 100 kHz voltage differs by $\pi$ between $(n + 1/4)\phi_0$ and $(n + 3/4)\phi_0$. The ac voltage is amplified and lock-in detected at 100 kHz (Fig. 4). The 200-kHz component is rejected by the resonant transformer and the lock-in detector. The output of the lock-in is linearly proportional to the amplitude of the 100 kHz signal across the SQUID, being positive (negative) when the SQUID voltage is in (out of) phase with the lock-in reference signal.
The output signal from the lock-in is integrated, filtered, and connected via a series resistor to the modulation coil to provide negative feedback. Thus, when a small flux change is applied to the SQUID, the feedback circuit produces an equal and opposite flux so that the SQUID becomes a null detector. The change in output voltage, $V_o$, is proportional to the flux change. In a typical device (enclosed in a superconducting shield to eliminate external flux noise), the spectral density of the SQUID noise is white, with a rms value of $4 \times 10^{-5} \phi_0$ Hz$^{-1/2}$, corresponding to $10^{-14} T$ Hz$^{-1/2}$ ($10 \mu T$ Hz$^{-1/2}$) at frequencies above $2 \times 10^{-2}$ Hz; at lower frequencies, the spectral density of the noise varies inversely with frequency. The flux sensitivity referred to the output is typically $30 \text{ mV} \phi_0^{-1}$, while the maximum allowed output voltage swing for the electronics is $=10 \text{V}$. Thus, the dynamic range of the system is about $10^7$ in a 1 Hz bandwidth. Under optimum conditions, the frequency response extends from dc to 35 kHz, and the slewing rate at frequencies up to 10 kHz can be as high as $10^5 \phi_0 s^{-1}$, corresponding to $3 \times 10^{-5} T s^{-1}$ ($3 \times 10^4 \text{Y} s^{-1}$).

In the magnetometer that we use for geophysics, each SQUID is mounted in a spiral tube consisting of a single (nonshorting) turn of 50 µm-thick Pb foil approximately 44 mm long and with an internal diameter of about 6 mm. This superconducting spiral ensures that only the axial component of the applied flux is incident on the SQUID. Three SQUIDs in their tubes are mounted orthogonally in a G-10 fiberglass block approximately 44 mm in diameter and 60 mm in length. This block is suspended by fiberglass rods or, in later designs, a 45 mm-diameter
fiberglass tube, at the bottom of a fiberglass cryostat containing liquid helium at atmospheric pressure. These commercially available fiberglass cryostats do not use liquid nitrogen, and contain many layers of superinsulation (aluminized mylar) in the vacuum space between the helium container and the outer vessel. The superinsulation is attached to a shield cooled by the cold helium gas before it is vented into the atmosphere, thereby greatly reducing the radiation losses. For the local magnetometer, we use a 5 l-capacity cryostat that has a hold-time of five days. The cryostat is 0.24 m in diameter and 0.81 m high, with an additional 0.25 m for the electronics; the total weight of the system (including shielos and electronics) is 22 kg. Figure 6 shows the cryostat as operated in the field. For the remote reference cryostat, which we move relatively infrequently, we use a 25 l cryostat 0.44 m in diameter and 1.02 m high (including electronics) with a total weight of 41 kg. This cryostat has a hold-time of about 20 days. A pair of twisted 75 μm diameter manganin wires inside a 0.75 mm outer-diameter CuNi shielding capillary carries the ac modulation current for each SQUID. Three 75 μm diameter manganin leads inside a similar CuNi capillary that serves as a common electrical ground supply the dc bias currents to the three SQUIDs. Signals from the secondaries of the three transformers are connected to the electronics by coaxial cables each consisting of a 100 μm diameter copper wire inside a 0.75 mm CuNi capillary. All of the necessary electronics are contained in two milled aluminum boxes mounted on top of the cryostat. The three SQUIDs share a common 100 kHz oscillator with individual amplitude adjustments, but each SQUID has its own amplification, lock-in detection, and feedback circuit. The total power requirements are +20 V at 100 mA, and -20 V at 90 mA; power is supplied by four 12 V car batteries that give a typical running time of a week.
In order to eliminate troublesome rf pick-up from radio, television, and radar stations, the cryostat is encased in a copper mesh that makes electrical contact with the electronics boxes. The cryostat is also surrounded by a 1 mm-thick copper can that acts as a single-pole low-pass filter with a typical 3 dB-frequency of 55 Hz. This filter minimizes the chances of overloading by distant fast lightning strikes. The roll-off of this filter is compensated in the subsequent data processing. Johnson noise currents in the can reduce the rms sensitivity of the magnetometer by an order of magnitude. The sensitivity and orthogonality of the SQUIDs are calibrated by means of ac fields generated by external Helmholtz coils. The SQUIDs are orthogonal to ± 0.5°, and the direction of one horizontal SQUID is scribed permanently on the mount at the top of the cryostat to which a magnetic compass can be attached.

The maximum output voltage of our SQUID magnetometers is about ±10 V, which usually corresponds to about 10^{-7} T (100 γ). Occasionally, the magnetic field variation exceeds this value, causing the magnetometers to saturate. To deal with this problem, an automatic reset circuit is incorporated into the electronics. The reset circuit consists simply of a comparator that senses the output voltage, and a normally open FET switch that shorts-out the capacitor across the integrator when the comparator fires.

The power spectral density of the magnetic signals increases roughly as T^{6} for periods, T, between 3 and 30s, and roughly as T^{2} from 30s to much higher periods. This means that the mean amplitude in constant-Q bands is at least 10^{4} times larger at 10^{3} s period than at 3 s. To reduce the dynamic range required of the processing electronics, we prewhiten the
magnetic signals at the output of the magnetometers with a single-pole high-pass filter with a 6 s time constant. This procedure also greatly reduces overlap from long-period to short-period spectral estimates. The capacitors used in the filters are of metalized polycarbonate to minimize the temperature sensitivity, and are matched between channels.

Whenever possible, the cryostats are buried below ground level. Each cryostat is leveled and aligned by means of a detachable compass, and earth is packed around the base to steady it. In the case of the 52-cryostat, three adjustable legs are sometimes used to brace it against the sides of the hole; the battery and output cables are connected and the cryostat covered with a sheet of plywood weighted down by earth or rock. These precautions ensure a stable cryostat that is almost unaffected by wind and reasonably well-protected against animals. When the ground is too rocky to bury the cryostat, we surround it with a wooden wind-shield, carefully arranged not to touch the cryostat. The local magnetometer is usually placed about 200m from the recording truck and is connected to it by three coaxial cables. This separation is sufficient to eliminate magnetic interference from the truck and operating personnel. The amplifiers and transmitter for the remote reference are typically 10m away from the reference magnetometer. We determine that both magnetometers are operating properly by observing the correlation of the micropulsations (at about ~30 s period) between corresponding channels of local and remote stations.

The SQUID magnetometers have proved to be exceptionally rugged and reliable; over the past five years they have been the least troublesome of
all our field equipment. They have been used successfully in the field by technicians who have had no previous experience with cryogenic techniques and only minimal training. The electronic gain, modulation amplitude, and dc bias level of each SQUID is set in the laboratory and is usually not adjusted at all during the entire trip. Thus, after setting up the cryostat, one merely switches on the power to make the system operational. Use of liquid helium has not proved to be an inconvenience. We initially transported our own supply of helium on field trips in a superinsulated 30 l storage dewar, but subsequently found it more convenient to have a commercial supplier ship helium to the nearest town. We refill the 5 l cryostat every fourth day, while the 25 l cryostat has a sufficient hold-time to last for the duration of most of our surveys. This is especially convenient if a single reference site is planned. For transportation, both cryostats must be fitted with overpressure blow-off valves; otherwise the cryostats will suck in air when they are transported from higher to lower elevations, which results in the formation of ice that may block the venting tubes.

We conclude this section with a discussion of several external sources of noise that can be particularly troublesome. In populated areas, 60 Hz and 180 Hz magnetic fields from power lines dominate. Even though noise at these frequencies may not interfere directly with the MT signals, it can limit the dynamic range of the magnetometer and may even saturate the instrument. In farming areas, it is not uncommon to have 60 Hz levels in excess of $10^{-8}$ T ($10 \gamma$) even when there are no power lines or underground cables within 500 m of the magnetometer. Presumably, this noise
comes from currents in the earth that are produced when pump motors and utility poles are electrically grounded at more than one point. Power-line noise is usually highly polarized, so that the pick-up is much larger on one axis than on another. Thus, it is sometimes possible to distribute the pick-up equally between the two horizontal axes by rotating the magnetometer appropriately so that neither channel saturates.

Low-frequency cultural noise is very unpredictable and can change abruptly from one minute to the next. At times the output of the magnetometer may wander about randomly, while at other times the noise may be in sharp bursts. Occasionally, we have seen bursts of perfectly sinusoidal noise at a few hertz. We have not been able to identify the sources of these noises but have found that moving the magnetometer to a new location less than 1 km away can often eliminate most of the interference.

Automobiles generate both magnetic and seismic disturbances. In most cases, the magnetic disturbances are negligible when the magnetometer is more than 200m from the road, although we have seen $10^{-9} \text{T (1 y)}$ signals from flat-bed trucks carrying steel girders at a distance of 300m. Seismic noise is transformed into magnetic noise primarily through the slight movement of the magnetometer in the uniform magnetic field of the earth. If $\theta$ is the angle between the sensor and the earth's field, $H$, rotation by an angle $d\theta$ is equivalent to a change in field $dH = H[\cos(\theta+d\theta) - \cos\theta]$. For the worst case, $\theta = 90^\circ$, a rotation of only 0.002° produces a field change of $10^{-9} \text{T (1 y)}$. Seismic noise often has a longer range than magnetic noise and therefore may be more difficult to
avoid. In wet muddy soil, we have observed large seismic oscillations at about 1 Hz caused by trucks over 500m away. Seismic interference is considerably reduced when the roadbed is on firm bedrock.

In certain locations, strong radar beacons can interfere with the operation of a SQUID magnetometer when it is above ground level, despite the existence of rf shields. To shield against these beacons, it is sometimes necessary to surround the wind screen of the magnetometer with aluminum foil. If this is done, extreme care must be taken to ensure that the foil is not free to move in the wind and that the wind screen is rigidly anchored. Any motion of the foil in the earth's magnetic field will produce currents that are detected directly by the magnetometer.

The largest and most common source of natural noise is wind, which can either move the magnetometer directly or induce seismic noise by blowing on trees or bushes whose roots then move the ground. If the ground is not firm, significant seismic noise can also result when the wind blows on a nearby earthen dike or hillock, even if there is no vegetation. Thus, ideally, the magnetometer should be in a flat area and buried so as to present the lowest possible cross-sectional area to the wind.

4. ELECTRIC FIELD MEASUREMENTS

We measure electric fields by burying three Cu-CuSO\(_4\) electrodes (Tinker and Razor model 3-A) in an L-shaped array and measuring the voltages between the distant electrodes, x and y, and the common electrode, c, that is typically 5m from the recording vehicle. The electrodes are placed as far as possible from sources of cultural noise, with x-c, y-c
spacings of either 100m or 200m. We use the longer separation when the anticipated apparent resistivity is less than about 5 Ωm to ensure that the MT signals will dominate the intrinsic electrode noise (~1 μV Hz−1/2) at most frequencies. Geography and topography often prevent the array from being perfectly orthogonal, and the legs of the array from being exactly equal. These factors are taken into account in the computer program that calculates the impedance elements. We determine the orientation of the array with a Brunton compass to within ±0.5°, and the electrode spacing with calibrated lengths of electrode wire to within ±1 percent. The electrode wire is 3 mm diameter steel-reinforced telephone wire that stretches negligibly when pulled taut.

The electrode wires are anchored to the ground with dirt or rocks at intervals of about 1m to minimize voltages that can arise when the wires are blown by the wind in the presence of the earth's magnetic field. A 1m-length of wire vibrating at 1 Hz with an amplitude of only 1 mm generates about 0.5 μV, a voltage that is comparable to the telluric signals at that frequency. Thus, considerable care is taken to anchor the wires securely. Signals from the electroodes are brought into the truck via isolated feedthroughs in a side wall. The feedthroughs are shielded from direct sunlight to minimize the possibility of generating thermal emfs at the contacts. It is very important that none of the wires (not even the one from the common electrode) are electrically connected to the frame of the vehicle, since the vehicle is grounded through its tires with a resistance that is often less than 100 kΩ. If a wire does short to the body of the truck, even the slightest motion of the vehicle can result in
enormous electrical noise as the contact resistance between tires and earth fluctuates. Inside the recording vehicle, the wires from the feedthroughs are ac coupled to a differential preamplifier with a gain of 50 and an intrinsic noise of 100 nV Hz\(^{-1/2}\) at 1 Hz. The signals then pass through a 60 Hz-notch filter followed by a high-pass filter with a 1000 s time constant and a gain of 20 at high frequencies. Prewhitening of the electric signals is not as important as it is for the magnetic signals. Over a homogeneous earth, the ratio of the electric to the magnetic-field power is proportional to the frequency. Thus, over the range \(10^{-3}\) to 40 Hz, the dynamic range of the electric fields is typically 200 times smaller than that of the magnetic fields. The electronics are powered by car batteries that provide at least a week of continuous operation without recharging.

To obtain the best data for the electric fields, we usually fill the electrodes with water at least one day prior to their use to ensure that the CuSO\(_4\) is saturated and that it has seeped through the porous ceramic pot. When the electrodes are placed in the holes (about 0.2 m deep), a small amount of water or electrolyte from the pot is used to moisten the soil and thereby reduce the contact resistance between the electrode and the ground. We check the continuity of the telluric dipoles with a high-impedance (greater than 1 M \(\Omega\)) ohm-meter so that no significant current is introduced by the measurement. In some types of soil, a few milliamps of current can quickly cause a significant polarization of the ground around the electrodes, leading to a long exponential decay of this potential with associated higher frequency noise. However, if a high-impedance ohm-meter is used, one should measure the resistance with
both polarities because the reading can be drastically affected by the self-potential voltage. Contact resistance of the telluric electrodes should be as low as possible; a typical value measured between the two electrodes is 1 kΩ, but the resistance can vary from 0.3 to 30 kΩ, depending on the local geology. The electrode is shielded from direct sunlight by a board placed over the hole to prevent drying of the soil and to reduce temperature gradients in the electrode that might alter the liquid junction potential. After planting the electrodes, we allow at least one hour for the electrodes to reach thermal and chemical equilibrium before recording data. To minimize the delay associated with equilibration, we plant the electrodes as soon as possible after arriving at an MT site so that they are ready for use by the time the remainder of the equipment has been set up and tested.

The two major sources of electric-field noise are cultural noise and self-potential voltages. We check the ac voltages with an oscilloscope to determine if there is danger of saturation of the preamplifiers; if the voltages are too large, we may have to relocate and/or shorten the telluric lines. If cultural noise is unavoidable, we take considerable care to minimize its influence on the data. Common sources of cultural telluric noise are power lines, irrigation pumps, and seismic vibrations arising from automobile traffic near an electrode. Noise from power lines is predominantly at 60 Hz, but a large 180 Hz component is often present if there is a nonlinear load on the power line. The amplitude of both the 60 Hz and the 180 Hz noises can be as large as 1 V near power lines, but we have generally been able to keep the amplitude below 200 mV by moving the
array a few hundred meters from the lines. In agricultural areas where equipment is powered by small generators, the line frequency may vary by a few Hz. Thus, the Q of the notch filters should be low enough to accommodate such variations. We have found that Q = 10 is adequate for most practical purposes. Electrical noise from irrigation systems has low-frequency components, as well as 60 and 180 Hz. The noise is often in the form of steps or square pulses in the electric field. We are not certain of the origin of these steps, but suspect that they are associated with the turning on and off of pump motors. The dc component of the noise may be caused by partial rectification of 60 Hz current flowing through a ground electrode of the pump. To minimize this form of interference when selecting MT sites in farming areas, it is advisable to inquire about the location of underground wires and the nearest operating irrigation system. Seismic vibration produces telluric noise through the modulation of the liquid junction potential between the electrode and the ground. Whenever possible, we keep our electrodes at least 300m from major highways to avoid noise from this source.

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Constant self-potential voltages can saturate the dc-coupled telluric preamplifiers. Thus, some provision must be made for either bucking out such voltages or reducing the gain of the preamplifier stage. We use a battery in the feedback circuit of the preamplifier to allow an input voltage range of ±0.4 V. This has proved to be inadequate on occasion, however, as we have observed steady voltages as large as 0.5 V on 100m lines. It is particularly important to minimize the contact resistance in the presence of large self-potentials. For example, if there
is a self-potential of 500 mV, a change in the contact resistance of 0.1 percent of the amplifier input resistance would cause a change in the measured voltage of 0.5 mV that may be comparable to the signal voltage.

5. TELEMETRY

We use conventional FM analog telemetry designed for seismic field work to transmit the remote measurements to the base station. Both sets of telemetry are powered by car batteries. The telemetry package (W.F. Sprengnether Instrument Co.) includes the voltage controlled oscillators (VCO), multiplexer, FM-VHF transmitters and receivers, and discriminators (see Fig. 1). The VCOs convert signal voltages into frequencies with a variation of ±125 Hz around center frequencies spaced at 340 Hz intervals. These signals are summed by the multiplexer and used to modulate the FM transmitter. At the base station, each discriminator selects the band of frequencies within 125 Hz of its center frequency from the output of the receiver and converts the frequency back into a voltage with a phase-locked loop.

The VCO-discriminator mode of signal transmission is less expensive than digital telemetry. Also, the noise characteristics are well-suited for magnetotellurics because the lowest noise is obtained in the 0.1-10 Hz band, where the magnetotelluric signal levels are lowest. Analog telemetry has the additional advantage over digital telemetry in that it requires far less bandwidth for a given dynamic range. To obtain a 60 dB dynamic range in a digital system, one would require ten-bit words, while to make measurements up to 50 Hz one would
require at least 100 samples per second. Thus, each channel would need at least 1000 bits per second, corresponding to a bandwidth of at least 2 kHz. In contrast, nine channels of analog signals with 340 Hz spacing can be fitted within the 3 kHz bandwidth of the FM transmitter and receiver.

In practice, we have found two modifications of the telemetry to be useful. First, we have observed drifts with temperature over a significant fraction of the full range. We reduced this drift somewhat by adding an extra channel, shorting the input to its VCO, and subtracting the output of its discriminator from the output of the signal channels. Second, when the 340 Hz spacing was used, we observed significant noise from the interaction between channels. We could reduce this noise by using more selective filters, but since we use only three channels we simply space them at 680 Hz.

We employ one of several rf frequencies in the vicinity of 170 MHz. The transmitters have 500 mW output power, and the horizontally polarized antennas are five-element Yagis. This combination provides a line-of-sight range of at least 80 km. However, in rough terrain the line of sight is often very limited—a problem that can be surmounted fairly easily by using a repeater station consisting of a transmitter, a receiver, and two antennas at a suitably high point. If difficulties are experienced in establishing communication, we have found the most important variable in the installation (other than orientation of the Yagis) to be the height of the antennas. They should be at least one wavelength (~2m) above the ground, above any large metal object, and, if possible, above any intervening vegetation. Attenuation by vegetation at VHF frequencies is
substantial, but generally less for horizontal than for vertical polarization. It is advisable to test that the FM carrier can be received at the base station before setting up the remote magnetometer.

6. DATA ACQUISITION SYSTEM

Two different systems operating simultaneously are used to collect the data. The low-frequency data (<0.1 Hz), requiring a long time to collect but little storage capacity, are amplified to an appropriate level and recorded on a Gould 6000 digital data logger. The sampling rate of the data logger itself is 1 Hz, but a third-order decimation is performed to produce a sampling rate of 0.33 Hz. This permits one to calculate accurate results at periods of 1000s with a transform length of 1024 points. A five-point triangular averaging window in the time domain is used to prevent aliasing during the decimation. We usually process the data in the laboratory by interfacing the logger with a Digital Equipment Corporation LSI-II minicomputer operating under RT-11. The high-frequency data (>0.01 Hz), which take little time to collect but for which a large number of segments are usually required to obtain accurate results, are analyzed in the field with the LSI-II minicomputer (see Fig. 7). In this real-time analysis, the signals are Fourier transformed and the autopowers and crosspowers, averaged over a suitable band of frequencies, are calculated. The in-field analysis not only reduces the amount of information to be stored by a factor of roughly 500, but also allows the operator to examine the quality of the data in the field. Preliminary interpretation of the soundings in the field is therefore possible immediately, enabling one to
decide when sufficient data have been collected at each station, and permitting one to select sites for further measurements. Figure 1 contains a block diagram of the data acquisition electronics. All seven channels are processed identically once the remote signals have been discriminated, and the base station signals have been preamplified.

The high-frequency signals are passed through matched Ithaco switchable band-pass filters to prevent aliasing and spectral overlap from large long-period signals, and are amplified to a level that makes the best use of the dynamic range of the LSI-11 digitizer (±5 volts). Figure 8 is a block diagram of the LSI-11 hardware. All of the major components are standard equipment available from DEC. Similar boards with satisfactory specifications are available from a number of other manufacturers, and all of the CPUs of the LSI-11 family are satisfactory. The hardware falls into three groups: 1) the CPU, 28K semiconductor RAM memory, and 512 words of ROM memory for loaders and memory diagnostics; 2) a programmable clock, a 12-bit 16-channel digitizer, and a D-to-A converter for data output; and 3) interface boards for communication and peripheral control. A few small custom circuits are employed in the interfacing, such as drivers for LEDs and a decoder for the thumbwheel switch, but these are available commercially or could easily be built for specific requirements by anyone with a knowledge of TTL logic.

Data and programs are stored on three-inch magnetic tape cassettes. These are read and written on two Phi-deck cassette decks controlled by a board designed by The Digital Group, Denver, Colorado. The status of the data collection is communicated to the operator through LED
displays and a four-digit hexadecimal display. The operator controls data collection through pushbuttons and a decimal thumbwheel switch. For presentation of the results, or for more sophisticated control of the computer, a thermal printer and a keyboard are used as a terminal.

Figure 9 is a flow diagram of the data through the computer. The computer can be used either for data collection or for calculating results. GGUT, the program for data collection, controls the sampling of the signals and produces all of the average auto- and crosspowers necessary for estimating the impedance tensor, the tipper, and statistical errors. The auto- and crosspowers are permanently stored on the cassette tape. We can program GGUT to accommodate any number of channels, subject only to the size of the memory. The largest number of channels we have used is nine, but one or two more should fit into the 28K memory with minor modifications without reducing the length of the Fourier transforms. This program is written entirely in the DEC Macro assembly language to minimize the size of the program and to maximize the processing speed. Final analysis and interpretation of the data are performed after returning from the field. However, we can get a summary evaluation of the data in the field from the RESULTS program; the Appendix lists the essential equations to be calculated. The program is written almost entirely in Fortran for ease of programming complex matrix inversion, and for high-level input/output control. With this program the operator can read the data tapes and select the blocks of data to be averaged together and processed. The operator uses the keyboard to enter the lengths and orientations of the telluric lines and the sensitivities of the magnetometers. The computer then
calculates and prints results selected by the operator. These results may include the signal-to-noise ratio of each of the measurement channels, the rotated apparent resistivities and phases with their probable error, and the tipper components and their probable errors.

The details of the various stages of data processing are as follows: The analog data are digitized with 12-bit resolution in 1024-point data segments. As the data are sampled, the digital words are converted back into analog form so that the operator can monitor them on a storage oscilloscope. The mean value of each data segment is subtracted and the ends tapered to zero by a linear taper over 256 points. The data are Fourier transformed using scaled 16-bit integer arithmetic that preserves approximately 11 bits of resolution in the Fourier coefficients. Products of the Fourier coefficients are computed and averaged over constant $Q = 3$ windows, using double-precision integer arithmetic. The averages are converted to floating-point form and added to the sum of the averages from previous data segments. While these calculations are being performed, collection of the next data segment continues on an interrupt basis, and the data are stored in the other input buffer. The last 25 percent of each data segment is used again as the first 25 percent of the subsequent data segment, so that the data in the tapered regions have the same statistical weight as data in the untapered regions. This procedure saves one-quarter of the data collection time when the sampling rate is low. When the sampling rate is so high that the second data segment is completed before the calculations on the first data segment are finished, the programmable clock is simply turned off until the
calculations are complete. Then the collection resumes, filling the first
buffer from the beginning.

The calculations require about 15 s per data segment when seven
channels of data are being collected, about 1.3 s for the tapering and
Fourier transformation of each data channel, and about 6 s for calculation
of the average powers. If the sampling rate is lower than 60 Hz, more than
15 s are required for collection of a data segment, and the segments will
be overlapped, so that all of the data are used. For higher sampling
rates, the segments will not overlap and not all of the available data will
be used, but analysis is still in real time. The maximum sampling rate is
limited by the software to about 6000 samples per second, or about 800 Hz
per channel for seven-channel operation. This rate is far in excess of our
needs since the analog electronics were designed to cover only the range of
frequencies below 60 Hz. A reorganization of GGUT for rapid data
collection could increase the sample rate to about 40,000 samples per
second.

The computer package requires +6 V at 18 A, and +12 V at 2 A; this
power is supplied by three 6 V 220 A-hr golf-cart batteries that provide
about 10 hours of continuous operation. The +6 V supply is reduced by
means of a variable series resistor to the lower end of the 5.2 ± 0.3 V range
in which the computer operates; because the battery voltage increases with
temperature, the supply could otherwise exceed the upper limit of the
allowed range.
7. COMMENTS ON DATA COLLECTION

Before recording low-frequency information on the data logger, we monitor the signals on all seven channels for about five minutes to estimate the maximum amplitude of the signals. The gain in each channel is adjusted so that the maximum signals are two or three times smaller than the saturation voltage. A conservative gain setting is important, since the peak signals may vary by factors of several over a period of hours. In particular, the signals are substantially larger at midday than they are early in the morning.

Next, we set the levels for in-field processing of the high-frequency data. Once data collection through the computer begins, the signals are monitored via the D-to-A output so that monitoring at other points in the signal path is not usually necessary. Nonetheless, it is convenient to have a slow-speed chart record, particularly of the remote channels, so that the operator need not monitor the signals continuously. We usually collect separately two bands of data in addition to the longer-period logger information: a high-frequency band sampled at 120 Hz, and a mid-frequency band sampled at about 3 Hz. These choices are easily modified by the operator. The high-frequency sampling rate is chosen to be a multiple of 60 Hz so that harmonics of 60 Hz will be aliased down to 0 and 60 Hz, outside the range of frequencies for which we obtain results from the high-frequency band. The low-pass or Nyquist filter cut-off is usually set at one-third the sampling frequency. This choice is not as conservative as is customary in some applications because in magnetotellurics one is interested only in the ratios of the electric and
magnetic fields, and any errors introduced by spectral aliasing are of second order. With this selection of filtering we maximize the frequency range covered by each band; at accuracies of better than one percent, we have not experienced any inconsistency between overlapping frequency bands that could be attributed to aliasing. We set the high-pass corner at the second harmonic of the fundamental frequency of the data segment, that is, at \( \frac{1}{512} \) of the sampling frequency. This eliminates spectral overlap from long period signals, but does not destroy any of the frequency coverage of the Fourier series since the third harmonic is the lowest harmonic used in the analysis. Thus, the high-frequency band covers approximately 0.4 to 40 Hz, while the mid-frequency band covers 0.01 to 1 Hz.

A final consideration in selection of the bands is that, even with prewhitening, the spectral density still increases steeply with decreasing frequency in the 0.03-0.3 Hz range. Resolution of the Fourier harmonics is proportional to the length of the data segments. In order to minimize errors due to spectral overlap in this range, it is therefore wise to use data segments long compared to 30 secs.

We collect data for at least 18 times the longest period for which results are desired. This quantity of data is necessary for the following reasons: The third harmonic is the lowest we use because the first two harmonics have very low spectral resolution and are very much affected by truncation errors. Thus, each segment must be at least three times as long as the longest period of interest. To obtain nondegenerate equations for the elements of the impedance tensor and the tipper, we must use at least two data segments. Finally, for the error estimates to have any validity,
we require at least three independent estimates of the impedance tensor and the tipper.

For periods longer than 30 s, the signals are generally sufficiently large so that collection of about 20 cycles of data ensures a sufficiently accurate estimate of the impedance. Thus, it is not really necessary to analyze the logged data in the field, although it is possible to do so. Usually, we carry out the final analysis of the logged data in the laboratory so that we have adequate time to edit the data records carefully. However, we do play back the tapes in the field at the end of each day to ensure that the recorder is functioning properly.

8. CONCLUDING REMARKS

The SQUID magnetometers have proved to be exceptionally rugged field-worthy instruments. The SQUIDs themselves have never failed despite the considerable physical abuse suffered by the cryostats. We experienced occasional failures in the electronics, but no more than in any of our other field equipment. On one occasion we rented a three-axis commercial magnetometer incorporating rf SQUIDs (manufactured by SHE Corporation), and found it to be as reliable as our own system.

Our data acquisition system has evolved to its present form over the course of six years, and represents what we feel is the simplest in-field processing system capable of handling MT data, yet versatile enough to be readily adapted to other types of geomagnetic measurement. While the system has been used successfully on many field trips, there are areas where we expect that significant improvements could be made as a
result of technological advances in the design of the equipment. One weakness in the system is the cassette tape recorder from which we load the computer programs. On occasion we have had difficulty loading the programs because of bit errors, and on hot days we have experienced problems with tape stretching during fast rewinds. We have greatly reduced these problems by writing the programs on the tape in triple redundancy (as we do when we store the average powers); but this, of course, has substantially increased the time required to read in the programs. If one could dispense with the flexibility in programming offered by the tape system, overall reliability could be enhanced by storing the programs in PROM memory rather than on tape. The data logger that we use to store low-frequency information has suffered repeated breakdowns and has required intensive maintenance. Moreover, it requires a continuous 12 V, 5 A power supply, which is a substantial amount of power in any remote area. To reduce power consumption, we would prefer to use a CMOS-based recorder, but, unfortunately, such recorders appear to be currently available with only a single track and a low sampling rate. Our thermal printer is only marginally satisfactory for presentation of results in the field. It is neither fast nor reliable, and the printer format is not easy to interpret. A graphic presentation of in-field results would be far more useful.

Our LSI-11 computer has functioned flawlessly in the field, with no breakdowns from overheating despite ambient temperatures in excess of 40°C. The computer has only two small cooling fans, and our vehicle is not air-conditioned. As this system has demonstrated, LSI technology has
advanced to the point where substantial computing power is available in a battery-powered minicomputer: The RESULTS program is almost identical to the program run on the CDC-7600 computer of the Lawrence Berkeley Laboratory.

Our MT data collection and processing system, although possibly not optimal for the needs of all magnetotelluric groups, has successfully satisfied our own particular requirements: immediate availability of results, flexibility in the scope of possible experiments, ease of operation by two individuals, and relative economy in cost. An MT group with different goals or resources might choose to construct their system somewhat differently. Regardless of emphasis, in-field processing of remote-referenced MT can now be performed accurately and reliably. The next logical extension of the in-field processing would probably be to perform approximate inversions of the measurements and to display graphically the results from several stations, simultaneously.

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APPENDIX

The in-field data collection and reduction program, GGUT, produces the cross- and auto-power spectral estimates, averaged over harmonics in a narrow frequency band and over data segments. We represent these, for example, as $E_x H^*_y$, as discussed in the text. The subsequent calculations are performed by the RESULTS program, either in the field or in the laboratory. The following sequence of calculations is followed in RESULTS to calculate the referenced impedance tensor, the apparent resistivities, the phases of the impedance elements, the skewness, the rotation angle of the coordinate frame for which $|Z_{xx} - Z_{yy}|^2$ is a minimum, the tipper, and the signal and noise powers of the magnetotelluric fields. Equations for the estimated errors are also presented. These results are derived in the paper by Gamble et al. (1979b), but are expressed here in slightly modified notation; minor errors that exist in Eqs. (23), (32) and (35) to that paper have been corrected.

Impedance Tensor

The referenced impedance tensor estimate is

$$Z = [E H_r] [HH_r]^{-1}, \quad (A1)$$

where

$$[AB] = \begin{bmatrix}
A_x B_x^* & A_x B_y^* \\
A_y B_x^* & A_y B_y^*
\end{bmatrix},$$

and $[AB]^{-1}$ is the matrix inverse of $[AB]$. The error $\Delta_{ij}$ in the $ij^{th}$
element of $Z$ is

$$\Delta_{ij} = \eta_i A_j / D, \ (i = x, y, j = x, y)$$  \hspace{1cm} (A2)$$

where

$$\eta_i = E_i - Z_{ix} H_x - Z_{iy} H_y,$$

$$A_j = H_{jr}(H^*_k H_{kr}) - H_{kr}(H^*_k H_{jr}), \ (k \neq j)$$

$$D = (H_{x x r})(H_{y y r}) - (H_{x y r})(H_{y x r}),$$

and $N$ is the number of independent Fourier harmonics of each field (number of data segments multiplied by the number of harmonics in the frequency window). The estimated variance in $Z_{ij}$ is

$$\text{var}(Z_{ij}) = <|\Delta_{ij}|^2> = \frac{1}{N} \frac{|\eta_i|^2 |A_j|^2}{|D|^2},$$  \hspace{1cm} (A3)$$

where $<$ > denotes an ensemble average.

The variances in quantities such as the apparent resistivities, phases, and skewness that are calculated from elements of $Z$ can be expressed in terms of the expectation values

$$<\Delta_{ij}^* \Delta_{nm}^*> = \frac{1}{N} \frac{|\eta_i \eta_n|^* (A_j^* A_m^*)}{|D|^2}.$$  \hspace{1cm} (A4)$$

The apparent resistivity matrix in $\Omega$ is
\[
\rho_{ij} = 0.2T|Z_{ij}|^2, \quad (A5)
\]

where \( T \) is the period in seconds, and the units of \( Z_{ij} \) are \( \text{(mV/km)/nT} \).

The variance in \( \rho_{ij} \) is

\[
\text{var}(\rho_{ij}) = 0.4T \rho_{ij} \text{var}(Z_{ij}). \quad (A6)
\]

The phase \( \phi_{ij} \) is defined as

\[
\phi_{ij} = \arg Z_{ij} = \tan^{-1} \left[ \frac{Z_{ij} - Z_{ij}^*}{i(Z_{ij} + Z_{ij}^*)} \right]. \quad (A7)
\]

The variance in \( \phi_{ij} \) is

\[
\text{var}(\phi_{ij}) = \left( \frac{1}{2|Z_{ij}|^2} \right) \text{var}(Z_{ij}). \quad (A8)
\]

The "skewness" is a parameter which serves as a measure of the three-dimensionality of the geology. It is defined as

\[
S = \left| \frac{Z_{xx} + Z_{yy}}{Z_{xy} - Z_{yx}} \right|. \quad (A9)
\]

The variance in \( S \) is

\[
\text{var}(S) = \frac{1}{2N|D|^2|Z_{xy} - Z_{yx}|^2} \left[ U + S^2V - 2\text{Re} \left\{ \frac{(Z_{xx} + Z_{yy})(Z_{xy} - Z_{yx})W}{|Z_{xy} - Z_{yx}|^2} \right\} \right], \quad (A10)
\]

where
\[ U = \left| n_x \right|^2 \left| A_x \right|^2 + 2 \text{Re} \left\{ n_x n_y A_x^* A_y^* \right\} + \left| n_y \right|^2 \left| A_y \right|^2, \]

\[ V = \left| n_x \right|^2 \left| A_y \right|^2 + \left| n_y \right|^2 \left| A_x \right|^2 - 2 \text{Re} \left\{ n_x n_y A_y^* A_x^* \right\}, \]

and

\[ W = \left| n_x \right|^2 A_x^* A_y + \left| n_y \right|^2 A_y^* A_x - n_x n_y \left| A_x \right|^2 - \left| n_y \right|^2 A_x^* A_y. \]

Traditionally, one usually calculates a rotation angle, \( \theta \), for each frequency window which minimizes \( |Z_{xx}' - Z_{yy}'|^2 \) or, equivalently, maximizes \( |Z_{xy}'|^2 + |Z_{yx}'|^2 \). The value of \( \theta \) is

\[ \theta = \tan^{-1} \left( \frac{2 \text{Re}[(Z_{yy}' - Z_{xx}')(Z_{xy}' + Z_{yx}')]}{|Z_{xy}' + Z_{yx}'|^2 - |Z_{yy}' - Z_{xx}'|^2} \right). \]  \( \text{(A1)} \)

The variance in \( \theta \) is

\[ \text{var}(\theta) = \frac{1}{8 N |D|^2} \left( \cos^4 4\theta \right) |a|^2 \left[ U' |Z_{xy}' + Z_{yx}'|^2 + V' |Z_{xx}' - Z_{yy}'|^2 \right. \]

\[ + 2 \text{Re} \left( W'(Z_{xy}' + Z_{yx}') (Z_{yy}' - Z_{xx}') \right) \],

where

\[ a = \frac{(Z_{xy}' + Z_{yx}')^2 + (Z_{yy}' - Z_{xx}')^2}{|Z_{xy}' + Z_{yx}'|^2 - |Z_{yy}' - Z_{xx}'|^2}, \]

\[ U' = \left| n_x \right|^2 \left| A_x \right|^2 - 2 \text{Re} \left\{ n_x n_y A_x^* A_y \right\} + \left| n_y \right|^2 \left| A_y \right|^2. \]
It has recently been suggested by Gamble et al. (1981) that a more useful presentation of MT results can be obtained by first calculating a regional strike direction, and then rotating $Z$ and calculating the above quantities in the new, fixed coordinate system. Gamble et al. (1979b) discuss the effect of such a rotation on the estimated variances.

**Tipper**

The referenced tipper estimate is:

$$\vec{T} = [H_z H_r][HH_r]^{-1}, \quad \text{(A12)}$$

where

$$\vec{T} = (T_x, T_y),$$

and

$$[H_z H_r] = (H_z H_r^*, H_z H_y^*).$$

The variance in each component of the tipper is:
\begin{align*}
\text{var}(T_i) &= \frac{2|\tau|^2 |A_{ij}|^2}{N|D|^2}, \quad (i = x, y) \quad (A13)
\end{align*}

where
\begin{align*}
|\tau|^2 &= |H_z|^2 + |T_x|^2 |H_x|^2 + |T_y|^2 |H_y|^2 - 2\text{Re}\left\{T_x(H_x H_z^*) \right\} \\
&\quad + T_y(H_y H_z^*) - T_x T_y(H_x H_y^*)
\end{align*}

As with the impedance tensor elements, the tipper elements may be presented in a coordinate frame the orientation of which is determined at each frequency, or in a frame that is fixed for all frequencies. In the first case, the tipper strike is aligned with the direction in which $|T_x|$ is a minimum, whereas in the second, the tipper regional strike is found by minimizing the sum of the appropriately weighted squares of $|T_x|$ at each frequency and station (Gamble et al., 1981).

**Signal and Noise Power Estimates**

The signal power matrices of the measured fields are best estimated by the following:

\begin{align*}
S_{\not E} \equiv [E_S E_S^*] &= \frac{1}{2}( [E^P E] + [EE^P]) , \quad (A14) \\
S_{\not H} \equiv [H_S H_S^*] &= \frac{1}{2}( [H^P H] + [HH^P]), \quad (A15)
\end{align*}

and
\[ S_{\text{HR}} \equiv [H_{sr}H_{sr}^T] = \frac{1}{2}([H_{r}^P H_{r}^T] + [H_{r}^P H_{r}^T]) , \quad (A16) \]

where

\[ E^P = [EH_{r}][HH_{r}]^{-1}H^r , \]

\[ H^P = [HH_{r}][EH_{r}]^{-1}E^r , \]

and

\[ H_{r}^P = [H_{r}E][HE]^{-1}H^r . \]

The subscript \( s \) refers to signal, and the superscript \( P \) implies a predicted field.

The noise power matrices are the differences between the measured and signal power matrices:

\[ N_{EE} = [EE] - S_{EE} , \quad (A17) \]

\[ N_{HH} = [HH] - S_{HH} , \quad (A18) \]

and

\[ N_{HR} = [H_{r}H_{r}^T] - S_{HR} . \quad (A19) \]

Analogous forms are obtained for the \( H_z \) power spectra:
\[ S_T \equiv |H_{zs}|^2 = \text{Re}(H_Z^PH_Z^*) = \text{Re}([H_ZH_P][HH_P]^{-1}[HH_Z]) , \]  
\hspace{1cm} (A20)

and

\[ N_T = |H_Z|^2 - S_T . \]  
\hspace{1cm} (A21)
REFERENCES


FIGURE CAPTIONS

Fig. 1  Schematic of remote reference magnetotellurics system.

Fig. 2  (a) Schematic of dc SQUID: * indicates a Josephson junction; a magnetic flux \( \phi \) threads the superconducting loop.
        (b) Current-voltage (I-V) characteristic of dc SQUID with
        \( \phi = n\phi_0, (n + 1/2)\phi_0 \); when the SQUID is biased with a constant
        current \( I_B \), the corresponding voltages are \( V_1, V_2 \).
        (c) \( V \) vs \( \phi/\phi_0 \) for SQUID at constant bias current \( I_B \).

Fig. 3  Cylindrical dc SQUID (not to scale).

Fig. 4  SQUID coupled to feedback electronics. Components within the
        dashed box are in liquid helium.

Fig. 5  Output voltage of SQUID in response to a 100-kHz flux, with a
        quasistatic applied flux of (a) \( n\phi_0 \) and (b) \( (n + 1/4)\phi_0 \).

Fig. 6  Three-axis SQUID magnetometer in 5l fiberglass cryostat.

Fig. 7  General view of LSI-11 minicomputer; on top are the tape drive
        (left) and thermal printer (right).

Fig. 8  Block diagram of LSI-11/3 system.

Fig. 9  Flow diagram of data through computer.
Fig. 2
Fig. 3
\[ \phi = n \phi_0 \]

\[ \phi = (n + 1/4) \phi_0 \]
CPU

28k RAM memory

ROM diagnostics and absolute loader

Programmable clock

16 channel digitizer

4 channel D/A converter

Interfaces for:
  terminal
  LED's and thumbwheel switch
  pushbuttons and hexadecimal display
  tape controller
  data logger controller

XBL 7910-13063

Fig. 8
RESULTS

locate and sum selected data blocks

correct for:
electrode locations
amplifier gains
filter response functions

print:
signal and noise powers
rotated apparent resistivities
tipper
certainty limits

Fig. 9
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