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CALCULATION OF THE STRAY FIELD OF MAGNETS WITH POISSON

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ABSTRACT

It is shown that application of conformal mapping to two dimensional magnets, containing iron with nonlinear B(H) characteristics, allows calculation of magnetic fields in all 2D space. For some magnets of finite length, this information can be used to get a good approximation for the stray fields in all of 3D space. It is furthermore shown that only minor modifications to POISSON are necessary to allow application of the same techniques to magnets with axial symmetry, leading in this case to solutions that genuinely and accurately describe the fields in all of 3D space.
I. INTRODUCTION

It is sometimes necessary to have information about the stray fields produced by two dimensional magnets. It is clear that the following consideration, being purely two dimensional, is valid at most up to a distance of the order of the physical length of the magnet. We will later describe a procedure that can give approximate information about three dimensional stray fields and will finally discuss application of the basic procedure to axially symmetric magnets. To simplify the description of the procedure, we discuss its application to a specific magnet that is typical for the kind of problem that arises in practice (and also leads to simple figures that are easily drawn). Although of general validity, the description of the method is tailored to the use of the magnetostatic analysis program POISSON. (1)

A. Stray Fields Produced By a Window Frame Magnet

Figure 1 represents the cross section of 1/4 of a window frame magnet. The field lines are perpendicular to the midplane 0-8, and the symmetry plane 0-6 has a constant vector potential. For calculation of the fields with a digital computer, one obviously has to limit artificially the grid that is used for the description of the problem. Even when saturation effects are of importance, it is a reasonably good approximation to limit the grid along the line 6-7-8 and put that line onto the same vector potential as the line 0-6. Whether one limits the grid in this way, or limits it farther outside, with air between 6-7-8 and the grid limitation, is immaterial for the method used to compute the stray fields.
When one wants to calculate the stray field at some point outside
the magnet, the grid does not only have to include that point, but should
go significantly beyond it in order to avoid falsification of the stray
field by the artificial grid limitation. This leads to an impractically
large total number of mesh points, since the magnet itself should still
contain a reasonable number of grid points in order to describe the
stray field-producing saturation of the iron adequately. The large number
of grid points and the errors resulting from the artificial grid limitation
are avoided with the following procedure.

One first solves the magnetostatic problem, without regard for
stray fields, in the configuration shown in Fig. 1, with the artificial
grid limitation along line 6-7-8. One then solves the same magnet again,
but in the geometry obtained by applying the conformal transformation
\[ w = -\frac{R^2}{z} \quad (w = u + iv; \quad z = x + iy) \]
to the original magnet. \( R \) is a suitably chosen scaling length, and Fig. 2
represents the transformed magnet, drawn to the same scale as the magnet
in Fig. 1, with \( R \) equal to the distance 0-5.

The minor program modifications necessary to analyse a magnet
with nonlinear iron in a conformally transformed geometry have been
described elsewhere (2) and are incorporated into POISSON.

To solve the magnetostatic problem in the transformed geometry,
we limit the grid in the \( w \)-plane along the map of a suitably chosen contour
inside the magnet in the original geometry, for instance line 1-3-4, or
alternatively the dashed circle. We obtain the vector potentials at the
grid points of that mapped contour from the analysis in the original geometry, and solve the resulting boundary value problem, or boundary value problem with currents, in the transformed geometry. The field components \( B_u, B_v \) inside the contour \( \alpha -6 -7 -8 \alpha \) are obtained from the vector potentials by standard numerical differentiation, and the field components \( B'_x, B'_y \) in the original geometry are obtained by application of equ. (10) of Ref. 2 and yield with equ. (1):

\[
(B_x - i B_y) = (B_u - i B_v) \cdot \frac{dw}{dz} = (B_u - i B_v) \cdot \frac{(R_z)^2}{R} = (B_u - i B_v) \cdot \left(\frac{w}{R}\right)^2
\]

It can also be practical to calculate from the vector potentials the multipole coefficients \( a_n \) of the complex potential describing the fields in the \( w \)-plane:

\[
P_w(w) = \sum_{n=0}^{\infty} a_n w^n
\]

This gives for the Laurent expansion of the stray field potential:

\[
P_z(z) = \sum_{n=0}^{\infty} a_n (-R^2/z)^n
\]

The only one step described above that is not routinely performed by POISSON is the transfer of the vector potentials from the contour 1-3-4 to the mapped contour 1'-3'-4' in the \( w \)-plane. The simplest way to accomplish this is to map the grid points on the outer contour in the \( w \)-plane into the \( z \)-plane and calculate the potentials there by interpolation of the vector potential field. Linear interpolation should in general be sufficient, since minute details of the vector potential distribution along the outer contour in the \( w \)-plane should have only a small effect on the
stray fields. An alternate method to solve for the stray fields would be to compute the scalar potentials along the contour 6-7-8 and use its map as the outer problem boundary in the w-plane. The first method is preferable, since scalar potentials are usually not computed by POISSON and the stray fields would be more sensitive to errors in the scalar potentials along 6'-7'-8' than they are to errors in the vector potentials along 1'-3'-4'.

It is clear that the computation of the stray fields in the w-plane allows the presence of ferromagnetic bodies in the stray field region as long as they satisfy the conditions implied by the two dimensional approximation.

When one is dealing with symmetrical multipoles (2n-poles), it has been shown in Ref. (2) that it is advantageous to analyse their fields in the geometry obtained by the conformal transformation \( w = k^{n-1} z^n \). Similarly, the stray fields of such a magnet should be computed in the geometry obtained by transforming the original geometry with \( w = -R^{n+1}/z^n \), and the stray fields in the original geometry are obtained from the fields in the transformed geometry with appropriately modified equivalents to equ. (2).

When one is evaluating magnets that saturate badly, or magnets with an open iron core (for instance C-magnets), it is not always obvious how much the solution in the magnet is influenced by the artificial grid limitation. To obtain a better solution, one can compute the stray fields as described above, then transfer the vector potentials obtained along the map of the grid-limiting-contour of the original magnet(6'-7'-8') to that contour (6-7-8) in the original geometry, and solve the problem again.
in the original geometry with these new values at that boundary. With this iterative process, which in general will not have to be repeated, one clearly obtains a very accurate all 2D-space solution for the magnet. There are obviously other configurations where these all 2D-space solutions might be useful. One might, for example, want to know how the field in an iron-free magnet is influenced by the presence of iron at a distance outside the magnet.

It is worthwhile to point out that it is possible to obtain reasonable approximations for the three dimensional stray fields at virtually any distance produced by magnets that have small end effects in the sense that they do not contribute significantly to the stray fields. One could derive the three dimensional stray fields from a vector potential that is obtained by superposition of finite length filamentary multipoles with strengths giving the "near field" multipole strengths described by equ. (3b).

B. Extension To Magnets With Axial Symmetry

Magnetostatic fields with axial symmetry can be derived from a vector potential that has only an azimuthal component which is, of course, independent of the azimuth. If we introduce the axial and radial coordinates respectively as the x and y coordinates of a Cartesian coordinate system and furthermore introduce a pseudo vector potential $\vec{\psi}$ with only a component in the $\hat{x} \times \hat{y}$ direction equal to $y$ times the vector potential, then the magnetostatic equations for the axially symmetric problem can be represented by:

\begin{align}
\vec{B} = \frac{1}{y} \vec{\nabla}_c x \vec{\psi} \\
\vec{H} = \gamma (|\vec{B}|, x, y) \cdot \vec{B}/\mu_o, \quad (\gamma = 1/\mu_{rel.}) \\
\vec{\nabla}_c x (\frac{1}{y} \vec{\nabla}_c x \vec{\psi}) = \mu_o \vec{J}
\end{align}
In these equations, \( J \) has only a component in the \( x \times y \) direction, equal to the current-density in the original problem, and \( \nabla_c \) is the Cartesian form of the \( \vec{\nabla} \) operator.

The only difference between eqn's (4) and the equations describing a genuine two dimensional problem in Cartesian coordinates is the extra factor \( 1/y \), and the integration routine is easily modified to take this into account. POISSON, just as its predecessor TRIM (as well as other programs), has this modification incorporated for the solution of problems with axial symmetry. It is clear from eqn's (4) and Ref. (2) that in order to solve eqn's (4) in a conformally mapped geometry, one needs to incorporate into POISSON the same modifications that are needed to solve genuine two dimensional problems in a transformed geometry, as well as the modification necessary to take the factor \( 1/y \) into account. Although this has not been done yet, it is clearly a simple matter to do so. To find the stray fields for an axially symmetric problem, the same transformation can be used that was employed in the 2D case, and the rest of the procedure is also identical, with only two modifications: the power expansion, eqn. 3, is not applicable, and the field components in the \( x-y \) (\( z-r \)) coordinate system are obtained from

\[
B_x - i B_y = \left( \frac{\partial V}{\partial v} + i \frac{\partial V}{\partial u} \right) \cdot \frac{dW}{dz}/y.
\]

In contrast to the two dimensional case, application of this procedure to an axially symmetric magnetostatic problem gives a solution that genuinely and accurately covers all space.
Footnotes and References

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1. POISSON is an improved version of TRIM (originally written by A. M. Winslow, Journal of Computer Physics 1, 149 (1967) and was developed by J. R. Spoerl, R. F. Holsinger, and K. Halbach. POISSON uses, like TRIM, the vector potential in an irregular triangular mesh.

FIGURE CAPTIONS

Fig. 1 Original geometry of magnet (complex z-plane).

Fig. 2 Conformally mapped geometry of magnet (complex w-plane).
Original Geometry of Magnet (complex z-plane)

fig. 1
Conformally Mapped Geometry of Magnet
(complex w - plane)
fig. 2
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