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Publication Date
1993-03-01
To be presented at the ASME National Heat Transfer Conference, Atlanta, GA, August 8–11, 1993, and to be published in the Proceedings

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March 1993
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This work was supported by the Office of Renewable Energy, Advanced Industrial Concepts Division, of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098.
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ABSTRACT

A theoretical investigation of laminar premixed flame quenching is carried out. Two orientations in which a flame may contact a cold wall are compared and contrasted by solving the conservation equations of mass, energy, and species utilizing a finite difference methodology. A one-step mechanism is used to specify the reaction rates. A simple analytical model, which is in qualitative agreement with the numerical results, is also presented. The results show that the heat transfer histories for one and two dimensional laminar flame quenching are similar.

NOMENCLATURE

\( a \) fuel reaction order
\( b_0 \) oxygen reaction order
\( c_p \) specific heat capacity
\( D \) mass diffusivity
\( E_a \) activation energy
\( h^o \) enthalpy of formation
\( H \) height of computational domain
\( H_f \) heating value of fuel
\( H_s \) Heaviside step function
\( k \) thermal conductivity
\( l_f \) flame length
\( L \) length of computational domain
\( Le \) Lewis number \( a/D \)
\( N \) number of species

\( P \) pressure
\( q \) \( Q/\rho \)
\( Q \) analytic model heat release rate value
\( R \) gas constant
\( S_w \) laminar flame speed
\( t \) time
\( T \) temperature
\( u \) velocity parallel to the wall
\( v \) velocity perpendicular to the wall
\( W \) molecular weight
\( x \) spatial coordinate parallel to wall
\( X \) dimensionless coordinate \( x/l_f \)
\( y \) spatial coordinate perpendicular to wall
\( Y \) species mass fraction
\( Z \) dimensionless coordinate \( y/l_f \)

Greek
\( \alpha \) thermal diffusivity
\( \beta \) constant approximating \( \rho^2 \alpha \)
\( \phi \) mass coordinate
\( \rho \) gas density
\( \theta \) nondimensional temperature
\( \tau \) time coordinate
\( \omega \) reaction rate

Subscripts
\( b \) burnt
The interaction of reacting gases with nonreacting, impermeable surfaces has both fundamental and practical significance. Fundamentally, the coupling of heat transfer processes with temperature sensitive chemical reactions is complex and poorly understood. Yet, many industries are dependent on the nature of these interactions; furnace, combustor, automotive, semiconductor and optical fiber technologies represent notable examples.

A number of pertinent studies on reacting-gas heat transfer have been carried out. Much of the work has primarily focused on modeling global properties of the geometry, flow field, and heat transfer. For most practical problems, flame quenching occurs on length scales far smaller than the characteristic dimensions of the system. Specifically, even at 1 atm, quenching phenomena typically have length scales less than 500 micrometers; this is often below the grid resolution of many global finite difference calculations. There are many applications where the flame quenching process significantly affects the global phenomena, system parameters, and performance; consequently, further investigation of the flame-wall interaction is required.

Flame quenching is experimentally investigated in either a two-sided (wall) or a single-sided (wall) configuration. In two-sided flame quenching studies, a quench distance is defined in terms of a minimum diameter channel through which a flame can propagate. This process is generally considered to be thermal in nature. The other configuration considered is single-sided flame quenching. This constitutes cases where a flame is effectively only in contact with a single wall. The mechanisms controlling this process are less well understood. There are a number of possible orientations that a flame can quench on a single wall. Limiting cases that have been investigated are a planar flame stagnating on a parallel wall (head-on quenching) and a flame sweeping over a perpendicular wall (side-wall quenching) (see Figure 1). Side-wall sweeping quenching has been primarily investigated as a two dimensional problem, while head-on stagnation quenching has been treated as a one dimensional problem.

A notable early numerical investigation of two-dimensional flame quenching was made by von Karman and Millan (1952). An integral method was used to track a "critical" ignition isotherm for the reacting gases above a cold wall. Reactions were followed by using a progress variable, corresponding to a reactant mass fraction. An infinite Lewis number assumption was made and diffusion of the reactants was neglected. This assumption tends to overpredict the reactant mass fraction downstream of the flame. Their work continues to influence thermal investigations of quenching; e.g. Aly and Hermance's (1981) study of two dimensional quenching is similar to that of von Karman and Millan. They solve the species and energy equations within a channel with the Lewis number, Le, as a parameter; von Karman and Millan present results for an infinite Le. In studying the thermal aspects of flame propagation in a channel, they found that flame extinction occurred for channel widths below a critical value. In both of these studies there is no discussion of the effects of a transverse velocity on the results. Carrier et al. (1980) studied the channel and the one-sided quenching problem using a formulation quite similar to Aly and Hermance (1981). However, they utilize an Oseen type approximation to model the convective terms in the conservation equations. A "Blasius function" is used to determine the velocity distribution everywhere and an eigenvalue problem is posed to satisfy mass conservation. Blant and Bechtel (1982) solve the energy, species and momentum equations for a flame near a cold moving wall. Their study was directed towards evaluating the effects of the relative velocity between the gas and the wall on the amount of unburned hydrocarbons (UHC's) in the quench region. They found that the level of UHC's in the quench zone was quite insensitive to the relative velocity. Their numerical results for the temperature and species profiles were consistent with their experimental findings.

There are a larger number of numerical investigations of one dimensional flame quenching than there are of two dimensional quenching. Unlike two dimensional studies, where heat transfer results are rarely presented, one dimensional studies often consider heat transfer issues. However, even in the one dimensional studies, the heat transfer is typically a secondary result. To date, the primary focus of most one and two dimensional studies has been on the consumption of UHC's in the quench layer. These investigations cover a range of chemical mechanism complexity. The range of approximations extends from an integral method using an ignition temperature chemistry mechanism (Huang et al. (1986)) to a finite difference formulation with a full chemical...
In a one dimensional study, Kurkov and Mirsky (1968) solve for the UHC distribution in the quench layer and also provide results for the heat flux as a function of the Lewis number and a dimensionless activation energy. They utilized a single step reaction mechanism to describe the chemistry, i.e. a thermal specification. Bush, Fendell and Fink (1980) solve a similar system of equations as Kurkov and Mirsky (1968) but consider the effects of various boundary thermal constraints on the quenching process. Previously, few studies had considered the effects of wall temperature on the heat transfer aspects of flame quenching. In varying the wall temperature with respect to the gas temperature, they found that the heat flux from the gas to the wall increased with respect to the (non-dimensional) wall temperature until a critical value of the (non-dimensional) wall temperature is reached. This value appears to correspond to the (non-dimensional) temperature at which the wall may ignite the unburned gas near it. Above this temperature, the wall heat flux from the gas to the wall decreases with increasing wall temperature. Ishikii and Nishiwaki (1974) investigated head-on (one dimensional) flame-quenching heat transfer using an approximate analytical model. They described the flame quenching process in terms of a reactive heating period followed by a diffusive cooling period. Much of the physics of one dimensional thermal quenching is contained in their model. Kiehne et al. (1986,1987) investigate flame quenching using both reduced and full chemical kinetics mechanisms. They concluded that some features of quenching, e.g. the evolution of intermediate hydrocarbons, are not adequately modeled by many reduced and single step kinetics mechanisms. They do note, however, that reduced mechanisms may be able to determine global features of the quenching process such as the rate of fuel consumption. It is unclear whether the heat flux is primarily related to the disappearance of the fuel species. Several studies have generally been successful in using single step mechanisms to model experimentally determined flame quenching heat transfer results (Voseen et. al 1984), Huang et. al (1986), and Lu et al. (1990)). However, at high wall temperatures it was shown by Ezekoye et. al (1992) that an analysis similar in approach and results to Bush et. al (1980) did not agree with their experimental data. It was found that the experimental peak heat flux decreased with respect to increases of the wall temperature which disagreed with the computational results. It is emphasized that at a wall temperature of 300K, there was good agreement between the experimental and numerical results over a range of equivalence ratios.

A study of one and two dimensional flame quenching is presented in this work. It is generally felt that although the one dimensional quenching orientation is only one of many, it retains most of the physical phenomena arising in more general configurations (Bush et. al. (1980)). However, to the authors' knowledge there has yet to be presented a comparative study, computational or analytic, of head-on and sweeping flame quenching. In the present study, a simulation of one and two dimensional quenching is made and the results for the heat transfer are compared for similar conditions. The one dimensional analysis of Ishikii and Nishiwaki (1974) is reformulated as a two dimensional problem to demonstrate the conditions under which one and two dimensional quenching are similar.

**ONE DIMENSIONAL MODEL**

Head-on or planar flame quenching may be modeled by solving the one dimensional conservation equations for mass, species, and energy:

\[
\frac{\partial \rho}{\partial t} + \frac{\partial (\rho \mathbf{v})}{\partial y} = 0 \tag{1}
\]

\[
\rho \frac{\partial \mathbf{v}}{\partial t} + \rho \mathbf{v} \cdot \frac{\partial \mathbf{v}}{\partial y} = \frac{\partial}{\partial y} \left( \rho D_i \frac{\partial Y_i}{\partial y} \right) - \mathbf{w}_i \tag{2}
\]

\[
\rho c_p \frac{\partial T}{\partial t} + \rho c_v \frac{\partial T}{\partial y} = \frac{\partial}{\partial y} \left( k \frac{\partial T}{\partial y} \right) - \sum_{i=1}^{N} h_i \mathbf{w}_i \tag{3}
\]

The transient pressure term is neglected in the energy equation because measurements have shown that the pressure does not vary significantly during the fast quenching event (of order millisecond duration). The following assumptions are made:

1. species diffusion is by Fick’s law
2. radiative heat transfer is negligible
3. the reaction is one-step and irreversible (specified by the fuel species)
4. the binary mass diffusivities of all species are equal
5. the specific heats of the constituent species are constant and equal.

The assumption relating to negligible radiative transfer is based on the experimental scenario which was modeled. A premixed propane-air flame quenched at nearly atmospheric pressure. In order to consider near atmospheric pressures, the flame kernel was generally less than 30 millimeter in diameter at the time of quenching. A mean beam length radiative analysis for such conditions indicates that the radiative flux from the flame to the wall is less than 3% of the conductive losses. The momentum equation is reduced to \( dp/dz = 0 \) for the low Mach number process. The equation of state is the ideal gas equation written as

\[
P = \rho R T \sum_{i=1}^{N} \frac{Y_i}{W_i} \tag{4}
\]

The reaction rate is specified from Westbrook and Dryer (1981) and is of the form:

\[
\mathbf{w}_f = A(\rho Y_f)^{a_f}(\rho Y_o)^{b_e} \exp \left( \frac{-E_a}{RT} \right)
\]

As in Ramos (1987), the governing equations are transformed from the space coordinate, \( y \), to a mass coordinate
system, \( \psi \), by use of the von Mises transformation. Transformations \( \rho = \partial \psi / \partial y \), \( t = \tau \), and \( \rho \psi = -\partial \psi / \partial \tau \) are used to satisfy the mass conservation equation; the energy and species equations are converted to diffusion equations of the form

\[
\frac{\partial Y_1}{\partial \tau} = \frac{\beta}{Le} \frac{\partial^2 Y_1}{\partial \psi^2} - \frac{w_1}{\rho} \tag{5}
\]

\[
\frac{\partial T}{\partial \tau} = \frac{\beta}{Le} \frac{\partial^2 T}{\partial \psi^2} + \frac{H c w_1}{\rho} \tag{6}
\]

At the initial time, a temperature profile is specified to be equal to the wall temperature for the majority of the unburnt gas region and equal to the burnt gas temperature at the edge of the computational domain. Though the mathematical problem is specified to occur over an infinite domain, it is possible to choose a sufficiently long computational domain over which disturbances at the wall do not affect the quenching results to model the infinite domain problem. The product \( \rho \alpha \) is simplified to be the constant \( \beta \). The Lewis number, \( Le \), is defined to be the ratio of the thermal diffusivity, \( \alpha \), to the mass diffusivity, \( D \). The wall is considered to be isothermal, impermeable and noncatalytic.

The governing conservation equations are solved by using an implicit finite difference method. Temporal derivatives are approximated by a backwards difference formula. Spatial derivatives are approximated by a central difference formulation. The nonlinear source terms are treated by using a Picard iteration formulation. The algebraic system of equations are solved by using a tridiagonal matrix solver to advance the solution in time.

**TWO DIMENSIONAL MODEL**

The two dimensional sidewall quenching model presented is a variant of the original model developed by von Karman and Millan (1952). A flame is assumed to move at an observed velocity \( w_o \); a fixed reference frame with respect to the flame is specified, \( z' = z - w_o t \). The velocities of the unburnt and burnt gases relative to the flame are \( u - w \) and \( w - u_t \), respectively. The difference between the unburnt gas velocity, \( u_t \) and the observed flame velocity, \( w_o \), is the laminar flame speed, \( S_u \); where \( S_u \) is specified by the rate of chemical reaction within the flame. In the transformed reference frame, transient effects are assumed to be negligible. It is also assumed that the transverse velocity component, \( v \), is small relative to the axial velocity component, \( u \). The conservation equations for quasi-steady flame propagation in two dimensions are:

\[
\rho \frac{\partial u}{\partial x} = \frac{\partial}{\partial x} \left( \rho \frac{\partial Y_1}{\partial x} + \frac{\partial}{\partial y} \rho \frac{\partial Y_1}{\partial y} - w_1 \right) \tag{7}
\]

The standard assumptions presented in the previous section are also incorporated in the development of the above equations and the ideal gas equation is again used to specify the state equation.

The boundary conditions are specified as:

\[
x \to -\infty : \quad T = T_w, \quad Y_1 = Y_{f,0}
\]

\[
x \to \infty : \quad \frac{\partial T}{\partial x} = 0, \quad \frac{\partial Y_1}{\partial x} = 0
\]

\[
y = 0 : \quad T = T_w, \quad \frac{\partial Y_1}{\partial y} = 0
\]

\[
y = H : \quad \frac{\partial T}{\partial y} = 0, \quad \frac{\partial Y_1}{\partial y} = 0
\]

The computational domain is a channel of sufficient width \((2H)\) that an one dimensional flame propagates through much of the channel; flame curvature due to wall effects is thus confined to a small portion of the wall. The centerline, \( y = H \), is at a value of \( H = 1.9 \text{ cm} \). The quenching region is typically confined to a region of thickness \( 1 \text{ mm} \). The gas velocity is determined by integrating the species conservation equation over the computational domain. The length of the computational domain was chosen such that the Neuman boundary conditions in the burnt gas regions were reasonable. The length of the channel, \( L \), was set to 1.96 cm. The result after utilizing the boundary conditions, is:

\[
\rho u = \frac{\int_0^H \int_0^L w_1 dy dx}{\int_0^H (Y_1(x = 0, y) - Y_1(x = L, y)) dy}
\]

It should be noted that the mass conservation requirement (i.e. \( \rho w = \text{constant} \)) is met in this specification by letting the streamwise velocity component, \( u \), accomodate the heat release expansion effects. The result is that \( u \) is a function of the transverse coordinate, \( y \), through the dependence of \( \rho \) on \( y \). Unlike the transient one dimensional simulation in which the cold-boundary difficulty is not an issue, for the quasi-steady two dimensional simulation, an ignition temperature is required in the specification of the reaction rate. This stems from the fact that over a long enough time or distance, even the relatively small cold gas reactant rates will have had sufficient time to consume the reactants. It has been shown, Williams (1981), that the problem specification of flame propagation in an infinite domain is mathematically ill posed. Consequently, a minimum temperature (ignition temperature) is specified below which reactions are precluded from occurring. Algebraic representations of the conservation equations are developed using a finite volume formulation. The nonlinear source terms are Picard linearized. Due to the different length scales present in this quenching process, a non-uniform computational mesh is used to resolve the various length scales; the computational mesh is \( 55 \times 500 \) with \( 10 \mu \text{m} \) being the smallest grid dimension. An Alternate Difference Implicit (ADI) method was used to solve the system of equations.

**RESULTS AND DISCUSSION**

Simulations of flames are typically appraised by their ability to predict quantities such as the flame speed and the
adiabatic flame temperature. The adiabatic flame temperatures, (2373K and 2372K) for the one and two dimensional calculations, respectively, are in good agreement with the value (2396K) calculated using the equilibrium chemistry code STANJAN (Reynolds (1986)). The calculated flame speeds, 35 cm/s and 33cm/s for the one and two dimensional calculations respectively, are in reasonable agreement with the experimentally determined value (38 cm/sec) of Metghalchi and Keck (1980) for a stoichiometric propane-air flame. One of the difficulties in analyzing flame quenching, as compared to steady flame propagation, is its transient nature. The flame speed, or the rate of reaction, is not constant during quenching and is quite sensitive to small changes in the quenching environment. Thus, the criteria for evaluating the accuracy of a quenching simulation are typically more stringent than those for an equivalent steady flame problem.

The transient temperature profiles for the one dimensional quenching simulation are shown in Figure 2. The initially, steadily propagating flame comes in contact with the impermeable wall. The flame heats the unburnt gas near the wall until reactions occur, and simultaneously loses heat to the wall. The losses to the wall constitute an enthalpy deficit and the adiabatic flame temperature drops. The reaction rate is nonlinearly dependent on the hot gas temperature and decreases. The nonlinear coupling between the decreasing burnt gas temperature and the decreasing available heat release rate (reaction rate) quenches the flame. The above description is for a Le greater than or equal to one; for this condition, the quenching process is thermally controlled. In this study isotherms are used to identify the flame, as compared to reaction rates, for ease in comparison with the works of Westbrook (1981) and Kiehne (1987).

The isotherms from the two dimensional quenching simulation are shown in Figure 3. As with the one dimensional quenching, it is assumed that thermal processes limit the extent to which reactions can occur near the cold wall. In increasing the dimensionality of the quenching model from one to two, an additional diffusion term is introduced into the energy and species equations (Equation 7). The increase in the number of terms in the conservation equations tends to obscure any similarities in the physics of one and two dimensional quenching. A simple model of one and two dimensional quenching is developed to consider where similarities between the two processes occur.

Ishihiki and Nishiwaki (1974) studied head-on (one dimensional) quenching by modeling the heat source as a linear combination of step functions. The energy equation is

$$\rho c_p \left( \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial y} \right) = -k \frac{\partial^2 T}{\partial y^2} + Q[H_f(t) - H_f(t - t_q)]$$

(8)

where $H_f(t)$ is the Heaviside step function, and $t_q$ is a quenching time which is scaled from the flame thickness, $l_f$, and the laminar flame speed, $S_f$; it is given by $l_f/S_f$. The equation is transformed to a mass coordinate system:

$$\frac{\partial T}{\partial \tau} = \beta \frac{\partial^2 T}{\partial \xi^2} + \frac{q}{c_p} [H_x(r) - H_x(r - \tau_q)]$$

(9)

The quantity, $q$, is assumed to be a constant value for a prescribed length/time, scaled as the flame length, and is zero elsewhere. The reaction region, $q \neq 0$, propagates towards an initially cold wall and then extinguishes on the wall after a time interval that is scaled by the flame length and the adiabatic flame speed. Ishihiki and Nishiwaki present the following result for the heat transfer during the quenching period:

$$q_w = 2q \sqrt{\frac{k_a \rho_0 l_f}{c_p}}$$

(10)

In determining the solution above, Ishihiki and Nishiwaki found that the initial temperature profile in the gas adjacent
to the wall did not significantly affect the heat transfer to the wall as much as the heat release rate, \( q \), did. Thus, the gas adjacent to the wall was assumed to be at the wall temperature.

In the period following quenching (source off), i.e. the conduction dominated period where the hot gas adjacent to the wall is cooled, they approximately determine the heat flux to the wall to be

\[
q_w = 2q \int \frac{k\rho c_p}{c_p} \exp \left( \frac{4t}{\tau q} \right) \text{erfc} \left( \frac{\sqrt{\frac{4t}{\tau q}}} {\tau q} \right) \quad (11)
\]

In the following discussion, a simplified model for two dimensional quenching will be solved and compared to the calculated quenching predictions as well as to equations (10,11). Quenching along a side wall may be studied by assuming that the heat conduction perpendicular to the wall is much greater than the conduction parallel to the wall. Referring to Equation (7) this yields

\[
\rho u c_p \frac{\partial T}{\partial x} = \frac{\partial}{\partial y} \left( k \frac{\partial T}{\partial y} \right) + H_\omega f \quad (12)
\]

The boundary conditions for this equation are specified as:

\[
x = 0 : \quad T = T_a \quad y = 0 : \quad T = T_b = T_s \quad y \to \infty : \quad \frac{\partial T}{\partial y} = 0
\]

Using the following expressions and nondimensional quantities:

\[
\rho S_u = \text{constant} \quad \rho S_u c_p (T_b - T_a) = H_\omega f l_f \quad X = x/l_f \quad l_f = \alpha / S_u \quad Z = y/l_f \quad \theta = (T - T_a)/(T_b - T_a)
\]

Equation (12) becomes,

\[
\frac{\partial \theta}{\partial X} = \frac{\partial^2 \theta}{\partial Z^2} + H_s(X) - H_s(X - 1) \quad (13)
\]

Here the source term, \( H_\omega f \), has been specified as a linear combination of step functions as noted previously (cf. Eq.(8)). A schematic diagram of the proposed interaction is shown in Figure 4. A source that is constant over a flame thickness and zero outside, translates along a cold wall. By the nature of the formulation, it is expected that the wall heat flux profile will be consistent with the results of Ishikawa and Nishiwaki (1974). However, it is also of interest to investigate the behavior of the temperature isotherms near the cold wall. The asymptotic solution for Equation (13) as \( Z \to \infty \) corresponds to a one dimensional flame propagating far from the wall and is given by

\[
\theta_{\infty} = \begin{cases} 
X & \text{if } X \leq 1. \\
1 & \text{if } X \geq 1.
\end{cases} \quad (14)
\]

The assumptions for the source term result in a linear temperature distribution in the reacting zone far from the wall.

Figure 4: Schematic diagram of analytically posed problem.

There, the temperature is assumed to increase from the unburnt gas temperature to the flame temperature. Using Laplace transform techniques, the solution, \( \theta_h \), to the resulting homogeneous equation (after subtracting the asymptotic solution, \( \theta_{\infty} \)) is found to be

\[
\theta_h = \int_{0}^{X} [H_s(X - 1) - 1] \text{erfc} \left( \frac{Z}{2\sqrt{X - X'}} \right) dX' \quad (15)
\]

The complete solution is given by

\[
\theta(X, Z) = \theta_{\infty} + \theta_h
\]

and the dimensionless wall heat flux is given by

\[
q_w(X, Z = 0) = \begin{cases} 
\sqrt{X} & \text{if } X \leq 1. \\
\sqrt{X - X} & \text{if } X \geq 1.
\end{cases} \quad (16)
\]

The values of the properties are evaluated at 1300K, which is the average of the unburnt (300K) and the burnt (2300K) gas temperatures. The flame length using the scaling definition, \( l_f = \alpha / S_u \) at 1300K, is 600 micrometers, a reasonable experimental value. It is seen that the two dimensional numerical results (Figure 3) are qualitatively similar to the two dimensional analytical results (Figure 5.). All of the isotherms reach minimum values as the integral of the heat release rate approaches its maximum value. Subsequently, the isotherms move away from the wall as conduction-controlled cooling of the adjacent hot-gas layer occurs.

The results of the analytical study do not suggest that axial conduction plays a small role in the two dimensional quenching process. In fact, it should be noted that the analytical model implicitly maintains the features of a one dimensional flame far from the wall (through the particular solution) where axial conduction defines the flame character. Thus, the similarities between the two dimensional numerical and analytical solutions imply that axial conduction is primarily responsible for maintaining the one dimensional flame structure far from the wall and transverse conduction is responsible for the flame curvature and heat transfer near the wall.

Since the two dimensional numerical results are (qualitatively) similar to the two dimensional analytical results
and the one and two dimensional analytical results are functionally similar, it is argued that under circumstances where the scaling law, \( t = z/S_{\infty} \), used to convert the one dimensional analytical equation to a two dimensional form is valid, the heat transfer results of one and two dimensional quenching are the same. The heat flux profiles from Ishiki and Nishiwaki's (1974) results (equations 10 and 11) are compared to the two dimensional heat flux results from equation 15 (cf. Figure 6). When both profiles are scaled by their respective maximum fluxes, the resulting profiles are in excellent agreement. There is a factor of three difference in the values of the dimensional maximum fluxes because the two dimensional analytic results are based on mean property values while Ishiki's results are based on variable property values. The heat transfer profile along the wall is presented in Figure 7 for the two dimensional numerical calculations.

Figure 5: Temperature profiles predicted by simplified analytical model

Figure 6: Comparison of heat flux expression of Ishiki and Nishiwaki with 2D analytical expression

Figure 7: Heat flux profile along the wall: (computational results).

Ahead of the flame, there is little interaction and the flux from the gas to the wall is zero. At a location on the wall coinciding with the axial position of the planar flame (far from the wall) sweeping past the wall, the wall heat flux suddenly rises. In the downstream region (post reaction zone) the wall cools the hot adjacent gases and the heat flux decreases. A comparison can be made between the two dimensional and one dimensional heat flux profiles if an observed flame velocity is known. Recall that the quasi-steady two dimensional quenching model (equation (7)) is formulated with an assumption that the reference frame chosen moves with the observed flame velocity. However, during flame quenching, the observed flame velocity may vary and take on a number of values. The observed flame velocity values for two limiting conditions are specified: flame expansion into the unburnt gas zone and flame expansion into the burnt gas zone. In general, a flame will propagate with an observed velocity, \( u_\infty \). The burnt gas behind the flame moves with a velocity \( u_\infty \) and the unburnt gas ahead of the flame moves at a velocity \( u_\infty \). The flame may be studied in a steady reference frame by subtracting its observed velocity from all other velocities. A property of laminar flame propagation is that \( u_\infty - u_\infty = S_\infty \). Thus, when a flame is propagating into a quiescent medium \( (u_\infty = 0) \), the observed flame velocity will be the laminar flame speed. The gas expansion necessary to satisfy mass conservation is directed towards the burnt gas region. However, if the burnt gas is constrained to be effectively stationary \( (u_\infty = 0) \), the flame must expand into the unburnt gas. In this case, the observed flame velocity can be shown to be \( \rho_\infty S_\infty / \rho_b \). These two values of the observed velocity, \( S_\infty \) and \( \rho_\infty S_\infty / \rho_b \) will be used to relate the distance along the wall to time by \( t = z/u_\infty \). In Figure 8 the heat flux profile for the one dimensional unsteady calculation is compared to the profiles from the two dimensional calculation with \( u_\infty = S_\infty \) and \( \rho_\infty S_\infty / \rho_b \). It is seen that for two dimensional quenching with expansion towards the unburnt...
Figure 8: Heat flux profiles for 1D and 2D (expansion to unburned and burned gas regions) quenching.

gas, the time to reach the maximum heat flux is closer to the result of the one dimensional calculation than to the result for expansion towards the burnt gas. It is also noted that the values of the maximum heat flux for one and two dimensional quenching differ by less than 10%.

The effect of the Lewis number, $Le$, on the maximum heat flux is considered for both head-on and sweeping quenching. In one dimensional or head-on quenching, increases in $Le$ correspond to increases in the (residual) reactants existing after quenching has occurred; i.e. the quenching is limited by thermal effects, in contrast to species depletion effects. Also with increasing $Le$, the flame is thinner and more reactants are present in the high temperature region, implying a higher heat release rate and higher flame speed. Accordingly, the maximum heat flux from the flame to the wall during quenching increases with increasing $Le$. For two dimensional or sweeping quenching, increases in the $Le$ correspond to a thinner one dimensional flame far from the wall and also to an increased reaction rate; as is expected, the wall heat flux increases. Figure 9 shows the two dimensional quenching isotherms for Lewis numbers of .5 and 2. The one dimensional flame is thinner in the $Le = 2$ calculation than in the $Le = .5$ calculation. For the $Le = .5$ calculation, the higher temperature isotherms do not approach the wall as closely as for the $Le = 2$ calculation. Also for $Le = .5$ the high temperature isotherms do not retreat from the wall as rapidly as they do for the higher Lewis number calculation. The variation of the maximum wall heat flux with respect to $Le$ is shown in Figure 10 for the one and two dimensional simulations. For both cases, the maximum flux increases with respect to $Le$. However, it is seen that for one dimensional quenching, the wall heat-flux increases more rapidly with respect to $Le$ than for two dimensional quenching. It is possible that the heat flux for one dimensional quenching is more sensitive to the species profiles than for two dimensional quenching. The difference

Figure 9: 2D quenching isotherms for Lewis number .5 (top) and 2. (bottom).

Figure 10: Variation of the maximum wall heat flux with Lewis number for 1 and 2D quenching.
in the heat flux dependence on Le may be associated with neglecting the transverse velocity component. It has been assumed that the transverse convection is small compared to the axial convection. However, in the case of sensitivity to Le, small changes in transverse convection (heat and species) could be responsible for the differences in the calculated wall heat fluxes between the one and two dimensional simulations. Near the wall, transverse convection would accommodate some of the expansion of the quench layer into the flame/reaction zone. It is possible that this effect becomes important when the Lewis number is varied. The values of Le for common mixtures do not deviate significantly from unity; the Le was varied over a wide range to gain insight into the quenching process.

CONCLUSIONS

Experimental observations of similarities in head-on and sweeping quenching heat fluxes have motivated a theoretical investigation of the phenomena. Simulations of head-on and sweeping quenching were performed. It is shown that the heat flux profiles for the two quenching configurations (at the same conditions) are similar when the flame propagates at a rate corresponding to expansion towards the unburnt gases. A simple analytic model of two dimensional quenching is shown to successfully predict features of two dimensional quenching that are obtained from a more complete numerical simulation. The analytic solution indicates that axial conduction is important in maintaining a one dimensional flame structure away from the wall but is less important in determining the temperature profiles and heat flux near the wall. The two dimensional solution was compared to a one dimensional analytic solution (Ishikii and Nishiwaki (1974)) and the heat flux profiles showed similar trends. When the quenching time, \( t_q \), is related to the ratio of the laminar flame thickness and the laminar flame speed, \( t_f / S_L \), it is shown that head-on and sweeping quenching have similar heat transfer characteristics. The effects of the Lewis number on quenching heat transfer were investigated. It was shown that one and two dimensional quenching heat fluxes have similar trends with respect to changes in the Lewis number. It is believed that the transverse velocity component may influence the heat transfer processes over the range of Le examined.

Acknowledgement

This work was supported by the Office of Renewable Energy, Advanced Industrial Concepts Division of the U.S. Department of Energy under contract No. DE-AC-03-76SF00098.

REFERENCES


