DORNBUSCH WAS WRONG:

THERE IS NO CONVINCING EVIDENCE OF OVERSHOOTING, DELAYED OR OTHERWISE*

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Several articles claim that Eichenbaum and Evans (1995) shows that nominal exchange rates experience a delayed version of Dornbusch overshooting. These same articles usually claim that impulse responses similar to those in Eichenbaum and Evans are evidence of such overshooting. But Eichenbaum and Evans never claim that their evidence implies overshooting, delayed or otherwise. More importantly, impulse response functions like those in Eichenbaum and Evans do not support overshooting. Three recent articles repeat this misinterpretation of the evidence. My objective is to use those articles to illustrate how the evidence about overshooting is widely misinterpreted. What is interpreted as supporting overshooting is at least as consistent with an efficient market as it is with overshooting.

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The seminal article by Eichenbaum and Evans (1995) that mentions the possibility of delayed overshooting has produced a large literature. But that literature has repeatedly misrepresented the evidence in Eichenbaum and Evans (1995) regarding delayed overshooting. In addition, contrary to repeated assertions in the literature, Eichenbaum and Evans never actually claim that they found evidence of a delayed version of Dornbusch overshooting. They only say that their evidence "...could be viewed as supporting a broader view of overshooting in which exchange rates eventually depreciate after appreciating for a period of time." Nowhere does their article say that they actually found evidence of delayed overshooting. More importantly, whatever Eichenbaum and Evans (1995) does or does not say about overshooting, impulse responses for nominal exchange rates like those found by Eichenbaum and Evans (1995) are not convincing evidence for any form of Dornbusch overshooting.

As examples of how the literature interprets what Eichenbaum and Evans (1995) says consider the following quotations: Kalyvitis and Michaelides (2001, 255) says that "In a seminal paper Eichenbaum and Evans (1995) have shown that in response to tighter US monetary policy, the US dollar exhibits a 'delayed overshooting' pattern of 2 to 3 years vis-à-vis the major currencies." Referring to Eichenbaum and Evans, Kim (2005, 775) says that "They report that (contractionary) monetary policy shocks lead gradual appreciation of the exchange rate, followed by gradual depreciation;..." Even more recently, in an article primarily on the forward-bias puzzle, Bacchetta and van Wincoop (forthcoming, 6) say that "Martin Eichenbaum and Charles L. Evans (1995) first documented that after an interest rate increase, a currency continues to appreciate for another 8 to 12 quarters before it starts to depreciate."

But Eichenbaum and Evans (1995) never claim that they found significant evidence of delayed overshooting. In their Conclusion they state that they found evidence of a "persistent" response to a monetary shock. "We found strong evidence that contractionary policy shocks lead to (i) significant, persistent appreciations in exchange rates, both nominal and real." They then go on to describe how those same shocks affect uncovered interest parity. With respect to exchange rates, they never go on to claim that the significant "persistent" appreciation is followed by depreciation, as is required by overshooting. Whatever Eichenbaum and Evans (1995) did or did not say, their impulse response functions suggest a gradual response to monetary shocks or undershooting, not a delayed version of Dornbusch overshooting.

With respect to the more important issue of claims that impulse responses like those found by Eichenbaum and Evans (1995) imply delayed overshooting, consider the following: Kim and Roubini

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1 In that quote, Kim (2005) is also referring to a 1995 unpublished paper by Grilli and Roubini. I have not seen that paper, but in their article published in 1996 that examines the effects of a contractionary monetary policy Grilli and Roubini never refer to overshooting. The missing "to" in what should be "lead to gradual" is missing in the original.
(2000, 583) find similar response functions and they conclude that "The currencies of all countries relative to the U.S. dollar depreciate on impact, but the depreciation is short lived in most cases as the exchange rate shows evidence of overshooting." Based on similar response functions, Kalyvitis and Michaelides (2001, 261) also claim to find support for overshooting: "We find that 'delayed overshooting' is eliminated and the US dollar appreciates instantaneously (except for the case of sterling which continues to display 'delayed overshooting'). This response is much closer to the classic 'overshooting' pattern described in the literature Dornbusch (1976)."

Panel C in Figure 4 from Bacchetta and van Wincoop (forthcoming), which appears here as Figure 1, is an excellent example of how impulse response functions like those in Eichenbaum and Evans (1995) are misinterpreted. Bacchetta and van Wincoop (forthcoming) interpret the impulse response in that panel as implying delayed overshooting. If Figure 1 described a step response, it would imply a delayed version of Dornbusch overshooting. But, as shown below, if impulse responses like the one in Figure 1 suggest anything, they suggest undershooting, not overshooting.

1.0 Interpreting Impulse Responses

Let \( y(t) \) be the output and \( x(t) \) the input. If there is a linear time invariant relationship between \( y \) and \( x \), it can be expressed as follows:

\[
y(t) = \int_0^\infty h(\tau)x(t-\tau)\,d\tau
\]

(1)

Where \( h(\tau) \) is the impulse response function. The unit step response, which is the response to a permanent one unit increase in \( x(t) \), is the appropriate integral of \( h(\tau) \).

Equation (1) is for continuous time. The impulse response in Figure 1 and impulse responses in the literature are for discrete time. Equation 2 is the discrete time analog of equation 1.

\[
y(t) = \sum_{\lambda=0}^{\infty} h(\lambda)x(t-\lambda)
\]

Now the impulse response is \( h(\lambda) \) and the step response is the appropriate summation of \( h(\lambda) \).

As illustrated below, the impulse response describes how \( \Delta y(t) \) responds to a unit pulse. The step response describes how \( y(t) \) responds to a unit step.

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2 The integral from zero to infinity rather than from minus infinity to plus infinity reflects the condition of physical realizability, which is appropriate for "news".

3 "…so the step response is the integral of the impulse response." Jenkins and Watts (1969, 37).
One way to estimate the relationship between x and y is to estimate the response to a unit pulse in x(t). Ideally a VAR estimate of the response to a unit pulse should produce an estimate of h(λ).

But impulse responses are not as easy to interpret as step responses. That is probably why Dornbusch (1976, 1168), when he discusses overshooting, describes how the exchange rate responds to a **permanent** monetary shock. "An increase in the nominal quantity of money that is expected to persist will cause a goods and asset market disequilibrium at the initial exchange rate and price." In the paragraphs after that quote he goes on to describe how the exchange rate overshoots in response to a unit step in the money stock. How the exchange rate responds to a unit step is what the literature interprets as Dornbusch overshooting. Figure 3 below clearly illustrates that interpretation.

Why do so many economists interpret impulse responses like the one in Figure 1 as indicating overshooting when, if they indicate anything, they indicate undershooting? I believe that there are two related reasons: First, they think about how the level of the output in the form of exchange rates responds to an isolated unit pulse rather than think about how exchange rates change in response to a unit pulse. Second, at least partly as a result of the first reason, they interpret impulse responses as though they were step responses.

1.1 Levels versus Changes

As illustrated below, impulse response functions are probably best interpreted in terms of how a unit pulse affects the change in the output. But the literature appears to interpret impulse response functions in terms of how a unit pulse affects the level of the output. For example, for an impulse response like the one in Figure 1, the literature appears to interpret it as saying that the level of the exchange rate falls to some minimum and then rises. When interpreted in terms of changes, Figure 1 says that the exchange rate only falls.

If one interprets "overshooting" in terms of how the output in the form of the level of the exchange rate behaves in response to a particular isolated pulse, I suppose that Figure 1 could describe "overshooting". But that interpretation robs the term "overshooting" of almost all useful meaning. Under that interpretation, every impulse response function for a stable system that is not all zeros implies some form of "overshooting". In addition, under that interpretation the same impulse response can imply both overshooting and undershooting depending on the nature of the "shock".

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4 For example Bjørnland (forthcoming, 10) interprets the impulse response in terms of levels. "Figure 1 shows the impulse responses for the interest rate and the level of the real exchange rate....."
A system or model is stable when bounded inputs produce bounded outputs. That condition for stability implies that the impulse response function converges to zero. If it does not converge to zero, a bounded input such as a unit step produces an unbounded output. Suppose an impulse response converged to 1.0 rather than zero. In that case, a bounded input like a unit step would produce an unbounded output. Beyond some point, in response to a unit step, the output would rise by one unit each time period.

Consider any stable model. Every temporary shock like a unit pulse that has any effect on the output will cause the output to first move above or below the original equilibrium and then eventually return to the original equilibrium. If one only considers how the level of the output responds to temporary inputs like a unit pulse, then all stable models imply overshooting.

In addition, with that interpretation, the same impulse response can produce both overshooting and undershooting. As an example, consider the impulse response in Figure 1. In response to an isolated positive unit pulse, the level of the exchange rate first moves above the original equilibrium and then falls back to the original equilibrium. But in response to a positive unit step, the exchange rate rises smoothly to a new equilibrium without ever reversing direction.

As a description of the impulse response function, the term "overshooting" is probably best interpreted as a convenient way of summarizing the important parameters of that function. For example, the Dornbusch model overshoots because the impact response to a unit step in the logarithm of the money stock $m$ equals $(1+\lambda\theta)/\lambda\theta$, which is greater than the steady state response, which equals $\lambda\theta\psi/\lambda\theta\psi$. Those parameters imply that the impulse response changes sign. It does not matter whether the shocks are unit pulses, unit steps or sine waves, that impulse response overshoots.

However, as pointed out below, an impulse response function that overshoots does not necessarily imply that foreign exchange markets over respond to news. An impulse response function that overshoots can be consistent with an efficient foreign exchange market. Whether or not such an impulse response function is or is not consistent with an efficient market depends on the nature of the monetary shocks.

1.2 Impulse Response versus Step Response

Impulse response functions in the literature are typically VAR estimates of how exchange rates respond to a negative unit pulse. As pointed out earlier, when Dornbusch (1976, 9) discusses the response of exchange rates to a monetary shock as illustrated in his Figure 2, it is a positive monetary shock that is expected to persist. When Dornbusch describes how his overshooting model responds to
money, he describes a step response, not an impulse response. As illustrated by Figure 3 below, when the
literature describes Dornbusch overshooting, it describes the response to a unit step.

Unfortunately the literature often interprets impulse response functions like the one in Figure 1 as
though they were step responses. In order to make it absolutely clear that impulse responses like the one
in Figure 1 imply undershooting within the context of the Dornbusch model, not overshooting, consider
the following mental experiment: Keeping within the spirit of Dornbusch (1976), let Δm(t) represent the
"news" about the logarithm of the money stock. With rational expectations, Δm(t) is white noise. At
each t, Δm(t) is essentially an isolated pulse.

To keep the computations in this mental experiment as simple as possible, consider the truncated
overshooting impulse response in equation (3). As in Dornbusch (1976), s(t) is the logarithm of the
domestic price of foreign exchange.

\[ s(t) = 1.2m(t) - 0.19m(t-1) - 0.01m(t-2) \]  

As in Dornbusch's overshooting model, the impact response to a unit step in m(t) is greater than 1.0 and
the steady state response is 1.0. I assume that the steady state response to a unit step is 1.0 only because it
is what Dornbusch (1976) assumes and because it simplifies the discussion. Doubling all the parameters
in equation (3) would double the impact and steady state responses. Exchange rates would still over
respond to news.

To see how exchange rates respond to Δm(t), write equation (3) in first differences.

\[ Δs(t) = 1.2Δm(t) - 0.19Δm(t-1) - 0.01Δm(t-2) \]  

The impulse response describes how Δs(t) responds when Δm(t) equals 1.0 at t=0 and is 0.0 for all
later t. Under those conditions, Δs(0) equals 1.2, Δs(1) equals -0.19 and Δs(2) equals -0.01. The impulse
response overshoots because it changes sign.

Note that I interpret the impulse response in terms of changes, not the level of the exchange rate.
Interpreting impulse responses in terms of levels may be why impulse responses like the one in Figure 1
are often interpreted as though they were step responses.

Suppose the initial steady state equilibrium is 100. The typical description of Dornbusch
overshooting refers to how the exchange rate responds to a unit step, not a unit pulse. For example,
equations (3) and (4) imply that s(t) responds to a unit step in m(t) at t equals zero as follows: s(-1)
equals 100.0, s(0) equals 101.2, s(1) equals 101.01, and s(2) and all subsequent s(t) equal 101.
The impulse response in equations (3) and (4) overshoots because it changes sign. Impulse responses like the one in Figure 1 and in Eichenbaum and Evans (1995) undershoot because they do not change sign.

Equation (5) contains a truncated version of the impulse response depicted in Figure 1. Like the earlier Dornbusch type model in discrete time, there are only two lags. As in the Dornbusch model, the steady state response to a unit step is 1.0. That is the sum of the impulse responses is 1.0. Whatever the short-run response, the long-run or steady state response is 1.0. A similar impulse response function with more lags in continuous time would produce similar results.

$$s(t) = 0.25m(t) + 0.5m(t-1) + 0.25m(t-2)$$  \hspace{1cm} (5)

Like the impulse response in Figure 1, this impulse response starts out relatively small, increases to a maximum and then declines. It never changes sign.

Now exchange rates respond to a unit step as follows: $s(t-1)$ equals 100.0, $s(0)$ equals 100.25, $s(1)$ equals 100.75, $s(2)$ and all subsequent $s(t)$ equal 101.0. The impact response is less than the steady state response.

To see how news about $m(t)$ changes exchange rates, write equation (5) in first differences.

$$\Delta s(t) = 0.25\Delta m(t) + 0.5\Delta m(t-1) + 0.25\Delta m(t-2)$$  \hspace{1cm} (6)

Now exchange rates respond to a unit pulse as follows: $\Delta s(0)$ equals 0.25, $\Delta s(1)$ equals 0.5 and $\Delta s(2)$ equals 0.25. The impulse response undershoots because the exchange rate moves smoothly to a new equilibrium without ever changing direction.

Perhaps the best way to see that equation (4) implies Dornbusch overshooting in response to news about $m(t)$ and that the impulse response in equation (6) undershoots in response to such news is to switch to the frequency domain. The spectrum for $\Delta s(t)$, denoted here as $\Gamma_{\Delta s}(f)$, describes how the variance for $\Delta s(t)$ is distributed by frequency or cycles per period. The area under $\Gamma_{\Delta s}(f)$ represents the variance of $\Delta s(t)$.

Since it is natural to identify the short run with high frequencies and the long run with low frequencies, it is natural to interpret the spectrum as describing the short-run, intermediate-run and long-run components of the variance. Of course such interpretation must be done carefully. What constitutes the short run for daily data is quite different from what constitutes the short run when the data are monthly or quarterly.

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5 For Dornbusch style overshooting, the change in sign takes place between $t=0$ and $t=1$. For a delayed version of that overshooting, the change in sign takes place later. For the steady state response to remain 1.0, the step response, the sum of the impulse responses, must converge to 1.0.

6 For the impulse response in continuous time $(1.0/T)(e^{-\tau/T})$ the impact response is 0.0 and the steady state response is 1.0.
Both models can be written in the time domain in a generic form as follows:

\[ \Delta s(t) = (a_0 + a_1 L^{-1} + a_2 L^{-2}) \Delta m(t) \]  

(7)

Where \( L^{-k} x(t) \) equals \( x(t-k) \).

As shown in the Appendix, equation (8) describes the generic spectrum for \( \Delta s(t) \) implied by equation (7).

\[ \Gamma_{\Delta s}(f) = [(a_0)^2 + (a_1)^2 + (a_2)^2 + 2.0(a_0 a_1 + a_1 a_2) \cos(2\pi f) + 2.0 a_0 a_2 \cos(4\pi f)] \Gamma_{\Delta m}(f) \]  

(8)

Where \( \Gamma_{\Delta m}(f) \) is the spectrum for \( \Delta m(t) \).

Expanding equation (7) to include more lags would not change the basic shape of the spectrum for \( \Delta s(t) \). As long as the impulse response function changes sign as it does in equations (3) and (4), exchange rates will over respond to news in the short run. The spectrum is large at the high frequencies and declines as frequency goes to zero. In the short run foreign exchange markets over respond to news about \( m(t) \).

As long as all the terms in the impulse response are positive as in equations (5) and (6) and as implied by impulse responses like the one in Figure 1, foreign exchange markets under respond to news in the short run. The details of the spectrum for \( \Delta s(t) \) will change as there are more lagged terms, but \( \Gamma_{\Delta s}(f) \) always will be smaller at high frequencies and rise as frequency goes to zero. That is \( \Delta s(t) \) under responds to news, not over responds.

As pointed out earlier in this mental experiment, \( \Delta m(t) \) is white noise. Assuming that \( \Delta m(t) \) is an uncorrelated and covariance stationary random variable simplifies the spectrum for \( \Delta m(t) \) denoted \( \Gamma_{\Delta m}(f) \). The spectrum for such a series is a constant. For simplicity, I assume that \( \Gamma_{\Delta m}(f) \) equals 1.0.

If the foreign exchange market is "efficient" and it does not either over or under respond to news, then in this mental experiment \( s(t) \) is a random walk and the spectrum for \( \Delta s(t) \) equals 1.0. If there is Dornbusch overshooting, then \( \Gamma_{\Delta s}(f) \) will be greater than 1.0 at the highest frequencies and converge to 1.0 as frequency goes to zero. If there is undershooting, that is the market under responds to news in the short run, then \( \Gamma_{\Delta s}(f) \) will be less than 1.0 at the highest frequencies and converge to 1.0 as frequency goes to zero.

Figure 2 shows three spectra. \( \Gamma_{\Delta s}(f) \) for the Dornbusch model described by equation (4) is labeled Dornbusch Overshooting. \( \Gamma_{\Delta s}(f) \) for an efficient market is labeled Efficient Market. Equation (6) describes a simple version of the model implied by the impulse response in Figure 1. The impulse response in Figure 1 is typical of the impulse responses found in Eichenbaum and Evans (1995) and in

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7 As an example, the Appendix derives a generic spectrum for an impulse response function with four lags.
most of the literature. \( \Gamma_{\Delta s}(f) \) for the model described by equation (6) is labeled Eichenbaum & Evans Undershooting.\(^8\)

The \( \Gamma_{\Delta s}(f) \) labeled Eichenbaum & Evans Undershooting equals 0.0 at the highest observable frequency 0.5. As frequency falls toward zero, the spectrum rises slowly and approaches 1.0 as frequency goes to zero. In the short run the market under responds to news about changes in the stock of money.

The spectrum labeled Efficient Market is constant at 1.0. In that case the market neither over nor under responds to news.

The spectrum labeled Dornbusch Overshooting equals about 2.0 when frequency is 0.5. As frequency falls toward zero, the spectrum falls until it equals 1.0 at the very lowest frequencies. With Dornbusch overshooting the market over responds to news in the short run. As the Appendix shows, the spectrum labeled Dornbusch Overshooting in Figure 2 is essentially the same as the spectrum implied by the original Dornbusch overshooting model when \( \Delta m(t) \) is white noise.

I believe that the spectra labeled Efficient Market and Dornbusch Overshooting in Figure 2 describe what economists usually mean by an efficient market and a market that overshoots. I believe that the spectrum labeled Eichenbaum & Evans Undershooting appropriately describes a market that undershoots. But note that these economic interpretations of foreign exchange markets do not depend solely on the impulse response functions. They also depend critically on the nature of the input \( \Delta m(t) \).

1.3 Monetary Shocks and Impulse Response Functions

As illustrated by Figure 2, when \( \Delta m(t) \) represents news about monetary shocks, impulse responses have simple economic interpretations. But when \( \Delta m(t) \) represents actual monetary shocks like changes in the interest rate for federal funds, that simple economic interpretation disappears.

Suppose a positive monetary shock today tends to be followed tomorrow by another positive shock. If markets are efficient, the response to today's shock will account for the fact that tomorrow's shock is also likely to be positive. In that case, the impulse response will "overshoot" because in an efficient market overshooting is necessary to convert positively autocorrelated monetary shocks into white noise.

When \( \Delta m(t) \) no longer represents news, the impulse response no longer has a simple economic interpretation. Whether or not foreign exchange markets overshoot now depends on both the impulse response and the nature of the observed monetary shocks.

As an example of the impulse response for Dornbusch overshooting being consistent with an efficient market, consider the following: Let \( e(t) \) be white noise with a spectrum equal to 1.0. Let

\[^8\text{As the Appendix shows, when } T \text{ is positive, } (1.0/T)(e^{-\tau T}) \text{ produces a spectrum for } \Delta s(t) \text{ similar to the Eichenbaum & Evans undershooting spectrum in Figure 2.}\]
\[ \Delta m(t) = \left[ \frac{1.0}{(1.2-0.19L-0.01L^2)} \right] e(t). \] (9)

Now if \( \Delta m(t) \) is positive today it is likely to be positive tomorrow. The spectrum for \( \Delta m(t) \) is lowest at the high frequencies and rises as frequency goes to zero.\(^9\) With this "monetary shock" and the impulse response function in equations (3) and (4), the spectrum for \( \Delta s(t) \) is a constant equal to 1.0. According to equation (10), the foreign exchange market is efficient. There is no undershooting or overshooting.

\[ \Gamma_{\Delta s}(f) = \left[ \frac{A}{A} \right] \Gamma_e(f) = 1.0 \] (10)

Where \( A \) equals \([a_0]^2 + (a_1)^2 + (a_2)^2 + 2.0(a_0a_1 + a_1a_2)\cos(2\pi f) + 2.0a_0a_2 \cos(4\pi f)\] and \( a_0 \) equals 1.2, \( a_1 \) equals -0.19 and \( a_2 \) equals -0.01.

It should be obvious from equation (10) that exactly the same point holds for "undershooting" impulse responses like the one in Figure 1 and the one in equations (5) and (6). If monetary shocks are sufficiently negatively correlated so that most of the power is at the highest frequencies, then those impulse response functions can imply an efficient market rather than undershooting.

The point I want to make is a simple one. It is not possible to interpret the economic implications of an impulse or step response in isolation. One must consider how observed monetary shocks and responses interact. But the literature inspired by Eichenbaum and Evans (1995) concentrates on the impulse responses and, to the best of my knowledge, never considers the interaction between the impulse responses and monetary shocks.\(^{10}\) As a result, the VAR estimates of impulse responses for nominal exchange rates in that literature are as consistent with an efficient market as they are with undershooting or overshooting. In spite of repeated claims about supporting overshooting, the articles inspired by Eichenbaum and Evans (1995) provide no convincing evidence of overshooting, delayed or otherwise.

This journal recently published two articles on overshooting that continue to misinterpret impulse responses. They are Scholl and Uhlig (2008) and Bjørnland (forthcoming). Both articles claim that Eichenbaum and Evans (1995) found evidence of a delayed version of Dornbusch overshooting. More importantly, both articles claim that their impulse responses imply such overshooting when their impulse responses are probably more consistent with an efficient market than with overshooting.

2.0 Scholl and Uhlig

Scholl and Uhlig (2008, 2) describe their findings as follows: “Thus, in contrast to Faust and Rogers (2003), who argue that delayed overshooting is a fragile finding, we restore the puzzle originally stated

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\(^9\) \( \Gamma_{\Delta s}(f) = \left[ \frac{1.0}{[a_0]^2 + (a_1)^2 + (a_2)^2 + 2.0(a_0a_1 + a_1a_2)\cos(2\pi f) + 2.0a_0a_2 \cos(4\pi f)]} \right] \Gamma_e(f) \) where \( a_0 = 1.2, a_1 = -0.19 \) and \( a_2 = -0.01. \)

\(^{10}\) Eichenbaum and Evans (1995, 1006) briefly consider this interaction.
by Eichenbaum and Evans (1995). We find sizeable and robust evidence in favor of a delayed overshooting of the US-German, the US-UK and the US-Japanese bilateral exchange rates…”

But what Scholl and Uhlig (2008) find is what Eichenbaum and Evans (1995) and others have found, impulse response functions that, in isolation, imply undershooting. Since Scholl and Uhlig (2008) never describe how these impulse responses interact with their monetary shocks, we cannot tell what they imply about foreign exchange markets.

Figure 1 in Scholl and Uhlig (2008) appears here as Figure 3. In Scholl and Uhlig that figure is labeled “A stylized representation of the delayed overshooting puzzle”. They refer to it later as Fig. 1. After describing Dornbusch overshooting, Scholl and Uhlig (2008, 2) says the following: "However, the empirical literature has found delayed overshooting in response to US monetary policy contractions, see Fig. 1."

[Insert Figure 3 about here.]

Their Fig. 1, which appears here as Figure 3, is an excellent example of how the literature routinely interprets impulse responses as though they were step responses. Scholl and Uhlig (2008) clearly interpret both response functions in Figure 3 as supporting overshooting. The step response in Figure 3 attributed to Dornbusch does overshoot. In response to a negative unit step, the exchange rate initially falls and then rises. The impulse response implied by that step response looks like the impulse response in equations (3) and (4). It is positive at t=0 and negative after that. The short-run response to $\Delta m(t)$ is larger than the long-run response.

If the response labeled Eichenbaum-Evans (1995) in Figure 3 were a response to a negative unit step, it would imply a delayed version of Dornbusch overshooting. In response to negative unit step, the exchange rate falls for several periods and then rises. The corresponding impulse response function would be negative for several periods and then turn positive. But the function labeled Eichenbaum and Evans (1995) is an impulse response function and it does not change sign. The corresponding step response says that the exchange rate falls for several quarters without ever rising. The short-run response to $\Delta m(t)$ is smaller than the long-run response.

When monetary shocks are white noise, impulse responses that do not change sign like the one in equations (5) and (6) and the one labeled Eichenbaum-Evans (1995) in Figure 3 imply undershooting not delayed overshooting. Those impulse responses imply that foreign exchange markets initially undershoot.
respond to information, not over respond.\textsuperscript{12} Without any information about the nature of the monetary shocks, it is impossible to determine what these impulse responses imply about how foreign exchange markets respond to news about monetary policy.

2.1 Confidence Intervals

Eichenbaum and Evans (1995) and most of the literature use confidence intervals that are about plus or minus one standard deviation.\textsuperscript{13} That corresponds to about a 70 percent confidence interval. Plus or minus two standard deviations would correspond to about a 95 percent confidence interval. Most of the literature apparently uses plus or minus one standard deviation, or its equivalent, because programs like RATS routinely report such confidence intervals. Scholl and Uhlig (2008) use intervals of 0.16 and 0.84 quantiles with their median impulse responses because they estimate their impulse responses using a Bayesian approach. Ignoring the econometric subtleties, these quantiles correspond roughly to the plus or minus one standard deviation often used in the literature.

But such narrow confidence intervals can lead both the unwary reader and the unwary author to assume that estimates of the impulse response function are significant when in fact they are not significant by standard criteria. However, for the impulse responses for nominal exchange rates reported in the first line of Figure 4 in Scholl and Uhlig (2008) the interpretation would be similar even with a more conventional "confidence interval".

2.2 Estimated Impulse Responses

Like Eichenbaum and Evans (1995) and most of the literature Scholl and Uhlig (2008) use VAR to estimate impulse response functions that describe how exchange rates respond to negative monetary shocks. Figures 4 through 6 show the impulse response functions for exchange rates reported in Figure 4 of Scholl and Uhlig (2008). The exchange rates are the dollar price of the German mark, pound sterling and Japanese yen. Since the exchange rate is the dollar price of another currency, appreciation means that the dollar price of the other currency falls.

Using their Figure 1 as an example of delayed overshooting, Scholl and Uhlig (2008, 7) interpret the three impulse responses as follows:

The first line of Fig. 4 shows the impulse response of the nominal exchange rate and should be compared to Fig. 1. The median impulse response for all three country pairs shows an appreciation of the exchange rate until a peak at 1 to 2

\textsuperscript{12} I suppose Scholl and Uhlig could claim that their impulse response implies that exchange rates over respond in the long run rather than the short run. But such delayed overshooting would not be consistent with a delayed version of Dornbusch overshooting and would not be consistent with what the literature appears to mean by "delayed overshooting".

\textsuperscript{13} Some exceptions include Kalyvitis and Michaelides (2001) and Kim (2003, 2005).
years after the shock followed by depreciation, surrounded by considerable posterior uncertainty in particular for the US-UK and US-Japan case.

[Insert Figures 4 through 6 approximately here.]

If the impulse responses in Figures 4 through 6 were step responses and their monetary shocks were white noise, their interpretation of the responses would be correct. But since the responses are impulse responses, if the monetary shocks were white noise, all significant estimates would suggest undershooting.\textsuperscript{14} Without any information about the monetary shocks, Figures 4 through 6 are at least as consistent with an efficient market as they are with overshooting. There is certainly no convincing evidence of overshooting in those figures.

3. Exchange Rates as Martingales

The evidence against overshooting is not just that the articles claiming to find such evidence are seriously flawed. There is also substantial evidence that neither undershooting nor overshooting helps explain the volatility of nominal exchange rates.

If foreign exchange markets were efficient, nominal exchange rates would be martingales. If undershooting explained an important part of the volatility of exchange rates, moves in one direction would be followed at some point by moves in the same direction. In the time domain that means positive autocorrelation for changes in exchange rates. In the frequency domain that means that the spectrum for changes in exchange rates should be higher at low frequencies than at high frequencies.

If overshooting explained an important part of the volatility of nominal exchange rates, exchange rates also should have some structure. In that case, moves in one direction should be followed at some point by moves in the other direction. In the time domain that means some evidence of negative autocorrelation for changes in exchange rates. In the frequency domain that means that the spectrum for changes in exchange rates should be higher at some high frequencies than at low frequencies.

But neither undershooting nor overshooting is consistent with the behavior of flexible exchange rates. As has been widely documented, exchange rates are approximately martingales. For some recent evidence see Cheung, Chinn and Pascual (2005) and Chung and Hong (2007). Unfortunately both of these articles and all earlier research cover periods when central banks intervened. However now it is possible to determine the behavior of daily exchange rates without intervention because several central banks now report information about daily intervention. For example, the Bank of Canada did not

\textsuperscript{14} In two cases \( h(0) \) is positive and then turns negative. Ignoring significance, that implies overshooting. But it is the opposite of Dornbusch overshooting which implies a negative \( h(0) \) followed by positive \( h(\lambda) \). Since neither \( h(0) \) is statistically significant, there is no significant evidence of overshooting.

Pippenger (2008) reports that, during these intervals, daily exchange rates are essentially martingales. There is no significant autocorrelation for first differences. The spectrum for first differences is flat. There is no significant deviation from white noise. The fact that nominal exchange rates are essentially martingales when central banks do not intervene is inconsistent with the claim that either undershooting or overshooting explains an important part of the behavior of exchange rates.

If daily exchange rates are martingales, then why do VAR estimates of impulse response functions using mostly monthly data typically produce impulse responses like the one in Figure 1? Given that changes in exchange rates are essentially white noise, the most likely explanation would seem to be that impulse responses like those are necessary to convert monetary shocks that are not white noise into changes in exchange rates that are white noise.

4. Bjørnland

The title for Bjørnland (forthcoming) says in part that "Dornbusch was right after all". In the Concluding Remarks, Bjørnland summarizes the key conclusions of that article as follows: "Contrary to the recent 'consensus', a contractionary monetary policy shock has a strong effect on the exchange rate, which appreciates on impact. The maximal impact occurs almost immediately (within 1-2 quarters) and the exchange rate thereafter gradually depreciates back to baseline." When Bjørnland (forthcoming) refers to a 'consensus', the consensus appears to be about delayed overshooting.

Bjørnland (forthcoming) makes the same mistakes described earlier. Like Scholl and Uhlig (2008) and most of the literature, Bjørnland uses unusually narrow confidence intervals. Like Scholl and Uhlig (2008) and most of the literature, Bjørnland interprets impulse responses as though they were step responses and also fails to account for the behavior of monetary shocks.

But Bjørnland (forthcoming) manages a new misinterpretation of the evidence. When Bjørnland refers to exchange rates overshooting, they are real exchange rates, not nominal exchange rates. The problem is that Dornbusch (1976) is primarily about nominal exchange rates overshooting. Overshooting in real exchange rates provides no convincing support for overshooting in nominal exchange rates.

Like Scholl and Uhlig (2008), Bjørnland (forthcoming) uses fractiles of 0.16 and 0.84. Scholl and Uhlig (2008) also estimates impulse responses for real exchange rates. Since those responses are very similar to the responses for nominal exchange rates, they concentrate on nominal exchange rates.
It is true that the standard interpretation of Dornbusch overshooting implies that real exchange rates overshoot unless relative price levels respond as rapidly to monetary shocks as nominal exchange rates, which seems highly unlikely. The problem with drawing any conclusions about nominal exchange rates from the behavior of real exchange rates is that, even if monetary shocks caused real exchange rates to overshoot, which Bjørnland (forthcoming) does not demonstrate, that overshooting does not imply that nominal exchange rates overshoot. Real exchange rates will overshoot when relative prices levels respond more slowly to monetary shocks than nominal exchange rates, which seems very likely.

Let me pose the issue this way: Suppose we must choose between three alternative models describing foreign exchange markets: in one model nominal exchange rates overshoot in response to monetary shocks, in another model they undershoot and in the third model they neither overshoot nor undershoot. How would evidence that real exchange rates overshoot help us discriminate between these three alternatives?

The answer is that evidence of overshooting for real exchange rates would not help us discriminate between any of the three alternatives. The reason is that overshooting in real exchange rates is consistent with all three alternatives because the sticky retail prices used in most measures of real exchange rates adjust far more slowly than the auction prices used for exchange rates. That relatively slow adjustment guarantees that real exchange rates will overshoot.

Equations (11) to (14) illustrate how overshooting for real exchange rates is consistent with all three alternatives for nominal exchange rates. For simplicity, s(t) is interpreted as the logarithm of the nominal exchange rate and p(t) is interpreted as the logarithm of the relative price levels. So the logarithm of the real exchange rate is s(t) - p(t). As long as one ignores any interaction between s(t) and p(t) that is not already captured by the estimated impulse responses, the impulse response function for the real exchange rate is just the response for s(t) minus the response for p(t). As in Figure 2, equations (11) through (14) assume that the monetary shocks are white noise.

As compared to the earlier impulse responses for s(t), in equation (9) p(t) responds relatively slowly to monetary shocks. But the sum of the impulse responses remains 1.0.

\[
p(t) = (0.2 + 0.3L^{-1} + 0.2L^{-2} + 0.2L^{-3} + 0.1L^{-4})m(t) \quad (11)
\]

When equation (11) describes the response of p(t) to m(t), equation (12) describes the real exchange rate with Dornbusch overshooting in equation (3). Given that relationship between p(t) and m(t), equation (13) describes the real exchange rate when there is neither overshooting nor

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undershooting. Equation (14) describes the real exchange rate when there is Eichenbaum & Evans undershooting as in equation (5).

\[
\begin{align*}
    s(t) - p(t) &= (1.0 - 0.49L^{-1} - 0.22L^{-2} - 0.2L^{-3} - 0.1L^{-4})m(t) \quad (12) \\
    s(t) - p(t) &= (0.8 - 0.3L^{-1} - 0.2L^{-2} - 0.2L^{-3} - 0.1L^{-4})m(t) \quad (13) \\
    s(t) - p(t) &= (0.05 + 0.2L^{-1} + 0.05L^{-2} - 0.2L^{-3} - 0.1L^{-4})m(t) \quad (14)
\end{align*}
\]

With nominal overshooting in equation (12), neither undershooting nor overshooting in equation (13) and nominal undershooting in equation (14), the real exchange rate overshoots. In all three equations the impulse response changes sign. In all three equations the impact response to a unit step is positive, but the steady state response is zero as is consistent with real exchange rates in Dornbusch (1976).

Even if Bjørnland (forthcoming) had demonstrated that real exchange rates overshoot, which it does not, that overshooting would not imply that nominal exchange rates overshoot. In other words, Bjørnland (forthcoming) provides no convincing evidence that nominal exchange rates overshoot.

5. Closing Comments

I have been studying the behavior of exchange rates for over forty years. I have often found convincing evidence of undershooting in daily exchange rates. That is the spectrum for changes in daily exchange rates has less power at high frequencies than at low frequencies. Usually that undershooting can be explained by official intervention.

I have never found convincing evidence of overshooting. Whenever I have investigated an article that claims to find evidence of overshooting I have always found the article seriously flawed. That is the case here.

Like most of the overshooting literature, both Scholl and Uhlig (2008) and Bjørnland (forthcoming) claim to find evidence of overshooting in impulse response functions estimated with VAR. Like other articles in the literature that claim to find such evidence, neither article does so.

Like most of the overshooting literature, these two articles misinterpret their impulse response functions. The first problem is that they interpret impulse response functions as though they were step response functions. The second problem is that they derive unwarranted economic conclusions from those impulse responses. In order to derive legitimate economic interpretations from impulse responses, one must consider how the impulse responses and monetary shocks interact. A given impulse response can be consistent with overshooting, undershooting or an efficient market depending on the nature of the monetary shocks. Since neither Scholl and Uhlig (2008) nor Bjørnland (forthcoming) consider that interaction, their impulse responses tell us nothing about how foreign exchange markets respond to news...
about monetary shocks. Of course the same can be said for essentially all the literature inspired by Eichenbaum and Evans (1995).

Since, in the absence of intervention, exchange rates are essentially martingales, the most plausible interpretation of the typical impulse responses found in that literature would seem to be that they convert monetary shocks that are not white noise into changes in exchange rates that are white noise. In other words, the impulse response functions are probably consistent with efficient foreign exchange markets. At the very least, neither Scholl and Uhlig (2008) nor Bjørnland (forthcoming) provide any convincing evidence of overshooting, delayed or otherwise.

REFERENCES


Bjørnland, Hilde, forthcoming, Monetary policy and exchange rate overshooting, Dornbusch was right after all, *Journal of International Economics*, forthcoming.


**Appendix**

Equation I is the generic time domain model for changes in exchange rates.

\[
\Delta s(t) = (a_0 + a_1 L^{-1} + a_2 L^{-2}) \Delta m(t)
\]  

(I)

The Laplace transform of \( L^{-k} \) is \( e^{-ik\omega} \) where \( i = \sqrt{-1} \) and \( \omega \) equals \( 2\pi f \). The highest observable frequency \( f \) is 0.5 cycles per period.

Equation (II) is the Laplace transform of equation (I).

\[
\Delta s(f) = [a_0 + a_1 e^{-i\omega} + a_0 e^{-i2\omega}] \Delta m(f)
\]  

(II)

where \( \Delta s(f) \) is the Laplace transform of \( \Delta s(t) \) and \( \Delta m(f) \) is the Laplace transform of \( \Delta m(t) \). If \( x(f) \) is the Laplace transform of \( x(t) \), then the spectrum for \( x(t) \) denoted \( \Gamma_x(f) \) equals \( s(f)s(f)^* \) where \( s(f)^* \) is the complex conjugate of \( s(f) \). Since \( e^{i\omega} \) equals \( \cos(k\omega) - \sin(k\omega) \) and \( e^{i2\omega} \) equals \( \cos(k\omega) + \sin(k\omega) \), equation (II) implies that the generic spectrum for \( s(t) \) equals equation (III).

\[
\Gamma_{\Delta s}(f) = [(a_0)^2 + (a_1)^2 + (a_2)^2 + 2.0(a_0a_1 + a_1a_2)\cos(\omega) + 2.0a_0a_2\cos(2\omega)] \Gamma_{\Delta m}(f)
\]  

(III)

The square root of \([(a_0)^2 + (a_1)^2 + (a_2)^2 + 2.0(a_0a_1 + a_1a_2)\cos(\omega) + 2.0a_0a_2\cos(2\omega)]\) is called the gain from the input to the output.

For the efficient market model, \( a_0 = 1.0 \) and \( a_1 = a_2 = 0.0 \). For the Dornbusch overshooting model and the Eichenbaum & Evans undershooting model, \( a_0, a_1 \) and \( a_2 \) are the coefficients from the appropriate impulse response.

Of course the Dornbusch overshooting model is in continuous time. Equation (IV) describes the spectrum for changes in the exchange rate produced by changes in the money stock implied by that model.
$$\Gamma_\Delta (f) = \frac{[(\lambda \theta v)^2 + ((1 + \lambda \theta)(2\pi f))^2]/[(\lambda \theta v)^2 + (\lambda \theta 2\pi f)^2]}{\Gamma_\Delta m (f)}$$  \hspace{1cm} (IV)

When $\Gamma_\Delta m (f)$ equals 1.0, $\Gamma_\Delta (f)$ looks like the Dornbusch Overshooting spectrum in Figure 2. For examples of such spectra, see Figures 6 and 7 in Pippenger (2008). For a formal derivation of equation (IV) and the full frequency domain implications of that model, see the Appendix to Pippenger (2008).

The fourier transform of $(1.0/T)(e^{-2/T})$ is $1.0/(1.0 + j2\pi fT)$. With the spectrum for $\Delta m(t)$ equal to 1.0, the spectrum for $\Delta s(t)$ equals that fourier transform times it's complex conjugate.

$$\Gamma_\Delta s (f) = \frac{1.0}{1.0 + (2\pi fT)^2} \Gamma_\Delta m (f)$$  \hspace{1cm} (V)

Except for the fact that here frequency runs from zero to infinity, this spectrum is very similar to the spectrum for undershooting in Figure 2.

Equation (VI) describes a generic impulse response function in discrete time with four lags.

$$H(\lambda) = a_0 + a_1 L^{-1} + a_2 L^{-2} + a_3 L^{-3} + a_4 L^{-4}$$  \hspace{1cm} (VI)

This impulse response implies the following spectrum for $\Delta s(t)$:

$$\Gamma_\Delta (f) = [(a_0)^2 + (a_1)^2 + (a_2)^2 + (a_3)^2 + 2.0(a_0a_1 + a_1a_2 + a_2a_3 + a_3a_4)\cos(\omega) +
2.0(a_0a_2 + a_1a_3 + a_2a_4)\cos(2\omega) + 2.0(a_0a_3 + a_1a_4)\cos(3\omega) + 2.0(a_0a_4)\cos(4\omega))\Gamma_\Delta m (f)$$  \hspace{1cm} (VII)

A simple way to see what this generic impulse response implies about undershooting is to consider the difference between the spectrum when frequency equals zero or $\Gamma_\Delta s (0)$ and when frequency equals 0.5 or $\Gamma_\Delta s (0.5)$. [Note that $\cos(0)$ equals 1.0. In addition, when $f$ equals 0.5, $\omega$ equals $\pi$ and $\cos(\pi)$ equals -1.0, $\cos(2\pi)$ equals 1.0, $\cos(3\pi)$ equals -1.0 and $\cos(4\pi)$ equals 1.0]

$$\Gamma_\Delta s (0) - \Gamma_\Delta s (0.5) = 4.0(a_0a_1 + a_1a_2 + a_2a_3 + a_3a_4) + 4.0(a_0a_3 + a_1a_4)$$  \hspace{1cm} (VIII)

It is obvious from equation (VIII) that, as long as the parameters are all positive, there is undershooting. $\Gamma_\Delta s (0)$ is larger than $\Gamma_\Delta s (0.5)$.

As an example of Dornbusch overshooting, consider the following impulse response function:

$$1.2, -0.1, -0.05, -0.03, -0.02.$$  

The impact response to a unit step is greater than 1.0 and the steady state response is 1.0. Given equation (VI), that impulse response implies the following spectrum for $\Gamma_\Delta s (0) - \Gamma_\Delta s (0.5)$:

$$\Gamma_\Delta s (0) - \Gamma_\Delta s (0.5) = -0.684$$  \hspace{1cm} (IX)

There is overshooting. The short-run response to news is larger than the long-run response.
Panel C: Impulse response exchange rate after one standard deviation drop in Foreign interest rate

Figure 1
Panel C of Figure 4 from Bacchetta and van Wincoop (forthcoming).

Figure 2
Spectra for $\Delta s(t)$
Figure 3
Figure 1 from Scholl and Uhlig labeled a stylized representation of the delayed overshooting puzzle.