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Dennis A. Reilly
(Ph. D. Thesis)

June 1, 1971

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LIGHT SCATTERING BY ION THERMAL FLUCTUATIONS
IN A DENSE PLASMA

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LIGHT SCATTERING BY ION THERMAL FLUCTUATIONS
IN A DENSE PLASMA

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1 June 1971

ABSTRACT

Laser scattering at 90 deg is used to obtain a local, detailed mapping in space and time of plasma development in a dense, relatively cold sheet pinch in deuterium gas. The observed plasma scattering parameter, $\alpha$, covers a range between 0.5 and 2.0 with plasma electron densities between $3 \times 10^{15}/\text{cm}^3$ to greater than $10^{17}/\text{cm}^3$, temperatures between 0.5 and 3 eV, and with gross plasma velocities up to 2 cm/$\mu$sec.

The scattering measurements are focused on observation of the central portion of the scattering spectrum, controlled by ion fluctuations, along with a segment of the wings, controlled by electron motions.

A numerical method for reducing scattering data is presented. Plasma parameters along with expected errors are deduced from each laser shot, with each spectral channel weighted according to its measured statistics and with full account being taken of the finite spectral width of both the laser and the spectrum measuring channels. The problem of parameter deduction is overdetermined, which allows for a test of the internal consistency.
of the scattering data.

Another laser scattering method is employed in which the spectrum of the "electron wings" is determined. Comparative determinations of density and temperature are also obtained using standard spectroscopic methods. All of those data and the results of photographic and magnetic probe measurements are combined in a simple analysis of the sheet pinch dynamics.

The different determinations of plasma parameters are in reasonable agreement except at times of rapid changes and/or high densities. In these instances the laser scattering data based on the ion feature do not satisfy the self-consistency test. Non-uniformities or turbulence in the plasma, heating of the plasma by the discharge current or by the laser pulse, Rayleigh scattering from electrons in the first excited atomic level, and distortions of the plasma dielectric function due to electrons in highly excited states or by photo-ejected electrons are investigated as possible causes of the discrepancy.

In particular it is shown that the ratio of the population density of the first excited hydrogenic state to the electron density can be significantly greater than the steady-state value whenever the plasma is at less than full steady-state ionization and that Rayleigh scattering by electrons in those levels will cause an enhancement of the ion feature.

Several mechanisms may be acting and a resolution of the problem would depend on more complete knowledge of the neutral atomic density and/or improved kinetic theory for the effect of
excited bound states with orbits on the order of the Debye radius.
I. INTRODUCTION

The development of diagnostic techniques has played an essential role in the progress of plasma physics. Traditionally, the experimenter has had the choice of local and possibly perturbing measurements utilizing physical probes, or of observational methods that average over at least one dimension of the plasma. Microwave and magnetic coil measurements, as well as those based on electromagnetic or particle emanations, are included in the latter category.

During the last decade, the laser beam has come into increasing use as a local nonperturbing plasma probe, allowing determination of electron densities ranging from less than $10^{13}/\text{cm}^3$ to greater than $10^{17}/\text{cm}^3$ and temperatures from $1 \text{eV}$ to the keV range. An excellent survey of a number of the experiments may be found in Ref. 1.

The present experiments are an extension of a method pioneered by O. A. Anderson\(^2,3\) in which laser scattering controlled by ion thermal fluctuations is used for determination of electron density, temperature, and directed ion velocities in a dense cold plasma. The primary objectives were a refinement of the technique allowing a detailed mapping in space and time of a sheet pinch plasma, followed by a scrutiny of the data thus obtained for internal consistency and for agreement with another laser scattering method ("electron feature" scattering), with independent measurements of plasma parameters, and with plasma dynamics.

The basic experimental arrangement is depicted in Fig. 1.
Fig. 1. Basic experimental arrangement. Plasma current flow is out of the page. The position of the focused laser beam could be shifted from the central plane ±6 mm towards or away from the spectrometer.
A giant pulsed ruby laser beam was brought to a 0.3-mm diameter focus at the approximate center of the discharge chamber. Light scattered at 90 deg from a 7-mm-long segment of the incident beam was focused on the entrance slit of an eight-channel spectrum analyzer (polychromator).

The sheet pinch discharge had a quarter cycle rise time of 3.1 µsec with a peak current of 150 kA in an initial fill of 0.73 torr deuterium gas. Current flow was in a direction normal to the incident and sampled beams. The general form of the discharge consisted at first of two thin, low-ionization fronts which proceeded towards a central plane from the two walls of the chamber normal to the observed scattering direction. The neutral fill gas was accelerated to the front velocity through ion-neutral charge exchange followed by ion-ion collisions. Collision of the fronts at the central plane resulted in thermalization of directed energy and an increase of ionization, with subsequent expansion of the compressed plasma.

Shifting of the laser focal spot allowed multipositional sampling and the giant pulse could be timed at any instant of the discharge.

The scattered laser beam samples an electron density fluctuation with wave number \( k \), which is equal to the vector difference of the incoming and scattered beam wave numbers. In these experiments \( k \) was fixed by the constant 90 deg scattering geometry and the 6943 Å ruby wavelength. The ratio of the inverse of \( |k| \) to the plasma Debye length, \( \lambda_D \), is conventionally
designated by the plasma scattering parameter, $\alpha$. In what follows, the general form of the scattering spectra and the information obtainable therefrom will be briefly introduced.

If $\alpha$ is less than about 0.5, simple Thompson scattering obtains and the spectra are Gaussian distributions (Maxwell distributions are assumed for the electrons and the ions), symmetrical about the scattering frequency with an e-folding angular frequency of $\Delta \omega = \omega_e = \left| k \right| \sqrt{2kT_e/m_e}$, the characteristic Doppler frequency of the thermal electrons with temperature $T_e$. Measurement of the spectral amplitude is necessary for a determination of the electron density. This may be done by calibrating the measured signals against Rayleigh scattering from a gas of known cross section, or by computation of the scattering volume, optical arrangement and photomultiplier response.

As the ratio of the sampled wavelength to the Debye length increases, electron-electron correlations come into play. In the region $0.5 < \alpha < 2$, germane to these experiments, the spectrum becomes flatter, and distinct shoulders develop on the wings. The reader is here referred to Fig. 3 (see p. 17). The location of these shoulders is given approximately by $\Delta \omega = \alpha \omega_p = \sqrt{2} \omega_p$, where $\omega_p$ is the plasma frequency. The frequency spread is a function of electron density only. However, the shape of the spectra is a function of $\alpha = \alpha(n_e, T_e)$, which allows the electron temperature to be determined. The amplitude of the spectral distribution is a function of both $T_e$ and $n_e$ and measurement of this factor can be used as a check on the self-consistency of the data.
Within the same range of $\alpha$, a new phenomenon, the "ion feature", governed by electron-ion and ion-ion correlations becomes apparent. A narrow scattering profile similar to the previously described "electron feature" is superimposed on the "electron feature". Its shape is characterized by a parameter $\beta = \beta(\alpha, T_e/T_1)$, which plays a role similar to $\alpha$ [see Eqs. (27) and (28)], and its amplitude is a function of $n_e, T_e$, and $T_e/T_1$.

If $\beta$ is less than about 0.5, the shape is again Gaussian with an e-folding frequency of $\Delta \omega = \omega_1 = |k| \sqrt{2\pi T_1/m_1}$, the ion Doppler frequency. The value of $\beta$ cannot be inferred from the shape in this case. If $\beta$ is between 0.5 and 2, the feature flattens in the center and shoulders develop at $\Delta \omega = \beta \omega_1$. In principle the value of $\beta$ can now be determined from the shape, but this is sometimes difficult in practice due to finite spectral resolution and statistical fluctuations.

It is to be noted that if the plasma has a gross velocity both the electron and the ion feature will be displaced as a unit from the laser frequency according to the appropriate Doppler shift. For velocities on the order of the ion thermal velocity, this effect is best observed using the ion feature since the asymmetry will be a significant fraction of the profile width.

If only the central ion feature, including its magnitude, were measured, it may be inferred from Table I that the 5 parameters $\alpha, \beta, n_e, T_e, T_e/T_1$ are underdetermined for $\beta < 0.5$, since the 5th relation, the value of $\beta$, cannot be obtained. The situation is improved if $T_e = T_1$ can be assumed.
Table I. Relations to be used in determining the five parameters \( \alpha, \beta, n_e, T_e, T_e/T_i \). \( 0.5 < \alpha < 2 \).

<table>
<thead>
<tr>
<th>Measure Ion Feature Only</th>
<th>( \beta &lt; 0.5 )</th>
<th>( 0.5 &lt; \beta &lt; 2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 ( \alpha = \alpha (T_e, n_e) ) definition</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 ( \beta = \beta (T_e/T_i, \alpha) ) definition</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 Measure amplitude = ( f_1 (n_e, T_e, T_e/T_i) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 Measure ( \Delta \omega = \omega_1 (T_i) )</td>
<td>Measure ( \Delta \omega = \beta \omega_1 (T_i) )</td>
<td>e-folding frequency.</td>
</tr>
<tr>
<td>5 ( \beta ) cannot be determined from shape.</td>
<td>( \beta ) can be determined from shape, in principle, but difficult.</td>
<td></td>
</tr>
<tr>
<td>6 ( T_e = T_i ). Assumption based on other data.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7 Measure ratio of &quot;ion feature&quot; to point on &quot;electron plateau&quot; = ( f_2 (\alpha, T_e/T_i) ).</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Anderson noted that the ratio of the central point on the ion feature to points on the flat part of the electron feature is a strong function of $\alpha$. Thus, by including a single spectral measurement of the "electron plateau", the problem is overdetermined (assuming $T_e = T_i$) for all values of $\beta$ and thus amenable to self-consistency checks.

Although the presentation of the parameter deduction problem has been in the spirit of the traditional "hand fit" methods, an essential portion of this work was the adaptation of known techniques to the proper statistical weighting of the factors involved, thus giving greater confidence in the consistency checks. A computer code was used for the solution of the non-linear "least-square fit" problem of parameter determination. Spectral data points were weighted according to experimentally determined statistics. A description of the method is presented in Sec. III. C. and Appendix A.
II. SCATTERING THEORY

The theory of scattering of laser light by a plasma has been treated in detail by a number of authors. In this section a brief heuristic derivation of these results will be given.

A. Form Factor

Consider the scattering geometry shown in Fig. 2 in which $\mathbf{k}_0$ represents the wave vector of an incoming monochromatic plane wave front and $\mathbf{k}_s$ represents a wave scattered toward a detector located at a distance $R$ from the plasma. The largest dimension of the scattering volume is assumed to be much less than $R$. It is also assumed that the frequency of the incoming radiation, $\omega_0 = c|\mathbf{k}_0|$, is much greater than the plasma frequency, $\omega_p = (4\pi ne^2/m)^{1/2}$, thus assuring penetration of the plasma with an index of refraction effectively equal to unity. The scattered electron field at the detector, $E_s$, may be written in terms of the incident field, $E_0$, cf. Jackson,

$$E_s = E_0 \frac{r_0}{R} \sin \psi \sum_j e^{i\mathbf{k}_0 \cdot \mathbf{r}_j(t') - i\omega_0 t'}, \quad (1)$$

where $r_0$ is the classical electron radius $e^2/mc^2$, $\psi$ is the angle between $\mathbf{E}_0$ and the scattering plane and will be assumed to be 90 deg in what follows, and $\mathbf{r}_j(t')$ is the position of the $j$th electron at the retarded time $t'$. The sum is over all the electrons in the scattering volume.

The retarded time is given approximately by
Fig. 2. General scattering geometry showing incident and scattered wave vectors and electric fields and sampled wave vector \( k \).
where \( \mathbf{k}_s \) is a unit vector in the direction of the detector. Substituting Eq. (2) and shifting the time scale by \( t - R/c \), the scattered field is

\[
\mathbf{E}_s = \mathbf{E}_0 \frac{R_0}{r} \sum_j e^{i(k \cdot \mathbf{r}_j(t) - \omega_0 t)} \tag{3}
\]

where

\[
k = k_0 - k_s
\]

\[
k_s = \frac{\omega_0}{c} \mathbf{k}_s. \tag{4}
\]

It is seen that the field depends on the incident and scattered wave numbers only through their difference, which is uniquely determined by the experimental choice of incident frequency and scattering angle \( \theta \). The magnitude of the scattering vector, \( k_s \), is \((2\omega_0/c) \sin \theta/2\) and for a given frequency it may be varied by changing the scattering angle. In the present experiment, \( \theta \) is fixed at 90 deg.

The Wiener-Khintchine theorem (cf. Born and Wolf)^10 is used to obtain the power spectrum in terms of the Fourier transform of the autocorrelation function of the electric field.

\[
I(k, \omega) = \frac{c}{4\pi} \int_{-\infty}^{\infty} dt \text{e}^{i\omega t} \frac{1}{T} \int_{-\infty}^{\infty} dt \langle \mathbf{E}_s^*(t) \cdot \mathbf{E}_s(t + \tau) \rangle. \tag{5}
\]
The limit $T \to \infty$ is understood. The brackets $\langle \rangle$ represent an ensemble average over identical scattering systems.

The light received by the detector is $I(k,\omega)R^2d\Omega\Delta\omega$, where $d\Omega$ is the solid angle and $\Delta\omega$ the bandwidth. On substitution of Eq. (3) into Eq. (5) and division by the incident flux $c|E_0|^2/4\pi$ and by the scattering volume $V$, the cross section per unit volume takes the form

$$\sigma(k,\omega) = \sigma_T S(k,\omega), \quad (6)$$

where $\sigma_T = r_0^2$ and the "form factor" is

$$S(k,\omega) = \frac{1}{VT} \int_{-\infty}^{\infty} \frac{1}{dte} \int_{-\infty}^{\infty} \frac{1}{dt} \left( \sum_i \sum_j e^{ik \cdot [\mathbf{r}_i(t) - \mathbf{r}_j(t+\tau)]} \right) \quad (7)$$

At this point, it is interesting to investigate the form of the scattered light if all the electrons are moving on straight uncorrelated orbits. Such an assumption was made by Bowles, using the theory of Gordon, in early scattering experiments from the ionospheric plasma. In this case the electron positions would be

$$\mathbf{r}_i(t) = \mathbf{r}_{i0} + \mathbf{v}_i t \quad (8)$$

and the velocities and initial positions are assumed to be independent of each other. Thus

$$S(k,\omega) = \frac{1}{V} \sum_{ij} (2\pi(\omega - \omega_0 - k \cdot \mathbf{v}_j)) \frac{1}{T} \int_{-\infty}^{\infty} dte e^{ik \cdot (\mathbf{v}_i - \mathbf{v}_j)t} \langle e^{-ik \cdot (\mathbf{r}_{i0} - \mathbf{r}_{j0})} \rangle \quad (9)$$
The last factor yields the Kroneker delta function $\delta_{ij}$ since it is simply a volume average over the difference vector
$$\varpi = \varpi_{i0} - \varpi_{j0}. \quad (A\ uniform\ plasma\ is\ assumed\ and\ boundaries\ are neglected.)$$
With $i = j$, $1/T$ times the integration over $dt$ gives unity, leaving

$$S(k, \omega) = 2\pi N/V \{\delta(\omega - \omega_0 - k \cdot \varpi)\}, \quad (10)$$

where $N = \Sigma$ is the number of electrons in $V$. The average over $j$ velocity is performed with a distribution function $f_e(v)$ normalized to unity

$$S(k, \omega) = \frac{2\pi n}{k} F_e \left( \frac{\omega - \omega_0}{k} \right) \quad (11)$$

$$F_e \left( \frac{\omega - \omega_0}{k} \right) = \int d^3v f_e(v) \delta(\omega - \omega_0 - k \cdot \varpi).$$

The spectrum shape is that of the velocity distribution function projected onto $k$, and it is centered about the incident frequency $\omega_0$. In what follows, $\omega$ will be used for the frequency difference $\omega - \omega_0$.

Returning to the more general case, the microscopic plasma density can be expressed as

$$n(r, t) = \sum_i n_i(r, t) = \sum_i \delta \left[ (r - r_i(t)) \right]$$

with spatial Fourier transform

$$n(k, t) = \sum_i e^{-i k \cdot r_i(t)}. \quad (12)$$
The form factor is then, from Eqs. (7) and (12)

\[ S(k, \omega) = \frac{1}{VT} \int d\tau e^{i\omega \tau} \int dt \langle n^*(k, t) n(k, t + \tau) \rangle = \frac{1}{VT} \langle |n(k, \omega)|^2 \rangle, \]  

(13)

where the last step makes use of the Faltung theorem.\(^{13}\)

B. Test Particle Picture

The electron density fluctuation spectrum including the effects of charged particle interactions may be done, in the case of complete thermal equilibrium, by recourse to the generalized Nyquist fluctuation dissipation theorem (cf. Dougherty and Farley\(^6\)). A more easily generalized approach based on plasma kinetic theory was used by Salpeter,\(^5\) whose notation will be followed, and others. Rosenbluth and Rostoker\(^7\) and Rostoker\(^14\) described plasma correlations in terms of test particle interactions, which leads to a physically intuitive picture of the scattering spectrum. The derivation will be continued in the spirit of the test particle theorems.

Consider a test charge q\(_S\) represented by number density

\[ n^T_S = \delta \left[ \mathbf{r} - \mathbf{r}^T_S(t) \right] \]

moving through the plasma on the known orbit \( \mathbf{r}^T_S(t) \). The test charge will polarize the plasma out to approximately the Debye radius, \( \lambda_D = (kT/4\pi n_0 e^2) \), producing deviations from the equilibrium density \( n^P_0 \) of \( n^P_{se} \) and \( n^P_{si} \), where \( e \) and \( i \) stand for electron and ion, respectively. From Poisson's equation, the Fourier-transformed electric field is
The induced charge densities are related to the field by the polarizability factors \( G_e \) and \( G_i \)

\[
4\pi (q_s n_s^T - e n_{se}^p + e n_{si}^p) = ik \cdot E. \tag{14}
\]

and thus the field is derived from \( n_s^T \) by

\[
4\pi q_n n_s^T = (1 - G_e - G_i) ik \cdot E. \tag{16}
\]

The factor \((1 - G_e - G_i)\) is recognized as the plasma dielectric constant \( K \). Combining Eqs. (15) and (16), the induced electron density is

\[
n_{se}^p = -\frac{q_s G_e}{e K} n_s^T. \tag{17}
\]

The total electron fluctuation is the superposition of all the test electrons (i.e. all the plasma electrons) and their induced electron densities plus the induced electron densities due to test ions of charge \( z \).

\[
6n_e = \left(1 + \frac{G_e}{K}\right) \sum_{s=\text{electrons}} n_s^T - z \frac{G_e}{K} \sum_{s=\text{ions}} n_s^T. \tag{18}
\]

In this picture, all correlations are accounted for in the polarization factors and the test particles are considered to be independent of each other. Placing the test particles on straight
orbits as in Eq. (8), transforming to \( \omega k \) space, and following the steps of Eqs. (9) through (11), the form factor becomes

\[
S(k,\omega) = \frac{2\pi n_0}{k} \left[ 1 + \frac{G_e}{K} F_e(\omega/k) + \frac{g_i}{k} F_i(\omega/k) \right]^2
\]

(19)

where \( F_i(\omega/k) \) is the projection of the ion distribution function.

C. Explicit Expression of Form Factor

The polarizability factors are readily obtained through solution of the Vlasov equation including the self-consistent field due to a test charge. For the case where the electron and ion velocity distributions are displaced Gaussians corresponding to temperatures \( T_e \) and \( T_i \) and to drift velocities \( v_{0e} \) and \( v_{0i} \), respectively, the factors are, following Salpeter,

\[
G_e = -\alpha^2 G(x_e), \quad G_i = -z \frac{T_e}{T_i} \alpha^2 G(x_i).
\]

(20)

\( x_e \) and \( x_i \) are frequency shifts normalized to the characteristic electron and ion thermal Doppler shifts

\[
x_e = \frac{(\omega - k \cdot v_{0e})/\omega_e}{\omega_e} = k(2\pi T_e/m)^{1/2} \]

\[
x_i = \frac{(\omega - k \cdot v_{0i})/\omega_i}{\omega_i} = k(2\pi T_i/M)^{1/2}.
\]

(21)

The "scattering parameter" \( \alpha \) is defined,

\[
\alpha = 1/(k\lambda_D) = \pi/\lambda_D,
\]

(22)

the ratio of the sampled plasma wavelength to the Debye length.

The function \( G(x) \) is defined
\[ \text{Re } G(x) = 1 - 2xe^{-x^2} \int_0^x e^{t^2} \, dt \]
\[ \text{Im } G(x) = \pi^{1/2}xe^{-x^2}. \]

The real part of \( G \) is plotted in Fig. 18, Sec. III. C. 2, p. 60, where an approximating function suitable for computer evaluation is presented. In terms of these definitions, the form factor Eq. (19) becomes

\[ S(k, \omega) = \frac{n}{2\pi \sqrt{\pi}} \left[ 1 + \alpha^2 G(x_1) \right] \alpha^2 e^{-x_1^2} \frac{dx_1}{\sqrt{\pi}} + \frac{e^{2z}e^{-x_1^2}}{\sqrt{\pi}} \right] \]

For the case \( T_e = T_i, \nu_0e = \nu_0i = 0, \) and \( M/m = 3672 \) (i.e. deuterium) Fig. 3 plots \( \text{H}(x_e) \) for a pertinent range of \( \alpha, \) where \( \text{H} \) is related to \( S \) by

\[ S(k, \omega, x) = n_e \frac{\text{H}(x)}{\sqrt{\pi} \omega_e}. \]

Salpeter\(^5\) has introduced an interesting approximation to Eq. (24) based on the fact that in the region \( x_1 \gg 1, G(x_1) \approx 0 \) and that for \( x_e \ll 1, G(x_e) \approx 1. \) Then, for \( T_e/T_i \approx 1, S(k, \omega) \) is represented over most of the spectrum by the sum of two terms of similar form.
Fig. 3. Theoretical scattering profiles for a deuterium plasma.

The terms "ion peak," "electron plateau," and "electron profile" are used in the text to refer to the regions 

\[ x_e \lesssim 0.03, \ 0.08 \lesssim x_e \lesssim 0.5, \ \text{and} \ 0.08 \lesssim x_e, \] 

respectively. 

\( x_e \) is a normalized frequency shift, \( x_e \equiv \Delta \omega / \omega_e \), and \( H(x_e) \) is related to the scattering cross section per unit volume by 

\[ \sigma(k, \omega) = \sigma_n \pi H(x_e) / \sqrt{\pi} \omega_e. \]
Fig. 3.
\[
S(k, \omega) \frac{d\omega}{2\pi} \approx n \left[ e^{-x_e^2} \frac{dx_e}{|1 + \alpha^2 g(x_e)|^2 \sqrt{\pi}} \right. \\
\left. + \frac{z \alpha^4}{(1 + \alpha^2)^2} e^{-x_i^2} \frac{dx_i}{|1 + \frac{\alpha^2}{\frac{e}{m} \frac{\alpha^2}{1 + \alpha^2} g(x_i)}|^2 \sqrt{\pi}} \right]. 
\] (26)

The similarity becomes more apparent if an "ion scattering parameter", \( \beta \), is defined

\[
\beta^2 \equiv z \frac{\frac{e}{m}}{\frac{T_1}{m} \frac{\alpha^2}{1 + \alpha^2}} 
\] (27)

in which case

\[
S(k, \omega) \frac{d\omega}{2\pi} = n \left[ \Gamma_\alpha(x_e) dx_e + z \frac{\alpha^2}{1 + \alpha^2} \Gamma_\beta(x_i) dx_i \right], 
\] (28)

\[
\Gamma_\alpha \equiv \frac{e^{-x^2}}{\sqrt{\pi} \ |1 + \alpha^2 g(x)|^2}. 
\]

The first term is generally referred to as the "electron contribution" and the second as the "ion contribution." Computer generated plots of \( \Gamma_\alpha(x) \) are available for "hand fitting" experimental data to the theory. 60

Salpeter was able to perform the analytic integral of \( \Gamma_\alpha(x) \), which yields the respective total contributions of the electrons and ions.
D. Physical Interpretation

The case of $\alpha << 1$ corresponds to the sampling of charge inhomogeneities in the plasma over lengths short compared to the Debye length. The test charges are not shielded over such short distances; i.e., $G_e, G_i \to 0$ and thus the form factor, Eq. (19), reduces to Eq. (10), the random scattering from uncorrelated test electrons.

On the other hand, when $\alpha$ is large compared to unity, completely shielded test particles are "viewed" and, except for frequencies at which $K \approx 0$, the factor $|1 + G_e/K|^2 \approx |1 + \alpha^2 G(x)|^{-2}$ is small. The "dressing" of a test electron is achieved through attraction of field ions, which do not scatter, and through repulsion of field electrons leading to a decrease of test electron charge plus field electron charge. In contrast, the test ions have up to one-half (for $T_i = T_e$ and $z = 1$) of their shielding provided by field electrons moving with velocities on the order of the ion thermal velocities. Thus scattering with a frequency spread characteristic of the ions is observed.

Due to a zero in the real part of the dielectric function, the electron feature does exhibit a spike near the electron plasma frequency at

\[
n \Gamma_e(k) = \frac{n}{1 + \alpha^2}
\]

\[
n \Gamma_i(k) = n \frac{z\alpha^4}{(1 + \alpha^2)^2} \frac{1}{1 + \beta^2} = \frac{n\alpha^4}{(1 + \alpha^2)^4 \left[1 + (1 + zT_e/T_i)\alpha^2\right]}.
\]
\[ \omega^2 = \omega_p^2 + \frac{(3kT_e/m)}{k^2}. \] (30)

That electron oscillations should be apparent for \( \alpha \gg 1 \) is, of course, consistent with the well known fact that Landau damping becomes negligible for wavelengths much greater than the Debye radius.* It is also noted that ion waves are observed if \( T_e \gg T_i \) or if the electrons have substantial drift velocities with respect to the ions.

In the more general case, \( \alpha \approx 1 \), such as observed in the experiments to be described, both the electron and ion features occur through the mechanisms described in their limits.

*The theory is, however, limited to wavelengths shorter than the mean free path for Coulomb collisions.
III. EXPERIMENTAL METHOD

The experimental design was essentially that used by Anderson. However, a number of refinements were carried out to achieve the experimental objective of a detailed mapping in space and time of the ribbon pinch development. Improvements in the stability of the laser's spectral purity, timing, and energy output were made. A new approach to the suppression of stray scattered light was demanded by the requirement of multipositional sampling. Discharge parameters were chosen and photon collection improved so that data on temperature, density, and plasma velocity could be obtained from the snowplow stage through to the expansion stage. Collection of the large amount of scattering spectrum data was facilitated by the development of memory units for the outputs of the photomultipliers. Magnetic probe and Stark broadening measurements were made for correlation and comparison with the laser scattering results. In the following subsections the experimental method will be reviewed with particular emphasis placed on the refinements mentioned.

A. Plasma Production

The sheet pinch and the related Triax device have received extensive treatment in the literature. Because of its high density, relative stability, and geometrical configuration the sheet pinch is a suitable subject for study using laser scattering as a diagnostic tool. Figure 4 gives a schematic view of the ribbon pinch, shown at the stage when snowplowing plasma sheets are approaching each other. A more accurate view of the
Fig. 4. Sheet pinch, showing conducting box (insides of side walls are insulated), insulated anode, imploding plasma sheets, magnetic field and current flow.
construction details may be had by reference to Fig. 5.

At the top of the chamber is a screen electrode whose apertures allowed photographic observations of the plasma. No differences in plasma behavior were observed when this screen was replaced by either a quartz plate or a solid copper plate. The upper electrode was used as the cathode. This was because the entrance window for the laser beam was closer to the upper electrode than to the bottom electrode and there was reason to suspect non-uniform plasma behavior in the anode region. Experiments with a parallel plate rail gun\textsuperscript{19} show electron density and current in the anode region (out to a distance of 2 cm from the anode) leading the main body of the current sheet. This was attributed to probably precursor ionization due to photoemission from the anode. Anderson\textsuperscript{20} noticed an anode effect with a copper anode in the ribbon pinch, but this effect was greatly reduced with a quartz anode. The transparency and different photoemissivity of the quartz may account for these differences.

The sequence of Kerr cell photographs, Fig. 6, taken through the cathode shows at times a considerable degree of non-uniformity. To remove ambiguity as to the source of the light emission, the apparatus shown in Fig. 7 was constructed to take photographs in stereo pairs. Image size and displacement was chosen such that a 3-dimensional effect could easily be obtained by direct observation of the photograph without a viewer. Observation of these pairs, Fig. 8, shows that a good deal, but not all, of the non-uniform light is coming from the anode region, i.e., from the
Fig. 5. Details of pinch discharge chamber construction. Stray rays from wall are prevented from scattering off of imperfections in viewing window by means of the window extension and baffles shown.
Fig. 6. Kerr cell photographs of pinch development taken through transparent cathode. The time after the start of current flow and the camera aperture are shown.
Fig. 7. Schematic diagram of optical arrangement for taking stereo photographs of pinch development.
Fig. 8. (a) Stereo pair of the discharge chamber with the interior artificially lighted. (b) Stereo pair of plasma light 1.8 μsec after start of current flow. A three-dimensional effect may be had by focusing one eye on each picture and staring until the two images converge. It then becomes evident that with the exception of the ends of the pinch, most of the light is from the electrodes.
bottom of the chamber.

The pinch chamber was pierced by four ports of 1-1/8 in. diameter to allow for laser beam entrance, laser beam dump, scattered light viewing, and viewing dump. The presence of these ports necessarily leads to some weakening of the magnetic field in the viewing region. It should be noted, however, that no differences in plasma behavior were noted when the viewing and plasma dumps were replaced by quartz windows that were flush with the inside surface of the alumina insulator. Conceivably, these dumps could have been the sites of secondary discharges or could have caused distortions to the plasma due to the reservoir of gas contained in them.

The current discharge was produced by means of a 33-μF capacitor bank with a 3.1 μsec quarter cycle rise. The circuit was heavily damped to avoid current reversals which could lead to more rapid erosion of the window surfaces by the plasma. Current switching was by means of a low-inductance, high-current switch. Current and voltage wave forms are shown in Fig. 9. Shock excitation and subsequent ringing of the length of RG 8U cables between the switch and the discharge tube led to a ±50 nsec uncertainty as to the beginning of the current flow.

The choices of discharge time constant, bank voltage, working gas, and gas pressure were made in view of the objective of mapping the plasma over a wide range of space time points. To this end a dense, relatively cool, slowly developing plasma was desired. As discussed in Sec. I the ion scattering feature
Fig. 9. Oscilloscope traces of (a) pinch current, 47 kA/div.,
and (b) discharge tube voltage, 650 V/div. The time scale
is 1 µsec/div.
becomes more prominent for \( \alpha = \sqrt{n_e/T_e} \geq 1 \) and, for a given \( \alpha \), the scattering intensity increases in proportion to \( n_e \) with consequent improvement in photon statistics. A slow discharge not only ensures lower plasma temperatures, but also allows for better time resolution. Deuterium gas was used rather than hydrogen because of its favorable characteristics in pinch discharges, and because the width of the ion feature in the scattering was best suited to the resolution of the dispersing instrument to be described in Sec. III. B.

A suitable combination of parameters satisfying these objectives was found with a gas fill pressure of 0.73 torr and bank voltage of 14 kV, giving a peak current of 150 kA at 3.1 \( \mu \)sec. The resulting pinch, to be discussed at length in Sec. IV is formed from imploding current sheets at 2.1 \( \mu \)sec and then expands without undergoing subsequent recompressions. The deuterium gas flow was maintained at a constant rate such as to ensure complete flushing of the plasma chamber between shots.

Some of the experiments utilized a preionization pulse which consisted of a 0.12 \( \mu F \) capacitor at \( \approx 9 \) kV switched by a hydrogen thyatron circuit. The main bank was fired with a delay of from 1 to 3 \( \mu \)sec after onset of the preheat current.

This low energy (maximum of 1.2 eV per \( D_2 \) molecule) slight preionization was found to have little effect on the plasma and was not used for the plasma mapping. A strong preionization and preheat, which is often the practice with pinch devices, was not used because of possible contamination, particularly in the
scattering volume, of the plasma by wall and electrode material. The theoretical effect of such impurities on the scattered light spectrum is discussed by Evans.\textsuperscript{22}

B. Optical Equipment

1. Laser

Observation of the ion scattering feature in the sheet pinch plasma places a number of demands on the light source which are only met over a rather restrictive range of parameters. Choice of wavelength is determined by the fact that $\alpha \propto \lambda / \sin \theta$ ($\theta$ is the scattering angle) must be on the order of unity or greater for significant contribution from the ion feature. For $\theta = 90$ deg (convenient for the experimental geometry) and the range of temperatures and densities encountered the ruby laser wavelength of 6943 Å satisfied this condition.

Plasma light from the bremsstrahlung process,\textsuperscript{*} which is proportional to $n_e^2$, competes with the scattered light signal which is proportional to $n_e I$, where $I$ is the laser intensity. Thus adequate signal-to-noise ratio places a lower limit on $I$. On the other hand, heating of the plasma by the laser beam through the inverse bremsstrahlung process (treated in Sec. V. A.), which increases with $n_e$ and $I$, places an upper limit on intensity.

\begin{itemize}
\item It should be noted that bremsstrahlung light is minimized by using the 90 deg scattering configuration since the view is through the thinnest section of plasma.
\end{itemize}
Unfortunately these upper and lower limits were in contact at
the high densities encountered and some plasma heating had to be
tolerated in order to obtain adequate signal-to-noise ratio, par-
ticularly in the electron plateau region where the scattered
signal is down by a factor of \( \sim 10^{-2} \) from the ion peak for
\( \alpha \sim 1.5 \).

With a 5/8 x 5 in. ruby in a 60 cm cavity and giant pulse
technique, a 1 joule, 40 nsec (full width at half maximum), \( \sim 2 \)
mrad divergence pulse was obtained. The 40 nsec pulse length
afforded good time resolution of the plasma, where significant
changes would take place in a time scale of \( \sim 100 \) nsec. The
small divergence factor, due in part to the long cavity, com-
bined with a 15.2 cm focal length lens optimized for minimum
spherical aberration resulted in a focal spot size in the plasma
of \( \sim 0.3 \) mm. The compressed plasma sheet was at times of approxi-
mately 1 mm thickness. Thus a small focal spot size was required
in order to insure that a relatively uniform volume of plasma
was being sampled.

Giant pulse operation was obtained with the use of a KDP
Pockels cell switch at 8.5 kV and triggered by a hydrogen thyra-
tron. Referring to the timing diagram, Fig. 10, voltage was
applied to the pinch tube after 1.1 msec of optical pumping of
the ruby. Onset of current flow to the pinch tube was sensed by
a current transformer whose output, after a chosen delay, trig-
ger the Pockels cell thyatron. With this scheme, any point
in the pinch discharge cycle could be chosen with a \( \pm 30 \) nsec
Fig. 10. Block diagram of timing sequence without preheat. Laser "giant pulse" is timed to pinch tube breakdown by means of a current transformer. Relay grounds residual photomultiplier dark current until ~100 μsec before the laser pulse.
jitter. Most of this jitter appeared between the time of the KDP switching and the giant pulse. Due to the low reflectance of the laser's front mirror, \( \sim 10\% \), the gain of the laser and thus the jitter and pulse output were highly sensitive to alignment of the laser's optical components.

Under the range of parameters encountered the spectral width of the ion feature was typically \( \sim 1.5 \text{ Å} \). Although the finite spectral-width of the laser pulse was accounted for in the final data reduction, it was desirable to have the laser width considerably less than the ion feature. Stability of the laser's central wavelength was also an important consideration. The velocity of a component of plasma in a direction along a line of sight of the polychromator results in a wavelength shift, \( \Delta \lambda \), of the scattered spectrum which is related to the velocity by

\[
v = c \left( \frac{\Delta \lambda}{\lambda} \right) \right) - 4.32 \times 10^6 \text{ cm/sec} \times \Delta \lambda,
\]

where \( \Delta \lambda \) is in Å units. After every other plasma shot, a calibration shot was made (without firing the plasma and with the stray background light artificially increased), from which the central laser wavelength was measured. Any drift in the laser wavelength, or the polychromator, results in an error in the inferred plasma velocity. The difficulty becomes apparent when it is noted that the ruby fluorescence has a temperature shift of \( \sim 1 \text{ Å per °C} \) and a temperature rise of tens of degrees centigrade takes place after the shot.

Control of ruby temperature was effected by circulating
filtered deionized water in a closed circuit through the ruby housing, which included the spiral flash lamp and reflectors. Water temperature was maintained within limits of ±0.05 °C by a commercial temperature bath. Five minutes between shots sufficed to allow the system to reach thermal equilibrium. The pumping efficiency of the spiral flash lamp, and thus the lamp life, was quite dependent on the presence of a close wrapped reflector. Quartz shells packed with MgO powder were found to offer superior performance to aluminum, which rapidly lost its polish due to corrosion in the water bath.

Spectral purity and wavelength stability were enhanced by using a temperature controlled resonant reflector for the front mirror of the cavity. The reflector was constructed from a 1.7 mm thick piece of optical glass, flat to λ/10 and with faces parallel to 1 arc sec. The resonator mode spacing of 0.9 λ is approximately the same as the ruby fluorescence width. The temperature of the mirror was adjusted so as to superimpose a reflectance peak on the ruby fluorescence peak (see Fig. 11). Without temperature tuning, the reflectance peaks would wander with a temperature coefficient of 0.06 λ per °C, leading to erratic laser performance. (The laser would lase on two wavelengths simultaneously when the fluorescence peak straddled the reflectance peaks.)

These measures resulted in a spectral width of ~ 0.2 λ and a short term wavelength stability of ~ 0.05 λ. The wavelength drift implied an uncertainty in velocity of ~ 0.2 x 10^6 cm/sec.
Fig. 11. (a) Ruby fluorescence spectrum straddled by two modes of the 1.7-mm resonant reflector. Photodiode monitor signal shows two giant pulses. (b) Reflector mode centered on ruby fluorescence. Photodiode shows single pulse.
The time integrated output of the laser showed a ±15\% variation shot to shot, but this caused no difficulties since the output on each shot was monitored through the rear mirror with a fast photodiode.

2. Stray Light Reduction and Alignment

Due to the small size of the Thomson cross section, $6.6 \times 10^{-25} \text{ cm}^2$, and the limitations of practical optical design, only a small fraction, $\sim 10^{-12}$, of the incoming energy is scattered and available for detection if plasma densities are on the order of $10^{16}/\text{cm}^3$. This figure, applicable in this experiment, has been typical of other experiments.\textsuperscript{23} Considering just the entrance window, it is evident that a 10 $\mu$m diameter imperfection scatters $10^{-6}$ of a 1-cm diameter beam's energy. Worse yet, double reflection from window surfaces coated for 99% transmission introduces $10^{-4}$ of the incoming beam as stray light, while reflection from an exit window is greater than 1%.

Stray light is seldom a serious problem when observing the electron portion of the spectrum, where the broadening is typically 100 $\AA$, as the dispersing device or passband filter discriminates against the unbroadened stray light. However, this is not the case with the narrow ion feature, and measures must be taken lest the desired signal is masked by the stray light.

The situation at the laser exit window is readily improved by using Brewster angle plates or, of more convenient construction, a Rayleigh horn. Best results were obtained by using a transparent horn (rather than an absorbing one) surrounded by a
large box lined with black velvet. With this method, energy absorption was spread over a large area and the backscattering from the absorbing surface into the plasma chamber was reduced.

It is common practice to situate the laser entrance window some distance from the plasma by means of an extension tube. This serves two purposes. First, it protects the inside window surface (coated if possible) from plasma bombardment. Second, it allows the placement of a series of apertures, the first one intercepting light scattered from the window that would otherwise strike the chamber walls; the second one intercepting similar rays scattered from the edge of the first, and so on. Scanning of the plasma in space can be achieved by mounting the tube assembly on a sliding vacuum seal.

For various practical reasons, such an assembly was not suited to the present experiment. Movement of a final focusing lens by, say, a factor of two back from the focal spot must be done in a manner such that the focal spot diameter is not changed in order to preserve spatial resolution and scattered light collection efficiency. This implies that the effective focal length of the system remain constant, which is achieved at the expense of the introduction of a second lens and a doubling of the diameter of the final focusing lens. Moreover, the sliding tube and diaphragm would have to be constructed of darkened ceramic with consequent mechanical, vacuum, and alignment difficulties.

Instead, the window and lens were placed as close as possible to the scattering volume. The window was periodically repolished
to remove pitting by, and deposits from, the plasma. Scanning of the plasma was achieved by the simple expedient of pivoting the optical bench. Stray rays from the laser were blocked from striking the chamber walls by placement of an aperture between the laser and the focusing lens in such a position that the lens projected its image to the plane of the exit window (see Fig. 12).

Since the interior of the plasma chamber was sprayed with a considerable amount of stray light, measures were taken to avoid seeing it. The optical train linking the plasma chamber to the polychromator was designed such that the acceptance "cone" of the polychromator terminated in a Rayleigh horn viewing dump without intercepting the chamber walls. A large part of the remaining stray light was traced to rays from the wall scattering off imperfections in the viewing window and into the acceptance cone. An extension tube with apertures, as shown in Fig. 5, served to block these rays.

Under the most extreme condition encountered, $n_e \sim 0.5 \times 10^{16}$, $\alpha = 0.7$, the detected stray light energy was 25% of the ion signal energy, but a more typical figure during the plasma development was about 1%.

The 300 $\mu$m diameter by 7.5 mm length scattering volume was focused on the 15 $\mu$m polychromator entrance slit with a magnification of two. Optimum scattering signal demanded placement of the focusing lens with a tolerance of $\sim \pm 50$ $\mu$m. Frequent alignment was demanded by the fact that the plasma chamber and the polychromator rested on different supporting structures separated by over
Fig. 12. Schematic view of beam defining apparatus. Stray rays scattered from the edge of the beam limiting aperture are either focused in the plane of the exit window or are intercepted by the lens aperture.
a meter in a room whose temperature fluctuations were at best ±1 °C. Multiple laser shots using nitrogen gas as a scattering target while the lens position was scanned was sometimes used for this purpose. A more rapid procedure (and more economical in terms of laser life) was to use a HeNe laser precisely aligned with the ruby laser axis. The discharge chamber was opened and a fine wire suspended at the focal point as a target. Continuous scanning of the lens for optimum photomultiplier signal from the polychromator was then easily achieved.

3. Polychromator and Data Recording

Multichannel recording of the ion scattering spectrum was dictated by a number of practical considerations besides convenience. Assuming perfect reproducibility of the plasma and precise timing of the laser pulse, thermal drifts of the laser wavelength and the dispersive device settings would make determination of the gross plasma velocity difficult at best if point by point readings of the spectrum were made on sequential shots. Moreover, the above assumptions could not be made. Thus such a method, even if a number of shots were taken at one wavelength setting, would reveal a spectrum that was a superposition of spectra corresponding to different plasma parameters.

The same objection, of course, applied to numerical averaging over a number of multichannel recordings. Photon collection efficiency had to be such as to give statistically significant information in one shot. The spectral width and the number of channels was also influenced by the fact that essentially all of
the ion peak must be recorded. Numerical analysis of the scattering spectra using the program to be described in Sec. III. C showed large errors, particularly in temperature and plasma velocity, if, say, only half the ion peak was detected.

The basic dispersion element was a Jaco Model 82-000, 0.5 meter focal length, f/10 monochromator. This instrument, when used with matching curved entrance and exit slits is capable of resolving 0.1 R. Spillman et al.25 made an adaptation to low resolution multichannel use. Modifications by Anderson2 and the author resulted in a resolution of 0.15 R using a 10 μ by 15 mm straight entrance slit. In the present experiment, seven adjacent channels of 0.26 R spectral width served to encompass most of the ion feature. Since less than maximum spectral resolution was required with this configuration, light gathering ability was increased 50% by using a 15 μ wide slit.

A short focal length cylindrical lens, with focal line parallel to the image of the entrance slit, projected the exit plane at high magnification, ~ 75, on a stack of light pipes which channeled the signal to individual photomultipliers (see Fig. 13). With this scheme alone, spectral resolution would be sacrificed because of the astigmatism inherent to off-axis placement of the entrance and exit apertures of the monochromator (Ebert type mounting). Practically full correction was obtained through use of a second cylindrical lens, with axis crossed to the first, which projected the exit plane to infinity. The mechanism and extent of the correction are shown in Figs. 14 and 15.
Fig. 13. Polychromator arrangement (schematic), showing crossed cylindrical lenses and Brewster angle beam splitter used to obtain signal for differential cancellation of bremsstrahlung signal into eighth photomultiplier. The scattered laser light is polarized in a direction normal to the long axis of the slit.
Fig. 14. Polychromator astigmatic images, for straight input slit. Lines in sketches represent images of distinct points.

(a) Image at exit slit region. (b) Vertical magnification only. (c) Horizontal correction is excessive. (d) Suitable ratio of focal lengths, properly adjusted.
Fig. 15. Images of 10-μm slit projected to the plane at the inputs to the light pipes. The light pipes were bent to match the profile (b). Illumination by He-Ne laser.
A monochromatic light source placed at the entrance slit produced the narrow curved pattern, shown in Fig. 15 (b), at the plane of entrance to the light pipes. The light pipes, consisting of 0.050 in. thick Lucite strips were bent to match this profile. A cylindrical lens shape was cut and polished on the entrance end of the pipes so as to direct the light down the strips without intercepting the edges where the losses were highest. It should be noted that image dissection by this method enjoys an advantage over the employment of fiber optics, which typically have element diameters of a few mils. This is due to the relatively few total internal reflections the light undergoes in the wide channel bounded by two planes. In fact, the ~ 10% loss of the 32 in. long gently bent strips could be attributed mainly to entrance and exit surface reflections.

The output of each RCA 7265, S20 cathode, photomultipliers was integrated with a simple RC network of 250 nsec time constant. Integration, as opposed to peak detection, afforded greater statistical accuracy and freedom from sensitivity to variations in the laser pulse shape. Since seven channels of information were to be recorded on each shot and signal strengths covered a range of \( \sim 10^3 \), detection with wide dynamic range memory circuits rather than with oscilloscopes was deemed to be more practical.

A voltage follower circuit with a differential electrometer tube front end detected the voltage on the 2nF integrating capacitor. The voltage follower output was fed to a track-and-hold memory circuit which had a sluing time of 30 msec. Output from
the memory was read with a digital voltmeter switchable between the seven units. Drifts in the outputs were held to less than 0.1% of full scale over a half hour period. High input impedance, \( > 10^{12} \ \Omega \), was necessary to allow for rise time of the memory. At this impedance level, special electrometer cables were required to connect the photomultipliers with the inputs to avoid the frictionally induced charges from standard coaxial cables. Measures were taken to prevent integration of photomultiplier dark current and plasma light. On a fast time scale, dynodes in the photomultipliers were pulsed to an "on" condition, by the method described in Ref. 26, for a 700 nsec time encompassing the laser pulse. When in an off condition, light signals were effectively blocked, but there remained a residual dark current of 0.5 nA. Relays, shorting this current to ground, were lifted at a time less than 1 msec before the laser pulse. The timing sequence is illustrated in Fig. 10. There are now MOS field effect transistors on the market with low enough leakage currents and "on" impedances to perform these gating functions in simpler and superior fashion.

An eighth photomultiplier was included in the polychromator to record a portion of the electron plateau region. This channel was removed by approximately 14 Å from the laser line center and had a spectral width of 4.6 Å in order to gain adequate signal strength. Poor resolution was not a problem, since the electron spectrum changes slowly in this area and those variations were accounted for numerically in the data fitting.
At high plasma densities, the bremsstrahlung light into this channel was of the same order of magnitude as the electron signal. Improvement in the signal-to-noise ratio was achieved with a variation of the differential technique suggested by Hughes. A pellicle type beam splitter was placed at the Brewster angle in front of the entrance slit to the polychromator, as shown in Fig. 13. Plasma light and scattered laser light passed through the pellicle, while only plasma light was reflected due to the polarization of the laser light. The reflected image was focused on a slit and passed therefrom through an interferometer filter having a 10 Å bandwidth centered on 6943 Å to a photomultiplier. The signal was integrated and combined in a Tektronix type 1A5 differential amplifier with a similarly integrated signal from the eighth channel. Oscilloscope pictures of the two signals and their algebraic difference are given in Fig. 16.

Calibration of the relative sensitivities of the eight polychromator channels was achieved by imaging a long spark gap enclosed in a capillary tube onto the entrance slit. A Kerr cell shutter and neutral density filters were placed between the gap and the slit. Illumination geometry, intensity, polarization, and pulse length were all adjusted so as to closely approximate the conditions encountered with scattered laser light. The spectral output of the gap was free of lines and flat in the region of interest. A number of shots were taken to determine the mean sensitivity of each channel and, of equal importance, the deviation from the mean to be expected on each shot. These
Fig. 16. Differential cancellation of bremsstrahlung. All four traces were obtained during a single laser shot. Time scale 1 μsec/div. (a) Integrated signal of bremsstrahlung photomultiplier showing plasma light only. Scale: 100 mV/div.
(b) Integrated signal from eighth photomultiplier showing scattered laser light plus plasma light. Scale: 50 mV/div.
(c) Integrated laser output signal. The laser fired at the time of the large jump in the trace. (d) Signal showing the algebraic difference between signal (b) and signal (a) (weighted for optimum cancellation). Scale: 20 mV/div.
measured statistics of the photomultiplier outputs served as basic inputs to the analysis of errors in plasma parameters determined from a scattering spectrum.

The "absolute" sensitivity of the system was measured by laser scattering from the plasma chamber filled with nitrogen gas, whose cross section is well known.* A relatively low fill pressure, 40 torr, avoided problems from suspended dust particles or laser-produced spark breakdown.

C. Treatment of Data

In practice, it is useful to normalize the scattering signal, $S_n'$, into a spectral channel, denoted by a subscript $n$ and of frequency spread $\Delta \omega_n$, to a total Rayleigh scattering signal, $S_R'$, from a gas of known density and cross section, $n_R$ and $\sigma_R$, using the same scattering geometry and signal detection equipment. Assuming equal intensities of the incident beam, the observed signal is related to the spectral function $H$, [Eq. (25)], by

$$S_n = S_R \frac{n_e \sigma_T}{n_R \sigma_R} \int_{-\infty}^{\infty} d\omega \frac{H(\omega/\omega_e)}{\sqrt{\pi} \omega_e} I_n(\omega).$$

*Experiments with several noble gases by George et al. 28,29 gave results at variance to the classical theory of Rayleigh scattering. Theimer 30 attempted to explain their results in terms of interference effects due to the coherence of the laser source. However, Watson and Clark 31 found no deviation from theory in $N_2$ gas.
$I_n(\omega)$ is an instrument function which has to be individually calibrated for each channel. It includes the finite spectral width and the central frequency of the channel, the measured sensitivity of the photomultiplier, the spectral resolution and wavelength setting of the dispersing instrument, and the nonzero frequency spread of the incident laser beam. The normalization of $I_n$ is such that in the hypothetical case of a complete set of adjacent channels covering the entire spectrum

$$\sum_{-\infty}^{\infty} I_n(\omega) = 1$$

$$\int_{-\infty}^{\infty} \, d\omega \, I_n(\omega) = \Delta \omega_n.$$

(32)

Using the Salpeter approximation, the total signal scattered by the plasma is

$$S = \sum S_n = S_R \frac{n_e \sigma_T}{n_R \sigma_R} \left[ \Gamma_e(k) + \Gamma_i(k) \right].$$

(33)

1. Choice of Parameters

The scattering signal of Eq. (31) depends on the plasma parameters $n_e, T_e, T_i, k \cdot v_0e, k \cdot v_0i$, and $\alpha$, which is a function of $n_e$ and $T_e$. For conditions obtained in the sheet pinch plasma under study, a reduction in the number of independent variables is possible. Following Spitzer$^{32}$ the equipartition time for electron ion populations of different temperature is on the order of 10 nsec for electron densities in the range of $10^{16}$/cm$^3$. 
and temperatures on the order of a few volts. Since the laser pulse width was 40 nsec and significant changes in the pinch occurred in a time scale no faster than 100 nsec, the assumption \( T_i = T_e \) was made. The 90 deg scattering was observed in a direction parallel to the sheet pinch contraction and expansion motion and the scattering vector \( k \) was normal to the discharge current vector. Assuming no other velocity components due to plasma turbulence, \( k \cdot v_{0e} = k \cdot v_{0i} = |k| v_p \sqrt{2} \), where \( v_p \) is the gross plasma speed.

The four variables, \( n_e, T_e, v_p, \) and \( \alpha \) were treated as free parameters in matching the theoretical spectra to the observed spectra. It will be noted that \( n_e \) appears explicitly only as a multiplicand in the fitting of theory, Eq. (31), to the data, and thus it will be referred to as the "scale factor density." However, as can be seen from Eq. (22), the three parameters \( n_e, T_e, \) and \( \alpha \) are not independent of each other. Using this last equation, a "self-consistent density", is defined:

\[
n_e' = \frac{|k|^2 \alpha^2}{4 \pi e^2 k T_e}.
\]

The "self-consistent density", \( n_e' \), is then compared with \( n_e \) as a check on the accuracy of the data and/or the suitability of the theory.

Eight spectral data channels were used, with seven adjacent channels covering the ion feature and an eighth on a portion of the electron wing. Anderson noted the value of the last channel
by pointing out the strong dependence of the ratio of the ion peak to the electron wing on the magnitude of $\alpha$ (see Fig. 3 and next section).

2. Computation

Determination of plasma parameters from the spectral profile was carried out in three stages. In the first step, an estimate was obtained from the cross features of the spectra. Using the Salpeter approximation, the following relations may be derived

$$\alpha^2 = \left( \frac{mS_c}{MS_{ep}} \right)^{1/2} - \frac{1}{2} + \left( \frac{1}{4} + \frac{mS_c}{MS_{ep}} \right)^{1/2}$$

$$\kappa_T \approx \frac{M_c}{4\pi} \left[ \frac{\Delta \omega_n}{\omega_0} \frac{1 + \alpha^2}{1 + 2\alpha^2} \sum S_n \right]$$

$$n_e = \frac{n_R \sigma_R}{\alpha_T} \frac{(1 + \alpha^2)(1 + 2\alpha^2)}{\alpha} \frac{\sum S_n}{S_R}$$

$$v \approx (n_c - n_0) \frac{\Delta \omega_n}{\omega_0} c$$

where $S_c$ is the signal into whatever channel lies at the center of the ion spectrum. $S_{ep}$ is the signal into the channel that samples the electron plateau. In deriving Eq. (35) it was assumed that $S_{ep}$ is the same as would be obtained in the Salpeter approximation at zero frequency shift. For deuterium with $\alpha > 0.5$ this approximation is reasonable. The factor $\sum S_n$ represents the sum of all the ion channel signals. It is assumed that the whole
ion peak is included. In the expression for the plasma velocity, Eq. (38), \( n_c - n_0 \) is the number of channels that the ion peak has shifted from the laser center, and the last factor, \( c \), is the speed of light. These crude estimates, which served as the starting point of the next stage of the calculation, generally turned out to be within ±20% of the final determination.

In the next stage, the parameters of the full theoretical spectra (not the Salpeter approximation) were adjusted to obtain a best fit in the least squares sense. An iterative computer code due to K. Halbach was used to this end. An outline of the method is presented in Appendix A.

In order to economize on computer time, the following substitution for the integral of Eq. (31) was made

\[
\int_{-\infty}^{\infty} d\omega I_n(\omega) H(\omega) \to \Delta \omega I_n H(\omega),
\]

i.e., a simple evaluation of the spectra in the center of the \( n \)th channel times the width of that channel. The appropriate corrections for relative sensitivities are represented by \( I_n \).

Upon convergence, the newly obtained parameters were used as input to the third stage, which was identical with the second stage with the exception that the trial spectra were integrated over an instrument function on each iteration. The form chosen for this function was such that
where \( Z \) is a coordinate in channel number space. The upper and lower limits of the \( n \)th channel are \( z_2(n) \) and \( z_1(n) \), respectively, and \( \Delta \omega_n \) is the angular frequency corresponding to \( z_2(n) - z_1(n) = 1 \), i.e., one channel width.

The significance of \( z_0 \) and \( \sigma \) may be seen by taking \( H(\omega) \) to be the Dirac delta function, which corresponds to a Rayleigh scattering shot. Then

\[
S_n \propto \int_{z_1(n)}^{z_2(n)} dz \frac{1}{2\sigma \cosh^2\left[\left(z - z_0\right)/\sigma\right]} H(\omega),
\]

\[
(40)
\]

i.e., the spectrum is centered at \( z = z_0 \) in channel number space and the width, in the units of that space, is \( \sigma \). The spread of \( \sigma \) is due to the finite spectral width of the laser and, to a lesser degree, diffraction by the entrance slit of the polychromator. The value of \( z_0 \) and of \( \sigma \) are determined experimentally from a Rayleigh scattering shot.

*Note that the choice of an instrument function that will "duplicate" the experiment by smearing out a theoretical spectrum, positioning its center, and breaking it into channels is somewhat arbitrary. However, the existence of a simple analytic expression for the integral of \( \cosh^{-2} \) makes that function more convenient than, say, a Gaussian.*
By interchanging the order of integration and integrating over \( dz \), Eq. (40) takes the form
\[
S_n = \int_\infty^{-\infty} d\omega \left[ \frac{1}{2} \tanh \left( \frac{z_2(n) - z_0 - \omega/\Delta \omega_n}{\sigma} \right) - \frac{1}{2} \tanh \left( \frac{z_1(n) - z_0 - \omega/\Delta \omega_n}{\sigma} \right) \right] H(\omega). \tag{42}
\]

The expression in the brackets is the instrument function of Eq. (31) except for the relative sensitivity corrections and it may be shown to satisfy the normalization conditions of Eq. (32).

In practice, the integration over \( d\omega \) was performed over finite limits using 6-point Gaussian quadrature.

Figure 17 presents an experimental spectra in the form of a histogram. The crosses are the best point fit (i.e., second stage) and the histogram of circles is the best fit using the instrument function (third stage). The improvement of fit in regions of strong curvature is to be noted.

The above calculations require multiple evaluations of \( G(x) \), Eq. (23), which is an integral. This function is closely related to the plasma dispersion function, \( Z(x) \), tabulated by Fried and Conte
\[
\text{Re} G(x) = 1 + x \text{Re} Z(x) \tag{43}
\]
where "Re" implies the real part, and to Dawson's integral.

For the purposes of computation, \( \text{Re} G(x) \) was approximated by the ratio of two polynomials,
Fig. 17. Straight lines: histogram of laser scattering shot corrected for photomultiplier sensitivities. Crosses: best fit with theoretical profile evaluated at the center of the channels. Circles: best fit with theoretical profile integrated over the finite channel widths. The parameters deduced from each fit are indicated.
The coefficients \( L_j \) were adjusted to give a least squares fit to 20 values of \( G(x) \) between \( x = 0 \) and \( x = 4 \). The parameters \( L_4 \) and \( L_9 \) were set equal to unity in order to make \( R e \ G(0) \) identically equal to one.

Because of the general usefulness of an approximating function for \( G \), the coefficients are presented in Table II. A plot of \( R e G(x) \) along with a graph of the percentage of error for the approximating function is given in Fig. 18. For \( x \leq 4 \) the maximum error is \(-1.3\%\).

In order to obtain the correct asymptotic behavior for large \( x \), the restraint \( L_2/L_1 = 2 \) would have to be made. Since this was not done, the error approaches \(+21\%\) for large \( x \), but this has negligible effect on the computed spectra.

3. Error Determination

From Appendix A it may be seen that a small change in a parameter \( p_i \) is related to a small change, \( \Delta S_j \), in a spectral data point through the relation

\[
\Delta p_i = \sum_j B_{ij}\Delta S_j,
\]  

(45)

where the coefficients \( B_{ij} \) are defined by Eq. (A-8). The ensemble average over identical scattering systems of the square of Eq. (45) expresses the uncertainty in the parameters in terms of the mean square fluctuations of the channel readings.
Table II. Coefficients used in approximating function \[ \text{Eq. (44)} \] for \( \text{Re} G(x) = 1 - 2xe^{-x^2} \int_0^x dt e^{t^2} \).

<table>
<thead>
<tr>
<th>( j )</th>
<th>( L_j )</th>
<th>( j )</th>
<th>( L_j )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.016704089</td>
<td>5</td>
<td>0.026463315</td>
</tr>
<tr>
<td>2</td>
<td>0.072792361</td>
<td>6</td>
<td>0.035835655</td>
</tr>
<tr>
<td>3</td>
<td>-1.2206521</td>
<td>7</td>
<td>0.33116055</td>
</tr>
<tr>
<td>4</td>
<td>1.0</td>
<td>8</td>
<td>0.77332125</td>
</tr>
<tr>
<td></td>
<td></td>
<td>9</td>
<td>1.0</td>
</tr>
</tbody>
</table>
Fig. 18. Upper curve: $\text{Re } G(x)$ [Eq. (23)]. Lower curve: error in approximating function [Eq. (44)] for $\text{Re } G(x)$. 
where the cross terms $\alpha(\delta S_j \delta S_k)$ have been set to zero under the assumption that different spectral channels are uncorrelated.

Because of nonperfect reproducibility of the plasma from shot to shot, a direct experimental determination of $\langle \delta S_j^2 \rangle$ would have implied an unrealistically high uncertainty in the parameters. An alternate approach allows probable errors to be inferred from the characteristics of a single plasma shot.

In the course of calibrating the relative sensitivities of the photomultipliers (cf. Sec. III. B. 3) an average signal $\langle c_j \rangle$ with mean square fluctuation $\langle \delta c_j^2 \rangle$ is observed for the $j$th channel. On the assumption that all fluctuations are due to photocathode statistics, the expected fluctuation for a laser scattering shot of amplitude $S_j$ is

$$\langle \delta S_j^2 \rangle = \langle \delta c_j^2 \rangle (S_j / \langle c_j \rangle).$$  \hfill(47)

Another term, $\langle \Delta S_j \rangle^2$, the square of the difference between the measured spectra and the best fit spectra was added to the random contribution, which yields a probable error of

$$\langle \delta F_i^2 \rangle = \sum_j (E_{ij})^2 \left[ \langle \delta c_j^2 \rangle (S_j / \langle c_j \rangle) + \langle \Delta S_j \rangle^2 \right].$$  \hfill(48)

Additional corrections involving background light from plasma bremsstrahlung and uncertainties in the relative calibra-
tion of the photomultipliers were employed, but they will not be detailed here.
IV. EXPERIMENTAL RESULTS

Data on plasma development were collected over a 1.4 μsec range of time beginning 1.4 μsec after the initiation of current flow. Three positions were sampled: along the central plane, 3 mm, and 6 mm removed from the central plane. At each position, a plot of density, temperature (θ is the temperature measured in eV), α, and velocity is given as a function of time in Figs. 9 through 24. At a given position and point in time, the parameters and their associated error bars are inferred from a single laser shot. Errors in the relative timing of the shots are estimated at ±30 nsec. For each shot, both the scale factor density, n, and the self-consistent density, n', are given. Possible causes for disagreement between these two measures of density will be treated in the next chapter.

In the following sections, these data are examined for consistency with other measurements and with some simple models of the pinch dynamics.

A. Auxiliary Measurements

Additional data on plasma temperature and density were obtained through spectrographic observation. One of the polychromator channels was used to scan the first two lines of the Balmer series, $H_\alpha$ and $H_\beta$, on successive plasma shots. The observed volume included that normally viewed during scattering, sighting along the same axis and using the same optics as in the laser scattering experiments.

Observation of emission spectra along a line of sight
Fig. 19. Sheet pinch development 6 mm (towards the spectrometer) from the central plane; density and temperature as a function of time after discharge initiation. Circles: parameters determined from observation of the ion feature and the electron plateau. Squares: test for self-consistency, n'. At a given position and time, all parameters represented by circles and their associated error bars were inferred from a single laser shot. Error bars are placed between circles and squares to facilitate judgment of self-consistency.
Fig. 20. Sheet pinch development 6 mm from the central plane continued: plasma velocity and scattering parameter, \( \alpha \). A positive velocity represents plasma motion towards the spectrometer.
Fig. 21. Sheet pinch development 3 mm (towards the spectrometer from the central plane: density and temperature.
Fig. 22. Sheet pinch development 3 mm from the central plane continued: plasma velocity and scattering parameter, $\alpha$. 
Fig. 23. Sheet pinch development on the central plane: density and temperature. Diamonds: parameter determination by observation of the electron feature using multiple laser shots. Truncated circles: spectroscopic measurements.
Fig. 24. Sheet pinch development on the central plane continued:

plasma velocity and scattering parameter, $\alpha$. The bias of
velocity data towards negative values is indicative of a dis-
crepancy between the geometric and the dynamic central plane
and/or systematic instabilities.
parallel to the laser axis was impractical due to the highly luminous "end effects" evident in Fig. 6. Likewise, electrode light, apparent in the stereo pairs of Fig. 6, precluded observation through a transparent cathode. (Photographs similar to those of Figs. 6 and 8 taken through filters selective to Hα and Hβ yielded the same emission patterns.) Since observation could only be done along a direction parallel to the gradients in the plasma parameters, unfolding of the data was not possible. It is in such situations that potentials of laser scattering are to be appreciated.

The Stark broadened Hα line was compared with theoretical profiles of Ref. 36 to obtain a measure of ion density. The results are in reasonable agreement with laser-determined density values along the central plane, as shown in Fig. 23. Although plasma density was at times quite nonuniform along the line of sight, agreement with central plane values is not unexpected. This is because the line wings, which are determined by the plane of maximum density, were given the greatest weight in curve matching.

Electron temperature was determined through the ratio of the Hα line intensity to that of the neighboring continuum. Temperature values, plotted in Fig. 23, are again in reasonable agreement with scattering data. Since temperature is fairly uniform across the plasma cross section, there is no serious folding question.

The line spectra showed excellent shot-to-shot reproduci-
bility for times up to 2.0 µsec after discharge initiation. Thereafter, signal variations of 30% were commonly observed. The scatter in laser data at these later times also indicate less than ideal reproducibility of the plasma parameters. On the other hand, magnetic probe traces, which are more indicative of gross dynamic effects, were almost indistinguishable from shot to shot (see Fig. 25). The magnetic field was obtained from the integrated output of a 6 turn, 2 by 6 mm cross section search coil encased in a quartz tube and inserted through the cathode. The field was measured as a function of distance from the central plane by moving the probe in 1 mm increments. Local current densities and magnetic pressures were derived from these readings.

Observation of laser scattering in the electron wing was used as another check on the data. Since the dispersion of the polychromator was not suited to multichannel recording of the electron spectra, single-channel readings of typically 40 laser shots were required. These data, taken along the central plane, are also plotted in Figs. 23 and 24. Within the experimental error, the scale factor density and the self-consistent density were in agreement for the electron spectra and the two are presented as a single point. Doppler shifts from gross plasma motion are small in comparison to the width of the electron feature, so velocity measurements are not obtainable by this method.

B. Snowplow Stage

The initial portion of pinch development, during which plasma
bility for times up to 2.0 μsec after discharge initiation. Thereafter, signal variations of 30% were commonly observed. The scatter in laser data at these later times also indicate less than ideal reproducibility of the plasma parameters. On the other hand, magnetic probe traces, which are more indicative of gross dynamic effects, were almost indistinguishable from shot to shot (see Fig. 25). The magnetic field was obtained from the integrated output of a 6 turn, 2 by 6 mm cross section search coil encased in a quartz tube and inserted through the cathode. The field was measured as a function of distance from the central plane by moving the probe in 1 mm increments. Local current densities and magnetic pressures were derived from these readings.

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B. Snowplow Stage

The initial portion of pinch development, during which plasma
is swept up by an incoming current sheet, can often be approximated by the so-called snowplow model. With the magnetic field varying according to $B_M \sin \omega t$, the time rate of change of momentum per unit volume (due to acceleration and mass accretion) equated to the magnetic pressure jump across the current sheet is

$$\frac{\rho d}{2 \frac{d}{dt}} (x - x_w)^2 = \frac{B_M^2}{8\pi} \sin^2 \omega t,$$

(49)

where $\rho$ is the initial fill mass density, $x$ is the distance measured from the central plane, and $x_w$ is the distance to the wall. Integrating,

$$x = x_w - \frac{1}{2} v_A t \left( 1 - \frac{\sin^2 \omega t}{(\omega t)^2} \right)^{1/2},$$

(50)

$$\frac{dx}{dt} = -\frac{1}{2} v_A \frac{1 - \sin \omega t \cos \omega t}{\left[ 1 - \sin^2 \omega t/(\omega t)^2 \right]^{1/2}},$$

(51)

where $v_A$ is the Alfvén velocity $B_M/(4\pi \rho)^{1/2}$. Figure 26 presents position and velocity corresponding to the experimental parameters of $B_M = 4.96$ kG, quarter cycle time of $3.1 \mu$sec, $x_w = 1.9$ cm, and an atomic fill density of $5.2 \times 10^{16}/cm^3$ deuterium atoms.

These estimates are in fairly good agreement with the scattering data. For instance, from Fig. 19, first evidence of the incoming mass at $x = 6$ mm is obtained at $t = 1.5 \mu$sec. The maximum observed velocities, on the order of $1.8 \times 10^6$ cm/sec, are likewise in agreement.
Fig. 26. Position and velocity of plasma front as predicted by "snowplow model" for a plane geometry using the experimental current waveform and gas fill density.
A more readily visualized picture of the plasma development may be obtained from Figs. 27, 28, and 29, in which electron density and current density are plotted as a function of position for different times. Early in time there is a well formed arc on the central plane, with electron density \( \sim 1 \times 10^{16} \). At 1.5 \( \mu \)sec, the leading edge of the current sheet may be seen at 6 mm. Subsequent time intervals up to 1.8 \( \mu \)sec indicate that the electron density is highest at the leading edge of the rather vaguely defined current sheet. At 1.9 \( \mu \)sec there is a jump in electron density at the central plane. Simultaneously, plasma velocity (but not density) experiences a sharp drop at 6 mm (see Fig. 20). Thus the snowplow structure is approximately 6 mm thick. If all the gas was swept up by the snowplow, the average heavy particle density in the structure was \( 16.5 \times 10^{16}/\text{cm}^3 \).

The ionized component of the snowplow is constantly accelerating due to the \( j \times B \) force. Previously accelerated particles that become neutrals through charge exchange would tend to drift behind the ionized front. Thus it is understandable that the electron density is greatest at the leading edge of the snowplow.

The efficiency of the snowplow may be judged on the basis of the fraction of neutral atoms that are able to pass through the ionized leading edge without undergoing a charge exchange collision (cross section \( \sim 40 \times 10^{-16}/\text{cm}^2 \) at 10 ev).\(^{38}\) At 1.5 \( \mu \)sec, the leading edge has an average electron density of \( \geq 0.5 \times 10^{15}/\text{cm}^3 \) and a thickness of 2 mm, which implies a mean free path of 0.5 mm and penetration by \( < 2\% \) of the neutrals. Ions formed from charge
Fig. 27. Sheet pinch development as a function of position at different times. Plasma electron density is interpolated from Figs. 19, 21, and 23. Current density is from magnetic probe measurements. Short vertical line indicates approximate boundary of plasma luminosity which extends outward from the central plane for times greater than 1.8 µsec. The insulated chamber walls are 19 mm from the central plane.
Fig. 27.
Fig. 28. Continuation of Fig. 27.
Fig. 29. Continuation of Figs. 27 and 28.
exchanged incoming neutrals are accelerated to the snowplow velocity primarily through ion-ion collision in a distance \( \sim 2 \times 10^{-2} \) mm (calculation based on Spitzer's "slowing down time").\(^{39}\) Thus, it is reasonable that most of the mass is swept up, as agreement with the snowplow model implies, even though the ionization is quite small.

C. Shock Stage

The two snowplow structures meet at the central plane approximately 1.9 \( \mu \)sec after initiation of the discharge. It is proposed that the subsequent dynamics may be described in terms of collisional shock model in which the gas moving with velocity \( v \) and with average mass density, temperature, and fractional ionization of \( \rho, \theta_1, \gamma_1 \), respectively, is compressed to density \( \rho_2 \) with temperature \( \theta_2 \) and ionization \( \gamma_2 \). The compressed gas is assumed to have zero velocity, and the shock moves away from the central plane with speed \( U \).

At the times of interest, the magnetic field energy density 3 mm off axis is on the order of \( 5 \times 10^{16} \) eV/cm\(^3\), while thermal and directed energy densities of the incoming gas are \( 25 \times 10^{16} \) eV/cm\(^3\) and \( 54 \times 10^{16} \) eV/cm\(^3\), respectively for \( v \approx 1.8 \times 10^6 \) cm/sec, \( \theta_1 \approx 1.5 \) eV, and \( \gamma_1 \approx 16 \times 10^{16} \) atoms/cm\(^3\) (as implied in the last section). Thus the magnetic field may be ignored if a 10\% accuracy is considered adequate.

For simplicity, the energy input due to resistive heating is also neglected. Based on Spitzer's expression for resistivity,\(^{40}\) \( \eta \), the power absorbed per unit volume is
\[ P = \eta j^2 = 3.5 \frac{\ln \Lambda}{\sqrt{\pi}} j_3^2 \times 10^{16} \text{ eV/cm}^3 \cdot \text{sec}, \]  \hspace{1cm} (52)

where \( j_3 \) is the current density in kA/cm\(^2\). For average values of \( \Theta_1 = 1.5 \text{ eV}, j_3 = 3, \) and \( \ln \Lambda \sim 5 \), the energy input during the approximately 0.2 \( \mu \)sec duration of the shock is \( 16 \times 10^{16} \text{ eV/cm}^3 \).

Conservation of mass, momentum, and energy fluxes across the shock gives the familiar equations in the shock frame

\[ \rho_1 (v + U) = \rho_2 U \]  \hspace{1cm} (53)

\[ p_1 + \rho_1 (v + U)^2 = p_2 + \rho_2 U^2 \]  \hspace{1cm} (54)

\[ e_1 + \frac{p_1}{\rho_1} \frac{1}{2} (v + U)^2 = e_2 + \frac{p_2}{\rho_2} \frac{1}{2} U^2 \]  \hspace{1cm} (55)

where the internal energy per unit mass and pressure

\[ e = \frac{3}{2} (1 + y) \frac{\Theta}{M} = y \frac{E_I}{M} \]  \hspace{1cm} (56)

\[ \frac{p}{\rho} = (1 + y) \frac{\Theta}{M} \]  \hspace{1cm} (57)

include the neutral atoms and the ionization energy \( (E_I = 13.6 \text{ eV}) \). The fractional ionization, \( y \), is defined

\[ y = \frac{\rho_{\text{ne}}}{\rho} = \frac{n_e}{n_e + n_a} \]  \hspace{1cm} (58)

where \( n_a \) is the atomic density. (It is reasonable to assume that the gas is largely dissociated by the snowplow or by the arc phase...
that precedes it.) The observables derived from the laser scattering data, \( v, \theta_1, \theta_2, n_{e1}, \) and \( n_{e2} \) may be regarded as inputs to the system. Defining the ratios,

\[
h_1 \equiv \frac{\theta_1}{Mv^2/2}
\]

\[
\ell \equiv \frac{E_i}{Mv^2/2}
\]

the incremental compression \( \eta, \)

\[
\eta \equiv \frac{n_{e2} - n_{e1}}{n_{e1}}
\]

may be plotted as a function of the electron density ratio through the derived relation

\[
\frac{n_{e2}}{n_{e1}} = \left[ \frac{2}{n} - \eta h_2 + 2 - h_2 + h_1 \right] \left[ \frac{5}{2} h_1 + \ell \right] - h_1 \left[ \frac{2}{n} + 1 - \frac{5}{2} (h_2 - h_1) \right].
\]

From this the shock velocity and the ionization fractions may be obtained.

\[
U = \frac{v}{\eta}
\]

\[
y_1 = \frac{2/\eta - \eta h_2 + 2 - h_2 + h_1}{(n_{e2}/n_{e1})h_2 - h_1}
\]

\[
y_2 = \frac{y_1(n_{e2}/n_{e1})}{1 + \eta}
\]
In Fig. 30 the variables \( \eta \), \( y_1 \), and \( y_2 \) are plotted for several combinations of \( \theta_1 \) and \( \theta_2 \) as a function of \( n_{e2}/n_{e1} \) to indicate their sensitivity to these variables. The velocity is assumed to be 1.77 cm/\( \mu \)sec (see Fig. 22), which corresponds to a directed energy of 3.2 eV per deuterium nucleus. Experimentally, a probable choice is \( n_{e1} \approx 2 \times 10^{16}/\text{cm}^3 \), \( \theta = 1.8 \text{ eV} \),

\[
n_{e2} \approx 12 \times 10^{16}/\text{cm}^3, \quad \theta_2 \approx 2.5 \text{ eV},
\]

which implies \( \eta \approx 2.2 \), \( y_2 \approx 0.40 \), and \( y_1 \approx 0.23 \). These results may be compared with an estimate of \( y_1 \approx 0.12 \) based on \( n_{e1} \approx 2 \times 10^{16}/\text{cm}^3 \) and \( n_{d1} \approx 16.5 \times 10^{16}/\text{cm}^3 \) (from the snowplow model). The atomic fill density corresponds to \( 20 \times 10^{16} \) atoms per square cm of chamber cross section. Integration of the electron density at 2.1 \( \mu \)sec from Fig. 29 gives \(~ 5 \times 10^{16} \) atoms per square cm, which implies \( y_2 \approx 0.25 \).

It is noted that a slight upward adjustment of \( \theta_2 \) and downward adjustment of \( \theta_1 \) would bring the different values of ionization into agreement. The main import of these calculations is to show that the plasma is considerably less than fully ionized.

Estimates of the expected ionization rate are presented in Appendix B. The snowplow structure, with a temperature of \(~ 1.5 \text{ eV} \), should have an ionization time of \(~ 0.5 \mu \text{sec} \), which is consistent with the low degree of ionization observed. However, the compressed phase of the shock is predicted to have an ionization time of \(~ 0.01 \mu \text{sec} \) with \( \theta_2 \approx 2.5 \text{ eV} \). In other words, complete ionization should be observed.

One explanation for the discrepancy would be a depletion of the tail of the electron distribution function due to previous
Fig. 30. Fractional ionization before shock ($y_1$), fractional ionization after shock ($y_2$), and incremental compression ($\eta$) as a function of the ratio of the electron density after the shock to the electron density before the shock, ($n_{e2}/n_{e1}$). Figures are plotted for several combinations of initial and final temperatures. Directed ion energy before the shock is assumed to be 3.2 eV per deuterium nucleus.
ionizing collisions. This would limit the ionization rate to somewhat less than the rate of collisional population of the tail to velocities greater than \( v_1 \), where \( v_1 \) corresponds to an energy of 10.2 eV. (Most electrons are first raised to the first excited hydrogenic level.) For velocities \( v_1 \) much greater than the thermal velocity, MacDonald et al. \(^{41}\) give a time for population to velocity \( v_1 \) based on the Fokker-Planck equation

\[
\tau(v_1) = \frac{m^2 v_1^3}{8 \pi n \Lambda \ln},
\]

which is on the order of the self-deflection time for particles of velocity \( v_1 \). For \( \frac{1}{2} m v_1^2 = 10.2 \) eV, \( \ln \Lambda = 5 \), and \( n_e = 10^{17} / \text{cm}^3 \), the "filling out" time is \( 8.2 \times 10^{-12} \) sec, so it is unlikely that this process is limiting the ionization rate.
V. SCATTERING ANOMALIES

An evaluation of the accuracy of plasma parameter determination based on scattering from ion controlled fluctuations can be made from a number of criteria. As presented in the last section, the results are in qualitative agreement with spectrographic observations, scattering from electron controlled fluctuations, and pinch dynamical considerations, although a folding of the spectrographic data over the plasma cross section and a detailed knowledge of the neutral density would facilitate comparison.

As an internal check on the scattering data, there is recourse to "goodness of fit", i.e., the chi-squared test, and to comparison of the scale factor density, \( n \), with the self-consistent density, \( n' \). As illustrated in Fig. 19, these two measures of density tended to disagree by more than the expected limits of error during times of rapid parameter change. Associated with such cases, but not altogether restricted to them, the \( \chi^2 \) test indicated rather poor fits of theory to data.

Any error in the density or cross section of the gas used for Rayleigh scattering calibration of the instrumentation would lead to an incorrect value of the scale factor density, since it appears along with \( n_R \) and \( \sigma_R \) as an amplitude factor in matching the theory to the data [cf. Eq. (31)]. It is true that published values of the nitrogen scattering cross section have been questioned (see footnote, Sec. III. B. 3), but such a discrepancy would imply \( n/n' = \) constant, which is not consistent with our
observation.

Another possibility is that the plasma was at times non-uniform within the scattering volume. Under such a circumstance, agreement between the two densities would be accidental. The data presented in the last section indicates that gradients in the plasma parameters across the laser beam diameter, \( \approx 0.3 \text{ mm} \), were generally small. However, the stereo Kerr cell photographs of Sec. II. A. give an indication of nonuniformity along the axis of the scattering "cylinder."

Experiments were conducted with a portion of the entrance slit to the polychromator blocked, which allowed observation of only a 1.5-mm length of the plasma. The determined plasma parameters, including \( n \) and \( n' \), were essentially the same as obtained with the usual 7.5-mm observation length. Thus, turbulence or gradients of a scale length less than 1.5 mm would be necessary to explain discrepancies. Availability of light precluded any finer spatial resolution.

In the following sections, several mechanisms that could lead to \( n' \neq n \) will be examined.

A. Preferential Electron Heating

A plasma absorbs energy from a laser beam through the inverse bremsstrahlung process

\[
\text{He} + e^- + H^+ \rightarrow e^{-*} + H^+
\]

where the energy is absorbed primarily by the electron. The cross section per electron is
\[ \sigma_{FP} = 2.43 \times 10^{-37} \frac{n e}{(kT)^{1/2}(\hbar \omega)^3} \left( g_{FP} \right) \text{ cm}^2 \]  

(66)

where \( kT \) and \( \hbar \omega \) are in eV and \( g_{FP} \) is the free-free Gaunt factor \( \approx 1.3 \).

In order to obtain an estimate of plasma heating, the non-linear heat flow equation for an infinite cylindrical geometry was solved numerically. The time and space dependent laser power flux was of the form

\[ P_L = E_L \frac{e^{-r^2/2\sigma^2} e^{-t^2/2\tau^2}}{2\pi \sigma^2 \sqrt{2\pi} \tau} \]  

(67)

where \( E_L \) was 1 joule and \( \sigma \) and \( \tau \) were 0.150 mm and 17 nsec, respectively. Spitzer's \[ ^{43} \] expression for thermal conductivity was used. Zero neutral density, equipartition of electron and ion energy, and constant electron density were assumed. Figures 31 and 32 present plasma temperature calculated as a function of space and time for various relevant plasma densities and initial temperatures.

From the figures it can be seen that the peak temperature rise is typically on the order of 20%, rising to 40% at the largest electron density considered. The resultant spatial and temporal gradients can be expected to produce distortions in the scattering spectrum. It is recognized that the assumption of constant density is weak, for the heated volume should expand on the order of the ion thermal velocity, \( \sim 10^6 \text{ cm/sec} \), covering the
Fig. 31. Laser plasma heating; plasma temperature as a function of position in the laser beam and at different times with respect to the peak laser power. Several conditions of electron density, \( n_e \), and initial temperature, \( \Theta_0 \), are shown. Maximum temperature rise generally occurred after the time of peak laser power (\( t/\tau = 0 \)). The effects of thermal conductivity are clearly evident at \( t/\tau = 2.0 \).
Fig. 32. Laser plasma heating continued.
laser beam radius in approximately 15 nsec. Although this effect would imply lower peak temperatures, the radial motions would themselves distort the spectrum.

The assumption of equal electron ion temperatures is also questionable. Following Spitzer\textsuperscript{32}, the equipartition time can be written

\[
t_{eq} \approx \frac{31.5 A \theta_e^{3/2}}{\ln \Lambda n_{16}} \times 10^{-9} \text{ sec}
\]

where \( \theta_e \) is the temperature in eV, \( n_{16} \) the density in units of \( 10^{16}/\text{cm}^3 \), \( A = 2 \), the atomic weight, and \( \ln \Lambda \approx 5 \). Since the equipartition time is a substantial fraction of the laser time, electron temperatures greater than the ion temperature can be expected. In the limit of infinite equipartition time, changes in electron temperature would be somewhat less than twice those obtained for zero equipartition time since \( \sigma_{\text{FF}} \) decreases with temperature and thermal conductivity increases with temperature.

It is interesting to see explicitly how a scattering spectrum corresponding to \( T_e \neq T_i \) can resemble a theoretical spectrum for which \( T_e = T_i \) is assumed, thus leading to the inconsistent result \( n \neq n' \). Assume that an experimental spectrum with \( T_{el} = 1.5 T_{il} \), scattering parameter, \( \alpha_1 = 1.5 \), and self-consistent density \( n_1 \) can be matched to a theoretical spectrum with \( T_{e2} = T_{i2} \) by proper choice of parameters \( \alpha_2, T_{e2}, \) and scale factor density \( n_2 \). Then

\[
\frac{n_1}{\omega_{el}} \frac{H_{\alpha1}(T_{el} \neq T_{il}, \omega/\omega_{el})}{\omega_{el}} \approx n_2 \frac{H_{\alpha2}(T_{e2} = T_{i2}, \omega/\omega_{e2})}{\omega_{e2}}
\]

(69)
Referring to Fig. 33, it is seen that a fair match * between the solid curve and the "crossed" curve can be made, or

\[
H_{\alpha_2}(T_{e2} = T_{12}, x) \approx \frac{n_1' / \omega_{e1}}{n_2' / \omega_{e2}} \frac{n_2'}{n_2} \frac{x}{T_{e1} = T_{11'}} \left[ \frac{x}{\sqrt{T_{e1} / T_{e2}}} \right] = \alpha_2 \frac{2/T_{e2}}{\alpha_1 \frac{2}{n_2}} \frac{n_2'}{n_2} \frac{x}{T_{e1} \neq T_{11'}} \left[ \frac{x}{\sqrt{T_{e1} / T_{e2}}} \right]. \tag{70}
\]

Experimentally, no correlation could be found between density discrepancy and laser energy, which varies between 0.4 and 1.6 joules. Successive shots using the same laser power and timing.

*No attempt was made to optimize the fit in the "least squares" sense. Note that the experimental spectrum is not normally observed in the region of large x near the electron resonance.
Fig. 33. Solid curve: $H(x)$ for $\alpha = 1.5$. Dashed curve: $0.38 \frac{H(x)}{H(x/1.45)}$ for $\alpha = 1.0$ and 0.1% of the electrons at temperature $T_e/3672$. Circles: $0.38 \frac{H(x)}{H(x/1.45)}$ for $\alpha = 1.0$ and atomic density in the $p = 2$ energy level equal to 1.3% of the electron density. Squares: $H(x/1.22)$ for $\alpha = 1.5$ and $T_e/T_i = 1.5$. 
would occasionally show large differences in density discrepancy.

The sampling units for the photomultiplier outputs have been modified by T. Jarboe to allow observation of the spectrum at two times during a single laser pulse. Preliminary results indicate no substantial differences between the first half of the scattering spectrum and the second half. It is concluded that laser plasma heating is not playing the major role in producing the discrepancies.

It is also of interest to examine resistive heating as a possible mechanism for enhancing the electron temperature over the ion temperature. A crude estimate of this effect may be made by equating the power input per unit volume, \( P \), given in Eq. (52), to the rate of energy exchange per unit volume between the electrons and the ions

\[
P = \frac{3}{2} n \frac{\theta_e - \theta_i}{t_{eq}}.
\]  

Substituting Eq. (68) for \( t_{eq} \) and solving for the electron ion temperature difference in eV

\[
\theta_e - \theta_i = 0.15 \left( \frac{j_3}{n_{16}} \right)^2 \text{eV},
\]  

where \( j_3 \) is the current density in kA/cm\(^2\) and \( n_{16} \) the electron density in units of \( 10^{16}/\text{cm}^3 \).

Figures 27 through 29 show that \( j_3/n_{16} \) is greater than unity along the central plane before the collision of the two plasma fronts. But, after the collision, when the most severe density
discrepancies are observed, the electron ion temperatures are expected to be sensibly equal on the basis of Eq. (73).

Effects due to microturbulence, i.e., non-thermal ion waves, seem improbable due to their strong damping if \( \theta_e = \theta_i \) and if electron ion streaming is at least than the electron thermal velocity.\(^4\) Although excitation of such waves might take place in the shock front postulated in Sec. IV. C, the shock front is not observed during most of the time that the density discrepancy is noted.

B. Rayleigh Scattering by Excited Neutrals

Another potential source of distortion to the ion spectrum is scattering of the Rayleigh type by neutral atoms. Röhr\(^4\) has pointed out that the ruby laser wavelength (6943 Å) is sufficiently close to that of the Balmer H\(\alpha\) line (6563 Å) as to cause near-resonance and consequently a large scattering cross section. Following Griem,\(^4\) the total cross section for scattering by electrons in the \(p\)th energy level of hydrogen is

\[
\sigma_{pp} = \sigma_T \left[ 1 - \sum_{p'} \frac{(E_{p'} - E_p)^2 f_{p'p}}{(E_{p'} - E_p)^2 - (\hbar \omega)^2} \right]^2, \tag{74}
\]

where \(E_p\) is the energy of the \(p\)th level, \(f_{p'p}\) is the absorption oscillator strength for \(p \rightarrow p'\), and \(\hbar \omega\) is the laser photon energy. The largest contributions are for \(p = 3, p' = 2\), and \(p = 3, p' = 2\). If all terms with \(p' > 4\) are neglected,

\[
\sigma_{22} = 27.6 \sigma_T
\]
and neglecting terms for which \( p' \neq 2 \)

\[
\sigma_{33} \approx 13.5 \sigma_T.
\]

At high densities the atom and ion temperatures can be assumed equal, which leads to a Doppler spread of the Rayleigh scattered light which is characteristic of the ion temperature. If \( n(2) \) is the population density of the \( p = 2 \) level, the net scattering spectrum will be proportional to

\[
\frac{n' \sigma_T}{\omega_e} \left[ n(2) \frac{\sigma_{22}}{\omega_e} \frac{\omega^2}{\omega_1^2} \right].
\]

Because of the low population density predicted by the Saha equation (verified experimentally, see next section), the \( p = 3 \) level has been neglected. The overall effect is an enhancement of the ion term.

As a concrete example, the case for \( \alpha = 1 \) and \( n(2)/n' = 1.3\% \) will be computed. These parameters have been chosen for purposes of comparison with the results of the last section and the following section. If the expression in the bracket of Eq. (75) is designated \( H_{\alpha\beta}[n(2)/n', x] \), then from Fig. 33 the approximate fit can be made

\[
H_{\alpha=1.5}[n(2)/n' = 0, x] \approx 0.38 H_{\alpha=1.0}[n(2)/n' = 1.3\%, x/1.45].
\]
etters is

\[ \alpha_1 = \frac{1}{1.5} \alpha_2 \]

\[ T_{e1} = 2.1 T_{e2} \]

\[ n_1' = 0.55n_2 \]

\[ = 0.93n_2' \]

with density discrepancy of

\[ n_2/n_2' = 1.69. \]

Discrepancies of this order are observed (Fig. 19), but the question remains as to whether such high values of \( n(2)/n \) are realizable. Since the plasma is opaque to Lyman \( \alpha \) radiation, direct spectrographic determination of \( n(2) \) is impractical. However, calculations based on the work of Bates et al. [48] (see Appendix B) show that during the ionization phase of plasma development the ratio \( n(2)/n \) can be orders of magnitude greater than the equilibrium value. From Fig. 36, with \( T_e = 2.8 \) eV, \( n(2)/n \) is equal to or greater than 1% for less than 15% ionization and \( n(2)/n > 0.1\% \) (which would still lead to observable effects) for ionization less than 65%. On the other hand, Fig. 35 shows that ionization times on the order of 0.1 \( \mu \text{sec} \) or less are to be expected. This implies that the Rayleigh scattering phenomenon should only play a significant role as the two "snowplow" fronts are colliding unless some unaccounted for factor is serving to clamp the ionization.
C. Cold Electron Hypothesis

Enhanced scattering in the ion peak as well as a possible unpredicted resonance in the electron wing have been observed by Ringler and Nodwell\textsuperscript{49} in hydrogen and by Neufeld\textsuperscript{50} in helium. As a possible explanation, Kegel\textsuperscript{51} has studied (theoretically) the effect on scattering of a small population of electrons with a temperature much less than the bulk plasma temperature.

Assume that a small fraction, \(a\), of electrons with temperature \(T' \ll T\) are added to the plasma. The electron distribution changes from \(f_e\) to

\[
f_e \rightarrow f_e + af_e'
\]

where \(f_e'\) is a Gaussian corresponding to \(T'\). The polarizability factor \(G_e\) (cf. Chapt. II) changes

\[
G_e \rightarrow G_e + aG_e'
\]

\[
= -\alpha^2 \left[ G(x_e) + a \frac{T}{T'} G(x_e') \right]
\]

where

\[
x_e' = (T/T')^{1/2} x_e.
\]

The small change in the ion distribution and polarizability factor of \((1 + a)\), necessary for charge neutrality, may be neglected without materially affecting the results.

On substitution into the form factor, Eq. (24), the modified scattering spectrum is
Evaluation of this expression at \( x = 0 \), where \( G(x) = 1 \), provides a test for enhancement of the ion spectrum

\[
H = \left\{ 1 + \alpha^2 G(x) \right\} \left[ e^{-x^2} + \lambda(T/T')^{1/2} e^{-x_1^2} \right]
\]

\[
+ \alpha^4 (M/m)^{1/2} \left| G(x) + \lambda(T/T') G(x') \right| e^{-x_1^2}
\]

\[
/ \left\{ 1 + \alpha^2 \left[ G(x) + \lambda(T/T') G(x') + G(x) \right] \right\}^2 \right\}. \quad (79)
\]

The peak is increased by a factor of approximately

\[
\frac{(1 + \lambda(T/T')^2}{1 + \alpha^2 (2 + \lambda(T/T')^2)^2}
\]

which is a monotonically increasing function of \( \lambda(T/T') \). For \( \alpha = 1 \) and \( \lambda(T/T') = 0.1 \), the enhancement is 14%. 

In the paper cited, Kegel did not speculate on the source of the cold electrons. In this connection, it is interesting to look at the possible role played by bound electrons in the highly excited states. The ratio of the electron number density of level \( p \), \( n(p) \), with \( p \geq 3 \) to the free electron density \( n \) may be determined from the Saha equation. \( \Theta \) is the electron temperature in eV.

\[
\frac{n(p)}{n} = 3.36 \times 10^{-22} \frac{np^2}{6^{3/2}} e^{13.6/p} \theta^2. \quad (81)
\]
A maximum value for \( p \) is obtained by cutting off the excited orbits at the Debye radius, \( \lambda_D \), or, as suggested by Ecker and Kröll,\(^{59}\) at the interatomic distance \( n^{-1/3} \), i.e.,

\[
\frac{13.6}{P_{\text{max}}^2} = \begin{cases} \\
e^2/\lambda_D \\
e^{2n^{1/3}} \end{cases}
\]

In Eq. (81) the sum over levels between \( p_0 \) and \( P_{\text{max}} > p_0 \) may be done approximately by integrating over \( p \) and neglecting the exponential factor. For \( p_0 > 3, \theta > 3 \text{ eV} \) this results in only a slight error. If \( P_{\text{max}} \) from Eq. (82) is then substituted, the fraction of excited states is

\[
\sum_{n} \frac{n(p)}{n} \approx \begin{cases} \\
-2.04 \times 10^{-6} n^{1/4}/\theta^{3/4} \quad \text{Debye cutoff} \\
1.02 \times 10^{-10} n^{1/2}/\theta^{3/2} \quad \text{nearest neighbor cutoff} \end{cases}
\]

As a numerical example for \( n = 3 \times 10^{16}/\text{cm}^3, \theta = 3 \text{ eV} \), the Debye cutoff gives \( P_{\text{max}} = 26, 1.2\% \) bound excited states, and the nearest neighbor cutoff gives \( P_{\text{max}} = 17, 0.33\% \) bound excited states. It may be easily shown that the number of excited bound electrons exceeds the number of free electrons with energy less than 0.1 eV.

One simplistic view of the role of bound electrons is to ignore the orbital velocity distribution and focus on the interactions of the quantum mechanically smeared out excited state electron density shell. The shells may be assigned a velocity
distribution characteristic of the ions to which they are attached. This implies a population of bound shells with an effective temperature \( T' = \frac{(m/M)T}{3672} \) since, ignoring small binding energies, the shell dynamical reaction to a potential is governed by the electron inertia.

If an excited atom is moving at approximately the same velocity as a test ion and if all or part of the electron shell passes through the test ion Debye sphere, the shell will be distorted, i.e., polarized, and thus it will contribute to the ion shielding. Since the number of these clouds exceeds the number of free electrons in the low velocity range and since that range has the greatest effect on ion shielding and hence on the ion feature scattering, their effect should be pronounced.

It would, of course, be desirable to calculate the polarizabilities of the excited atoms, but since the inverse wave number of the polarizing potential (on the order of a Debye length) is of the same order as the bound electron orbit, quantum mechanical dipole calculations will not suffice. Thus treatment of the slightly bound cloud as a cold free point electron [implicit in the use of Eq. (79)] is but a first approximation.

Figure 34 is a plot of \( H(x) \) for \( T/T' = 3672 \), \( \alpha = 1 \), and several ratios of cold electrons. A strong ion feature enhancement is noted for \( n_{\text{cold}}/n > 0.01\% \). When \( n_{\text{cold}}/n > 1\% \) a resonance appears in the neighborhood of \( x = 0.55 \). No evidence of this resonance was obtained experimentally, which, in this theory implies less than 1% excited states.
Fig. 34. Theoretical scattering profiles for a deuterium plasma assuming various fractions of the electrons at a temperature $T' = T_e/3672$. The selected computation points did not include the peak of the new resonance obtained for 1% cold electrons.
By way of illustration, Fig. 33 shows that a match can be made

\[ H_{\alpha=1.5}(x) = 0.38 H_{\alpha=1.0}(n_{\text{cold}} / n = 0.1\%, T/T' = 3672, x/1.45), (84) \]

which leads to the same relation between the true and inferred parameters as the example of Rayleigh scattering from the p = 2 state in the last section.

The possibility that the excited states are photoionized by the intense laser flux must be investigated. If that were the case, the upper excited states could no longer serve as a source of cold electrons. On the other hand, the ejected electrons have an initial energy equal to the difference of the laser photon energy and the ionization energy. For instance, electrons ejected from the p = 3 level have 0.28 eV energy. Their effective temperature would be less than this, due in part to their preferential emission in the direction of the laser polarization.\(^5\) Such electrons are candidates for free cold electrons.

Detailed calculations presented in Appendix C predict a 42% depopulation of the p = 3 level and progressively smaller depopulation for the higher states whenever maximum laser output was used. Measurements were made in which the \( H_{\alpha} \) and \( H_{\beta} \) lines were observed during the laser pulse. By consideration of the ratios of the observed laser beam volume to the observed plasma volume, a 5% drop in signal would be expected if these levels were totally depleted. (Calculations show the plasma to be optically thin for these lines. Measurement of the absolute intensity of the \( H_{\alpha} \)
emission $\propto \int n(3) dx$ gives results in good agreement with the assumption of optical transparency and a Saha population.) Although random fluctuations of the $H_\alpha$ on the order of 5% were observed, there was no correlation with the time of laser firing. Sensitivity was not good enough to verify a 42% depletion of the $p = 3$ level.

Appendix C also predicts that the electron continuum experiences steady state (i.e., during the laser pulse), narrow band enhancements corresponding to approximately 1.2% of the Saha population of the energy bands. This holds for source levels 3 through to 6. The total contribution of these bands seems to be too low to produce a product $n_{\text{cold}}/n \times T/T'$ large enough for observable effect unless very low effective temperatures $T'$ are assumed.
VI. SUMMARY AND CONCLUSIONS

Measurement of laser scattering spectral profiles and intensities has proven to be an effective means for detailed mapping of a dense cold plasma with scattering parameter \( \alpha \) ranging between 0.6 and 2. Data based on the electron scattering feature are self-consistent and in agreement with independent spectroscopic determinations. Data derived from the combination of the ion feature and a section of the electron plateau are dynamically reasonable and (where comparison was practical) in agreement with spectroscopic analysis for those points that satisfy self-consistency.

However, during periods of rapid electron density change, the later data do not satisfy self-consistency. Of the proposed mechanisms for this effect, large scale nonuniformities, errors in intensity calibration, selective laser or resistive heating of the electrons, nonthermal ion waves, and photoionized cold electrons have been rejected on theoretical or experimental bases. Computations were presented to show that the second energy level of atomic hydrogen rises to a population considerably above thermal during ionization and that Rayleigh scattering by atoms in this level could give rise to the density discrepancy. But the same theory predicts ionization time shorter than the period over which the discrepancy is observed. Finally, a crude model was proposed to link bound electrons in highly excited states with distortions in the plasma polarizability factors.

Some potentially difficult work in kinetic theory remains to
be done in order to clearly define the role of bound electrons moving on orbits of the dimensions of the Debye radius. Experimentally, a dense steady state plasma arc would be more suitable for detailed study of neutral atoms in the $p = 2$ and $p = 3$ state and Rayleigh scattering from those levels.

It is concluded that laser scattering from the ion feature can be a useful tool for plasma diagnostics over the range of parameters cited if self-consistency is verified.
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A. Least Squares Optimization

Let \( S_n \) represent the observed spectral data point for channel \( n \) and let \( S_n(p) \) represent the corresponding theoretical value for a choice of the set of parameters \( p \). The symbol \( p \) may be regarded as a column vector of arbitrary dimensionality and with components consisting of the chosen parameters. \( S_n(p) \) may be a function of an arbitrary number of other variables whose value is known. The optimum choice of parameters in the least-mean-square sense is obtained by minimizing the quantity

\[
L(p) = \sum_n \left[ S_n - S_n(p) \right]^2 W_{nn}
\]

(A1)

with respect to \( p \) where the weights \( W_{nn} \) are related to the expected mean square fluctuations of \( S_n \) by

\[
W_{nn} = \frac{1}{\langle \delta S_n^2 \rangle}.
\]

(A2)

The functions \( S_n(p) \) are generally nonlinear, so the optimization proceeds in small steps. Expanding \( S_n(p + \Delta p) \) about \( p \) and retaining first order terms in \( \Delta p \)

\[
S_n(p + \Delta p) \approx S_n(p) + \sum_m M_{nm} \Delta p_m
\]

(A3)

\[
M_{nm} \equiv \frac{\partial S_n(p)}{\partial p_m}
\]

(A4)

or, in matrix notation
\[ S(p + \Delta p) = S(p) + M\Delta p \]  
(A5)

where the components of \( M \) are defined in Eq. (A4) and \( S \) is a column vector.

If \( \Delta S \) is a vector with components

\[ \Delta S = S_n - S_n(p) \]  
(A6)

Eq. (A1) takes the form

\[ L(p + \Delta p) \approx L_{lin}(p + \Delta p) = (\Delta S - M\Delta p)^T \mathbf{W}(\Delta S - M\Delta p) \]  
(A7)

where \( \mathbf{W} \) is a diagonal matrix with components given in Eq. (A2) and the superscript \( T \) denotes a transpose. \( L_{lin} \) is the value of \( L \) that would be obtained if \( S(p) \) were a linear function of \( p \).

If the partial derivative of \( L \) with respect to \( \Delta p_m \) is set to equal zero for each \( m \) and the resulting matrix equation is inverted, the following is readily obtained:

\[ \Delta p = (M^T \mathbf{W} M)^{-1} M^T \mathbf{W} \Delta S. \]  
(A8)

This is the change in \( p \) that would minimize Eq. (A1) if \( S_n(p) \) were linear in \( p \). The direction of the vector \( \Delta p \) is in the direction of "steepest descent" from the point \( p \), but its magnitude is generally too large to satisfy Eq. (A3). With Eq. (A8) inserted

*Note that matrix \( M \) is generally not square. Hence its inverse is not defined. However, the matrix \( M^T \mathbf{W} M \) is square and thus has an inverse if its determinant is non-zero.
In the case of nonlinear $S_n(p)$, an iterative scheme is used for optimization. An initial guess at the parameters $p$ is made and the vector $\Delta p$ is calculated from Eq. (A8). The parameters are then changed to $p + \epsilon \Delta p$, where the relaxation factor $\epsilon$ ($\epsilon \leq 1$) is adjusted so as to satisfy Eq. (A3). This is accomplished by computing the actual change in $L$ using the given nonlinear functions $S_n(p)$

\[
\Delta L = L(p + \epsilon \Delta p) - L(p)
\]

and comparing it with the change in $L$ that would obtain if the problem were linear

\[
\Delta L_{\text{lin}} = L_{\text{lin}}(p + \epsilon p) - L(p)
= - (2\epsilon - \epsilon^2)\Delta p^T N^{-1} W M \Delta p
\]

[where the last line follows from substitution of $\epsilon$ times Eq. (A8) into Eq. (A7)]. Comparison is effected through use of the quantity $R$,

\[
R \equiv 1 - \frac{\Delta L}{\Delta L_{\text{lin}}} .
\]

If $0 < R < 0.075$, $\epsilon$ is increased by a factor of 2 for the next step. If $0.075 < R < 0.30$, $\epsilon$ remains the same. If $0.30 < R$, $\epsilon$ is decreased by a factor of 2 for the next iteration and if
0.60 < R the last evaluation of the system is rejected and then repeated with \( \varepsilon \) reduced by a factor of 2.

An initial choice of \( \varepsilon = 1/2 \) is made. After each step, the matrix elements are recalculated* and the anticipated least square sum, \( L_{\text{ALSS}} \), is determined. The criteria for convergence are that \( \varepsilon > 1/4 \) and that last sum, \( L \), be within 10\% of the last \( L_{\text{ALSS}} \).

If \( \varepsilon \) drops below 1/128, the problem is taken as too nonlinear for practical solution with the initial choice of parameters and the program is terminated.

*The matrix elements may be determined either analytically or numerically. Due to the complicated dependence of the scattering spectrum on its parameters, the latter procedure was used.
B. Ionization Rate and Excited State Population

For a hydrogen plasma opaque to Lyman line radiation, Bates et al. show that the electron density, \( n(e) \), can be related to the population densities of the ground state, \( n(1) \), and the first excited state, \( n(2) \), through the coupled rate equations

\[
\dot{n}(1) = a_1 n^2(e) + P_{21} n(2) n(e) - R_1 n(1) n(e) \quad (B1)
\]

\[
\dot{n}(2) = a_2 n^2(e) + P_{12} n(1) n(e) - R_2 n(2) n(e) \quad (B2)
\]

where the coefficients \( a_1, a_2, P_{21}, P_{12}, R_1, \) and \( R_2 \) are tabulated in Ref. 48 as functions of \( n(e) \) and temperature \( T \). If \( n(0) \) is the total density of neutral atoms plus ions, and if excited states above the second level are neglected,

\[
n(0) \approx n(1) + n(2) + n(e) \quad (B3)
\]

If \( n(2) \) is perturbed from an equilibrium value, it will relax with a time constant \( \tau_2 = \frac{1}{R_2 n(e)} \). Similarly, \( n(1) \) will relax with a time constant \( \tau_1 = \frac{1}{R_1 n(e)} \). For the temperatures and densities of interest, \( \tau_2 \ll \tau_1 \) and thus Eq. (B2) may be set equal to zero and \( n(2) \approx n^Q(2) \), a quasi-equilibrium value.

If \( n(0) \) and the temperature are constant with time, the system of equations may be put in the form

\[
\frac{n^Q(2)}{n(e)} = \frac{1}{(1 + P_{12}/R_2)} \frac{1}{R_2} \left[ \frac{a_2}{n(e)} n(0)y + P_{12} \left( \frac{1}{y} - 1 \right) \right] \quad (B4)
\]

\[
\frac{dy}{dx} + y^2 - yy_s = 0 \quad (B5)
\]
where \( y \) is the fractional ionization, 

\[
y = \frac{n(e)}{n(0)}
\]

(B6)

and \( y_s \) is defined

\[
y_s = \frac{n(0) - n_s(1) - n_s(2)}{n(0)}
\]

(B7)

where \( n_s(1) \) and \( n_s(2) \) are the steady state values of \( n(1) \) and \( n(2) \) obtainable by setting both Eq. (B1) and (B2) equal to zero, and \( x \) is a normalized time, \( x = t/\tau \), where

\[
\tau = \frac{1}{n(0)} \frac{R_2 + P_{12}}{R_1 R_2 - P_{21} P_{12}}
\]

(B8)

The time constant \( \tau \) is independent of electron density for \( n(e) \geq 10^{15}/\text{cm}^3 \) and is plotted in Fig. 35 as a function of temperature for \( n(0) \) equal to the atomic fill density used in the experiment \( n(0) = 5.2 \times 10^{16}/\text{cm}^3 \), and for the presumed snowplow density \( 20 \times 10^{16}/\text{cm}^3 \). The factor \( y_s \) appearing in Eq. (B7) is a function of \( y \), but for the given values of \( n(0) \) and \( T > 1.4 \text{ eV} \) it may be treated as a constant \( (\approx 1) \) during the integration of Eq. (B5), which yields

\[
y = y_s \left( \frac{y_s(x-x_B)}{1 + e^{y_s(x-x_B)}} \right)
\]

(B9)

\[
x_B = \frac{1}{y_s} \ln \frac{y_s - y_0}{y_0}
\]
Fig. 35. Ionization time as a function of electron temperature.

Solid circles: atomic fill density, $n(0) = 5.2 \times 10^{16}/\text{cm}^3$.

Open circles: $n(0) = 20 \times 10^{16}/\text{cm}^3$. 
i.e. after a "breakdown time", $\tau_B$, the fractional ionization rises from an initial value, $y_0$, to a steady state value $y_s \approx 1$, with time constant $\tau$.

In Eq. (B4) the ratio $a_2/n(e)$, as well as the other coefficients, is independent of $n(e)$ for the conditions cited. Thus the ratio of the first excited level to the electron density can be plotted as a function of temperature and fractional ionization, cf. Fig. 36. It is seen that the ratio can greatly exceed the stationary state value (where $y \approx 1$). In effect, the electrons must "percolate" up through the first excited level during the ionization process.

Drawin and Emard\textsuperscript{56} have carried out detailed rate calculations for both optically thin and thick hydrogen plasmas in which the "instantaneous" (corresponding to our quasi-equilibrium) density of electrons in any level $p \geq 2$ is expressed

$$n(p) = \left[ r_p^{(0)} + r_p^{(1)} \frac{n(1)}{n_e} \right] \frac{Saha}{n(1)} n(p). \quad \text{(B10)}$$

The coefficients $r_p^{(0)}$ and $r_p^{(1)}$ are tabulated as a function of electron density and temperature for various degrees of reabsorption of the Lyman series.

If the assumption of Eq. (B3) is again made, an equation for $n(2)/n(e)$ with the same functional dependence on $n(0)$ and $y$ is obtained. The numerical results, within the range of parameters discussed, are in agreement to a factor of two.
Fig. 36. Ratio of electrons in first excited hydrogenic energy level to free electrons as a function of fractional ionization and for several electron temperatures assuming $5.2 \times 10^{16}$ nuclei/cm$^3$. Steady state values are off scale for $T_e = 2.8$ eV and 5.5 eV.
C. Laser Induced Photoionization

The ruby laser photon (energy $\hbar \omega = 1.79$ ev) is capable of ejecting electrons from excited states of the hydrogen atom with principal quantum number $p \geq 3$. In this section the effect of the intense (i.e. strong compared to blackbody radiation at the plasma temperature) laser photon flux on the excited state and continuum population densities will be investigated.

Let $R(p)$ represent the rate of laser induced photoionization from an energy level $p$ with an electron population density $n(p)$ and statistical weight $g(p)$ to a group of continuum levels, $c$, with energy $\hbar \omega$ greater than level $p$. The group $c$ may be characterized by a population density $n(c)$, statistical weight $g(c)$, and energy spread $\Delta E(c)$. The rate $R$ is, as is usual in absorption calculations, averaged over the initial states $p$ and summed over the final states $c$.

Let $C_1(p)$ be the effective population rate of level $p$ due to collisionally induced transitions to and from states $p' > p$, and let $C_2$ be the rate at which an excess population in the group $c$ is diminished through collisions. Then the relevant equations governing $n(p)$ and $n(c)$ are

$$\frac{dn(p)}{dt} = -R(p) \left[ n(p) - \frac{g(p)}{g(c)} n(c) \right] - C_1(p) \left[ n(p) - n_s(p) \right] \quad (C1)$$

$$\frac{dn(c)}{dt} = R(p) \left[ n(p) - \frac{g(p)}{g(c)} n(c) \right] - C_2 \left[ n(c) - n_s(c) \right] \quad (C2)$$

*Normalization of the continuum is given in Eq. (C10).*
where the subscript "s" denotes the Saha equilibrium level. Thermal equilibrium implies

\[
\frac{n_s(c)}{n_s(p)} = \frac{g(c)}{g(p)} e^{-\Delta \mu/kT}.
\]  

(C3)

The first term in Eqs. (C1) and (C2) include the effects of induced emission through the presence of \( n(c) \). The factor \( g(p)/g(c) \) reverses the role of initial and final states. For equilibrium populations, the effective photoionization rate coefficient is then \( R(p)(1 - e^{-\Delta \mu/kT}) \).

Figure 37 illustrates the processes included in Eqs. (C1) and (C2). Neglected processes include interaction with the ground state through trapped radiation, radiative decay of states with \( p' > p \) into \( p \), radiative decay of \( p \) into states with \( p' < p \), and collisional interaction with states \( p' < p \). In the order of their introduction, these terms can be put in the form

\[
\begin{align*}
- \left[ n(p) - n_s(p) \right] \frac{A(p,1)}{1 - e^{-E(p)/kT}} + \sum_{p' > p} n(p')A(p',p) \\
- \sum_{p' < p \land p' \neq 1} n(p)A(p,p') - \sum_{p' < p} \left\{ n(p) - \left[ 1 + A(p') \right] n_s(p) \right\} D(p,p')
\end{align*}
\]

where \( A(m,n) \) is the spontaneous transition rate from \( m \) to \( n \) and \( n(1) = n_s(1) \) has been assumed. The largest radiative rate is from the first term with \( A(3,1) = 5.58 \times 10^7/\text{sec} \), which is 5 orders of magnitude less than the corresponding laser photoioniza-
Fig. 37. Schematic energy level diagram illustrating processes involved in photoionization of the pth hydrogenic energy level. Photon-induced recombination rate is less than direct photoionization rate by a factor

\[ r = \left[ \frac{n(c)}{g(c)} \right] \left[ \frac{g(p)}{n(p)} \right]. \]
tion rate (see Table CI). In the last term,

\[ \Delta(p') = \left[ n(p') - n_s(p') \right] / n_s(p'), \]

the fractional departure from equilibrium for level \( p' \). Assuming for simplicity that \( \Delta(p') = 0 \), this term becomes

\[ - \left[ n(p) - n_s(p) \right] \sum_{p' < p} D(p, p'). \]

The rate \( \sum_{p' < p} D(p, p') \) is always less than \( C_1(p) \) since collisional transitions become more likely as the level separation decreases.\(^{53}\)

Since all the rates involved in Eqs. (Cl) and (C2) are considerably faster than the inverse of the laser pulse width, the time derivatives are set to zero. Solving for the steady state populations in terms of their fraction deviations from equilibrium,

\[ \frac{n(p) - n_s(p)}{n_s(p)} = \frac{1 - e^{-\hbar \omega / kT}}{1 + \frac{g(p)c_1(p)}{g(c)c_2} + \frac{c_1(p)}{R(p)}} \quad (C4) \]

\[ \frac{n(c) - n_s(c)}{n_s(c)} = \frac{e^{\hbar \omega / kT} - 1}{1 + \frac{g(c)c_2}{g(p)c_1(p)} + \frac{c_2}{R(p)}} \quad (C5) \]

Griem\(^{57}\) gives the photoionization cross section for hydrogen

\[ \sigma(p) = \frac{6\alpha}{3^{3/2}} \pi a_0^2 \left( \frac{E_H}{\hbar \omega} \right)^3 \frac{1}{p^5}, \quad (C6) \]

where \( \alpha = 1/137 \) (fine structure constant), \( a_0 = 0.53 \times 10^{-8} \) cm, and \( E_H = 13.6 \) eV. From the experimental parameters of a 1.8 joule
laser pulse (maximum) in 40 nsec with a beam diameter of 0.3 mm, a photon flux of \( I = 2.2 \times 10^{29} \) photons/cm²-sec may be deduced. Then the photoionization rate coefficient is

\[ R(p) = I\sigma(p). \quad (C7) \]

The rate of population for the \( p \)th level due to scattering of electrons from levels \( p' > p \) may be deduced from Griem

\[ C_1(p) \approx 2.3 \times 10^{-3} \left( \frac{E_H}{kT} \right)^{1/2} \rho^n n_e. \quad (C8) \]

This expression can be considered as approximately valid only if the levels \( p' > p \) are not themselves significantly depopulated by the laser pulse. Fortunately, this turns out to be the case (thus avoiding a more complicated system of coupled equations).

The rate \( C_2 \) is estimated from Spitzer's electron-electron self-collision time,

\[ C_2 = t_c^{-1} = 3.02 \times 10^{-6} \left( \frac{\ln \Lambda n_e}{\theta_e} \right). \quad (C9) \]

Using "box" normalization for free electrons of energy \( E(c) \) (measured from the ionization limit) and a volume per electron of \( n_e^{-1} \), the density of states in the continuum is

\[ g(c) = \frac{1}{\sqrt{2}} \frac{(mc^2)^{3/2}}{(\hbar c)^3} \frac{E^{1/2}(c)}{n_e} \Delta E(c). \quad (C10) \]

The energy spread \( \Delta E(c) \) may be taken as the Stark broadened
width of the origin level \( p \) (considerably greater than the laser spectral width) which is estimated by Griem:

\[
\Delta E_p = 22 e^2 a_0 p^2 (2n_e)^{2/3}.
\]

\[
= 264 \times 10^{-16} p^{2/3} eV
\]

where \( a_0 \) is the first Bohr radius. Combining Eqs. (C10) and (C11) with the density of states for level \( p \), \( g(p) = 2p^2 \), the ratio of density of states is

\[
\frac{g(c)}{g(p)} = 450 \frac{E(c)^{1/2}}{(n_e \times 10^{-15})^{1/3}}
\]

where \( E(c) \) is in eV.

Calculated rates and population density for \( \theta_e = 3 \) eV and \( n_e = 3 \times 10^{16} \) are presented in Table C1.
Table CI. Changes in population of level \( p \) and the continuum due to laser photoionization and relevant rates. Electron self-collision rate is \( C_2 = 9.1 \times 10^{10} \text{/sec} \).

<table>
<thead>
<tr>
<th>Level</th>
<th>Photo-ion. Rate ( 10^{10} \text{/sec} )</th>
<th>Population Rate ( 10^{10} \text{/sec} )</th>
<th>Stat. Weight Ratio ( g(c)/g(p) )</th>
<th>Fractional Change in Population ( \frac{n(p) - n_s(p)}{n_s(p)} )</th>
<th>Fractional Change in Population ( \frac{n(c) - n_s(c)}{n_s(c)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p )</td>
<td>( R(p) )</td>
<td>( C_1(p) )</td>
<td>( g(c)/g(p) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>276</td>
<td>12</td>
<td>77</td>
<td>-0.42</td>
<td>0.013</td>
</tr>
<tr>
<td>4</td>
<td>65</td>
<td>37</td>
<td>140</td>
<td>-0.28</td>
<td>0.015</td>
</tr>
<tr>
<td>5</td>
<td>21</td>
<td>90</td>
<td>160</td>
<td>-0.10</td>
<td>0.012</td>
</tr>
<tr>
<td>6</td>
<td>8.8</td>
<td>180</td>
<td>170</td>
<td>-0.022</td>
<td>0.005</td>
</tr>
<tr>
<td>10</td>
<td>0.67</td>
<td>1400</td>
<td>190</td>
<td>(-2.2 \times 10^{-4})</td>
<td>0</td>
</tr>
</tbody>
</table>
REFERENCES


4. See Ref. 2, p. 89.


20. See Ref. 2, p. 61.
23. See Ref. 1, p. 237.


40. See Ref. 32, p. 139.


43. See Ref. 32, p. 145.

44. T. R. Jarboe, Lawrence Radiation Laboratory, Berkeley, Calif., personal communication.


47. See Ref. 37, p. 37.


54. See Ref. 37, p. 125.


57. See Ref. 37, p. 112.

58. See Ref. 37, p. 146 and 148.

59. See Ref. 32, p. 133.
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