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Nuclear Equation of State from Nuclear Masses, Relativistic Collisions, Supernova Explosions and Neutron Star Masses.

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Abstract

Data on the nuclear equation of state from a number of different sources, from nuclei, high energy nuclear collisions, supernova and neutron stars is analyzed. The current situation concerning supernova simulations is studied and it is concluded that supernova explosions of the prompt kind do not provide a constraint on the nuclear equation of state, and that explosions due to the late-time neutrino heating mechanism of Wilson, may do so. Evidence from the other sources favor a high compression modulus and stiff equation of state.

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1 Introduction

The equation of state of nuclear matter impinges on a number of areas of physics, but unlike the situation at lower density, cannot be directly measured. This is so because one cannot construct a machine out of matter in the solid state composed of atoms and molecules to compress matter to densities higher than the material of the machine. Consequently determinations of the nuclear equation of state must be very indirect, and involve not only complex experimental measurements, but also complex theoretical interpretations of the measurements. Until recently, it had been assumed that the compression modulus, at least, was reasonably well known, \( K \approx 220 \pm 20 \text{MeV} \), through a theoretical analysis of the giant monopole resonance in heavy nuclei, employing the random phase approximation and a variety of phenomenological two-nucleon interactions [1]. Even this has been called into question by developments in other areas. In this work we shall examine some of the evidence and interpretations concerning the equation of state from such diverse areas as nuclear masses to neutron star masses.

2 Nuclear Masses

The droplet model of average nuclear properties involves two parts, a smooth macroscopic and an oscillating microscopic part [2]. The former is represented by a refined liquid drop model and the latter by shell corrections computed as the deviation of nuclear energy levels from uniformity. The liquid drop model is an approximate solution of the nuclear many-body
The problem valid for saturating thin-skinned systems and is a systematic expansion in two small parameters. The coefficients of various terms in the expansion have such significance as the volume energy, symmetry energy, compression modulus, etc., and the dozen or so such parameters in the expansion are able to represent the masses of thousands of nuclei to very high accuracy. In Fig. 1 we show a section of the surface of the rms deviation in mass about the minimum as a function of the compression modulus, the calculations for which were kindly provided by Möller [3]. The region of the minimum is very broad, but suggests a value $K \approx 310\,\text{MeV}$. However, one should note that the behaviour of the rms deviation as a function of $K$ depends on the precise formulation of the model. The macroscopic model that is studied here is the finite-range droplet model. This model combines the droplet model with the folding model surface and Coulomb energy integrals. It also incorporates a new exponential term, that has a large influence on how the model describes nuclear compressibility.

Charge distributions could also be analysed for their information content on $K$, but this has not been done yet with the improved droplet model of [3]. An older version applied to a combined fit of masses and charge radii gives $K > 180\,\text{MeV}$, with a strong prejudice (i.e. large $\chi^2$) against smaller $K$ [4]. Charge distribution differences have been studied in the framework of the Landau-Migdal theory of finite Fermi systems [5]. The approach has not been applied in the global manner of the droplet model [4,6], but only

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**Figure 1:** Section of the rms mass deviation of the droplet model fit to 1593 experimental masses (P. Möller) as a function of compression modulus [3].
to Pb isotopes. It is interesting to note that the best fit is obtained with $K \approx 350\,MeV$.

3 Relativistic Nuclear Collisions

Since the seminal experiments of Stock et. al. [7] many researchers have contributed to the effort of determining the equation of state as a function of nuclear density. Numerous hypotheses and collision models, have been invoked to infer from measurement of pion yields or flow angles what density and compression energy is achieved [8]. The issues have been further clouded by recent findings that a momentum dependance of the mean field experienced by nucleons moving in the compressed matter have an effect on the flow angle [9,10]. The presence of a momentum dependance in the mean field experienced by a nucleon in a nucleus has not been in doubt since Weisskopf pointed out this consequence of nuclear saturation and the approximate independent-particle structure of nuclei, but its form is by no means known [11]. The degree to which the computed pion yields and flow angles reflect the momentum dependance will depend radically on the form assumed for it. No doubt the particular results found in [9] depend also on the effective mass assumed for the nucleons, which, if less than unity will correspond to a greater collision frequency, and hence flow angle.

The equation of state, $W(\rho, T = 0)$, which is the object of all these efforts is then usually interpreted or quoted in terms of $K$, the compression modulus of nuclear matter at saturation, even though the measurements are at supernuclear density, and do not comprise a measurement of the curvature of $W(\rho)$ at $\rho_0$. The connection between $K$ and determinations of $W(\rho)$ at $\rho > \rho_0$ is therefore model dependent. In some cases the model is a simple parameterization that is not based on any underlying theory of the response of matter to compression and heating. An alternative to such parameterized equations of state that we prefer is the relativistic nuclear field theory [12]. Once its coupling constants have been fitted to bulk nuclear properties, it is then able to account for an increasing body of data on finite nuclei [13]. This may be interpreted as attesting to the general correctness of its form, as an effective theory. Depending on how well the coupling constants are determined, it provides a more or less unique way of extrapolating to the domain of densities that are believed to be probed in the experiments, within the assumed validity of the theory. We emphasize
that unlike the frequently used parameterizations of the equation of state, the high density behavior of the theory is determined, as is its saturation properties, by the coupling constants. These are fixed by the binding energy of nuclear matter, its saturation density, the effective nucleon mass at saturation, the symmetry energy and the compression modulus, which we shall vary to obtain agreement with experiment. We use two methods and find that they agree roughly with each other.

3.1 Missing Potential Energy

The first method employs the values quoted by Harris et. al. [14] for the energy per nucleon of compressed matter, obtained as the missing potential energy in a comparison of the observed pion yield at a given energy, and the energy at which the pion yield computed from the fireball model of an ideal gas of nucleons, pions and delta's, has the same value. The assumptions underlying this method are enumerated by Sano et. al. [15], to which should be added that the delta's are assumed to decay promptly. The secondary data obtained in this way is shown in Fig. 2. The equation of state computed from nuclear field theory [12] is compared for several values of the assumed compression modulus and effective nucleon mass at saturation, which most likely lies in the range \(0.7 \leq m^*/m \leq 0.8\). The smaller is the compression modulus, the more sensitive is the equation of state to this property. We see here that within the uncertainty in \(m^*\), a wide range in \(K\) is compatible with the data when interpreted in the way described above. The conclusion from this comparison is that \(K \approx 250 MeV\) to \(800 MeV\).

3.2 Field Theory Fireball

Next we compute the pion yield from primordial pions and deltas produced in hot dense matter that is described by relativistic nuclear field theory. The Lagrangian is,

\[
\mathcal{L} = \sum_B \bar{\psi}_B (i \gamma_\mu \partial^\mu - m_B + g_{\sigma B} \sigma - g_{\omega B} \gamma_\mu \omega^\mu - \frac{1}{2} g_{\rho B} \gamma_\mu \gamma_3 \rho^\mu) \psi_B \\
+ \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2) - \frac{1}{4} \omega_{\mu \nu} \omega^{\mu \nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu \\
- \frac{1}{4} \rho_{\mu \nu} \cdot \rho^{\mu \nu} + \frac{1}{2} m_\rho^2 \rho_\mu \cdot \rho^{\mu} - \frac{1}{3} b m_n (g_\sigma \sigma)^3 - \frac{1}{4} c (g_\sigma \sigma)^4 + \cdots \quad (1)
\]
Here, $\psi_B$ denotes a baryon spinor and the sum is over all charge states of $N$, and $\Delta$, in this particular nuclear application and over $\Lambda, \Sigma, \Xi, ...$ in addition for neutron stars. The $\sigma$ and $\omega$-mesons are Yukawa coupled to the baryons and the $\rho$-meson is coupled to the isospin current. The tensors, $\omega_{\mu\nu}$ denote $\omega_{\mu\nu} = \partial_{\mu} \omega_{\nu} - \partial_{\nu} \omega_{\mu}$, and similarly for the isovector $\rho$-meson. The ellipsis represent the Lagrangians of the mesons that are treated as thermal populations, in this case, pions and kaons.

We use the Rankine-Hugoniot relations to compute the conditions of matter in the shock zone, the temperature, compression, composition etc, and otherwise make the same assumptions enumerated by Sano et. al. [15].

$$\gamma = \frac{\epsilon / \rho}{\epsilon_0 / \rho_0}, \quad \left( \frac{\rho}{\rho_0} \right)^2 = \frac{\epsilon (p + \epsilon)}{\epsilon_0 (p + \epsilon_0)}$$

(2)

Here $\gamma$ is the Lorentz factor, $p$ and $\epsilon$ refer to pressure and energy density in the shock zone, and $\epsilon_0$ to the energy density of matter in the ground state of the incoming nuclei, (represented in all calculations that employ these equations as semi-infinite slabs).

The total pion yield is calculated from the particle densities as,

$$\frac{n_\pi}{A} = \frac{\rho_\pi + \rho_\Delta + \bar{\rho}_\Delta}{\rho_N - \bar{\rho}_N + \rho_\Delta - \bar{\rho}_\Delta}.$$  

(3)

For three assumed values of the compression modulus the calculated yields are compared in Fig. 3 with the measured pion yields [14]. Here,
under the above assumptions we conclude that $K > 800 \text{MeV}$. We show these computed results for only one value of the effective mass because at the higher compression modulus, the computed pion yields differ by less than five percent over the range $0.7 \leq m^*/m \leq 0.8$.

Clearly, the possibility of inconsistency exists between the above two treatments, the one that attributes the missing potential energy to the low pion yield, when compression is neglected, and the other which requires a very stiff equation of state to account for the yield when it is included. Their domain of overlap is for very large $K$, but the agreement of the calculated pion yield with the data for the second method is not very good. The assumption that the pion number produced in the early high density stage, coming from both thermal pions and pions from the decay of the delta, is frozen, seems suspect. For without this assumption, pions will be reabsorbed during the expansion and cooling. More likely is that some pions escape by evaporation during the expansion, while some of those remaining are reabsorbed.

4 Supernova Explosions

In the late stages of the evolution of a star, thermonuclear combustion burns to the endpoint, or minimum mass possible for the number of baryons
present. The star cools and the drop in internal pressure then fails to support the star against gravitational collapse. As the iron core collapses, gravitational energy is converted into heat and kinetic energy of mass transport. Baade and Zwicky [16] realized already more than fifty years ago that the coupling of even a fraction of this energy to the mantle and envelope would lead to an enormous explosion that could explain the energies and luminosities of type II supernovae, whose progenitor stars have masses $M \approx 8M_\odot$. Such supernova eruptions are believed to be the birthplace of intermediate-mass elements in the universe and of neutron stars. However numerical simulations have not, until recently, produced a successful scenario in which most of the imploding material from the collapse of a massive star is ejected as a result of the bounce and the subsequent shock wave that are produced when the infalling matter compresses to supernuclear density. Failure to eject means that the stellar material will once more be accreted by gravity, and the massive remnant will subside into a black hole rather than a neutron star, as must be the case whenever the mass of the accreted material exceeds a critical value of several solar masses.

4.1 Prompt Bounce

The prompt bounce scenario in which mass ejection occurs on the time scale of a few hydrodynamical crossings of the iron core ($\approx 10\text{ms}$) is a tenuous one. On the one hand, stellar evolution calculations of the pre-collapse configuration of the star find that the iron core mass is an increasing function of progenitor mass with a lower bound of $\approx 1.3M_\odot$ for the core mass of the lightest progenitors of type II supernovae ($\approx 10M_\odot$), while numerous simulations of the subsequent evolution find that the mass of the iron core cannot exceed $\approx 1.35M_\odot$ and still allow a successful prompt explosion [17]. Otherwise the shock is dissipated by neutrino losses and photodisintegration, and stalls at the order of 100 km and the star does not explode. Within this narrow window, Baron et. al. [18] find that if they choose an equation of state that is sufficiently soft at high densities above nuclear density, a successful prompt ejection can occur. If this were the whole story, then a tentative conclusion could be reached that the equation of state must be sufficiently soft at high density to produce type II supernovae. Incidentally, a corollary of this scenario, because of the narrow window for iron core masses for which it is successful, is that neutron star masses fall in a narrow range around $1.3M_\odot$.
Very recently [19] it has been discovered by Woosley and collaborators and by Nomoto that a very small correction to the Coulomb energy in the pre-supernova, corresponding to the lower energy of a lattice compared to a free electron gas, lowers the iron core mass by about $\frac{1}{10}M_\odot$. This permits the collapsing matter to sink deeper into the gravitational potential before the bounce, releasing a greater energy to the shock. Consequently a successful ejection by prompt bounce does not require a soft equation of state at high density, as was previously claimed.

4.2 Late-Time Shock Revival

A different scenario has been recently discovered which leads to successful ejection for a wide range of pre-collapse cores arising from the wide range of star masses that occur in nature. By continuing his simulation for more than ten times longer after the bounce than had been done previously, Wilson [20] found that the stalled shock was revived by reheating due to absorption of a neutrino shower emitted by the cooling neutron star. This scenario, if confirmed, requires the failure of the bounce to promptly eject most of the stellar material. Since it leads to successful explosions for a wide range of progenitors it is a robust mechanism that may in fact describe all type II supernova. It is possible that further study may provide some constraint on the nuclear equation of state, since the distance at which the shock stalls ($\approx 100\text{km}$) affects the opacity of the material exterior to the neutrinosphere. This will influence the energy of the explosion and also the distribution of elements produced.

The conclusion to be reached therefore is that the occurrence in nature of supernova explosions does not provide a strong constraint on the equation of state, as might have been concluded, had the only successful scenario been the prompt one without the correction for coulomb lattice energy. Because of the delicate conditions that must be met for the prompt explosion to occur at all, it may be that no supernova explode promptly. It would appear moreover, that the prompt scenario, having such a narrow window in iron core mass for which it can happen, would give too low a frequency of supernova events and too low a mass to account for the more massive neutron stars. The prompt bounce mechanism, requiring as it does a light iron core mass, seems already to be in trouble with the accurate mass measurements on PSR 1913+16 of $M = 1.42 \pm 0.03M_\odot$ [21].
5 Neutron Stars

As mentioned above, the prompt bounce scenario produces neutron stars in a narrow mass range. The late-time explosion scenario, because it works for a wide range of progenitor star masses, produces neutron stars from very light up to the limiting neutron star mass and beyond to black holes. As we discuss below this does have important implications for the equation of state.

We analyse neutron stars in the framework of relativistic nuclear field theory \[12\], generalized to beta-stable charge neutral neutron star matter, including all baryon species that are required to achieve equilibrium over the relevant density range \[22\]. The hadronic part of the Lagrangian for this theory was written earlier in eq.\((1)\) and details of the theory and its solution can be found in ref.\[22\]. When the field equations are solved subject to the subsidiary condition of isospin symmetry, the solution corresponds to symmetric nuclear matter. It is this solution whose properties we quote and which are used to fix the coupling constants of the theory, including the \(\rho\)-meson coupling which is determined mainly by the symmetry energy coefficient. When this same theory, with coupling constants so determined, is solved with subsidiary conditions of charge neutrality and beta equilibrium, we get the solution for neutron star matter. These solutions, by convention, will be denoted always by the properties of the corresponding solutions of symmetric matter. We also emphasize that the high density behavior of the equation of state is determined by the coupling constants which are fixed by the properties of matter at saturation, and unlike parameterizations, there is no further freedom.

In this analysis of neutron stars, we treat the compression modulus of symmetric matter as an unknown, and place lower bounds on it from observations on neutron star masses. The reason why a lower bound is imposed by observation is that, for a given equation of state, there is a maximum or limiting mass that a neutron star can attain. The limiting mass is an increasing function of the stiffness of the equation of state. An acceptable equation of state must have a limiting mass at least as large as the largest known neutron star mass. Hence the lower bound. Since there is uncertainty in the effective nucleon mass at saturation, which uncertainty is reflected in different behavior of the equation of state at high density, we quote a theoretical error on the determination of the lower bound on the compression modulus.

More than three hundred neutron stars are known through their pulsed
radiation. However mass determinations are possible for only those that occur as a companion of another star. In such a case the orbital motion introduces characteristics into the observed radiation that permit mass measurements [23]. So far only half a dozen or so determinations have been made. If on the basis of theoretical prejudice, arising from the narrow mass window of the iron core for which the prompt supernova explosion can successfully eject the mantle and envelope, the observations are analysed as if they were taken on representatives of a population all having the same mass, then the most probable mass is $1.4M_\odot$ [23]. As we have shown elsewhere [24], this value places a lower limit on the nuclear compression modulus of symmetric matter of $\approx 200\, MeV$. However, in view of the success of the late-time explosion mechanism discussed above, for a wide range of progenitor star masses, and hence of neutron star masses, we should accept the dispersion in mass determinations as representative of neutron stars of different masses. These range from $1.05M_\odot$ to $1.87M_\odot$, generally with large errors. The most probable mass in the case of one of the largest mass determinations is $1.85^{+0.35}_{-0.30}M_\odot$ for 4U0900-40. In Fig. 4 we show our calculation of limiting neutron star mass as a function of nuclear compression modulus of symmetric nuclear matter. The above mass is seen to place a lower bound of $K \geq 335 \pm 65\, MeV$. We note that the sequence of limiting masses as a function of $m^*$ at fixed $K$ changes at $K \approx 260$ so that lower $m^*$ than used here will not effect the conclusion on the range of $K$ that is needed to

**Figure 4:** Limiting neutron star mass as a function of compression modulus for several nucleon effective masses (at saturation density) computed in the relativistic nuclear field theory [22]. Horizontal line represents the mass of neutron star 4U0900-40.
account for the mass of this star.

6 Summary

We have discussed evidence on the equation of state coming from a wide range of sources, from nuclear masses to neutron stars. In no single case would the evidence be considered convincing, at least at the present stage. Taken together, it favors a large nuclear compression modulus and a stiff equation of state. The results are,

Nuclear masses: $K \approx 310\text{MeV}$.

High energy nuclear collisions: $K \approx 250 - 800\text{MeV}$, stiff equation of state.

Supernova: determination not possible for prompt bounce mechanism, but constraint may be possible from late-time mechanism.

Neutron star masses: $K \geq 335 \pm 60\text{MeV}$, stiff equation of state.

The wide range of $K$ quoted for nuclear collisions reflects the uncertainty in the hypotheses and methods of analysis.

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