Introduction
This paper demonstrates the effectiveness of children's own verbalization on their conceptual understanding of why they do what they do to solve a simple arithmetic problem. The problem was solvable by the interaction with the external resources, and the externalized answers could be described verbally as they were seen. Verbalization, however, in its essence, could include talks of their own interpretation or explanation of the externalized records (Pine & Messer, 2000; Shirouzu, Miyake & Masukawa, 2001).

I conducted a small-case learning experiment, asking six sixth-graders in a class to cut out the 3/4 of 2/3 of the origami paper's area. They were of roughly the same performance on the math and had already mastered the fractional multiplication. Initially, all of them manipulated the paper directly to solve the task. Yet, gradually guided by a teacher-experimenter through multiple collections of the solutions and explicit comparisons among them, four students actively worked out the answer, 2/3 * 3/4 = 1/2. Six months later, these students described the task by mentioning the paper's area. They were of roughly the same performance on the math and had already mastered the fractional multiplication.

Analysis
Overall, the performance of this class "appears" to be quite high. Everyone solved the given task actively and correctly. Newer interpretations were frequently made and easily shared under "one voice." Hidden by such seemingly one voice, however, crucial differences in their understanding occurred through chances of verbalizing their own interpretations.

If a student replied to the question about the same areas of the answer as, "They are the same not in form but in area," instead of only as, "Different," I coded that he verbalized more than what was seen actually. When the others only consented to such interpretation, I defined that they did not take initiative of explicit verbalization. In this way I coded what child mentioned what interpretation. A through space prevents me from describing the entire shifting-process of interpretation, the interpretations they made and articulated in the lesson appeared in their reports in the follow-up inquiry clearly.

Child 1, for example, answered to why all the solutions were the same as, "If I multiply these two fractions, we can see the answer in the form of the whole, which equals one-half. So, all of these are equal to the one-half of original." In the follow-up, he tied his experience of using origami into the fractional multiplication. On the other hand, Child 2 consented vigorously to Child 1's explanation above, but answered to the inquiry, "I rembered two-thirds and three-fourth," which reveals his remembrance of fragmental facts. Child 3 explained her solving step of the second try as, "I rembered two-thirds and three-fourths," which reveals her remembrance of fragmental facts. Child 3 explained her solving step of the second try as, "If a part of the rest (2/3 of 2/3) of my first answer is combined with this area (1/3), I can get these folded rectangles. This (the answer, 3/4 of 2/3) has also the three rectangles. So if I folded the paper into the half of six parts, the three-sixths, I thought that I could make the 3/4 of 2/3." Even though this early reference to the one-half-ness could not be shared among others, she was explainable at the end of the lesson why the answer was one-half based on her diagrammatic understanding, which could be also recognized in her follow-up report.

The result implies that the students who verbalized their interpretations could produce durable abstract understanding. This process promotes a dialogue among teachers in which interpretations could produce durable abstract understanding. This process promotes a dialogue among teachers in which interpretations could produce durable abstract understanding.