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A LIMITATION ON THE NUMBER OF PARTICLES GENERATED BY BOOTSTRAP DYNAMICS

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December 3, 1963
A LIMITATION ON THE NUMBER OF PARTICLES
GENERATED BY BOOTSTRAP DYNAMICS

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This note calls attention to the constraint imposed in bootstrap
dynamics by the simultaneous requirement of unitarity in the different
reactions connected by analytic continuation. We explore a simple aspect
of this constraint: that the total attractive force in any set of
coupled channels shall not be so strong as to raise any Regge trajectory
above the Froissart limit of \( J = 1 \) at zero energy.\(^1\) It will be argued
here that this requirement can eliminate candidates for the solution of
the self-consistent "bootstrap." In particular, since each particle
contributes to the forces, a limitation on force strength may limit the
number of particles that can exist.\(^2\)

It might appear, since forces from the exchange of individual
particles are sometimes attractive and sometimes repulsive, that an
arbitrarily large number of particles could contribute to the force in
a given channel without raising the net attraction above the Froissart
limit. There exists a set of coupled two-body channels, however, in
which all individual force components appear to be attractive. These
are the particle-antiparticle channels with the quantum numbers of the
vacuum, from which emerge the famous Pomeranchuk Regge-trajectory.
Chew, Frautschi, and Mandelstam pointed out that in the approximation of keeping only \( \pi \pi \) channels, the SU\(_2\) crossing matrix for the three multiplets \( I = 0, 1, \) and 2 \( (B = Y = 0) \) gave purely attractive forces in the singlet \( (I = 0) \) state and that these forces were uniformly stronger or at least as strong as those in the \( I = 1 \) and \( I = 2 \) states.\(^3\) This favored status of the vacuum quantum numbers was conjectured to have a general basis, and was put forward as an explanation of the top position of the Pomeranchuk trajectory on the plot of Re\( \cdot J \) vs energy.\(^1\) Recently Neville has published a general 7-by-7 crossing matrix for the multiplets reached by combining two regular representations of SU\(_n\).\(^4\) Pignotti has pointed out that in this matrix the feature is maintained for any \( n \) that all forces acting on the singlet are attractive and larger in magnitude than the forces produced in any other multiplet.\(^5\) For equal masses and coupling constants each exchanged particle makes the same contribution to the singlet force. There is temptation to see here a general rule; in any event one has a firm basis for discussing current bootstrap models.

Let us now apply the notion of a ceiling on force strength to the observation of Neville that, among the SU\(_3\) multiplets formed from a combination of two pseudoscalar octets, it is possible to have self-sustaining vector-10 and -10 resonances as well as the vector octet. We shall argue that the existence of such vector decuplets of particles probably would furnish excessive attraction to the vacuum singlet and violate the Froissart limit. Only the vector octet seems permitted by the force-strength constraint.
In Table I is reproduced the portion of Neville's crossing matrix relevant to our argument. The simple model of the vector octet arises from the diagonal element \( \frac{1}{2} \) coupling the antisymmetric \( \mathbf{8} \) representation to itself, and Neville observed that the \( \mathbf{10} \) and \( \overline{\mathbf{10}} \) representations can similarly be self-supporting, since they also have a diagonal element equal to \( \frac{1}{2} \). Note, however, the element \( \frac{5}{2} \) giving the force on the singlet state produced by the exchange of \( \mathbf{10} + \overline{\mathbf{10}} \). This is the key to our argument.

If the \((\mathbf{10}, \overline{\mathbf{10}})\) vector particles exist, then in the model considered they should have masses and coupling constants close to those for the vector octet. But then the total attractive force acting on the singlet state would be seven times that acting on the vector octet (or the \( \mathbf{10}, \overline{\mathbf{10}} \)). Knowing what we do about the Regge trajectories for the vector octet (see Fig. 1), it seems probable that such a factor would raise the singlet trajectory well above the Froissart limit. A variety of estimates have indicated that a factor of two in force strength is about right to account for the observed spacing of \( \rho \) and Pomeranchuk trajectories; such a factor occurs if the vector octet exists but the decuplets do not. Note that, because of the zeros in Neville's crossing matrix, the absence of decuplets does not cause any inconsistency in the bootstrap mechanism.

At this point a complicating factor must be drawn to the reader's attention. An examination of Neville's \( SU_n \) crossing matrix reveals that, while the attractive forces on the singlet are proportional to
the number of contributing particles, they are inversely proportional to the number of channels communicating with the singlet. Evidently there are as many such particle-antiparticle channels as particles that exist, so the two effects tend to cancel. Starting with no knowledge of the experimental situation, therefore, it might seem difficult to make a priori statements about particle multiplicity on the basis of the force-strength constraint. What has been done above is to exploit experimental knowledge about the location of the \( p \) trajectory, which of course reflects the overall particle multiplicity that actually exists.

Where, then, does hope lie for making fundamental inferences from the constraint on force strength? It lies in the circumstance that, if any particles exist with nonvacuum quantum numbers, there must be corresponding trajectories (other than the Pomeranchuk trajectory) rising close to if not above \( J = 0 \) at zero total energy. Now we expect that, as in our example above, the spacing between the Pomeranchuk and the next highest trajectory will in general increase with the total number of particles. The reason is that only for the vacuum singlet are each of the individual contributing forces attractive. If such is the case, then, since the next highest trajectory must rise at least to \( J = 0 \), an indefinite increase in the total number of particles will necessarily at some point push the Pomeranchuk trajectory above the Froissart limit. In this way, the combination of unitarity and analyticity may place an upper limit on the number of possible strongly interacting particles.
I am deeply grateful to Dr. D. Neville and Mr. A. Pignotti for enlightening discussions.
FOOTNOTES AND REFERENCES

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2. By a "particle" we mean a pole of the S matrix lying sufficiently close to the physical region so that its effect is substantial. We suppose that an infinite number of poles exist but that the vast majority are so distant from the physical region that they individually play a negligible role.


8. Experimentally, the limit seems to be just reached, as shown in Fig. 1. We are not attempting here to explain this "saturation" of the Froissart condition.
Table I. Neville's crossing matrix for certain representations formed from the product of two SU$_3$ octets. The parentheses (a) or (s) denote symmetric or antisymmetric.

<table>
<thead>
<tr>
<th></th>
<th>1 (s)</th>
<th>8 (a)</th>
<th>10, 10 (a)</th>
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<td>1 (s)</td>
<td>$\frac{1}{8}$</td>
<td>1</td>
<td>$\frac{5}{2}$</td>
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<tr>
<td>8 (a)</td>
<td>$\frac{1}{8}$</td>
<td>$\frac{1}{2}$</td>
<td>0</td>
</tr>
<tr>
<td>10, 10 (a)</td>
<td>$\frac{1}{8}$</td>
<td>0</td>
<td>$\frac{1}{2}$</td>
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</tbody>
</table>
FIGURE LEGEND

Fig. 1. The $p$ trajectory, if we assume that it is parallel to the Pomeranchuk trajectory of Ahmadzadeh and Sakmar [A. Ahmadzadeh and I. Sakmar, Phys. Rev. Letters 2, 439 (1963)].