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MAGNET SEPARATION REQUIRED FOR AN IRON-FREE COLLIDER

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**High Field Magnet Development Analysis**

**Title**
Magnet Separation Required for an Iron-Free Collider

**Introduction**

The problem is to see how the fields produced by two parallel, iron free dipole coils (as in an collider), separated horizontally, affect each other, and to dope out what to do about it.

The coils are assumed to be thin cosine-theta windings.

The fringe field from one coil seen at the second coil looks like a small dipole component plus higher-order terms of all orders.

We use parameters for the LBL Nb$_3$Sn, 10 tesla magnet now under construction as an example.

First: We determine how far the two coils must be separated in order to reduce the quadrupole component to an acceptable amount, assumed to be $1 \times 10^{-4}$ of the dipole field, and we calculate higher-order multipole coefficients.

Second: We assume that the quadrupole field is backed out by a compensating winding, and then separate the coils to where the sextupole component is $1 \times 10^{-4}$, calculate higher-order multipoles, and determine the amp turns required for the compensating winding.

The whole thing isn't very long - I won't try to summarize the results.
For a thin coil of radius $a$, with a linear current density of $J = J_0 \cos \theta$, the field along the $x$-axis is

For $x < a$ \[ B = B_0 \left( \frac{a}{x} \right)^{N-1} \]

For $x > a$ \[ B = B_0 \left( \frac{a}{x} \right)^{N+1} \]

where $B_0$, the field just inside or outside the winding is

\[ B_0 = -\frac{\mu_0 J_0}{2} = -\frac{\mu_0 I}{4\pi} \]

where $I = 2J_0a$ is the total ampere turns for all poles.

Consider two dipole coils as shown

\[ \text{Coil A} \quad \text{Coil B} \]

The field within Coil B produced by Coil A is

\[ B_A = B_0 A \left( \frac{a}{x} \right)^2 \]

or in terms of an $x', y'$ coordinate system centered on Coil B $(x = x' + s)$:

\[ B_A = B_0 A \left( \frac{a}{x' + s} \right)^2 = B_0 A \left( \frac{a}{s} \right)^2 \left( 1 + \frac{x'}{s} \right)^{-2} \]

\[ = B_0 A \left( \frac{a}{s} \right)^2 \left[ 1 - 2 \left( \frac{x'}{s} \right) + 3 \left( \frac{x'}{s} \right)^2 - 4 \left( \frac{x'}{s} \right)^3 + \ldots \right] \]
The field produced inside coil B by itself is \( B_{0B} \), and the total field within coil B is, then

\[
B = B_{0B} + B_A
\]

\[
= B_{0B} + B_{0A} \left( \frac{a}{5} \right) \left[ 1 - 2\left( \frac{a}{5} \right) + \ldots \right]
\]

which can be re-organized

\[
B = \left[ B_{0B} + B_{0A} \left( \frac{a}{5} \right) \right] \left[ 1 - 2K \frac{a}{5} + 3K \left( \frac{a}{5} \right)^2 + \ldots \right]
\]

where

\[
K = \frac{B_{0A} \left( \frac{a}{5} \right)}{B_{0B} + B_{0A} \left( \frac{a}{5} \right)}
\]

We can also express the field in coil B on the x-axis in terms of "multipole coefficients":

\[
B = B_{0B} \left[ 1 + \frac{C_2}{C_1} \frac{a}{5} + \frac{C_4}{C_1} \left( \frac{a}{5} \right)^2 + \ldots \right]
\]

\((C_n \text{ is the magnitude of the field vector associated with a } 2n\text{-pole magnet at } r = \rho, \rho \text{ being an arbitrary reference radius.})\)

Upon equating terms we get

\[
B_0 = B_{0B} \left[ 1 + \frac{B_{0A} \left( \frac{a}{5} \right)}{B_{0B}} \right]
\]

\(\text{Quadrupole: } \quad \frac{C_2}{C_1} = -2K \frac{a}{5}\)

\(\text{Sextupole: } \quad \frac{C_4}{C_1} = -3K \left( \frac{a}{5} \right)^2\)

\(\text{Octupole: } \quad \frac{C_6}{C_1} = 4K \left( \frac{a}{5} \right)^3\)

\(\text{2n-pole: } \quad \frac{C_{2n}}{C_1} = (-1)^{n+1} n! K \left( \frac{a}{5} \right)^{n-1}\)
For the LBL 10-T Nb₃Sn magnet now under construction:

\[ a \approx 50 \text{ mm} \]
\[ \rho \approx 20 \text{ mm} \] (bore radius)

If we assume that the contribution of coil A to the dipole field of coil B is small, and let the coils be of equal strength, then

\[ K \approx \left( \frac{8}{3} \right) \]

Let's demand that \( \left| \frac{C_2}{C_1} \right| \leq 10^{-4} \). Then

\[ \frac{C_2}{C_1} = -2K \frac{\rho^3}{a} = -2 \left( \frac{8}{3} \right)^2 \frac{20}{5} \]
\[ a^3 = 2 \frac{a^2 \rho}{C_2/C_1} \]
\[ = 2 \frac{50^2 \cdot 20}{10^{-4}} = 10^9 \text{ mm}^3 \]
\[ s = 10^3 \text{ mm} = 1 \text{ m} \]

Then, for the other higher order components we get

\[ \frac{C_3}{C_1} = \pm 3 \left( \frac{50}{1000} \right)^2 \left( \frac{20}{1000} \right)^2 = 3 \cdot 10^{-6} \]
\[ \frac{C_4}{C_1} = 4 \left( \frac{\rho}{a} \right)^4 \left( \frac{\rho}{a} \right)^3 = 8 \cdot 10^{-8} \]
\[ \frac{C_5}{C_1} = 5 \left( \frac{\rho}{a} \right)^5 \left( \frac{\rho}{a} \right)^4 = 2 \cdot 10^{-9} \]

Alternatively, let's back out the quadrupole field with a quad trim coil, and demand that \( \left| \frac{C_3}{C_1} \right| \leq 10^{-4} \). Then

\[ \frac{C_3}{C_1} = 3K \left( \frac{\rho}{a} \right)^2 = 3 \left( \frac{8}{3} \right)^2 \left( \frac{20}{5} \right)^2 \]
\[ s^4 = 3 \frac{a^2 \rho^2}{C_3/C_1} = 3 \times \frac{50^2 \cdot 20^2}{10^{-4}} = 3 \times 10^{10} \text{ mm}^4 \]
\[ s = 416 \text{ mm} , \, 416 \text{ m} \]
Then the remaining higher-order multipole coefficients are
\[ C_4/C_1 = 4 \left( \frac{50}{416} \right)^2 \left( \frac{10}{416} \right)^3 = 6 \times 10^{-6} \]
\[ C_5/C_1 = 5 \left( \frac{15}{416} \right)^4 = 4 \times 10^{-7} \]
\[ C_6/C_1 = 6 \left( \frac{10}{416} \right)^5 = 2 \times 10^{-8} \]

The ampere turns required for this quad compensator are calculated as follows:
\[ I = \frac{4a B_0}{\mu_0} \]

or using sub-Q for quad and D for dipole
\[ \frac{I_Q}{I_D} = \frac{a_Q B_{0Q}}{a_D B_{0D}} \]

The radius of the quad coil, if it is outside the dipole coil, is \( a_Q = 160 \)

At a radius \( r = 20 \text{ mm} \), \( B_{0Q}/B_{0D} = C_2/C_1 = 10^{-4} \)
so at 160 mm it would be
\[ \frac{160}{20} \times 10^{-4} = 8 \times 10^{-4} \]
\[ \therefore \ \frac{I_Q}{I_D} = \frac{160}{50} \times 8 \times 10^{-4} = 2.4 \times 10^{-3} \]

etc, etc, etc.
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