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Sampled Data Feedback-Feedforward Control of Structures with Time Delays

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Abstract: A sampled data control algorithm taking into account external excitations was developed for structural engineering applications. The structure was modeled as a continuous system controlled by a digital control system. In addition to the feedback gain obtained using a conventional control design method, a linear feedforward gain was proposed to improve control efficiency. In order to ensure a satisfactory control performance, an adaptive scheme was developed to adjust this feedforward gain in real time according to the measurement history. The conventional sampled data linear quadratic regulator design was used as a baseline to evaluate the effectiveness of the proposed control algorithm. Some practical issues such as the effects of sampling frequency, time delay and actuator dynamics were addressed. This method was validated on the examples of two multistory shear beam lumped mass models. The numerical examples show that combining both the feedforward and feedback terms can improve the control efficiency significantly as compared to the case with feedback control only. It is also shown that the improvement on the control efficiency obtained with the proposed method is still noticeable when time delay is considered and compensated properly.

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Introduction

The control of building structures subject to seismic excitations has been an active research area in recent decades. Many research efforts have been devoted to the development of such control algorithms and devices (Soong 1990; Spencer et al. 1994; Yang et al. 1995; Dyke et al. 1998; Yi et al. 2001; Ohtori et al. 2004). These feedback control systems and their algorithms implicitly consider the external excitations by output feedback control or state feedback control to reach the control goal because the dynamic states reflect the external excitations. However, the seismic excitation can sometimes be measured with relatively high accuracy and inclusion of such measurement information explicitly in the control design process may enable tuning of controllers in real time to achieve an improved control performance (e.g., Grimble 1999, 2001a,b). To this extent, some structural control methods have been developed to take the excitation information into account in order to achieve an optimal control performance; for example, Yang (1975) proposed a method in which the excitation is modeled as a filtered white noise and this excitation model is combined with the governing equations of the structure to solve for the optimal control signals. This procedure gives an exact analytical solution, however it is practically very difficult to accurately model the seismic excitations using filtered white noise, and thus the optimality of the solution may not be readily guaranteed.

In a previous study, an adaptive structural control algorithm taking seismic excitation into account was developed (Ma and Yang 2004). The fundamental assumption for this method is that the measurements of structural response and control signals are available at any time instant. Thus an analog sensor-actuator system is required. Due to the rapid advances in the computer technology, digital control systems are becoming preferable in many engineering applications due to their higher reliabilities, easier operation and maintenance procedures, and lower costs, etc. Although the algorithms developed for analog systems can be transformed to be applicable in discrete time for digital control applications (e.g., Antsaklis and Michel 1997), the efficiency of these algorithms may not be preserved by such a transformation since the standard techniques require the discretization of not only the control algorithm, but also the physical plant which is a continuous system in real world. Preferably, a digital control system should be obtained by direct digital design, treating the controlled system as a sampled-data system operating in continuous time. Classical solutions to this type of feedback control problem as well as their applications in civil engineering can be found in the literature (e.g., Levis et al. 1971; Ankireddi and Yang 1999). In this study, an adaptive sampled data controller using both feedback and feedforward loops was developed. The measurements of the structural response and the external excitations were assumed to be sampled at every T seconds (sample rate). An adaptive scheme was used to adjust the feedforward gain in real time to ensure high control efficiency. The effects of the sample rate T, time delay, and actuator dynamics on the control performance were studied and the compensation strategies were presented.
Control Algorithm Development

The governing equation for an n-degree-of-freedom actively controlled building structure subject to external excitations can be written as (Soong 1990)
\[ M_{\text{mass}} \dddot{q}(t) + C_{\text{damp}} \dot{q}(t) + K_q q(t) = D_{\text{loc}} u(t) + E_w w(t) \]  
(1)
where \( q(t) \) = n-dimensional displacement vector; \( \dddot{q}(t) \) and \( \dot{q}(t) \) = first and second time derivatives of \( q(t) \), respectively; \( u(t) \) and \( w(t) \) = m-dimensional control force and external excitation vectors, respectively; \( M_{\text{mass}}, C_{\text{damp}}, \) and \( K_q = n \times n \)-dimensional mass, damping and stiffness matrices, respectively; and \( D_{\text{loc}} \) and \( E_w = n \times m \)-dimensional control force and external excitation location matrices, respectively. The state space representation of the structure in Eq. (1) can be written as (Soong 1990)
\[ \dot{x}(t) = Ax(t) + Bu(t) + Fw(t) \]
\[ y_{\text{perf}} = C_{\text{perf}} x(t) + D_{\text{perf}} u(t) \]
\[ y_{\text{meas}} = C_{\text{meas}} x(t) + D_{\text{meas}} u(t) + D_s w(t) \]
where \( x(t) \) = state vector associated with the displacements and velocities relative to the ground motion. \( y_{\text{perf}} \) contains the structural responses desired to control, and \( y_{\text{meas}} \) = the measurement vector and
\[ A = \begin{bmatrix} 0 & I \\ -M_{\text{mass}}^{-1}K_q & -M_{\text{mass}}^{-1}C_{\text{damp}} \end{bmatrix} \]
(3)
\[ B = \begin{bmatrix} 0 \\ M_{\text{mass}}^{-1}D_{\text{loc}} \end{bmatrix} \]
(4)
\[ F = \begin{bmatrix} 0 \\ M_{\text{mass}}^{-1}E_w \end{bmatrix} \]
(5)
The matrices \( C_{\text{perf}}, C_{\text{meas}}, \) and \( D_{\text{meas}} \) are determined according to the definition of \( y_{\text{perf}} \) and \( y_{\text{meas}} \). The items \( 0 \) and \( I \) stand for zero and unit matrices, respectively, with appropriate dimensions. The objective of the structural control algorithm proposed in this paper is to find a piecewise-constant control force vector for a structure subject to external excitations such that a predefined cost function is minimized. Generally, the cost function takes the form of (Soong 1990)
\[ J = \frac{1}{2} \int_0^T \left( y_{\text{perf}}^T Q y_{\text{perf}} + u^T R u \right) dt \]  
(6)
In this study, the value of the terminal time \( t_f \) is assumed to be large enough such that the steady state form of the Riccati equation can be used.

Let the sampling time set be (Levis et al. 1971)
\[ \{ t_k \} = \{ t_k; t_k = t_0 + kT; k = 0, 1, 2, \ldots \} \]
(7)
Although not necessary, the terminal time \( t_f \) is assumed to be an integral multiple-\( N \) of the sampling period \( T \) for simplicity. The piecewise-constant control vector \( u \) can be realized by the sample and hold operation as
\[ u(t) = u(t_k) = u_k \text{ for } t_k \leq t < t_{k+1} \]
(8)
The state equation described by Eq. (2) can be transformed to discrete time representation with a sample rate of \( T \)
\[ x_{k+1} = \Phi(T)x_k + \xi(T)u_k + \Psi(T) \]
(9)
where
\[ \Phi(t) = e^{Ax} \]
(10)
\[ \xi(t) = \int_0^t e^{A(t-s)} B dt \]
(11)
\[ \psi_k(t) = \int_0^t e^{A(t-s)} F \cdot w_k(s) dt, \]
(12)
\[ w_k(t) = w(t^k + s) \text{ for } 0 \leq s < T \]
The cost function defined in continuous time as shown in Eq. (6) can be further written in discrete time as
\[ J = \frac{1}{2} \sum_{k=0}^{N-1} \left( x_k^T Q x_k + 2x_k^T M u_k + x_k^T V w_k + 2u_k^T W w_k + u_k^T R u_k + G_k \right) \]
(13)
where
\[ Q = \int_0^T \Phi^T(t) \tilde{Q} \Phi(t) dt \]
(14)
\[ M = \int_0^T \Phi^T(t) \tilde{M} \Phi(t) dt \]
(15)
\[ V = \int_0^T \Phi^T(t) \tilde{V} \Phi(t) dt \]
(16)
\[ W = \int_0^T \Phi^T(t) \tilde{W} \Phi(t) dt \]
(17)
\[ R = \int_0^T \Phi^T(t) \tilde{R} \Phi(t) dt + R \]
(18)
\[ \tilde{Q} = C_{\text{perf}}^T Q C_{\text{perf}} \]
(19)
\[ \tilde{G} = \int_0^T \tilde{V} \Phi(t) \tilde{Q} \Phi(t) dt \]
(20)
Therefore, the optimal control problem can be stated as follows: Find a piecewise-constant control sequence described by Eq. (8) such that the cost function \( J \) defined by Eq. (13) is minimized under the constraint described by Eq. (9). This optimization problem can be solved analytically by using the Lagrange multiplier technique. By defining a co-state vector \( \lambda_k \), which has the same dimensions as the state vector \( x_k \), an augmented cost function can be constructed as Eq. (21) and a Hamiltonian can be defined as Eq. (22)
\[ J_{\text{aug}} = J + \sum_{k=0}^{N-1} \lambda_k^T \left[ \Phi(T)x_k + \xi(T)u_k + \psi_k(T) - x_{k+1} \right] \]
(21)
\[ H_k = \frac{1}{2} \left( x_k^T Q x_k + 2x_k^T M u_k + x_k^T V w_k + 2u_k^T W w_k + u_k^T R u_k + G_k \right) + \lambda_k^T \left[ \Phi(T)x_k + \xi(T)u_k + \psi_k(T) - x_{k+1} \right] \]
(22)
The necessary condition of optimality can be obtained as follows:
\[
\frac{\partial H_k}{\partial \lambda_k} = \ddot{Q}_k + \ddot{M} \ddot{u}_k + \ddot{W}_{ik} + \Phi^T(T)\lambda_{k+1} - \lambda_k = 0
\]
\[
\frac{\partial H_k}{\partial \lambda_k} = \ddot{M} \ddot{u}_k + \ddot{R}_u_k + \ddot{W}_{2k} + \ddot{\xi}(T)\lambda_{k+1} = 0
\] (23)
\[
\lambda_k = 0
\]

Therefore, the optimal control vector can be solved as
\[
u_k^o = -\dddot{R}^{-1}(\ddot{M} \ddot{u}_k + \ddot{W}_{2k} + \ddot{\xi}(T)\lambda_{k+1})
\] (24)

Let
\[
\lambda_k = P_k \ddot{u}_k + f_k
\] (25)
where \(P_k\)=so-called Riccati matrix and \(f_k\)=vector which has the same dimension of the states. Then the following equations can be obtained:
\[
P_k = \Theta^T(p_{k+1} - P_{k+1}(T)|\ddot{R} + \ddot{\xi}(T)P_{k+1}(T)|^{-1}\ddot{\xi}(T)P_{k+1})\Theta + \Gamma
\] (26)
\[
f_k = \Omega f_{k+1} + \Lambda_k
\] (27)
where
\[
\Theta = \Phi(T) - \ddot{\xi}(T)\dddot{R}^{-1}\ddot{M}^T
\] (28)
\[
\Gamma = \ddot{Q} - \dddot{M}\dddot{R}^{-1}\ddot{M}^T
\] (29)
\[
\Omega = \Theta^T(I - \ddot{P}_{k+1}(T)|\ddot{R} + \ddot{\xi}(T)P_{k+1}(T)|^{-1}\ddot{\xi}(T))
\] (30)
\[
\Lambda_k = \Theta^T(P_{k+1} - P_{k+1}(T)|I + \ddot{P}_{k+1}(T)\ddot{R} + \ddot{\xi}(T)P_{k+1}(T)|^{-1}\ddot{\xi}(T)P_{k+1})[\ddot{\xi}(T)\dddot{R}^{-1}\ddot{W}_{2k}] + \dddot{W}_{1k} - \dddot{M}\dddot{R}^{-1}\dddot{W}_{2k}
\] (31)

The terminal condition of Eqs. (26) and (27) are \(P_k=0\) and \(f_k=0\). Eq. (26) has the same form of the well-known matrix Riccati difference equation (Levis et al. 1971) which can be solved with little difficulty (Lasiecka and Triggiani 1991). In this study, a steady state solution \(P\) of Eq. (26) is used if not mentioned otherwise. Hence, matrix \(\Omega\) is the same for all time index \(k\). Eq. (27) is generated by including external excitation into controller design. The optimal control vector can then be expressed as
\[
u_k^o = -[K_{\ddot{R}} \ddot{u}_k + L \dddot{W}_{2k} + N(P_{k+1}\ddot{\xi}(T) + f_{k+1})]
\] (32)
where
\[
K_{\ddot{R}} = \dddot{R}^{-1}\{\dddot{M}^T + \dddot{\xi}(T)\dddot{R}^{-1}\dddot{\xi}(T)\}_k \dddot{P}_{k+1}\Theta
\] (33)
\[
L = \dddot{R}^{-1} - \dddot{R}^{-1}\dddot{\xi}(T)\dddot{R}^{-1}\dddot{\xi}(T)\}_k \dddot{P}_{k+1}\ddot{\xi}(T)\dddot{R}^{-1}
\] (34)
\[
N = \dddot{R}^{-1}\dddot{\xi}(T)\dddot{R}^{-1}\dddot{\xi}(T)\}_k \dddot{P}_{k+1}\ddot{\xi}(T)\dddot{R}^{-1}
\] (35)

It is seen that the calculation of \(\nu_k^o\) involves the calculations of \(\dddot{W}_{1k}, \dddot{W}_{2k}, \dddot{\xi}(T)\), and \(f_{k+1}\) which require the whole information of external excitation be known a priori. It is obviously not possible in reality. However, if the sample rate is short enough, the external excitation within each sampling interval can be approximated as constant and assigned the value of zero outside the interval as (Ma and Yang 2004)
\[
\nu_k(T) = \begin{cases} 
\nu(kT), & \tau \in [kT, (k+1)T] \\
0, & \tau \in [(k+1)T, t_f] 
\end{cases}
\] (36)

Then \(\dddot{W}_{1k}, \dddot{W}_{2k}\), and \(\dddot{\xi}(T)\) can be estimated as
\[
\dddot{W}_{1k, \text{est}} = \eta \nu_k
\] (37)
\[
\dddot{W}_{2k, \text{est}} = \mu \nu_k
\] (38)
\[
\dddot{\xi}(T) = \zeta(T) \nu_k
\] (39)

where
\[
\zeta(s) = \int_0^s e^{A^s} F d\tau
\] (40)
\[
\eta = \int_0^T \Phi^T(\tau) \ddot{\xi}(\tau) d\tau
\] (41)
\[
\mu = \int_0^T \dddot{\xi}(\tau) \ddot{\xi}(\tau) d\tau
\] (42)

and \(f_{k+1}\) can be express as
\[
f_{k+1} = \sum_{i=1}^{N-k-1} \Omega_i \lambda_{k+i}
\] (43)

The calculation of \(f\) of current time requires knowing the future values of the excitation, as indicated by Eq. (31), which is unrealistic. It is thus assumed that \(\lambda_k = \lambda_k\) for all \(k < l\), and \(\lambda_k = 0\) for \(v > l\), where \(l\) is the number of time steps that is arbitrarily chosen, thus
\[
f_{k+1} = S_l \lambda_k
\] (44)
\[
S_l = I + \Omega + \Omega^2 + \cdots + \Omega^{l-2}
\] (45)

Using Eqs. (32)–(45), the optimal control vector can be written as
\[
\nu_k^o = -(K_{\ddot{R}} \ddot{u}_k + K_{\dddot{\xi}(T)} + \dddot{\xi}(T)P_{k+1}(T)\nu_k + \eta - \dddot{M}\dddot{R}^{-1}\mu)
\] (46)

where \(K_{\ddot{R}}=\text{feedforward gain obtained by the sampled data linear quadratic regulator (LQR) design; and } K_{\dddot{\xi}(T)}=\text{feedforward gain which can be written as Eq. (47) by using the steady state solution P of Eq. (26). It is noted that the controller expressed by Eq. (46) may not be the optimal solution. Since the external excitation is explicitly included, the control performance may be improved when the feedforward gain is tuned in real time according to the information provided by past measurements. The feed-forward gain \(K_{\dddot{\xi}(T)}\) is apparently related to \(l\), thus a set of \(K_{\dddot{\xi}(T)}\) can be obtained for various values of \(l\). For any given excitation, there is a value of \(K_{\dddot{\xi}(T)}\) that is most effective at reducing the cost function defined in Eq. (6) among all possible values of \(K_{\dddot{\xi}(T)}\). Unfortunately, this optimal \(K_{\dddot{\xi}(T)}\) cannot be determined unless
the excitation is known in advance. In order to address this problem, the adaptive control scheme proposed by Ma and Yang (2004) is adopted here. The most effective gain is found by testing all the precalculated gains with the available measurements and selecting the gain that gives the smallest value of the cost function for the measurement history. Fig. 1 shows a block diagram of this sampled data control system.

**Time Delay Compensation**

Time delay is one of the important problems in control systems and its effects on controller design have been studied (e.g., Kondo and Furuta 1985 and Chung et al. 1995). In reality, time delays may be generated from data acquisition, signal processing, and control command calculation, etc. It is usually desirable to make such delays as small as possible, especially in controlling an unstable mechanical system (Furuta et al. 1984). In this study, time delays less than one sample rate are considered because there are few cases where the time delay is larger than the sample rate if the time delay is mainly from the computation of control laws (Kondo and Furuta 1985).

Let the time delay be denoted by \( \Delta \). Then the control command can be expressed as

\[
u(t) = u(t_k) = u_k \quad \text{for} \quad t_k + \Delta \leq t < t_{k+1} + \Delta \quad (49)
\]

The state equation can be written as

\[
x_{k+1} = \Phi(T)x_k + \Phi(T - \Delta)\xi(\Delta)u_{k-1} + \xi(T - \Delta)u_k + \psi_k(T) \quad (50)
\]

Let

\[
z_k = \begin{bmatrix} x_k \\ u_{k-1} \end{bmatrix} \quad (51)
\]

and using Eqs. (37), (38), (39), the new state space representation of the structure with time delay is found to be

\[
z_{k+1} = \hat{A}z_k + \hat{B}u_k + \hat{F}w_k
\]

\[
y_{\text{meas}} = \hat{C}_{\text{meas}}z_k + D_w w_k
\]

where

\[
\hat{A} = \begin{bmatrix} \Phi(T) & \Phi(T - \Delta)\xi(\Delta) \\ 0 & 0 \end{bmatrix} \quad (53)
\]

\[
\hat{B} = \begin{bmatrix} \xi(T - \Delta) \\ 1 \end{bmatrix} \quad (54)
\]

Then the control vector can be obtained by estimating the control command needed at time \( t_k + \Delta \) using current measurements and then send the control signal at current time \( t \). The states at time \( t_k + \Delta \) are

\[
x(t_k + \Delta) = \Phi(\Delta)x(t_k) + \xi(\Delta)u_{k-1} + \xi(\Delta)w_k
\]

Then the control command can be written as

\[
u_k^* = -\{K_{fb}\Phi(\Delta)x_k + K_{fb}\xi(\Delta)u_{k-1} + [K_{fb} + K_{fb}\xi(\Delta)]w_k\} \quad (58)
\]

In traditional time delay compensation algorithms, the information of the excitation is not included, only the structural response measurements and the past values of control force are used. The corresponding control command becomes (Kondo and Furuta 1985)

\[
u_k^* = -\{K_{fb}\Phi(\Delta)x_k + K_{fb}\xi(\Delta)u_{k-1}\} \quad (59)
\]

However, if the excitation is measured and such measurement data are included in the time delay compensation, the overall control performance may be improved since the estimation of the states may be more accurate in this case.

**Actuator Dynamics**

In reality, control forces required can not be obtained instantaneously; every actuator has its own dynamic response to the control command input. Therefore, to make a control algorithm feasible in the real world, actuator dynamics needs to be compensated. In this section, linear actuators are considered.

Let the state space model of an actuator be expressed as

\[
\dot{x}_u = A_u x_u + B_u u_c
\]

\[
u = C_u x_u + D_u u_c
\]

where \( x_u = \text{state vector of the actuator} \); \( u = \text{control signal sent to the actuator} \); and \( y = \text{desired control output—in this study, the control force generated} \).

Eq. (60) can be combined with Eq. (2) to construct a new control system as

\[
\ddot{x} = A \dot{x} + B u + F w
\]

\[
y_{\text{meas}} = C \dot{x} + D_{\text{meas}} u_c + D_w w
\]

\[
y_{\text{perf}} = C \dot{x}
\]

\[
A = \begin{bmatrix} A & BC_u \\ 0 & A_u \end{bmatrix}
\]

\[
B = \begin{bmatrix} BD_u \\ B_u \end{bmatrix}
\]
\[
\begin{align*}
\vec{F} & = \begin{bmatrix} F \ 0 \end{bmatrix} \\
\vec{C} & = [C_{\text{meas}} \ D_{\text{meas}} \ C_{\alpha}] \\
\end{align*}
\]

and

\[
\vec{C}_1 = [C_{\text{perf}} \ 0]
\]

Then the optimal control problem becomes the one to find the optimal control command signal sequence \( u \) in order to minimize the cost function defined previously. Thus this problem has exactly the same form of the problem as discussed in the previous sections, and can be solved by using the same procedure. In the following section, numerical examples are presented to illustrate the performance and the effectiveness of the proposed algorithm.

**Illustrative Examples**

In order to demonstrate the applicability and assess the effectiveness of the proposed control method, two shear beam models with three (Yang et al. 1995; Ma and Yang 2004) and five stories (Yang et al. 1993; Brown et al. 1999) were chosen as numerical examples. For both models, the cost function was chosen to be the same as in Ma and Yang (2004). The “tradeoff” parameter, \( \beta \), was fixed as 0.01 unless mentioned otherwise. The earthquakes considered are the N-S components of the ground acceleration from the 1979 Imperial Valley earthquake, Imperial County, Calif. and the E-W components of the ground acceleration from the 1995 Kobe earthquake, Kobe, Japan.

**Results**

The presentation of the numerical results in this paper is focused on the five-story model because: (1) the results obtained in this study for both models are similar and they provide the same conclusions; and (2) the structural properties of the five-story model are closer to those from an actual building, while the three-story model more likely corresponds to a scaled building model. In the following sections, the results presented are for the five-story model unless stated otherwise.

In this example, the case with no time delay or actuator dynamics was first considered. The profile of the first component of the feedforward gain, \( K_{ff} \), is shown in Fig. 2 for different sampling frequencies. The other components show the same trend and are omitted in this figure for simplicity. It is seen that the feedforward gain converges to a constant value with similar convergent rate for every sampling frequency considered. More specifically, in this example, when the time horizon \((l \cdot T)\) used to calculate \( K_{ff} \) is greater than 0.4 s, \( K_{ff} \) does not obviously vary. It is also seen in Fig. 2 that more values of \( K_{ff} \) can be obtained for larger sampling frequencies, thus more choices of \( K_{ff} \) are available for the adaptive algorithm discussed in the previous section and consequently, the control performance may be expected to be better in these cases.

Fig. 3 shows the effect of sampling frequency on the normalized cost function that determines the control efficiency of the proposed method as compared to an LQR design. The cost function was normalized such that it is unity for the LQR design for all sampling frequencies studied. Thus for a controller shown in the subsequent figures, the normalized cost function represents a percentage of the cost function as compared to the corresponding LQR design for every sampling frequency. It is seen that about 40% or more reduction in cost function can be obtained with the proposed feedback-feedforward algorithm when the sampling frequency is greater than 10 Hz. For the sampling frequencies lower than 10 Hz, the cost function of the proposed method could be larger than that from the LQR design. Therefore the system should be sampled at a relatively higher frequency in order to achieve a better performance with the proposed method. In practice, the sampling frequencies of most vibration sensors are on the order of 100–500 Hz, thus they are sufficient for the application of the proposed method.

Fig. 4 shows the normalized root-mean-square (RMS) structural responses and control forces as well as the normalized peak values at different floors with the sampling frequency of 50 Hz. The normalized RMS responses and control forces are shown in Fig. 4(a), while the normalized peak values are shown in Fig. 4(b). These values were once again obtained by normalizing those values for the LQR design to be unity. It can be seen in this example that the normalized interstory displacements are almost

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**Fig. 2.** Feedforward gain profile (first component) with various sampling frequencies.

**Fig. 3.** Normalized cost function for different sampling frequencies.
constant for every floor while the normalized RMS control forces and accelerations are smaller in higher floors. It is observed that significant reduction in interstory displacement can be achieved by using the proposed method, as indicated by the normalized RMS and peak values. Particularly for the RMS displacement, the normalized values range from 65 to 69%. The performance on the peak values of the proposed method is slightly better as indicated by the normalized peak values. These values are between 60 and 64%. The percentage of such improvement becomes slightly smaller as the floor becomes higher. It is also observed that for this example, the proposed method and the LQR design require comparable control force from the actuator on each floor. The normalized RMS control force at the first floor is 1.08, and decreases approximately by 0.05 for each higher floor, and becomes 0.93 at the fifth floor, whereas the normalized peak control forces are 1.08, 0.96, 0.94, 0.91, and 0.89 for the first to the fifth floors, respectively. The trend exhibited in the normalized accelerations is similar to that of the control forces. However, all these values for acceleration were found to be below 1.0 in this example, which indicates that improvement on control performance in acceleration was also achieved. Such improvement is not as significant. Only about 5–10% improvements were achieved. One can find the reason for this observation by noticing that the absolute acceleration of every floor is the summation of two terms as shown in the following equation:

\[ y_{acc} = a_x + a_u = C_{acc} x + D_{acc} u \]

where \( a_x \) and \( a_u \) = contributions from the state vector \( x \) and the control force vector \( u \) to the accelerations, respectively. Generally more reductions in displacements achieved through linear LQR controllers would result in smaller values of \( a_x \), but higher values of \( a_u \), thus accelerations may not be reduced as significantly as displacements, which are part of the state vector. Fig. 5 shows these values of the acceleration at the fifth floor for the LQR design and the proposed method. It is seen that \( a_u \) is more dominant than \( a_x \) for both methods, indicating that the significant reduction in \( a_x \) obtained from the proposed method does not affect the accelerations noticeably. Therefore, the reduction in acceleration is determined by the improvement on \( a_u \) only. In this example, the objective of the controller designed with the proposed method was to achieve relatively large improvement on displacement without using larger control forces as compared to the LQR design; the values of \( a_u \) were thus similar for both methods due to the similar forces they required. As a result, less significant improvement on acceleration was achieved.

Fig. 6 shows the time histories of the interstory displacement and control force from the actuator at the first floor for this case. The results for the other floors are similar and thus are omitted in this paper. It is seen that the interstory displacement is reduced dramatically by the proposed adaptive scheme as compared to the LQR design with comparable control force.

It was of interest to consider the case when the focus of the control is placed on accelerations. The controller obtained from the LQR design remained the same in this example, while the controller from the proposed method was designed such that the displacements would be similar for both methods. This was achieved by increasing \( \beta \) in the proposed method. The results for this case are shown in Fig. 7. It is seen that by design, the normalized RMS displacements are all close to 1.0, however, both the normalized RMS accelerations and control forces are noticeably less than 1, indicating that less control forces were required in this case with the proposed method and more reductions in acceleration were achieved. Similar to the results shown in Fig. 4, both the normalized RMS accelerations and control forces exhibit...
the same trend. They decrease as the floor becomes higher, suggesting that more improvements were achieved at higher floors. In this example, the normalized RMS accelerations are 0.87, 0.86, 0.81, 0.78, and 0.76 for floors 1–5, respectively, while the corresponding values for control forces are 0.85, 0.80, 0.79, 0.77, and 0.76 for the first to fifth floor, respectively. It is seen that 24% improvements on acceleration and control forces at the top floor were achieved with the proposed method while maintaining the same performance in displacement as compared to the LQR design.

Time delay and actuator dynamics were then added into the control system and their effects were studied. The traditional LQR design was once again used as a baseline to evaluate the proposed method. Time delay in the traditional design is compensated by Eq. (59). The range of time delays considered was assumed to be 20–80% of the sample rate and the sample rate was fixed as 25 Hz for this example. Although in practice, actuator itself may need a separate control system to achieve certain desired dynamic functions and thus may be of higher order and/or nonlinear, a first order linear actuator model was considered as in Samali et al. (1985) and Brown and Yang (2001) to illustrate the effectiveness of the proposed method. Complete analysis of the effects of actuator dynamics on the control performance is beyond the scope of this current paper and thus is not included. This first order actuator has the form of

\[ \dot{x}_u = \frac{1}{\tau}(x_c - u) \]

where \( \tau \)= time constant of the actuator and was assumed to be 15 ms as assumed in the previous study (Brown and Yang 2001).

The controllers were designed to focus on displacement reduction in this case. The values of the normalized cost function obtained under the situations with different time delays are shown in Fig. 8. Method I refers to the method with only one feedforward term in time delay compensation, i.e., \( K_{ff}=0 \) in Eq. (58), while Method II corresponds to the method with two feedforward terms, as shown in Eq. (58). The traditional LQR algorithm with no feedforward term in time delay compensation as shown in Eq. (59) was used as a baseline to evaluate the proposed methods. The cost functions were normalized the same way as in the previous examples so that those for the traditional method are unity. It is seen that both methods improve the control performance noticeably, even when time delay is relatively large. For the method with one feedforward terms (Method I), such improvement increases as the time delay increases, ranging from 10% when time delay is at 20% to 35% when time delay is at 80%. For the method with two feedforward terms (Method II), such improvement decreases as the time delay increases. For example, when the time delay is 20% of the sample rate, about 70% improvement in cost function reduction can be achieved, whereas when the time delay is 80% of the sample rate, such improvement reduces to about 40%.

Fig. 9 shows the normalized structural responses for these two methods, respectively. The time delay was chosen to be 60% of the sample rate. The results for Method I are shown in Fig. 9(a). It is seen that the normalized RMS displacements obtained are all

![Fig. 6. Time histories of interstory displacement and control force at first floor](image)

![Fig. 7. Normalized structural responses and control forces at various floors with focus on accelerations in controller design](image)

![Fig. 8. Effects of time delays on control efficiency of different algorithms (Method I: one feedforward terms, Method II: two feedforward term)](image)

![Fig. 9. Normalized RMS Values by Story Number](image)
less than one, which indicates that the more reductions in displacement were achieved for all floors as compared to the traditional method. The improvement again decreases for higher floors, with 16% improvement at the first floor and 4% at the fifth floor. The normalized RMS accelerations and control forces are all close to one in this case, and these values decrease as the floors become higher. The normalized values in this example are 1.12, 1.04, 0.94, 0.93, and 0.93 for the accelerations at floors 1–5, respectively, and 1.01, 1.00, 0.93, 0.91, and 0.89 for the control forces at floors 1–5, respectively. Fig. 9(b) shows the results for Method II. It is noted that controller designed with this method requires slightly larger control force at the first floor as compared to Method I, and the control forces at the rest of the floors are all noticeably less. For example, the normalized RMS control force is 1.04 for the actuator located at the first floor, and decreases to 0.93, 0.85, 0.83, and 0.83 for floors 2–5, respectively. Even with less control forces, the reductions in the interstory displacement are still significant in this case. The normalized RMS displacements are 0.73, 0.71, 0.73, 0.78, and 0.76 for the first to the fifth floors, respectively. Similar to the results presented in Fig. 4, the normalized RMS accelerations show a similar trend to that of the control forces. In this case the normalized RMS accelerations are 1.26, 1.09, 0.89, 0.84, and 0.84 for floors 1–5, respectively. The acceleration obtained using Method II at the first floor is 26% higher than that of the traditional method and about 14% higher than that of Method I. This phenomenon may be explained by providing the force balances due to interaction between each floor and the two adjacent tendons. In this study, active tendons are used as actuators such that the net force acting on a certain floor is obtained as the difference between the horizontal components of the axial forces of the two adjacent tendons (e.g., Ma and Yang 2004). Using the traditional method, the RMS values of the actual forces acting on the first to fifth floors in this example were obtained as: 24.94, 44.31, 64.88, 79.92, and 76.83 kN, respectively.

The corresponding values using Method I were obtained as: 31.72, 58.99, 66.43, 75.17, and 69.04 kN, respectively. For Method II, these values were obtained as: 50.18, 65.38, 63.95, 68.04, and 64.65 kN, respectively. It is seen that the accelerations of the first two floor are larger in Method II as compared to those obtained by Method I or the traditional method because of the larger forces acting on the these two floors. It is however, noted that the acceleration at the first floor is the smallest as compared to those at higher floors. In this example, the RMS acceleration at the first floor is 0.42 m/s², which is 16% less than that of 0.48 m/s² at the fifth floor. The time histories of the structural responses and control forces at the selected floors for the three methods are shown in Fig. 10. It is seen that the method with two feedforward terms (Method II) requires slightly larger control force at the first floor but provides the most reduction in interstory displacement. The control forces at the first floor of the method with one feedforward term (Method I) and the traditional method are almost not distinguishable; however, the values of the interstory displacement of Method I are noticeably lower.

As mentioned previously, a lumped mass shear beam model with three floors was also considered in this study. Only the results for the case with time delays are shown here. The sampling frequency was assumed to be the same as in the example of the five-floor model, with a time delay to be 80% of the sampling period. In real structural applications, it is easier to measure acceleration than the full states. Thus, acceleration feedback was considered in this example using Kalman filtering techniques (e.g., Antsaklis and Michel 1997). A 10% measurement noise was also considered to simulate real world situations. A different earthquake record, the E-W component of the 1995 Kobe earth-
earthquake, was used in this example to show the robustness of performance. The results are shown in Fig. 11. Similar to the case of the five-floor model, both Methods I and II show a better control performance as compared to the traditional method in terms of interstory displacement reduction. For example, the control forces required by both methods are very close to those required by the traditional method; however, the displacement reductions are obviously improved. More specifically, the normalized control forces for Method I are 0.97, 1.00, and 0.91 for floors 1–3, respectively; whereas those for Method II are 1.05, 1.01, and 0.90, respectively. The normalized displacements for Method I are 0.84, 0.85, and 0.88 for floors 1–3, respectively, and those for Method II are 0.73, 0.87, and 0.94, respectively. Acceleration response was also improved for the proposed methods in this example. The normalized RMS accelerations for Method I are 0.84, 0.85, and 0.88 for floors 1–3, respectively, and those for Method II are 0.73, 0.87, and 0.94, respectively. Acceleration response was also improved for the proposed methods in this example. The normalized RMS accelerations for Method I are 0.84, 0.86, and 0.90 for the first to third floors, respectively. The corresponding values for Method II are 0.73, 0.95, and 0.98, respectively. In this example, Method II is able to achieve 10% improvement over Method I in acceleration reduction for the first floor. However, such improvement for the third floor is more noticeable for Method I, which may be more important since the acceleration of the top floor is the largest as compared to those of the two lower floors.

Concluding Remarks

A sampled data control algorithm taking external excitation into account has been developed. This algorithm can be realized as a controller with both feedback and feedforward links. For the examples studied, it has been found that adding a feedforward term to the feedback controller seems to effectively improve the control performance over the feedback control alone.

Numerical examples have been presented to demonstrate the improved efficiency achieved by the proposed method as compared to the conventional LQR design. When the sampling frequency of the system is not very low, i.e., greater than 10 Hz, the control efficiency has been shown in the current examples to be improved significantly and independent of such frequency. This observation may suggest that the proposed method can be applied currently with better efficiency using standard digital control devices since the sampling frequency over 100 Hz can be easily obtained through modern sensor technologies.

When reducing the interstory displacement is chosen as the focus of the controller design, the improvements on reduction of these quantities have been shown to be significant using the proposed method in the examples studied. However, the improvements on accelerations are very limited. This is possibly because achieving larger reductions in displacements usually requires larger control forces, and as a result, the accelerations, which are mainly determined by the control forces in the cases of large displacement reductions, are not reduced as significantly. On the other hand, if the controller design objective is modified toward more reductions in acceleration, which is equivalent to loosen the constraints on displacement and is realized by increasing $\beta$ in this study, the proposed method has also shown a better performance than the LQR design as shown in Fig. 7.

The practical situations of time delay, actuator dynamics, and output feedback have also been considered in this study. A time delay compensation scheme including excitation measurement has been developed. It is found through simulation results that including such measurement in the time delay compensation process does improve the control efficiency. This is because the excitation actually affects the states of the system and thus using the excitation measurement can improve the accuracy of the state estimation, leading to a better control performance. It is also found that the control results are preserved when output feedback and measurement noise are included.

For the proposed method, its applications need not be restricted to the current examples. It can also be applied in other cases where external excitations cannot be ignored in the synthesis of optimal controller and can be measured in real time.

In this study, actuator saturation and model uncertainties are not considered. It seems to be a logical next step to include in the studies these effects, as well as the generality with more representative structures and parameters.

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References


