PUBLIC VS. PRIVATE GOOD RESEARCH AT LAND-GRANT UNIVERSITIES

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ABSTRACT. The basic concern of this paper is the effect of private sponsorship of university research on the allocation of expenditures between public good research and commercial applications. Throughout the land-grant university system, there is much concern that as a result of reduced government funding, fundamental research will be neglected at the expense of research that is geared toward commercial applications. This paper attempts to shed some light on the relationship between research priorities and the availability of public funding for university research. In particular, we use both a static and a dynamic model to investigate the conditions under which university/private research partnerships can “crowd-in” or “crowd-out” basic science research as public funding becomes scarcer.

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1. Introduction

The land-grant university system, which is the centerpiece of the agricultural science establishment in the United States, has been one of the most successful innovations in the history of education (Kerr, 1987; Rasmussen, 1989). The future viability of the system is, however, in jeopardy. Bruce Gardner has shown that while the U.S. agricultural sector was once fairly uniform, composed largely of family farms, and the benefits of new technologies were widely distributed, its benefits have become increasingly more concentrated while its costs have remained widely dispersed (Gardner, 2002). Since the Smith-Lever legislation augmenting the land-grant university system in 1914, the farm share of the population has declined from 33% to 2% and the farm size distribution has become highly skewed.\(^1\) Correspondingly, the agricultural science establishment has received a declining share of the public research budget, roughly in line with the farm sector’s declining share of overall economic activity: agriculture received almost 40% of the federal R&D budget in 1940 but by 2007 this share had declined to 1.4%.

Since the rate of return on agricultural \(R\&D\) has been estimated at well over 40 percent (Alston et al., 2000), it is not surprising that private agricultural research has more than offset declining government funds (Huffman and Evenson, 1993). Many land-grant administrators, looking to supplement their shrinking public budgets, have found eager partners in the private sector to participate in research collaborations and partnerships. These administrators have become increasingly successful forming industry research partnerships; in 2006 land-grant

\(^1\) The largest 7.1% of farms produces 75.1% of output value and the smallest 78.7% produces only 6.8% of output value (Department of Agriculture, 2002).
universities accounted for more than half of the top 20 schools in attracting industry R&D sponsorship.

This corporate sponsorship has caused controversy in some states where deans and directors have tended to “direct” faculty to perform short-term profit-enhancing service (Beattie, 1991). Moreover, choice of research projects is often controlled by internal competitive grants, and/or the administration of extension has been separated from the administration of teaching and research, presumably to concentrate more directly on serving special interests. In other states, a rift has developed between extension and teaching/research faculty. As a consequence, extension activities have been slow to evolve beyond the farm sector in the same way that research activities have. In both cases, the fundamental premise of the land-grant system is violated as research conducted for the private sector has tended to “crowd-out” public good research (Greenberg, 2007).

University administrators increasingly encourage researchers to replace declining formula funding by research grants, many of which are motivated by private interests either directly through private grants or indirectly through public research grants generated by private lobbying and pork-barrel politics. As a consequence, researchers’ marginal research time and the generation of ideas are increasingly focused more on specific private interests. The ultimate concern is that private interests will leverage their funds to crowd-out public good research at land-grant universities.

The motivations to acquire private funding have been amplified by changes in intellectual property (IP) ownership and any royalty distribution within the university (Washburn, 2005). Following the passage of the Bayh-Dole Act, IP rights were assigned to the universities and research scientists have typically received some fraction of the royalty stream associated
with the commercialization of their research. This allocation of royalty revenues provides university scientists with incentives to pursue lines of research that are likely to lead to commercially profitable discoveries. Moreover, since private industry funds are directed to commercially appropriated research, university researchers can increase the likelihood that their research will be licensed by pursuing private sponsorship. Again, this research funded by the private sector crowds-out public-good research that is not generated elsewhere. Under-emphasized research products include fundamental scientific knowledge and, in the field of social science, research on new institutions and policies, analysis of labor displacement effects of new technologies, and safety and environmental research on new biotechnologies and chemicals.

As land-grant universities work more collaboratively with private interests, questions are raised about the need for continued public funding and, more fundamentally, public justification for the land-grant research system. Some argue that privatization of university activities offers new potential for encouraging socially relevant research and facilitating transfer of technology. However, without proper policies and incentives, universities can become pawns of powerful private interests, and the unique and separable contribution that universities can make to the public good may be lost (Just and Rausser, 1993). Public-private partnerships cannot be allowed to leverage university resources and divert research from public-good outputs not produced elsewhere.

The capture of land-grant universities can be expected to lead to increased public criticism and possibly more dramatic reductions in funding. However, capture by the private sector is not the inevitable outcome of public-private research collaboration. In fact, it is conceivable that with proper policies and incentives, universities can use public-private partnerships to
leverage industry resources to crowd-in public good research. Without such partnerships, there is little prospect that the private sector will replace the public-good research that would otherwise take place at land-grant universities. Sufficient incentives simply do not exist for the private sector to effectively replace fundamental investments in public research. Even though private investment in agricultural research is substantial, it quite obviously is aimed toward applied commercial research with transparent potential for profitability: chemicals, hybrid or genetically engineered seeds.

In this paper, we concentrate on the bargaining that must necessarily take place in establishing public-private joint ventures and the incentives among the two agents: research administrators and private industry representatives. The focus is on the potential for crowding-in or crowding-out of public-good research. In addition to private-sector incentives, a governance structure is specified for university research administrators. Crowding-in and/or crowding-out is shown to depend critically on the structure of the bargaining problem between these two parties.

Two alternative frameworks will be used to model university research: static and dynamic with feedback effects. The single-shot static research framework assumes research funding only affects the sponsored research, that is, there are no knowledge spill-overs. This ‘worst-case scenario’ in which there is no feedback between public good and private good research is overly simplistic. An alternative framework is then employed that admits the nonlinearities and feedback between public good research and applied research that characterizes the research process at land-grant universities. By accounting for knowledge spill-overs this alternative framework blurs the distinction between between public good research and commercial
research and, as a result, also blurs the boundary between public land-grant universities and the private sector research.

2. MODEL I: SINGLE-PERIOD WITH NO FEEDBACK STOCK ACCUMULATION

At the center of our model is a public research institution. To fix ideas we’ll label this research institution “the university.” However, the model could apply with few, if any, modifications to a wide variety of public research institutions at any level of governmental organization.

The university produces two kinds of research: theorems and mousetraps. Theorems, denoted by $k$, are basic research that generates public goods. As such, they can not be appropriated for direct commercial benefit. Mousetraps, denoted by $m$, are technologies and products resulting from applied research which are private goods that can be fully appropriated for direct commercial benefit.

In this model the technology available to the university to produce new theorems and mousetraps is simply a function of expenditure, denoted by $e$. These production technologies have the associated cost function $C(m, k) = m^{\beta_m} + k^{\beta_k}$, where $\beta_m, \beta_k > 1$. This cost function gives rise to a production possibilities frontier for the university which describes the set of feasible combinations of theorem and mousetrap research for a given level of expenditure, $\bar{e} \geq m^{\beta_m} + k^{\beta_k}$. This production possibilities frontier is shown in Fig. 1.

The university’s research allocation decisions are made by an administrator who is given a performance function by the university’s regents. This function is used to evaluate the quality of the administrator’s decisions. Initially we presume a simple linear performance
Figure 1. The production possibilities frontier and expansion path.

function, viz.,

\[ P(m, k) = \mu m + (1 - \mu)k \quad \mu \in [0, 1] \]  

(1)

This performance function gives the administrator \( \mu \) performance units for each mousetrap she produces and \((1 - \mu)\) performance units for each theorem she produces. This function establishes a relative price between mousetraps and theorems equal to the ratio \( p = \mu/(1 - \mu) \).

For a fixed level of expenditure, the administrator will select the research mix along her production possibilities frontier defined by the usual condition that the rate of product transformation between mousetraps and theorems just equal their relative price, \( p \). We consider later the effect of generalizing the performance measure to allow for imperfect substitution between mousetraps and theorems.
To produce any research the university must obtain funding. The university has both public (government) and private commercial company funding sources. Government funding, $g$, is presumed to be costlessly and exogenously obtained. Funding from the commercial company, denoted by $f$, is given in exchange for property rights over the produced mousetraps and is obtained through a bargaining process. The outcome of this bargaining process is an $(m, k)$ pair which is produced using funding from the commercial company and government, $C(m, k) \leq f + g$. If we normalize the price of mousetraps to unity, the commercial company’s profit function is given by

$$\pi(m, k) = m - f$$

In the current static model, any investment in theorems is worthless to the commercial company; they only capture value from the mousetraps. However, depending on the bargaining outcome, they may also have to produce funding support for some theorems.

We use the Nash cooperative bargaining solution concept to solve this problem. Undeniably, the Nash solution is a good place to start due to its simplicity. In particular, we will be able to neatly define a precise measure of what we mean by “crowding-out” or “crowding-in” and provide simple geometric interpretations.

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2We broaden our analysis to consider investment dynamics in the second model and consider a third funding source, viz., the royalties associated with any university mousetraps that have been commercialized.

3In reality the university would not fully cede its property rights to the mousetraps, but would instead negotiate a royalties agreement. This complication is addressed in the second model.

4We assume the commercial company can costlessly transform the university’s mousetrap research into a marketable product. For simplicity, we assume every unit of the mousetrap research produced by the university can produce a single unit of marketable product for the commercial company. Relaxing this assumption by allowing for a costly, nonlinear transformation of mousetrap research does not significantly alter our conclusions.

5There are good reasons to doubt the validity of this solution concept for modeling collective decision making processes in general. We won’t discuss these here, but instead refer the reader to Rausser, Simon and van ’t Veld (1995).
Recall that the Nash solution is computed as the point in the bargaining set, \( B \), that maximizes the product of the players’ utility gains from cooperating:

\[
(\hat{m}, \hat{k}) = \arg \max \{[\pi(m, k) - \pi_d] [P(m, k) - P_d] : \forall (m, k) \in B\}
\]

(3)

\( \pi_d \) and \( P_d \) represent the disagreement outcomes (or threat points) for the commercial company and university, respectively. In the event of disagreement, the commercial company gets nothing—its threat point \( \pi_d \) is zero. In the event of disagreement, the university still has a funding level of \( g \) from governmental grants and can produce any allocation in the feasible set labeled \( g \) in Fig. 2. The iso-performance line associated with this level of governmental funding is labeled \( P_d \). The administrator would allocate this funding in accordance with the price, \( p \), induced by its performance function. Let the chosen allocation be denoted by \((m_g, k_g)\). The threat point for the university is \( P_d = \mu m_g + (1 - \mu) k_g \).

For the problem in (3) to be well-defined, we must specify the bargaining set \( B \). \( B \) is the set of efficient points that lie between the ideal research allocations for the university and company. This set is constructed as follows. First consider commercial company’s iso-profit curves labeled \( \pi \) in Fig. 2. The profit associated with these iso-profit lines is decreasing along the vertical axis. The commercial company’s ideal point is on the iso-profit curve with the highest profit, which is the allocation \((m^*_c, k^*_c)\), where \( k^*_c = 0 \) and the level of mousetraps is defined by the standard marginal condition \( \frac{df}{dm} = 1 \). The university has no global ideal point, as its preferences are monotonic, but does have a local ideal point for every iso-profit line. This local ideal point is determined by the tangency between the administrator’s iso-performance line and the company’s iso-profit line. For the commercial company to fund university research, the administrator must offer an allocation that gives the company at
least its zero-profit disagreement outcome, \( \pi_d \). Such allocations are found in Fig. 2 along the iso-profit line labeled \( \pi_d \) that begins at the origin. The university’s local ideal point along this iso-profit curve is the allocation \( (m_u^*, k_u^*) \) that lies at the tangency with its iso-performance line. The bargaining set \( B \) is the set of all such efficient points between this local ideal point for the university, \( (m_u^*, k_u^*) \), and the company’s global ideal point, \( (m_c^*, 0) \). This set of points is also the \textit{core} of the bargaining problem, in which the solution must lie. It is relatively straightforward to map this bargaining set from \( m-k \) space to utility space to produce (a slight variant of) the well-known Nash bargaining picture, as depicted in Fig. 3.\(^6\)

\(^6\)To maintain concavity of the Nash product requires some mild restrictions on the cost functions which essentially guarantee that the contract curve does not rise out of the commercial company’s ideal point too quickly.
2.1. **Measuring the Degree of Crowding-Out.** As noted in the introduction, a basic concern of this paper is the allocation of research expenditures between public good research (theorems) and commercial applications (mousetraps). There is concern that as a result of reduced government funding, public good research will be neglected and research will be expanded that is more short-term in orientation, and more geared toward commercial applications. This paper attempts to shed some light on the relationship between research priorities and the availability of public funding for university research. In particular, we use the model to investigate the claim that privately funded research can crowd-out or, in
the alternative, crowd-in fundamental research, and that this can occur even when public funding becomes scarcer.

Two notions of crowding-out are easily captured, one a research ratio measure, the other a negotiating leverage measure. The research ratio measure focuses on the ratio of expenditure on mousetrap research to expenditure on theorem research. This ratio will be referred to as the m-k ratio.

Definition: A fall in the level of government funding exacerbates the research ratio measure of crowding-out if it results in an increase in the m-k ratio. That is, research ratio crowding-out exists if \( \frac{d(m/k)}{dg} < 0 \). Research ratio crowding-in exists if \( \frac{d(m/k)}{dg} > 0 \), i.e. a decrease in the level of government funding decreases the m-k ratio.

This is a very natural notion that seems, at first blush, to capture the essence of the crowding-out metaphor. The first result from our model is that it exhibits research ratio crowding-out.

Proposition 1: In the context of the specified model, a decline in the level of government funding will increase the ratio of mousetraps to theorems produced by the university.

The intuition for this result is quite straightforward. Under extremely weak restrictions, publicly funded research in our model will generate a lower mousetrap to theorem ratio than privately funded research. The m-k ratio compares total expenditure on mousetrap research to total expenditure on theorem research, aggregating across privately-funded and publicly-funded activities. As public research funds decline, so does the contribution of its relatively theorem-rich research mix to the aggregate mix, and so the m-k ratio declines.

The above result is hardly surprising. Avoiding this research ratio form of crowding-out seems to be too much to expect. While administrators might wish that private agencies would share their enthusiasm for fundamental research, they can hardly expect this result. More realistically, administrators should expect that some dilution of the purity of their
research activities is a necessary price that they must pay if they are to augment their public funds with private ones. For this reason, the research ratio measure of crowding-out result is of little practical use as it tells us nothing about how policy makers could restructure incentives to ameliorate the deleterious effects of crowding-out.

For the remainder of this section we focus on a more subtle notion of crowding-out, which is intimately connected to the view that the research fund raising process should be viewed as a bargaining problem. The fundamental issue is: how will the decline in government funding affect the relative bargaining strength of the university administrator as she enters into negotiations with private funding sources for research contracts. A very natural concern is that as government funding declines, the administrator’s need for funds will become more desperate, and hence more willing to compromise in the bargaining process in order to secure funding. In this context, compromise will take the form of skewing research contracts in favor of mousetraps rather than theorems.

To operationalize this idea, we introduce the following negotiation leverage measure of crowding-out. The very best that the administrator can hope for in a negotiation with a private funding source is to drive the other bargainer to her reservation utility, i.e., to extract all of the surplus from the bargaining relationship so that the other party is just indifferent as to whether or not an agreement is reached. We have already defined this as the administrator’s local ideal point, \((m_u^*, k_u^*)\). In general, of course, such a good bargain will never be struck, and the commercial company will secure some of the surplus. A natural way to measure the university’s negotiation leverage, then, is to consider the gap between the realized bargaining outcome and the best possible outcome, as a fraction of the total
potential surplus that is available to the administrator from the bargaining relationship (Rausser and Zusman, 1992).

**Definition:** Formally, this measure of the university’s leverage is:

\[ \zeta = \left[ \frac{P^* - \hat{P}}{P^* - P_d} \right] \]  

(4)

where, as illustrated in Fig. 3, \( P^* \) is the highest utility that the administrator can obtain from the bargaining relationship, \( \hat{P} \) is the utility she obtains from the actual solution to the bargaining and \( P_d \) is the administrator’s default utility.

This measure of leverage ranges between zero and one. When \( \zeta = 0 \), all the negotiation leverage resides with the administrator, while if \( \zeta = 1 \), the administrator is completely at the whim of the commercial company.

Given this definition of leverage, a negotiation leverage measure of crowding-out may be defined that relates directly to the incentive structure of the underlying bargaining problem that can generate the crowding-out phenomenon:

**Definition:** The degree of crowding-out is measured by the change in the administrator’s negotiation leverage as government funding falls. If \( \frac{d\zeta}{dg} = 0 \), we say the bargaining problem is neutral. The bargaining problem exhibits negotiation leverage crowding-out if \( \frac{d\zeta}{dg} < 0 \). Conversely, the bargaining problem exhibits negotiation leverage crowding-in if \( \frac{d\zeta}{dg} > 0 \).

3. Neutrality

The key issue to be addressed is: can a decline in the level of government research funding increase the university administrator’s negotiation leverage? We begin by constructing a “neutrality” result. That is, we identify conditions under which the extent of negotiation leverage crowding-out is independent of the level of government funding. The theorem below
is intended as a benchmark rather than as a positive result. Because the three assumptions
upon which the result depends are all extremely restrictive, we can conclude that in reality,
the extent of crowding-out is indeed quite sensitive to the level of government funding.

**Proposition 2:** The following three separability conditions are necessary and sufficient
for the bargaining problem to exhibit neutrality, i.e., for the degree of negotiation leverage
crowding-out to be independent of the level of government funding.

(a) **Benefit Separability:** The commercial company only has property rights over the mousetraps that result from research that it funds. The university retains property rights over the mousetraps it produces with government funds.

(b) **Cost Separability:** The cost functions for privately and publicly funded research are independent of each other.

(c) **Performance Separability:** Theorems and mousetraps are separable in the administrator’s performance function.

Under condition (a), the commercial company derives utility from any mousetraps resulting
from the research program that it funds, but derives none from mousetraps resulting from
publicly funded research. More generally, this condition reflects the idea that association
with the university provides the commercial company with no externality whatsoever. In
reality, of course, this is not the case. Private funding sources generally gain a great deal
from these associations, over and above the utility that the research outputs generate. These
benefits take many forms, ranging from prestige and the benefits of public visibility through
access to research ideas and potential employees at all levels.

An implication of condition (b) is that the marginal cost of producing an additional
mousetrap or theorem depends only on the number of mousetraps and theorems already being
produced within the given research program as opposed to depending on the outputs of other
research projects. In particular, privately-funded and publicly-funded research programs
cannot competing for the same scarce resources. Once again, this is an extremely restrictive
assumption. In fact, it is typically the case that some fraction of the private research funds
are earmarked for operating expenses rather than for infrastructure, so that at the very least, the university’s opportunity cost of publicly funded research *does* increase with the level of privately funded research.

Condition (c) will be satisfied if and only if theorems and mousetraps are perfect substitutes in the administrator’s performance function. To the extent that the performance function is intended as a realistic proxy for “social welfare,” this condition is just as restrictive as any other assumption about perfect substitutability.

**Figure 4.** A geometric view of neutrality.
The proof of this neutrality result is illustrated by Fig. 4. The figure depicts the two bargaining problems confronting the administrator and the commercial company, with different levels of public funding, as exact translates of each other. The neutrality result then follows immediately from the fact that the Nash Bargaining Solution is translation invariant.\footnote{A monotonic decrease in government funding simply shifts the bargaining problem and the properties of the Nash Bargaining Solution do not change.}

The key intuition for the proof lies in the demonstration that the problems are indeed translates of each other. To see this, note that under assumption a, the commercial company obtains no benefit whatsoever from the outputs of publicly-funded research. Nor do its costs of doing business depend in any way on the level of public research. Hence, from the commercial company’s point of view, all aspects of the bargaining problem are completely unchanged as the level of public funding decreases from $g$ to $g'$. Similarly, the administrator benefits from publicly-funded research by exactly the same amount, irrespective of whether or not it reaches an agreement with the commercial company, and irrespective of the nature of this agreement. In particular, note that by condition (c), the manner in which the administrator allocates public funds between theorem and mousetrap research is completely independent of whether or not, and how, it negotiates with the commercial company.

4. Nonneutrality

In this section, we relax each of the assumptions of Proposition 2 in turn, and consider the effect of reduced government funding on the degree of crowding-out. We begin by omitting condition (a). For concreteness, imagine that the university administrator is authorized to offer to the commercial company property rights over all government-funded mousetraps as
a “side-payment” that may induce more private participation. Specifically, consider a bar-
gaining contract which assigns the commercial company property rights to every mousetrap
produced, provided that some agreement is reached. If no agreement is reached, then the
commercial company gets nothing.

As government funding declines, the size of the “pot” of bonus mousetraps available for
side-payments also declines. What is the implication of this for crowding-out?

**Proposition 3: Nonseparable Benefits** If conditions (b) and (c) hold but not (a), then
a decrease in the level of government funded research may either increase or decrease the
degree of crowding-out.

This indeterminacy is at first sight unexpected. Intuitively, it would seem obvious that
the larger is the pot of bonus mousetraps, the more the commercial company has to lose in
the event that it fails to reach an agreement. It would seem that this device for enticing
the commercial company into a bargaining relationship should strengthen the administrator’s
hand in the bargaining process. After all, the more she brings to the table, the more leverage
she has in bargaining with the commercial company. Hence, we would expect that a decrease
in the level of government funding would decrease the degree of crowding-out. Clearly, this
is not the case, however.

The intuition for Proposition 3 is provided by Fig. 5 and Fig. 6 for a nonlinear univer-
sity performance function. To highlight the problem, we consider two very extreme cases in
which the university administrator’s iso-performance lines are convex. Note however that
the bargaining problems illustrated in these figures do not correspond to the problem at
hand. In particular, their Pareto loci are not smooth. On the other hand, it will be appar-
ent to the reader that we could construct smoothed versions with the same properties that
would be consistent with our model. In Fig. 5, the original bargaining frontier is the line
Figure 5. Nonseparable benefits can lead to crowding-out

Figure 6. Nonseparable benefits can lead to crowding-in

\( \hat{\pi} = \pi^* \)  
\( \hat{\pi}' = \pi'^* \)  

\( abc \). After government funding has been reduced, the new frontier is \( cde \). The explanation
for this difference is that before government funding is cut, the university can offer a side-payment to the commercial company that the company values at $\pi^* - \pi^*$ but the university values at zero. The commercial company receives this side payment in full, provided some agreement is reached, but receives nothing in the event of disagreement. Note that the bargaining frontier is initially downward-sloping but then vertical. After the cut in government funding, government-funding research dries up entirely, and the administrator cannot offer the commercial company a side-payment. Consequently the entire bargaining frontier shifts down by precisely the amount $\pi^* - \pi^*$. Note that because of the extreme specification of the frontier, the the maximal utility level that the administrator can obtain from the bargaining relationship, $P^*$, is unaffected by the decline in funding. On the other hand, the realized outcome shifts back down along the ray through the default outcome, so that $\hat{P}$ exceeds $\hat{P}'$. Clearly, this example exhibits crowding-out.

Now consider Fig. 6. The structure is exactly parallel except that the bargaining frontier starts out horizontal and becomes downward sloping. In this case, however, $P^*$ exceeds $P^*$ while $\hat{P}$ is unaffected by the decline in funding. This example exhibits crowding-in.

Since the “intuitive” result proved to be false, the reader will not be surprised to learn that the effects of relaxing the other two conditions are also indeterminate. First, consider what happens when costs are nonseparable. For concreteness, assume that research costs for mousetraps (similarly theorems) strictly increase with the aggregate number of mousetraps (theorems) produced. Clearly, in this case an issue of cost-sharing arises: should private or public funding have access to the flatter part of the cost curve? To simplify the computations,

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8While this extreme specification is useful for heuristic purposes, the argument clearly holds for more general specifications.
we shall assume that public research has first access to the technology. That is, we will assume that public research costs are independent of private research activity, but the reverse relationship does not hold. In extensions to this work, we will consider the effect of including cost-sharing as an additional dimension along which bargaining can occur.

**Proposition 4: Nonseparable Costs** If conditions (a) and (b) hold but not (c), then a decrease in the level of government funded research may either increase or decrease the degree of crowding-out.

Finally, consider the effect of introducing curvature into the administrator’s performance measure. Specifically, assume that the administrator’s performance measure is Cobb Douglas in aggregate mousetraps and aggregate theorems. In this case, intuition strongly suggests a determinate result. We have already noted in Proposition 1 that as the level of public funding declines, the aggregate theorem to mousetrap ratio will decline also. Once we move to a Cobb Douglas performance measure, a decline in public funding will have the effect of increasing the administrator’s “hunger” for theorems. Being more “needy” in this regard, we would expect that the administrator’s bargaining position would be weakened by the decline, and that crowding-out would increase. Once again, however, this intuition proves to be spurious. Specifically,

**Proposition 5: Nonseparable Performance** If conditions (a) and (c) hold but not (b), then a decrease in the level of government funded research may either increase or decrease the degree of crowding-out.

5. **Model II: Multi-Period With Feedback Effects**

The success of the land-grant university system is attributed to two-way interaction whereby research scientists are informed of field problems and relevant solutions based on
scientific developments are communicated to the field (Bonnen, 1986). Phenomenal pro-
ductivity has been due to dispersing and commercializing these technologies widely in the
agricultural sector of the economy (Ruttan, 1982; Evanson, Waggoner and Ruttan, 1979).
Moreover, the system has supplied products with public good characteristics such as new
crop varieties, improved breeding stock, and improved management practices. These prod-
ucts have been easily reproducible and thus have not lent themselves to private market
development and appropriation. The following model focuses on the joint dependencies
between basic and applied research.

In our dynamic model, the production functions of mousetraps and theorems are dependent
on the stocks of both basic and applied research. Since money spent on research in the current
period increases the stock of theorems available for future research, this funding should be
considered investment. The creation of theorems will also increase the stock of knowledge
available to the creators of mousetraps; thus the stocks are public goods. However, there is
also potential feedback from the accumulated mousetrap stocks to new discoveries in basic
science.

Each period the university administrator chooses $m_t$ and $k_t$ and output is determined by:

$$M_t = \alpha M_{t-1} + m_t + h(K_{t-1})$$

(5)

$$K_t = \gamma K_{t-1} + k_t + l(M_{t-1})$$

(6)

Now the number of mousetraps (theorems) produced in period $t$ is a function of the stock of
mousetraps (theorems) available to researchers, $M_{t-1}$, mousetrap (theorem) research funding,
\( m_t \), and feedback from the stock of theorems (mousetraps), \( h(K_{t-1}) \). The feedback effects have the following properties for all \( M, K > 0 \):

\[
    h(K), l(M) > 0 \quad (7)
\]

\[
    h', l' > 0 \quad (8)
\]

\[
    h'', l'' < 0 \quad (9)
\]

Conditions (7), (8), and (9) capture the strictly positive, convex nature of feedback effects in university research.

Each period the administrator’s performance is evaluated by a performance function, now stated in terms of the stocks of both applied \( (M_t) \) and basic \( (K_t) \) knowledge, i.e.

\[
    P_t(M_t, K_t) = \mu M_t + (1 - \mu) K_t \quad \mu \in [0, 1] \quad (10)
\]

Thus over \( T \) periods the administrator’s objective is to maximize

\[
    P = \sum \beta^t P_t + \beta^T \psi(M_T, K_T) \quad (11)
\]

where \( \beta \) is the administrator’s discount factor and \( \psi(M_T, K_T) \) is the value to the administrator of research conducted during the final year. This objective function establishes the relative price between mousetraps (applied research) and theorems (basic research) in every period is \( p = \mu/(1 - \mu) \).
As in the single-period model, the university must obtain funding to sponsor research, but now the university has three potential funding sources: the government, a commercial company, and royalties from the university’s mousetraps that have been commercialized by the company. As before, government funding, $g_t$, is presumed to be costlessly and exogenously obtained. Private funding, denoted by $f_t$, is obtained through a bargaining process that also determines royalties, $R(M_t)$. The outcome of this bargaining process is a pair of vectors $(m = m_1, \ldots, m_T; k = k_1, \ldots, k_T)$ and a royalty rate, $R(M_t)$: the company agrees to provide a level of funding sufficient to produce a portion of this allocation, $f_t$, and to pay the university some fraction of the revenue from the commercialization of mousetrap research from the previous period as a royalty. We assume the administrator will only have $\theta \in [0,1]$ of the royalties paid by the company at their disposal, while the remaining $(1 - \theta)$ is distributed to other groups in the university and has no impact on research. Note, the price of mousetraps is again normalized to unity.

The production of theorems and mousetraps in period $t$ again has the following cost function: $C_t(m_t, k_t) = m_t^{\beta_m} + k_t^{\beta_k}$, where $\beta_k, \beta_m > 1$. This cost function gives rise to a production possibilities frontier for the university which describes the set of feasible combinations of theorem and mousetrap research for a given level of expenditure in period $t$,

$$C(m_t, k_t) \leq g_t + f_t + R(M_{t-1}) \tag{12}$$

where $R(M_{t-1})$ are the royalties paid by the company to the university in period $t$ for the commercialization of mousetrap research in period $t-1$.

If a commercial company reaches a funding agreement with the university, then each period that company sells mousetraps licensed from the university, funds university research, and
pays royalties based on the commercialization of mousetrap research in the previous period. The company’s profit in period $t$ is therefore

$$\pi_t(m_t, k_t) = M_t - f_t - \mathcal{R}(M_{t-1})$$

(13)

where the private funding level in period $t$, $f_t$, is equal to the university’s cost of producing the company’s share of the total research output. The company’s total profit is the sum of the profits from each period and the scrap value of an agreement, $W(M_T)$, discounted by the company’s weighted average cost of capital, $r$:

$$\Pi = \sum_{t=0}^{T-1} \left( \frac{1}{1+r} \right)^t \pi_t(m_t, k_t) + \left( \frac{1}{1+r} \right)^T W(M_T)$$

(14)

Again, we use the Nash cooperative bargaining solution concept to solve this problem. Using the multi-period period extension of the Nash bargaining framework from the first model, the solution is the pair of vectors $\hat{m} = \hat{m}_1, \ldots, \hat{m}_T$ and $\hat{k} = \hat{k}_1, \ldots, \hat{k}_T$ in the bargaining set, $\mathcal{B}$, that maximize the product of the players’ utility gains from cooperating:

$$(\hat{m}, \hat{k}) = \arg \max \left[ \Pi(m_1, \ldots, m_T, k_1, \ldots, k_T) - \Pi_d \right]$$

$$(P(m_1, \ldots, m_T, k_1, \ldots, k_T) - P_d)$$

$$: \forall (m_1, \ldots, m_T, k_1, \ldots, k_T) \in \mathcal{B}$$

(15)

Again, $\Pi_d$ and $P_d$ represent the disagreement outcomes for the commercial company and university, respectively. The company earns no revenue if it does not reach an agreement with the university, since no mousetraps are sold, and has no costs, since no research is
funded, so its threat point, $\Pi_d$, is zero. The university still has a funding level of $g$ from the government in the event of disagreement, which it will allocate in accordance with the price, $p = \mu/(1 - \mu)$, induced by its performance function. Let the allocation chosen by the administrator for a funding level of $g$ be denoted by $(m_{g_1}, \ldots, m_{g_T}; k_{g_1}, \ldots, k_{g_T})$. In the event no agreement is reached, the university administrator’s performance function will be

$$P_d = \sum_{t=0}^{T-1} \beta^t [\mu M_{g_t} + (1 - \mu) K_{g_t}] + \beta^T \psi(M_{g_T}, K_{g_T}).$$

A new bargaining set, $B$, must be defined for this multi-period game for (15) to be well-defined. The company’s ideal allocation is now the set of points $(m_{c_1}^*, \ldots, m_{c_T}^*, k_{c_1}^*, \ldots, k_{c_T}^*)$ which are determined by the first-order conditions given in the appendix. Again, the university administrator has no global ideal point and must only offer the company an allocation that gives the company at least its disagreement outcome, if there is to be an agreement. The set of efficient points between the company’s global ideal allocation and disagreement outcome form the bargaining set $B$, in which the solution must lie.

5.1. Measuring Crowding-Out With Feedback. We will use the two notions of crowding-out from section 2.1 to determine the conditions under a research partnership with the private sector can improve a university’s public good research. Recall, the first notion, the research ratio measure of crowding-out, focuses on the ratio of applied research to basic research. We extend the single-period definition to this multi-period model by comparing the ratio of total expenditure on mousetrap research, $(m_1 + \ldots + m_T)$, to total expenditure on theorem research, $(k_1 + \ldots + k_T)$.

Definition: A fall in the level of government funding exacerbates the research ratio measure of crowding-out if it results in an increase in the $M-K$ ratio. That is, crowding-out exists if \[
\frac{d((m_1 + \ldots + m_T)/(k_1 + \ldots + k_T))}{d g} < 0.
\]
Though the first-order conditions, see appendix, indicate the profit-maximizing company will want to invest in some basic research because of the feedback loops, this model still exhibits research ratio crowding-out.

**Proposition 6:** For the specified model, there exists $\delta, r > 0$, such that for all $\beta > 1 - \delta$, a decline in the level of government funding will increase the ratio of total expenditure on mousetraps to theorems produced by the university.

The university administrator’s linear performance function guarantees that for any $\mu \in [0, 1]$, the administrator values the production of theorems more than the commercial company, which leads the administrator to always prefer more basic research than the company (see equations (20) and (26) in the appendix). Once again, public funding leads to a relatively theorem-rich research mix and, as private funding increases, relatively fewer theorems are produced. Even with large feedback effects the private sector will not value basic research as much as the administrator.

As before, the negotiation leverage measure of crowding-out, in which private fund raising is viewed as a bargaining problem, focuses on change in the university administrator’s leverage in her bargaining negotiations with the private sector as government funding falls. Note that we extend the negotiation leverage measure of crowding-out to this multi-period problem by simply using the weighted sum of the administrator’s utility in the local ideal, actual, and default bargaining outcomes:

$$P^* = \sum_{t=0}^{T-1} \beta^t (\mu M^*_{ut} + (1 - \mu) K^*_{ut}) + \beta^T \psi(M^*_{u_T}, K^*_{u_T})$$  \hspace{1cm} (16)
\[
\hat{P} = \sum_{t=0}^{T-1} \beta^t (\mu \hat{M}_{u_t} + (1 - \mu) \hat{K}_{u_t}) + \beta^T \psi(\hat{M}_{u_T}, \hat{K}_{u_T})
\] (17)

\[
P_d = \sum_{t=0}^{T-1} \beta^t (\mu M_{g_t} + (1 - \mu) K_{g_t}) + \beta^T \psi(M_{g_T}, K_{g_T})
\] (18)

The university’s leverage, (4), is again measured as the distance between the realized bargaining outcome and the local ideal outcome, as a fraction of the administrator’s total potential surplus from the bargaining outcome.

Again, negotiation leverage crowding-out occurs if \( \frac{d \zeta}{dg} < 0 \) and negotiation leverage crowding-in occurs if \( \frac{d \zeta}{dg} > 0 \). If \( \frac{d \zeta}{dg} = 0 \), the bargaining problem is neutral.

6. Neutrality

Two more neutrality conditions are needed to establish the “neutrality” result for the multi-period model with two feedback loops. These assumptions, like conditions (a), (b), and (c) from Proposition 2, are extremely restrictive, so the extent of crowding-out in this multi-period model is likely sensitive to the level of governmental funding.

Proposition 7: When conditions (d) and (e) are added to conditions (a), (b), and (c), the separability conditions are necessary and sufficient for the multi-period bargaining problem to exhibit neutrality, i.e., \( \frac{d \zeta}{dg} = 0 \).

(d) Equal Weighting: The university administrator’s performance function assigns equal weight to basic and applied research. That is, \( \mu = 1/2 \).

(e) Equal Feedback: The feedback functions are equal, \( h(\cdot) = l(\cdot) \).

Conditions (d) and (e) ensure the symmetry required for the neutrality result. Under condition (d) the administrator derives an equal amount of utility from the production of basic and applied research. Because the administrator’s performance function is a proxy
for “social welfare,” constraint (d) on \( \mu \) is just as restrictive as the other substitutability assumptions. Condition (e) ensures the administrator’s production possibilities frontier is symmetric about the 45° line. Though necessary to guarantee neutrality of governmental funding, this assumption is highly unlikely to be satisfied in practice.

Fig. 7 illustrates why conditions (d) and (e) are needed to prove Proposition 7. The symmetry imposed by these neutrality conditions ensures that the administrator’s leverage does not change as governmental funding changes. With feedback effects, a decrease in funding from the government has two effects: conditions (a), (b), and (c) guarantee the end points of the production possibilities frontier shift down along their respective axis and condition (e) guarantees the frontier becomes less convex in a symmetric way so that the derivative of the frontier at the 45° line is \(-1\). Condition (d) establishes that the allocation
that lies at the intersection of the production possibilities frontier and the 45° line is optimal for the administrator. Fig. 7 depicts a change in the university’s $m - k$ space as funding decreases. Since the ray along which the administrator’s preferred research allocations lie does not change with changes in the level of governmental funding, the bargaining problem is neutral.

7. Nonneutrality

We now consider the effect of relaxing condition (d) of Proposition 7 on the degree of negotiation leverage crowding-out.

**Proposition 8:** If conditions (a), (b), (c), and (e) hold but not (d), a performance function that weights applied research more heavily than basic research, $\mu < 1/2$ will increase the degree of negotiation leverage crowding-out, and a performance function that weights basic research more heavily than applied research, $\mu > 1/2$ increases the degree of negotiation leverage crowding-in.

The effect of changing the relative weights of basic and applied research in the administrator’s performance function is illustrated in Fig. 8 and Fig. 9. In Fig. 8 the administrator’s performance function gives more weight to theorems, $\mu > 1/2$. A decline in funding leads to crowding-in as the university administrator’s optimal allocation includes relatively more basic research. Fig. 9 illustrates that for an administrator with $\mu < 1/2$, a similar decline in government funding leads to crowding-out. The magnitude of crowding-in/out increases with the size of the drop in governmental funding.

Relaxing condition (e) of Proposition 7 can similarly lead to either negotiation leverage crowding-in or out.

**Proposition 9:** If conditions (a), (b), (c), and (d) hold but not (e), then a decrease in the level of government funded research will increase the degree of negotiation leverage crowding-out if $h(\cdot) > l(\cdot)$, or increase the degree of negotiation leverage crowding-in if $l(\cdot) > h(\cdot)$.
**Figure 8.** Decrease in governmental funding leads to crowding-in

**Figure 9.** Decrease in governmental funding leads to crowding-out
First, assume the feedback from mousetraps to theorems is larger than the feedback from theorems to mousetraps, \( l(\cdot) > h(\cdot) \). Fig. 10 illustrates the effect of a decrease in government funding. For a university administrator who weights basic and applied research equally (\( \mu = 1/2 \)), it is optimal to fund relatively more basic research when government funding declines. The intuition for this crowding-in of basic research is simple; the university administrator gets relatively more bang for every buck invested in applied research and, as funding declines and the asymmetric feedback effect becomes less pronounced, relatively more basic research is funded.

Fig. 11 illustrates that a decrease in governmental funding has the opposite effect when the feedback from theorems to mousetraps is larger than the feedback from mousetraps to theorems, \( h(\cdot) > l(\cdot) \). Since basic research gives relatively more bang for the buck, the administrator will fund relatively more applied research as governmental funding declines and the asymmetry becomes less pronounced.
Concerns that commercial sponsorship of university research will crowd-out basic research conducted for the public good, “theorems”, are legitimate. Both models of the university research process used in this paper show that commercial sponsorship agreements can lead
to the crowding-out of public good research. The likelihood of crowding-out depends on the framework used to model university research. In the first model, there is an implicit linear evolution from public good or basic research to applied or private good research (Bush, 1945).

For the case of no dynamic relationships crowding-in only occurs under a set of highly
restrictive assumptions. When there exists a linear decomposition of public good research and commercial or private good research, it is difficult for university administrators to form partnerships with profit-maximizing companies do not crowd-out public good research.

In the second model we use a dynamic framework to model university research that admits nonlinearities and recognizes the chaotic nature of the research and development processes. Unsurprisingly, crowding-in becomes more likely as we allow for feedback loops from discoveries in applied science to expand the opportunity set for public good research. Astute university administrators can form partnerships that magnify these feedback loops and increase public good research while working with the private sector. University administrator’s should consider evaluating commercial sponsors based on the potential for significant feedback effects. For example, administrators can increase feedback effects by forming partnerships with companies that allow university researchers to access proprietary knowledge that is otherwise unavailable to the public. Identifying these complementarities will help university administrators create partnerships with the private sector that improve both public good and commercial research (Rausser, 1999).

We have structured the models presented in this paper under the simplifying assumption that university researchers simply follow direction from the administrator and do not respond to other incentives. In practice, the potential for royalties from commercialized research entices university researchers away from conducting public good research (Greenberg, 2007; Washburn, 2005). We expect these internal incentives to lead to more researchers working on applied science with commercial benefits than is socially optimal. Given these incentives underlying research pursuits, it is all the more important for administrators to acknowledge
and exploit feedback loops by forming commercial research partnerships that enhance public good research.

The importance of feedback effects in scientific research has long been acknowledged. In 1871, Louis Pasteur wrote, “There does not exist a category of science to which one can give the name applied science. There are science and the applications of science, bound together as the fruit to the tree which bears it.” By establishing research partnerships with the private sector that maximize feedback from applied to basic research, administrators at land-grant universities can take advantage of this unique feature of scientific research and promote the public good while managing their funding problems.
The administrator’s choices of $m_t$ and $k_t$ in the second model are determined by solving the following constrained optimization problem:

$$L = \sum_{t=0}^{T-1} \left[ \beta^t P_t(M_t, K_t) + \beta^T \psi(M_T, K_T) + \lambda_{1t}(\frac{1}{1 + r})^t(\theta R(M_{t-1}) + \bar{e}_t - m_t^{\beta m} - k_t^{\beta k})ight.$$  

$$+ \lambda_{2t}(K_t - \gamma K_{t-1} - k_t - h(M_{t-1})) + \lambda_{3t}(M_t - \alpha M_{t-1} - m_t - h(K_{t-1})) \right]$$  \hspace{1cm} (19)
The first-order conditions for the administrator’s optimization problem in the second model are

\[
\frac{dL}{dk_t} = \left[ \sum_{j=0}^{T-1} (\beta^{t+j} \frac{dP_{t+j}(M_{t+j}, K_{t+j})}{dK_{t+j}} \frac{dK_{t+j}}{dk_t}) \right]^{(T-1)-t} + \sum_{j=0}^{(T-2)-t} (\beta^{t+j} \frac{dP_{t+j+1}(M_{t+j+1}, K_{t+j+1})}{dM_{t+j+1}} \frac{dM_{t+j+1}}{dh_{t+j+1}} \frac{dh_{t+j+1} + dK_{t+j}}{dk_t})] \\
+ \sum_{j=0}^{(T-2)-t} (\beta^T \frac{d\psi(M_{T}, K_{T})}{dK_T} \frac{dK_T}{dk_t} + \frac{d\psi(M_{T}, K_{T})}{dM_T} \frac{dM_T}{dh_T} \frac{dh_T + dK_{T-1}}{dk_t})] \\
+ \sum_{j=0}^{(T-2)-t} (\lambda_1(t+j+2)(\frac{1}{1+r})^{t+j+2} \frac{dR_{t+j+2}}{dM_{t+j+1}} \frac{dM_{t+j+1}}{dh_{t+j+1}} \frac{dh_{t+j+1} + dK_{t+j}}{dk_t})] - \lambda_1t(\frac{1}{1+r})^t \beta_k k_t^{-1}] \\
+ \left[ \sum_{j=0}^{(T-2)-t} (\lambda_2(t+j+2) \frac{dK_{t+j}}{dk_t}) \right] - \left[ \sum_{j=0}^{(T-2)-t} \lambda_2(t+j+2) \frac{dK_{t+j}}{dk_t} \right] - \lambda_2t] \\
- \left[ \sum_{j=0}^{(T-3)-t} \lambda_2(t+j+2) \frac{dM_{t+j+1}}{dM_{t+j+1}} \frac{dM_{t+j+1}}{dh_{t+j+1}} \frac{dh_{t+j+1} + dK_{t+j}}{dk_t} \right] \\
+ \sum_{j=0}^{(T-2)-t} (\lambda_3(t+j) \frac{dM_{t+j+1}}{dh_{t+j+1}} \frac{dh_{t+j+1} + dK_{t+j}}{dk_t}) \\
- \sum_{j=0}^{(T-3)-t} (\lambda_3(t+j) \frac{dM_{t+j+1}}{dh_{t+j+1}} \frac{dh_{t+j+1} + dK_{t+j}}{dk_t}) \\
- \sum_{j=0}^{(T-2)-t} (\lambda_3(t+j) \frac{dK_{t+j}}{dk_t}) \leq 0 \quad (20)
\]
\[
\frac{dL}{dm_t} = \sum_{j=0}^{(T-1)-t} \left( \beta^{t+j} \frac{dP_{t+j}(M_{t+j}, K_{t+j})}{dM_{t+j}} \frac{dM_{t+j}}{dm_t} \right) \\
+ \sum_{j=0}^{(T-2)-t} \left( \beta^{t+j} \frac{dP_{t+j+1}(M_{t+j+1}, K_{t+j+1})}{dK_{t+j+1}} \frac{dK_{t+j+1}}{dl_{t+j+1}} \frac{dl_{t+j+1}}{dM_{t+j}} \frac{dM_{t+j}}{dm_t} \right) \\
+ \left[ \beta^T \left( \frac{d\psi(M_T, K_T)}{dM_T} \frac{dM_T}{dm_T} + \frac{d\psi(M_T, K_T)}{dK_T} \frac{dK_T}{dl_T} \frac{dl_T}{dM_T} \frac{dM_{T-1}}{dm_t} \right) \right] \\
+ \sum_{j=0}^{(T-2)-t} \left( \alpha_1(t+j+1) \left( \frac{1}{1+r} \right)^{t+j+1} \theta dR_{t+j+1} \frac{dM_{t+j}}{dm_t} \right) - \lambda_{1t} \left( \frac{1}{1+r} \right)^t \beta_m \lambda_{m-1} \\
+ \sum_{j=0}^{(T-1)-1} \left( \alpha_2(t+j) \frac{dM_{t+j}}{dm_t} \right) + \sum_{j=0}^{(T-2)-t} \left( \alpha_2(t+j) \frac{dM_{t+j}}{dm_t} \right) - \lambda_{2t} \\
- \sum_{j=0}^{(T-3)-t} \left( \alpha_2(t+j+2) \frac{dK_{t+j+1}}{dl_{t+j+1}} \frac{dl_{t+j+1}}{dM_{t+j}} \frac{dM_{t+j}}{dm_t} \right) \\
+ \sum_{j=0}^{(T-2)-t} \left( \lambda_3(t+j) \frac{dK_{t+j+1}}{dl_{t+j+1}} \frac{dl_{t+j+1}}{dM_{t+j}} \frac{dM_{t+j}}{dm_t} \right) \\
- \sum_{j=0}^{(T-3)-t} \left( \lambda_3(t+j) \frac{dK_{t+j+1}}{dl_{t+j+1}} \frac{dl_{t+j+1}}{dM_{t+j}} \frac{dM_{t+j}}{dm_t} \right) \\
- \sum_{j=0}^{(T-2)-t} \left( \lambda_3(t+j) \frac{dl_{t+j+1}}{dM_{t+j}} \frac{dM_{t+j}}{dm_t} \right) \leq 0 \tag{21}
\]

where

\[
\frac{dK_{t+j}}{dk_t} = \alpha^j + I_{j \geq 2} \alpha^{j-i} \frac{dl(M_{t+i-1})}{dM_{t+i-1}} \frac{dM_{t+i-1}}{dk_t} \frac{dK_{t+i-2}}{dk_t} \tag{22}
\]

\[
\frac{dM_{t+j}}{dm_t} = \gamma^j + I_{j \geq 2} \gamma^{j-i} \frac{dh(K_{t+i-1})}{dK_{t+i-1}} \frac{dK_{t+i-1}}{dm_t} \frac{dM_{t+i-2}}{dm_t} \tag{23}
\]
In the second model the commercial company wants to maximize the sum of the profits from each period, \( \pi_t(m_t, k_t) = M_t - f_t - R(M_{t-1}) \), and the scrap value of an agreement, \( W(M_T) \), discounted by the weighted average cost of capital, \( r \):

\[
\Pi = \sum_{t=0}^{T-1} \left( \frac{1}{1+r} \right)^t \pi_t(m_t, k_t) + \left( \frac{1}{1+r} \right)^T W(M_T)
\]  

(24)

where \( f_t \) is the funding the company provides to the university to produce the company’s share, \( (m_{ct}, k_{ct}) \), of the university’s research allocation each period, \( f_t = C(m_{ct}, k_{ct}) \).

The company’s first-order conditions are

\[
\frac{d\Pi}{dm_t} = \sum_{j=0}^{(T-1)-t} \left( \frac{1}{1+r} \right)^{t+j} \frac{dM_{t+j}}{dm_t} - \left( \frac{1}{1+r} \right)^t \frac{dC_t(m_{ct}, k_{ct})}{dm_t}
\]

\[
- \sum_{j=0}^{(T-2)-t} \left( \frac{1}{1+r} \right)^{t+j+1} \frac{dR(M_{t+j+1} dM_{t+j+1})}{dm_t} + \left( \frac{1}{1+r} \right)^T \frac{dW(T)}{dm_t} = 0
\]  

(25)

\[
\frac{d\Pi}{dk_t} = \sum_{j=0}^{(T-2)-t} \left( \frac{1}{1+r} \right)^{t+j+1} \frac{dM_{t+j+1}}{dh_{j+t+1}} \frac{dh_{j+t+1}}{dk_t} - \left( \frac{1}{1+r} \right)^t \frac{dK_t+j}{dk_t}
\]

\[
- \sum_{j=0}^{(T-2)-t} \left( \frac{1}{1+r} \right)^{t+j+1} \frac{dR_{t+j+1} dM_{t+j+1}}{dm_t} \frac{dh_{j+t+1}}{dk_t} + \frac{dK_{t+j}}{dk_t} + \left( \frac{1}{1+r} \right)^T \frac{dW(T)}{dk_t} = 0
\]  

(26)
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