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COMPENSATION UNDER UNCERTAINTY:
EX ANTE VERSUS EX POST RULES

by

Gordon C. Rausser and David Zilberman

GIANNINI FOUNDATION OF
AGRICULTURAL ECONOMICS
LIBRARY

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California Agricultural Experiment Station
Giannini Foundation of Agricultural Economics
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1. Introduction

There is a growing awareness that democratic societies are faced with numerous internal conflicts among various interest groups. This view has been articulated by Thurow (1980), who notes:

"Our society has reached a point where it must start to make explicit equity decisions if it is to advance. The implicit, undefended, unanalyzed equity decisions that have been built into our tax, expenditures, and regulatory policies of the past simply won't carry us into the future. To implement public policies in the future, we are going to have to be able to decide when losers should suffer income losses and when losers should be compensated. We have to be able to decide when society should take actions to raise the income to some group and when it should not take such actions. If we cannot learn to make, impose, and defend equity decisions, we are not going to solve any of our economic problems."

Economists have generally provided the intellectual ammunition for not imposing or defending equity decisions. The difficulty of implementing compensation schemes has led to the concept of "potential" compensation. This concept originated with Kaldor (1939) and Hicks (1939) as a solution for the interpersonal comparison problem in welfare economics. Following the corrections of the original Kaldor-Hicks potential compensation principle offered by Scitovsky (1941) and Samuelson (1950), a number of economists have used this concept to determine if it is possible for gainers of a particular public policy to compensate the losers.
In a world of certainty, the operational manifestation of the principle of potential compensation has assumed the form of benefit-cost analysis. As Harberger (19 ) has shown, public programs should be supported if their net benefits are positive. Programs with negative net benefits should be discarded. Under uncertainty, the analysis provided by Arrow and Lind (1970) has shown under a wide variety of circumstances that it is safe to assume risk neutrality on the part of public decision-makers. The resulting criteria for the selection of public projects under uncertainty is expected net benefits.

The purpose of this paper is to investigate alternative compensation rules for the public sector under uncertainty. It will be shown that the criterion of expected net benefits is, in general, suboptimal. An ex post compensation rule will be shown to be optimal. It dominates the ex ante compensation rule resulting from maximizing expected net benefits. Moreover, in a world of uncertainty, it dominates the principle of potential compensation (i.e., no actual compensation) from both an equity and efficiency standpoint.

2. Ex Ante vs. Ex Post Compensation

In determining a rule for compensation, we must first deal with its exact timing. Should the compensation rule be executed on an ex ante basis; namely, should winners compensate losers according to expected losses (plus some risk premium) before the program takes place or should compensation be ex post where the gainers compensate losers according to their actual losses? An obvious danger of using strict ex post compensation rules is that some groups may modify their behavior as information becomes available on the progress of the research program. There is some incentive for such groups to minimize the compensation payment.

The above "moral hazard" can be partially overcome by specifying compensation levels for each contingency prior to the initiation of the program.
These compensation levels would be based on the best estimates of costs and benefits across various affected groups. In what follows, a modified ex post rule will be examined. This rule will be determined at the beginning of the planning horizon for each possible state of nature but actually paid only after the state of nature is revealed.

While the moral hazard problem is the Achilles' heel of ex post compensation rules, this approach nevertheless has three distinct advantages over ex ante compensation rules. First, it is far easier to specify an operational ex post compensation rule than an equivalent ex ante rule. The latter is computationally more demanding than the former. For an ex post rule, we must compute the losses for each possible state of nature. These losses are then fully compensated by the beneficiaries where each gainer's contribution to the compensation fund is proportional to its gain. For example, for a certain state of nature, suppose there are \( n \) beneficiary groups, and the benefit of the \( i \)th group is denoted by \( B_i \). Let the sum of losses be given by \( L \) and for the state of nature in question; thus, the level of compensation imposed upon the \( i \)th beneficiary group is

\[
C_i = L \frac{B_i}{\sum B_i}.
\]  

(1)

The computation of an ex ante compensation rule requires determining for each loser group a certainty monetary equivalent to the uncertain losses. Since risk aversion parameters of the various parties are unknown and difficult to estimate, it is operationally impossible to design a simple compensation rule that reflects risk aversion. A workable but yet unsatisfactory solution to the risk-aversion measurement problem is simply to pay losers
their expected losses and require that gainers contribute to the compensation fund in proportion to their expected gains. The remaining two disadvantages of the \textit{ex ante} compensation rule emanate from this plausible solution.

Given that potential compensation exists, there are two possibilities that must be recognized for both \textit{ex post} and \textit{ex ante} compensation rules. First, for all states of nature, total benefits from the research program may exceed total cost. The second possibility is that, for some states of nature but certainly not all, total benefits fall short of total costs. For the first possibility, the expected net benefits for all groups are identical under both \textit{ex ante} and \textit{ex post} compensation systems. Nevertheless, if the various groups are risk averse, the \textit{ex ante} compensation rule will always be inferior from the losers' standpoint. In addition, in some instances, the \textit{ex ante} compensation rule will prove inferior for the beneficiary groups. The reason is simply that, if losers are risk averse, the distribution of net benefits under the \textit{ex post} compensation rule will stochastically dominate the distribution of net benefits under an \textit{ex ante} compensation rule. In other words, the \textit{ex post} compensation rule distribution of net benefits can be transformed to a distribution of net benefits under the \textit{ex ante} compensation rule using a mean-preserving and risk-increasing transformation [Rothschild and Stiglitz (1970)].

The above result is also obtained for the distribution of net benefits of beneficiary groups so long as there is positive correlation between gains and losses. For example, suppose we have the simple case of two groups—one which loses and another which gains from the adoption of a new technology. Let us suppose in addition that there are two possible states of nature with respect to this technology. Let the probability of the first state of nature be $\alpha$, the gain of the beneficiary group for state of nature $j$ be $B_j$, and the loss to the losing group be $L_j$. Given this specification, the \textit{ex ante} compensation will be
\[ C = \alpha L_1 + (1 - \alpha) L_2. \] (2)

The distribution of benefits and costs under the \textit{ex ante} compensation rule is recorded in table 1 along with the distribution of benefits and costs under the \textit{ex post} compensation rule.

As table 1 reveals, the loser under the \textit{ex post} compensation rule faces no loss. However, under the \textit{ex ante} compensation rule, for one state of nature, he has a gain and for the other he faces a loss with an average loss of zero under the \textit{ex ante} rule. Obviously, risk-averse individuals will prefer the \textit{ex post} compensation rule. Similarly, if large gains are correlated with large losses, \( B_2 > B_1, L_2 > L_1 \) and \( B_2 - L_2 > B_1 - L_1 \), then

\[ B_2 - L_2 < B_2 - C \] (3)

while

\[ B_1 - L_1 > B_1 - C \] (4)

and, hence, risk-averse beneficiaries will prefer, under these circumstances, the \textit{ex post} to the \textit{ex ante} compensation rule.

One of the most attractive features of the \textit{ex post} compensation rule is that losers will always prefer it to an \textit{ex ante} rule. This is the case since losers are expected to be the greatest obstacle to forming an effective coalition to support public intervention. The \textit{ex post} compensation approach preserves the aggregate net-expected benefit to society while simultaneously being preferred by those who suffer from the introduction of public intervention.

In addition to the above advantages of the \textit{ex post} role, an even more important advantage relates to the influence of these rules on the aggregate expected net benefits of a program. Specifically, under very plausible circumstances, the aggregate expected net benefits are higher under an \textit{ex post}
Table 1
Losses and Gains Under Ex Post and Ex Ante Compensation Rules

<table>
<thead>
<tr>
<th>State</th>
<th>Gain</th>
<th>Loss</th>
<th>Probability</th>
<th>Ex-ante rule</th>
<th>Ex-post rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>B - C</td>
<td>L - C</td>
<td>α</td>
<td>B - L</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>B - C</td>
<td>L - C</td>
<td>1 - α</td>
<td>B - L</td>
<td>0</td>
</tr>
</tbody>
</table>
compensation rule than an *ex ante* compensation rule. This will occur when, for some states of nature, the net benefits of public projects are negative. If an *ex post* compensation rule is employed, the new innovation will not be adopted and, under this circumstance, the aggregate actual net benefit for the research output of the project will be zero. On the other hand, if—under the realized state of nature—there are gainers, the new program should be introduced under an *ex ante* rule and the net benefit associated with this outcome will be negative. This observation can be illustrated using the notation that appears in Table 1. If the gain resulting from a realization of the first state is smaller than the loss, then the net expected benefit prior to the initiation of the research program and under the *ex post* rule will be $(1 - \alpha) (B_2 - L_2) - R$, where $R$ denotes costs of the research program while, under the *ex ante* rule, the net benefit will be $\alpha(B_1 - L_1) + (1 - \alpha) (B_2 - L_2) - R$. Clearly,

$$\alpha(B_1 - L_1) + (1 - \alpha) (B_2 - L_2) < (1 - \alpha) (B_2 - L_2).$$

(5)

Given a viable public project, namely, the existence of potential compensation, each beneficiary group may provide funding to finance the costs of the program in proportion to its net expected benefit from the project. Given the *ex post* compensation rule, this computation needs to be done only for states of nature with positive aggregate net benefits (relevant states). Hence, if $G_i$ is a net expected benefit across the relevant states of nature, and $R$ is the cost of project, the $i$th beneficiary group must contribute

$$s_i = \frac{G_i}{\sum_{i=1}^{n} G_i} R$$

(6)
to finance the research. In addition to this cost, each beneficiary group
must also pay the level of compensation determined by (1) once the state of
nature on the research output is realized.

The _ex post_ compensation rule has direct implications for the measure of
aggregate expected net benefits, on the set of projects that will be selected,
and on the distribution of research cost among the beneficiary groups. This
can be illustrated by the example that has been constructed in table 2. This
table assumes three groups, A, B, and C; two states of nature with equal like-
lihood; and various net benefits (losses) for each group and each state of
nature. The total cost of the research program is assumed to be $100.

Given the adoption of the new innovation resulting from the research out-
put under state 1, the aggregate net benefit is negative. Hence, under the
_ex post_ compensation rule, the innovation will not be adopted under state 1.
No damage is incurred under this rule if state 1 occurs, and the expected
aggregate benefit from the research program is $250. Since the cost of the
research program is presumed to be $100, the program is economically viable.
Under the _ex ante_ compensation rule, the output of the research program will
be adopted regardless of the state of nature. Hence, the expected aggregate
net benefit under the _ex ante_ compensation rule will be $150. Hence, if the cost
of the research program were to exceed $150, the program would not be econo-
mically viable under the _ex ante_ rule but would be viable under the _ex post_ rule.

The incidence of burden is obviously different under the two systems.
Under the _ex post_ rule, they are smaller since they need cover only the re-
search cost. Moreover, the distribution of cost differs since the expected
benefits under the two rules differ. Notice that in the example of table 2,
on average, all parties will benefit if the _ex post_ compensation rule is
employed; and all must share in the burden of financing the public project.
### Table 2
Incidence of Burden Under *Ex Post* and *Ex Ante* Compensation Rules

<table>
<thead>
<tr>
<th>State</th>
<th>Party</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>State 1 (.5)</td>
<td></td>
<td>-400</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>State 2 (.5)</td>
<td></td>
<td>100</td>
<td>200</td>
<td>200</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Research costs</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Ex ante</em> rule</td>
<td></td>
<td>0</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td><em>Ex post</em> rule</td>
<td></td>
<td>20</td>
<td>40</td>
<td>40</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Compensation</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Ex ante</em> rule</td>
<td></td>
<td>150</td>
<td>-75</td>
<td>-75</td>
</tr>
<tr>
<td><em>Ex post</em> rule</td>
<td>State 1</td>
<td>400</td>
<td>200</td>
<td>200</td>
</tr>
<tr>
<td></td>
<td>State 2</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

*Under this outcome and the *ex post* rule, the research output will not be adopted.*
References


Harberger,


Scitovsky, T., 1941, A note on welfare proposition in economics, Review of Economic Studies 9, 77-78.