Lawrence Berkeley National Laboratory
Recent Work

Title
FINITE TEMPERATURE CALCULATION OF ANGULAR VELOCITIES AND MOMENTS OF INERTIA IN ROTATING NUCLEI

Permalink
https://escholarship.org/uc/item/7ft4f7f9

Author
Moretto, L.G.

Publication Date
1973-03-01
FINITE TEMPERATURE CALCULATION OF ANGULAR VELOCITIES AND MOMENTS OF INERTIA IN ROTATING NUCLEI

L. G. Moretto

March 1973

Prepared for the U. S. Atomic Energy Commission under Contract W-7405-ENG-48

For Reference

Not to be taken from this room
DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.
FINITE TEMPERATURE CALCULATION OF ANGULAR VELOCITIES AND MOMENTS OF INERTIA IN ROTATING NUCLEI

L. G. Moretto

Department of Chemistry and
Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

March 1973

The rotational properties of nuclei in the statistical region are predicted on the basis of a simple BCS model. The model can be described in terms of three reduced variables. A back-bending, which disappears as the temperature increases, is observed in the plot of the moment of inertia versus the square of the angular velocity.

The experimental discovery of strongly varying moments of inertia in the ground state rotational bands of deformed even-even nuclei [1,2] has prompted a variety of theoretical developments [3-5] for the interpretation of this feature. A rather appealing model explains the observed increase and back-bending of the moment of inertia versus the square of the angular velocity in terms of the disappearance of the pairing superfluidity due to the increasing angular momentum [3,4].

In this work a very simple pairing model, introduced in detail in a previous paper for different purposes [6], is used to calculate the zero and finite temperature behavior of the angular velocity and of the moment of inertia as a function of angular momentum.

* Work done under the auspices of the U.S. Atomic Energy Commission.
The model is characterized by three parameters: the density $g$ of the equidistant, doubly degenerate, single particle levels; the average $z$ projection $m$ of the single particle angular momentum; the gap parameter $\Delta_0$ corresponding to zero temperature and zero angular momentum.

The properties of the system are contained in the grand potential $\Omega$:

$$\Omega = -\beta g \int_{-S}^{+S} \, d\varepsilon \left[ \varepsilon - E + \frac{1}{\beta} \{ \ln[1 + \exp - \beta(E - \gamma m)] + \ln[1 + \exp - \beta(E + \gamma m)] \} \right] - \frac{\beta \Delta_0^2}{2}$$

where $\beta = \frac{1}{T}$ is the reciprocal of the temperature, $\varepsilon$ is the energy of the single particle levels measured with respect to the constant chemical potential, $E = [\varepsilon^2 + \Delta^2]^{1/2}$, $\gamma$ is the angular velocity and $g$ is the pairing strength.

The pairing cut-off $S$ is taken to be much larger than the gap parameter throughout the calculation: $S \gg \Delta_0$.

By suitable differentiation of the grand potential, all the other thermodynamical equations are obtained:

The gap equation:

$$\frac{3}{2 \beta} \frac{d\Omega}{d\Delta} = 2g \int_{0}^{S} \, d\varepsilon \frac{1}{2} \left[ \tanh \frac{\beta}{2} (E - \gamma m) + \tanh \frac{\beta}{2} (E + \gamma m) \right] = 0 \, ; \quad (2)$$

the angular momentum:

$$I = \frac{1}{\beta} \frac{3}{2} \frac{d\Omega}{d\gamma} = 2mg \int_{0}^{S} \, d\varepsilon \left[ \frac{1}{1 + \exp \beta(E - \gamma m)} - \frac{1}{1 + \exp \beta(E + \gamma m)} \right] \, ; \quad (3)$$

the energy
The case in which $T = 0$ can be worked out analytically and is considered first. The angular momentum equation yields:

$$I = 2mg \left[(\gamma m)^2 - \Delta^2\right]^{1/2}$$  \hspace{1cm} (5)

The gap equation can also be integrated explicitly, producing the following relationship between $\Delta$ and $\gamma$:

$$\frac{1}{2} \frac{\Delta^2 - \Delta_0^2}{\Delta_0} = \left[(\gamma m)^2 - \Delta^2\right]^{1/2}$$  \hspace{1cm} (6)

By means of eqs. (5) and (6) one obtains the following important relations:

$$\frac{\Delta}{\Delta_0} = \left(1 - \frac{I}{I_{cr}}\right)^{1/2} \quad \text{for } I \ll I_{cr} \quad \Delta = 0 \text{ for } I \gg I_{cr}$$  \hspace{1cm} (7)

$$\gamma = \frac{I_{cr}}{\sigma_R} \left(2 - \frac{I}{I_{cr}}\right) \quad \text{for } I \ll I_{cr} \quad \gamma = \frac{I}{\sigma_R} \quad \text{for } I \gg I_{cr}$$  \hspace{1cm} (8)

$$\sigma = \frac{I}{\gamma} = \sigma_R \left(\frac{I/I_{cr}}{2 - \frac{I}{I_{cr}}}\right) \quad \text{for } I \ll I_{cr} \quad = \sigma_R \quad \text{for } I \gg I_{cr}$$  \hspace{1cm} (9)

where $I_{cr} = g m \Delta_0$, $\sigma_R = 2 m^2 g$.

The eq. (7) gives the dependence of the gap parameter $\Delta$ upon angular momentum. The gap parameter decreases as the angular momentum increases and vanishes at the critical angular momentum $I_{cr}$. The effect of superfluidity
on the moment of inertia is quite visible in eq. (9). The moment of inertia vanishes at zero angular momentum, and it increases monotonically as the angular momentum increases from zero to its critical value. Above the critical angular momentum the superfluidity disappears and the moment of inertia attains its rigid value. More peculiarly, the angular velocity \( \gamma \), given by eq. (8), decreases with increasing angular momentum in the interval \( 0 \leq I \leq I_{cr} \). For values of the angular momentum larger than \( I_{cr} \) the angular velocity decreases proportionally to the angular momentum. This behavior generates a very pronounced back-bend in the plot of the moment of inertia versus the square of the angular velocity.

By integrating eq. (4) and subtracting the ground state energy one obtains the yrast function, namely the region of the highest angular momentum states at a given energy, or conversely, the region of the lowest energy states at a given angular momentum:

\[
\begin{align*}
\frac{I^2}{\partial R} &\frac{I}{I_{cr}} (2 - \frac{I}{2I_{cr}}) \text{ for } I \leq I_{cr} \\
E_y &= \\
\frac{I^2}{\partial R} + \frac{I^2}{2\partial R} \text{ for } I \geq I_{cr}
\end{align*}
\]

Above the critical angular momentum, the yrast line is a parabola typical of an object rotating with a rigid moment of inertia. Below the critical angular momentum the yrast line is also a parabola but it has a negative instead of a positive second derivative. The consistency of eq. (10) and eq. (8) can be tested by remembering that:
If the system is characterized by a moment of inertia depending upon a variable which is allowed to take its equilibrium value, one obtains:

\[ \gamma = \frac{\partial E}{\partial I} \]

It is easy to verify that by applying eq. (12) to eq. (10) one does indeed obtain eq. (8).

The case in which \( T > 0 \) will now be considered. It is well known that at zero angular momentum the gap parameter decreases with increasing temperature and vanishes at the critical temperature \( T_{cr} \) given by:

\[ T_{cr} = \frac{2\Delta_o}{3.5} \]

It follows that the system is characterized by three critical quantities \( I_{cr} \), \( T_{cr} \), \( \Omega_R \) which can be related to the physical parameters of the system as follows:

\[ I_{cr} = \frac{g m \Delta_o}{2} \; ; \; \frac{\Omega}{I_{cr}} = \frac{2 g m}{2} \; ; \]

Since these three critical quantities contain all the physical information on the model, it is possible to scale every specific physical situation by means of the following three reduced variables: the reduced angular momentum \( I/I_{cr} \), the reduced temperature \( T/T_{cr} \) and the reduced moment of inertia \( \Omega/I_{cr} \).

The dependence of the critical temperature upon angular momentum as well as the overall dependence of the gap parameter upon angular momentum and
temperature are shown in fig. 1. It can be observed that at large angular momenta, an increase in temperature has an antiblocking effect resulting in an increase of the pairing correlation. The same effect is responsible for an increase of the critical angular momentum up to a value $I/I_{cr} = 1.22$ at a temperature $T/T_{cr} = 0.47$. This effect, called thermally assisted pairing correlation, has been discussed in detail elsewhere [6].

In fig. 2 the yrast line is shown together with higher energy-angular momentum isotherms and with the line of the critical energies. One can see that the yrast line has negative second derivative everywhere below the critical angular momentum; the lower isotherms have positive second derivatives at low angular momentum and negative second derivatives at high angular momentum; the higher isotherms have positive second derivatives everywhere. This behavior is better appreciated in fig. 3 where the angular velocity is plotted as a function of angular momentum for various temperatures. At $T = 0$, as predicted by eq. (8), the angular velocity decreases as the angular momentum increases. At higher temperatures the angular velocity goes from zero to a maximum and then decreases. At the highest temperatures the angular velocity increases monotonically with angular momentum. At and above the critical angular momenta, the angular velocity becomes independent of temperature and increases proportionally to the angular momentum.

The dependence of the moment of inertia on angular momentum is shown in fig. 4. At zero temperature such a dependence is given by eq. (9). At larger temperatures the moment of inertia starts with a finite value and increases until it reaches the rigid value at the corresponding critical angular momenta.
More impressive is the dependence of the moment of inertia on the square of the angular velocity shown in fig. 5. The back-bending is extremely pronounced at \( T = 0 \), it is attenuated as the temperature increases until, at a temperature \( \frac{T}{T_{cr}} \approx 0.53 \), it disappears. Above this temperature and below the critical temperature the moment of inertia increases regularly with the square of the angular velocity. The back bending is associated with the presence of negative second derivatives in the energy angular momentum isotherms and with the presence of negative first derivatives in the angular velocity angular momentum isotherms. The kinks appearing in figs. 3, 4, 5 at the critical angular momenta are due to a phase transition which should be real for macroscopic systems, but which becomes an artifact of the theory for small systems where fluctuations are large. This difficulty can be eliminated by accounting for the fluctuations as it has been done in ref. 7. Calculations which allow for fluctuations and which are based on a more realistic single particle model are in progress.
References

FIGURE CAPTIONS

Fig. 1. Contour map of the reduced gap parameter as a function of the reduced temperature and of the reduced angular momentum. The lines of constant gap parameter are spaced 0.05 $\Delta_0$.

Fig. 2. Yrast line and higher isotherms. The energy scale is expressed in units of condensation energy. The isotherm next to the yrast line is at $T/T_{cr} = 0.158$ and the following isotherms are in steps of 0.0526 $T_{cr}$. The dashed line represents the yrast line of the unpaired system. The upper line crossing the various isotherms represents the boundaries between the paired and unpaired region.

Fig. 3. Reduced angular velocity as a function of the reduced angular momentum for various temperatures. The rising diagonal line represents the angular velocity of an unpaired system. The upper line corresponds to $T/T_{cr} = 0$. The next isotherm is at $T/T_{cr} = 0.158$ and the following isotherms are in steps of 0.04 $T_{cr}$.

Fig. 4. Moment of inertia as a function of angular momentum for various temperatures. The $T/T_{cr} = 0$ curve reaches the rigid value at $I/I_{cr} = 1$. The following isotherm is at $T/T_{cr} = 0.158$ and the spacing between the remaining isotherms is 0.08 $T_{cr}$.

Fig. 5. Moment of inertia versus the square of the angular velocity for various temperatures (solid lines). The $T/T_{cr} = 0$ isotherm is incomplete on the right. The next isotherm is at $T/T_{cr} = 0.158$ and the following isotherms are in steps of 0.04 $T_{cr}$. The dashed lines are lines of constant angular momentum in steps of $1/7 I_{cr}$.
Fig. 1
Fig. 3
This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Atomic Energy Commission, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.