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Labor Adjustment and Gradual Reform: Is Commitment Important?¹

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ABSTRACT

We analyze a model in which a government uses a second best policy to affect the reallocation of labor, following a change in relative prices. We consider two extreme cases, in which the government has either unlimited or negligible ability to commit to future actions. We explain why the ability to make commitments may be unimportant, and we illustrate this conjecture with numerical examples. For either assumption about commitment ability, the equilibrium policy involves gradual liberalization. The dying sector is protected during the transition to a free market, in order to decrease the amount of unemployment. Our results are sensitive to the assumptions about migration.

Key words: adjustment costs, dynamic tariffs, time inconsistency, Markov perfection

JEL classification numbers: F13, J20, J24

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1. Introduction

The transition toward market economies in Central and Eastern Europe (CEE) has been associated with high unemployment, creating the potential for social instability and threatening continued liberalization. Although the historical evidence on the relation between economic liberalization and unemployment is ambiguous\(^1\), the theoretical relation is straightforward. A change in relative prices requires that some sectors shrink and others grow, and this requires the adjustment of labor. For many workers, the move from one sector to another involves a period of unemployment. The loss in production during this period is an important economic cost of the transition. Since beginning transition, Bulgaria, Hungary and Poland have experienced unemployment rates of approximately 15\%, and East Germany’s rate has been twice that. The labor markets of CEE have been described by, *inter alia*, Akerlof *et al.* (1991), Begg and Portes (1992), Blanchard *et al.* (1993), Bofinger and Cernohorsky (1992), Dornbusch (1992), and Svejnar (1993).

High unemployment during transition may be inevitable. It may be impossible to avoid the costs associated with labor adjustment, except by maintaining an unacceptable *status quo*. However, many economists and policymakers think that the market solution involves unnecessary and unacceptable costs, and that governments should intervene to save jobs in the short run. Abel and Bonin (1992) and McKinnon (1991, 1993) discuss arguments in favor of gradualism. Methods of protecting the dying state sector include trade restrictions,

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\(^1\) Papageorgiou *et al.* (1991) study 36 liberalization episodes in 19 countries and fail to find a significant relation between liberalization and increased unemployment. Edwards (1988) and Greenaway (1993) provide other interpretations of this evidence. Some of the episodes in the study involved very mild forms of liberalization, from which it is difficult to make any inference. In most cases it is difficult to isolate the effect of trade reform from the effects of other policy changes.
soft budget constraints, and direct subsidies. There is an obvious political and social rationale for these policies. They may be justified on efficiency considerations, if market failures lead to "excessive" unemployment. However, the question remains whether governments should attempt to reduce unemployment by supporting dying industries. If they should, we would like to know the optimal path of intervention.

We study a model in which some workers leave the dying state sector and pass through the pool of unemployed workers in order to obtain better jobs in the growing private sector. There is congestion in the labor market, which means that an increase in the number of unemployed decreases the probability that any single worker will get a job. In making their migration decisions, workers fail to internalize this congestion cost. The decentralized solution therefore leads to excessive migration and unemployment, and a role for government intervention. This model is adapted from Lapan (1976, 1978, 1979), Cassing and Ochs (1978) and Ray (1978). Workers have rational (point) expectations, which in this deterministic model implies perfect foresight.

In principal the government could correct the distortion by using a migration tax or a wage subsidy cum tax, and thereby achieve the first best (social planner’s) migration trajectory. In practice, however, any policy that affects migration is likely to introduce other distortions, so available policies are only second best. We consider the case where the government’s only policy instrument is a tariff, which results in a consumption and (possibly) a production distortion. This modeling choice is motivated by the fact that tariffs remain an important means of protecting (former) state sectors. For our theoretical results, the first important assumption is that any available policy instrument introduces a secondary distortion.
Given the state of the tax system in CEE, this is plausible. The second important assumption is that workers are forward looking, and therefore base their migration decision on their expectation of future policies.

These assumption imply that the second best policy trajectory is time-inconsistent.² Outside of a steady state, the government would like to announce, and promise to follow, a particular policy trajectory; agents know that in subsequent periods the government would want to revise that trajectory. In the absence of a mechanism for commitment, agents do not believe that the announced policy will be followed, so it is ineffective. The difficulty of making credible commitments about future policy is particularly acute in reforming countries, because of the absence of an institutional structure that supports policy continuity.

Recognition of the commitment problem is one reason for opposing government intervention. A common scenario is the following: A government wants to announce a gradual reform trajectory, e.g. a reduction in tariffs, which balances the benefits of adjustment and the costs of unemployment. If workers recognize that in the future the government will have an incentive to deviate by increasing (or failing to decrease) the tariff, a suboptimal amount of migration occurs. (See, e.g., Staiger and Tabellini, 1987). This increases the government’s incentive to renege, because not reneging leads to high migration and adjustment costs. Consequently, a proposal to reduce gradually the level of protection leads instead to continued protection and the failure to make needed adjustments. In this case, the government might do better to eschew intervention altogether, adopting the "big bang"

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² The issue of time consistency in trade models has been widely studied. See, for example, the papers by Brainard (1993), Karp (1987), Maskin and Newbery (1989), Matsuyama (1990), Staiger and Tabellini (1987), and Tornell (1991).
approach to reform. Proponents of this view presumably believe that it is easier to commit to a trajectory of non-intervention, than to a trajectory of phased liberalization.

We consider two extreme possibilities: the government has either perfect or zero commitment ability. In the former case it announces - and follows - the second best trajectory. In the latter case, the government can set the policy at any level it chooses in the current period, but cannot make promises about the future. In reality, a government's ability to commit is neither perfect nor non-existent, but analysis of the extreme cases enables us to determine whether the outcome is sensitive to the degree of commitment ability. This has important policy implications. In circumstances where commitment ability is relatively unimportant, one argument against an interventionist policy (e.g., favoring the big bang) is less compelling. Our objective is to develop a framework for answering the following two questions: How important is the government's ability to commit to future actions? How does the answer to that question depend on the form of the labor migration process?

Karp and Paul (1994) address these issues using a different model of migration. For that model, a government with perfect commitment begins with complete liberalization, but then phases in and later phases out protection. The big bang is used, but is short-lived. In the same model, nonintervention is the equilibrium outcome when the government has zero ability to commit; the big bang is permanent. In the current model, governments with either perfect or zero commitment ability follow qualitatively similar policy trajectories. Liberalization is gradual and monotonic. The relation between these contrasting results and the contrasting assumptions about labor migration in the two models provides a key insight.

The plan of the paper is as follows: Section 2 sets out the model. Section 3 analyses
the properties of the private adjustment and derives the first best subsidy. Section 4 compares the second best tariff policy under two extreme assumptions regarding the government’s ability to make binding commitments. Section 5 discusses the role of the labor migration constraint. The last section contains concluding remarks.

2. The Model

Our theoretical discussion uses a continuous time, infinite horizon model. This makes it straightforward to present the model under either extreme assumption about the government’s commitment ability. We use a discrete stage, finite horizon approximation for numerical results. For the zero-commitment case, we assume that agents have Markov expectations. Agents condition their actions and beliefs about the future on the payoff-relevant state variable(s), which in this case is the stock of labor in the growing sector. The Markov assumption is adopted in many models that study time-consistency and government policy (e.g., see references in note 2). In infinite horizon models, there typically exist many non-Markov (reputational) subgame perfect equilibria [Chari and Kehoe, 1990]. One reason for studying those is to show that when "reputation" is introduced to a model, the government (or other decision-maker) is able to achieve nearly the full-commitment outcome even with negligible commitment ability. This is possible because "punishment" strategies can be used to support "good behavior", in much the same manner as is done in repeated games.

However, we show that even with the Markov restriction, the lack of commitment ability is not very important (in our particular model). The question of which is more empirically relevant, Markov or reputational equilibria, depends on the specific situation being modeled. We think that the Markov assumption is reasonable in CEE, where uncertainty
about government longevity (e.g. the rapid turnover of Prime Ministers in Poland), makes a reputation harder to establish.

2.1 Model Basics

We consider a small open economy with two sectors, each of which produces a single good using labor under diminishing returns to scale. The fixed supply of labour is $L$. Good 1 is both imported and domestically consumed. Liberalization leads to a decline in sector 1's price, and we call this the declining sector. The restricted revenue function of sector $i$ is $R^i(p_i, L_i)$, where $p_i$ and $L_i$ are output price and amount of labour for sector $i$. Letting subscript $j$ denote a partial derivative with respect to the $j$'th argument, we have $R^i_j$ equal to the sector $i$ output supply and $R^i_j$ equal to the sector $i$ wage. Hereafter we set $p = p_1/p_2$, the free trade relative price of sector 1 output; $L = L_2$, labor in sector 2; and $U$ equal to the amount of unemployment. The labor employed in sector 1 is then $L - L - U$. When the dying sector is protected by a unit tariff of $\tau$, the relative price of commodity 1 is $p + \tau$.

There is a continuum of workers in each sector, and a residual claimant (e.g. a capitalist or government ministry). All agents have the identical homothetic expenditure function $E(p+\tau, u^j)$, where $u^j$ is agent $j$'s utility. With homothetic preferences, we can write the expenditure function as $Y^jE(p+\tau)$, where $Y^j$ is defined as agent $j$'s real income and $E(p+\tau)$ is the unitary expenditure function.

2.2 Dynamics

Workers who leave the dying sector pass through the pool of $U$ unemployed workers before being employed in the expanding sector. The probability that an individual worker gets a job over an interval of time $dt$ is $f(U)dt$. We assume that $f(U)$ is positive for all $U$ and that
If(0) is finite. Therefore, the expected duration of unemployment for the "last unemployed worker", 1/f(0), is positive. We define η as the elasticity of f with respect to U, η = Uf'(U)/f(U). The assumption that there is congestion in the labor market implies that η < 0. This assumption is supported by empirical evidence and can be given a formal justification, as in Cassing and Ochs (1978).

Workers have rational expectations regarding future wages and future levels of unemployment. The present discounted value at time t of the wage differential in the two sectors is q_t = \int_0^\infty e^{-rs} [R_2^2(t+s) - R_2^1(t+s)] ds, where r is the discount rate. This implies

\[ \dot{q} = rq + R_2^1 [p + \tau, \tilde{L} - L - U] - R_2^2 [L] \] (1)

We suppress the time index when this does not lead to confusion. The variable q is the private value (to a worker) of a successful job search.

Following Lapan (1979), the equilibrium migration decision is given by the complementary slackness relations R_2^1 -fq ≥ 0, U ≥ 0, and [R_2^1 -fq]U ≡ 0. Outside the steady state, where U > 0, we have

\[ R_2^1 (p + \tau, \tilde{L} - L - U) - f(U)q = 0. \] (2)

The opportunity cost of looking for a job over a unit of time is the foregone wage, R_2, and the expected benefit is the probability of getting a job during that interval, f(U), times the present value of having a job in the growing sector, q. The opportunity cost of migration can not be less than the expected benefit. If it were less, migration would increase; this would increase U, resulting in an increase in the wage in the dying sector and a decrease in the
probability of finding a job. Equation (2) states that if unemployment is positive, the opportunity cost of migration equals the expected benefit.

We can use (2) to write unemployment as $U = U(L, q, \tau)$. Totally differentiating (2), shows that $U_L < 0$ and $U_q > 0$. An increase in the level of workers employed in the expanding sector decreases the wage there, making jobs in that sector less attractive. This decreases the level of unemployment by making workers in the dying sector less willing to leave their jobs. An increase of the present value of the future wage differentials makes workers more eager to leave the declining sector and causes a rise in unemployment.

Implicit in (2) is the assumption that when an unemployed worker wants to give up his search, he instantly gets a job in the dying sector at the going wage: unemployment is voluntary. There clearly does exist involuntary unemployment in CEE. Nevertheless, the model captures the important stylized fact that labor mobility is asymmetric. Workers can move more easily into the dying sector, with which they are familiar, than into the unfamiliar expanding sector.

Given a continuum of unemployed workers of measure $U$, and the probability, $f(U)$, that an individual worker gets a job, then by the law of large numbers the measure of workers moving into the growing sector is $Uf(U)$. This implies

$$\frac{dL}{dt} = Uf(U) \tag{3}$$

We assume that the elasticity of congestion with respect to unemployment, $\eta$, satisfies $0 > \eta > -1$, so that higher unemployment leads to higher engagement rates in the expanding sector.

Equations (1), (2) and (3) define the dynamics of the economy. The stock of workers
L is the predetermined variable of the system. The private value of migration, q, is a jump variable; its initial value is endogenous.

3. Equilibrium under Non-Intervention and with a First Best Policy

As a preliminary to studying the model with and without commitment, using a second-best tariff, we first summarize the characteristics of the model under non-intervention and using a first best, nondistortionary policy. In this section, therefore, we set $\tau = 0$.

3.1 Private adjustment without government intervention

From (5), a steady state requires that $U = 0$. From (1) and (2), steady state values of L and q, denoted with superscript "*", satisfy

$$q^* = \frac{R_1^2(L^*) - R_1^1(\bar{L} - L^*)}{r} = \frac{R_2^1(\bar{L} - L^*)}{f(0)}.$$  \hspace{1cm} (4)

The relation between the steady state wages is given by

$$R_2^1(p, \bar{L} - L^*) = \frac{f(0)}{r + f(0)} R_1^2(L^*).$$  \hspace{1cm} (5)

The quantity $L^*$ is the smallest steady state; there exists a range of values greater than $L^*$ that are also steady states. However, provided that the initial value of L satisfies $L_0 < L^*$, as we assume, the economy converges to $L^*$. There is full employment at steady state. Given the assumption that it takes a positive amount of time to get a job at full-employment ($f(0)$ is finite), the steady state wage in the growing sector is higher than in the declining sector. The ratio of the steady state wages, $f(0)/(r + f(0))$, is a decreasing function of $r$ and an increasing
function of \( f(0) \).³

If the economy is originally at the steady state \( L^* \) consistent with an initial relative price, and the relative price of good 1 falls (e.g., as a result of liberalization), there is a sudden jump in unemployment. Workers leave sector 1, the dying sector. Thereafter, unemployment is monotonically decreasing, and the growing sector monotonically increasing as the higher steady state is approached. If all quits occur in the first instant, the dying sector is monotonically increasing, as some unemployed workers return to that sector. However, if congestion is sufficiently important, workers may leave the dying sector more slowly, in which case the size of that sector (after the initial jump) falls for a time, and then rises.

In Appendix A we discuss in greater detail the properties of the private adjustment equilibrium.

3.2 The first best policy

Suppose that the government is able to control migration directly, and can therefore prevent it from occurring too rapidly. The government wants to maximize the present discounted value of revenue, \( \int_0^\infty e^{-s} [R_1(p, \bar{L}, \dot{L}(t+s)-U(t+s)) + R_2(L(t+s))] \, ds \) subject to (3).

The first order conditions for its problem include

\[
R_2^1 - \mu [f(U) + Uf'(U)] = 0
\]

\[
\dot{\mu} = r\mu + R_2^1 - R_2^2; \lim_{t \to \infty} \mu(t) e^{-rt} = 0
\]

The variable \( \mu \) is the social shadow value of labor in sector 2. Integrating the differential

³ This model has an interesting relation to the familiar Harris-Todaro model, in which an exogenous wage determines the probability of unemployment; the causation is reversed here, with the exogenous probability, \( f(0) \), determining the wage differential.
equation in (7), using the transversality condition, implies that μ = q, for a given trajectory of L and U. Therefore, comparison of (6) and (2) establishes that (during adjustment) private decisions are socially optimal if and only if f'(U) = 0, so there is no congestion (Lapan, 1978, Cassing and Ochs, 1978). However, even with congestion, the steady state under private adjustment and the socially optimal steady state are equal. This is due to the fact that in both cases unemployment vanishes in the steady state, so congestion is unimportant.

When there is congestion, the socially optimal trajectory can be decentralized by giving a wage subsidy φ(t) to workers in sector 1. Those workers receive the wage R_2 + φ. The private value of migrating is now ̇q, which is the solution to dq/dt = ṙq + R_2 + φ - R_2.

Using (6) and (2), with q replaced by ̇q, and simplifying, we have that the optimal φ satisfies

$$\phi(t) = f(U) (q - μ) - uf'(U) μ = -f(U) \left[ \int_0^\infty e^{-rs} φ(r+s) \, ds + η μ \right]$$

where U and μ are the solutions to (6) and (7). The wage subsidy at time t equals the sum of two terms: -f(U)ημ is positive, and is due to the presence of congestion; the sign of f(u)(q - μ) is negative if the trajectory of future subsidies is positive. The anticipation of future subsidies makes the private value of migration less than the social shadow value of labor in sector 2; in order to partly offset this disincentive to migrate, a lower current subsidy is used.

We note four properties of the optimal subsidy. First, in the steady state φ = 0. This can seen by substituting the steady state values for μ and ̇q into (8). Second, the optimal φ is positive during a final phase of the trajectory. This can be shown using a proof by contradiction, as follows. Suppose that φ(τ) ≤ 0 for all τ ∈ [t, ∞), for some t. Then the last expression in (8) implies that φ(t) > 0, which contradicts the hypothesis. Consequently, over
a final part of the adjustment phase, the subsidy is positive, and it decreases to 0.4

A third property of the optimal wage subsidy is that it is subgame perfect (and therefore time-consistent), despite the fact that it is obtained by solving a control problem in which forward-looking agents have rational expectations. The government's control problem has a "jump state", i.e. one whose initial condition is endogenous, a situation associated with time-inconsistency problems. Here, however, under the optimal program the value of the jump state is optimal for the government at every point and not just at the initial time.5 The intuition for this result is simply that the wage subsidy enables the government to achieve the

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4 Lapan (1976) derives the optimal wage subsidy in the case where wages are required to be equal in the two sectors because of an institutional constraint. In that model, migration is an exogenous function of the unemployment rate, but does not depend on workers' expectations about the future. The subsidy in that model corrects two distortions: the fact that workers do not internalize the congestion they cause, and the distortion that arises from the wage equalization constraint. The first distortion disappears in the steady state, but the second persists. Therefore, the steady state subsidy is positive in Lapan's model.

5 To show this formally, we can set up the government's control problem when it chooses $\phi$ to maximize the present discounted value of national revenue. First, define $H^*$ as the government's Hamiltonian when it chooses $U$ directly: $H^* = R^1 + R^2 + \mu U f(U)$. This is the Hamiltonian used to derive (6) and (7). Define the function $g(\hat{q}, L, U, \phi) \equiv r\hat{q} + R^1_\phi + \phi - R^2_\phi = \frac{d\hat{q}}{dt}$, and define $\beta$ as the government's shadow value of the jump state $\hat{q}$. The variable $\hat{q}$ was defined in the text; it is simply the private benefit of migration, given the subsidy. We want to show that on the optimal trajectory, $\beta \equiv 0$; the interpretation of this identity is that the value of the jump state is optimal for the government at every point. The equilibrium unemployment is given by the relation $R_\phi + \phi = f(U)\hat{q}$, which we write as $U = U(L, \hat{q}, \phi)$. When the government chooses $\phi$ as the control variable, its Hamiltonian is $H \equiv H^* + \beta g$, where we have used the constraint $U = U(L, \hat{q}, \phi)$ to eliminate $U$. At an interior solution, the first order conditions are (i) $\partial H / \partial \phi = 0$, (ii) $-\partial H / \partial L = \mu - rL$, and (iii) $-\partial H / \partial \hat{q} = \beta - r\hat{q}$. If the reader writes these three equations out, and evaluates them on the first best trajectory, where (6) and (6) are satisfied, (s)he will see that $\beta \equiv 0$, as was to be shown.
first best equilibrium; the government has no incentive to renege at a future date.6

The fourth property is that the social marginal value of unemployment is identically 0 on the optimal trajectory. We have seen that the wage subsidy supports the first best outcome. On that trajectory the shadow value of labor in sector 2 is μ. The marginal unit of unemployment generates a flow of \( f(U) + Uf'(U) \) units of labor into sector 2, for a marginal benefit of \( μ[U + Uf'(U)] \). The cost is the foregone wage, \( R_1 \). By equation (6), the marginal benefit and cost are equal. This equality does not hold under a second best policy.

4. The Second Best Tariff

In empirically relevant circumstances, the government is seldom able to correct a distortion without introducing other, secondary distortions. It is well known in this case that the full-commitment (open loop) trajectory is time-inconsistent. Therefore, if the government cannot make credible commitments about future behavior, it loses some of its ability to influence private agents. We wish to know if this is likely to be is an important problem. We present evidence, for this model, that the lack of ability to make credible commitments is not very important. We have biased the model in favor of finding that commitments are important, by considering the two extremes were ability to commit is either perfect or non-existent.

We assume that the government can use a tariff. If this protects the dying sector, it

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6 Solving the control problem described in footnote 5 shows that the government has no incentive to renege from the optimal wage subsidy on the equilibrium path. However, it is straightforward to show that the optimal subsidy can be supported as a subgame perfect equilibrium. Denote the optimal subsidy as \( φ^*(t) \) and the optimal state trajectory as \( L^*(t) \). Since the latter function is monotonic, we can invert it and then write the optimal subsidy as \( φ^*(L) \), i.e., in "feedback form".
raises the wage there, as would a wage subsidy. This slows migration, reduces unemployment, and reduces congestion costs. However, it also causes a consumption distortion. In a more general model with other mobile factors, the tariff would also cause a production distortion.\footnote{We have also studied a similar model in which the government is able to use a wage subsidy, but it costs $\$(1+c)$ to raise $\$1$ of government revenue, $c > 0$. The qualitative results are unchanged. The existence of a secondary distortion, not its source, is what matters.}

4.1 Perfect commitment

At time $t$ the tariff is $\tau(t)$. Real income for the economy is $Y$; using the notation in Section 2.1, total expenditure is $Y\bar{E}(p + \tau)$. We assume that there is no international borrowing, so that the deficit in the balance of trade, $D$, must be 0 at all times. We write this as

$$D(Y, \tau, L, q, U) \equiv 0 \equiv Y\bar{E}(p + \tau) - R^1(p + \tau, \bar{L} - L - U) - R^2(L) - \tau \left[ Y\bar{E}_1(p + \tau) - R^1_1(p + \tau, \bar{L} - L - U) \right]$$

The first term on the right hand side of (9) is total expenditure, the next two terms give the value of total production, and the third gives the tariff revenue, assumed to be redistributed in a lump sum. The government wants to choose a trajectory of $\tau$ and $Y$ to maximize the present discounted value of real income, $\int_0^\infty Y e^{-\rho s} ds$, subject to (1), (2), (3) and (9). This is a standard control problem with a free initial condition for the jump state, $q$. The Hamiltonian, necessary conditions, and definitions, are collected in Table 1. The costate variables for $L$ and $q$ are $\beta_1$ and $\beta_2$; the constraint multiplier for (9) is $\lambda$, which is the marginal utility of income.

The open loop steady state tariff is 0, and the optimal steady state size of sector 2 is
the same as under private adjustment or the wage subsidy. This is not surprising, since the externality vanishes in the steady state.\(^8\)

Outside the steady state, the optimal tariff solves equation (12). This expression decomposes the effect of the tariff at time \(t\) into three terms, the "import effect", the "announcement effect" and the "unemployment effect"; optimality requires balancing these. The first term in (12) is the static import loss (measured in units of utility); this equals the change in imports (the decrease in demand, plus the increase in production of good 1) times the tariff. The import loss includes the consumption distortion in standard static models. Since we have assumed that labor is the only mobile factors (\(R_{11}^1 = 0\)) the production distortion of those static models is absent here. However, there is a production effect, since the tariff reduces migration, which increases the amount of labor and hence production in sector 1.

The second term in (12) is the announcement effect. The tariff changes the wage in sector 1 by \(R_{21}^1 - U_1R_{22}^1\), which equals the change in the value of marginal productivity caused by both the change in producer price, and the change in the amount of labor employed. From equation (1), this changes the evolution of \(q\); the shadow value of \(q\) is \(\beta_2\). We designate \(\beta_2[R_{21}^1 - U_1R_{22}^1]\) as the "announcement" effect because this term captures the dependence of the optimal tariff on the forward looking variable, \(q\).

The initial value of \(q\), \(q(0)\), is determined by the trajectory of tariffs, \(\{\tau(t)\}_{t=0}^{\infty}\). The

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\(^8\) Steady state values (denoted by "\(*\"\)) \((\tau^*, \beta_2^*) = (0, 0)\) imply that \(U^* = 0\) and \(\beta_2^* = \lambda^*R_{11}^1/f(0)\), using (12) and (15). Using (13), we have \(\beta_2^* = \lambda^*d/r\), where \(d\) is the wage differential. Setting these two expressions for \(\beta_2^*\) equal to each other gives the equation for the steady state (5) under private adjustment.
Hamiltonian

\[ H = Y + \lambda \left[ -Y \bar{E}(p + \tau) + R^1(p + \tau, \bar{L} - L - U) + R^2(L) + \tau \left[ Y \bar{E}_1(p + \tau) - R^1(p + \tau, \bar{L} - L - U) \right] \right] \]

Maximisation conditions

\[ \lambda = \frac{1}{E - \tau \bar{E}_1} \] (11)

\[ \tau \lambda \left[ Y \bar{E}_{11} + U_i R_{21}^{1} \right] + \beta_2 \left[ R_{21}^{1} - R_{22}^{1} U_t \right] + U_t \lambda s = 0 \] (12)

Adjoint conditions

\[ \beta_1 = r \beta_1 - \lambda \left[ d + \tau R_{12}^{1} (1 + U_t) \right] - U_i \beta s + \beta_2 [h + R_{22}^{1} U_t] \] (13)

\[ \beta_2 = R_{22}^{1} U_i \beta_2 + \lambda U_i \tau R_{12}^{1} - U_i \lambda s \] (14)

where

\[ s = \frac{\beta_1}{\lambda} \left[ f(U) + U_L f'(U) \right] - R_{22}^{1} \] (15)

\[ U_t = \frac{R_{21}^{1}}{R_{22}^{1} + f'(U) q} < 0 \] (16)

\[ d(p + \tau, L) = R_{22}^{1} (L) - R_{22}^{1} (p + \tau, \bar{L} - L - U) \]

\[ h(p + \tau, L) = R_{22}^{1} (p + \tau, \bar{L} - L - U) + R_{22}^{2} (L) < 0 . \]

Boundary values and Transversality conditions

\[ L(0) = L_0 \]

\[ \lim_{t \to \infty} q(t) e^{\pi t} = 0 \]

\[ \lim_{t \to \infty} \beta_1 (t) e^{\pi t} = 0 \]

\[ \beta_2 (0) = 0 \]

\[ \begin{align*}
L &= \text{stock of labor in sector 2} \\
q &= \text{PV of future wage differentials} \\
\lambda &= \text{marginal utility of income} \\
\beta_1 &= \text{costate variable of } L \\
\beta_2 &= \text{costate variable of } q \\
U &= \text{unemployed} \\
d &= \text{current wage differential} \\
h &= \text{sum of the slopes of marginal productivities} \\
\tau &= \text{tariff} \\
s &= \text{net social value of migrating} \\
r &= \text{discount rate}
\end{align*} \]
government chooses this trajectory and therefore indirectly chooses \( q(0) \). The boundary condition \( \beta_2(0) = 0 \) states that the equilibrium \( q(0) \) is optimal. This boundary condition implies that the announcement effect equals 0 at the initial time. Since \( q \) depends on future events, the tariff at time \( t \) affects previous values of \( q \). At the initial time, \( t = 0 \) there are - by definition - no previous values of \( q \) that can be altered; since \( q(0) \) is optimal, a change in that value results in no first order improvement in welfare. Consequently, the announcement effect does not contribute to the initial tariff.

The third term in (12) is the unemployment effect. This equals the value (in units of utility) of the tariff’s effect on migration. This value is the product of \( U_\tau \) (the effect of the tariff on unemployment) and \( s \) (the net social value of unemployment). An increase in unemployment increases migration to the second sector by the amount \( f(U) + Uf'(U) \); the value (in units of money) of the marginal migrant is \( \beta_\tau/\lambda \) (the shadow value of labor in sector 2 divided by the marginal utility of money). The cost of a unit of unemployment is the value of foregone production, \( \xi_2 \). Since \( \lambda U_\tau < 0 \), the sign of the unemployment effect equals the sign of \(-s\).

The sign of the initial tariff equals the sign of \(-s\). This can be seen from the slope of the Hamiltonian, evaluated at time 0, at a 0 tariff: \( \frac{\partial H}{\partial \tau} |_{t=0,\tau=0} = U_\tau \lambda s \Rightarrow \text{sign}{\frac{\partial H}{\partial \tau} |_{t=0,\tau=0}} = \text{sign}{-s} \). Under private adjustment, migration occurs too quickly, leading to excessive unemployment; consequently we expect \( s < 0 \), and the initial optimal tariff to be positive. At the initial time, the optimal tariff is chosen to balance the import effect and the unemployment effect.

If the government announced the optimal tariff trajectory at time 0, and was able to
re-optimize at a later time $t > 0$, it would want to change its original plan. The optimal tariff is time-inconsistent. The reason for this is related to the announcement effect, the second term in equation (12). Define $\tau'(t)$ as the value of the optimal tariff in effect at time $t$, which is announced at time $t'$. The optimal time $t$ tariff announced at the initial time, $\tau^0(t)$, affects the migration decisions over the interval $[0,t)$. At time $t$, those decisions are "bygones"; they cannot be affected by anything done at time $t$ or in the future. If the government re-optimizes at time $t$, it ignores previous migration decisions in choosing the tariff trajectory over $[t,\infty)$. In other words, the choice of the tariff to be in effect at time $t$ is governed by different considerations at times 0 and at $t$, so $\tau^0(t) \neq \tau(t)$. Another way to see this is to note that the shadow value of the jump state, $\beta_2$, is not identically 0, in contrast to the first best wage subsidy. If the government is allowed to re-optimize at time $t > 0$, optimality of the subsequent trajectory requires that $\beta_2 = 0$ at $t$.

4.2 Infinitesimal Commitment

We now consider the case where the government is unable to make binding commitments; we restrict attention to Markov Perfect Equilibria (MPE). At time $t$ all agents (the government and workers) observe the state, $L_r$, and condition their point expectations about future variables, such as tariffs, wages, and levels of $L$, on $L_r$. All agents have the same information at $t$ and the same point expectations. For example, we denote $\tau^e(L_r,t')$ as agents’ conditional expectation of the tariff at time $t+t'$, for $t' \geq 0$, given $L_r$. These expectations

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9 Suppose, to the contrary, that $\beta_2 = 0$. Then equations (14) and (16) imply that $\tau = 0$; This identity and (14) then imply that $s = 0$. However, in our discussion of the first best wage subsidy we noted that there $s = 0$ (the fourth property), and this requires a non-zero subsidy over at least part of the trajectory (the second property). Therefore, non-intervention ($\tau = 0$) is not consistent with $s = 0$. 
(functions) induce an expectation of the forward looking variable q. We write this as $q_{t,t+t'}^e = q^e(L,t')$, which says that at time $t$, agents' point expectation of the value $q$ will take at time $t+t'$, depends only on the current value of $L$ and on $t'$. In order to write this relation, we require only that the underlying expectations functions are stationary, and that they are sufficiently regular so that when substituted into equation (1), the resulting expression has a solution; in addition, we assume that $\partial q^e/\partial L_0$ is continuous. We define the function $Q(L) = q^e(L,0)$.

\begin{table}[h]
\centering
\caption{MPE Tariff Policy}
\begin{tabular}{|c|c|}
\hline
\textbf{Hamiltonian} & \\
\hline
$H = Y + \lambda \left[ -Y\bar{E}(p + \tau) + R^1(p + \tau, \bar{L} - L - U) + R^2(L) + \tau \left[ \bar{Y}\bar{E}_1(p + \tau) - R_1^1(p + \tau, \bar{L} - L - U) \right] \right] + \beta Uf(U)$ & (17) \\
\hline
\textbf{Maximization conditions} & \\
\hline
$\lambda = \frac{1}{E - \tau E_1}$ & (18) \\
\hline
$\tau \lambda \left[ \bar{Y}\bar{E}_{11} + U_{\tau}R_{21}^1 \right] + U_{\lambda} \lambda s = 0$ & (19) \\
\hline
\textbf{Adjoint equation} & \\
\hline
$\beta = r\beta - \lambda \left[ d + \tau R_{12}^1(1 + U_L) \right] - U_{\lambda} \lambda s$ & (20) \\
\hline
\textbf{Boundary values and Transversality conditions} & \\
\hline
$L(0) = L_0$ & \text{lim} $\beta(t)e^s = 0$ \\
\hline
\end{tabular}
\end{table}

\textit{Consistency Condition}

$$\frac{dQ}{dL} = \dot{q} = rQ(L) - d(\tau, L)$$  \hspace{1cm} (21)

\text{lim}_{t \to \infty} Q(L_t) e^s = 0
For any function $Q(L)$, the government's problem is to choose policy rules that determine $Y$ and $\tau$ in order to maximize the present discounted value of real income, subject to (2), (3) and (9). The variable $q$ which appears in these constraints is now replaced by the function $Q(L)$. This is a standard control problem, except that the function $Q(L)$ is unknown. However, we can write the necessary conditions to the control problem, given an arbitrary function $Q(L)$, and then impose a consistency condition to find the unknown function.

The Hamiltonian and necessary conditions are given by equations (17) - (20) in Table 2. The costate variable for $L$ is $\beta$; other terms used in the Table 2, $\lambda$, $s$, $U_1$, $d$ and $h$, have the same definitions as in Table 1, although their values will of course be different. There is a single state variable, $L$, in this control problem; in the open loop problem there are two state variables, $L$ and $q$. In solving the MPE, we lose a state variable, but gain an unknown function.

In comparing the necessary conditions in the two Tables it is important to remember that in the MPE the function $Q(L)$ replaces the variable $q$ in the government's constraints. Equilibrium unemployment outside the steady state in both cases is given by equation (2), which for the MPE we can invert to obtain $U = U(\tau, L, Q(L))$. In Table (1) the expression $U_L$ is obtained by differentiating (2) with respect to $L$ and $\tau$, holding $q$ constant; in Table (2), however, we have $U_L = \partial U/\partial L + (\partial U/\partial Q)(dQ/dL)$.

Keeping in mind this qualification, we note the similarity between the necessary conditions (11) - (13) for the full commitment trajectory, and (18) - (20) for the zero commitment trajectory. In particular, comparison of (12) and (19) shows that the announcement effect is absent in the MPE. When the government is unable to make
commitments about future tariffs, the tariff in place at time $t$ is chosen without regard to its effect on previous migration decisions. The other effects of the tariff, the import effect and the unemployment effect, are present and take the same form in both equilibria. In both cases a tariff lowers utility because it causes a consumption distortion (the import effect), and increases utility because it lessens the gap between the social marginal benefit and cost of unemployment, $s$ (the unemployment effect). The import effect and the unemployment effect influence welfare in the opposite direction. Therefore, the importance of the ability to make commitments depends on the relative importance of the announcement effect 	extit{vis a vis} the the net effect of the import and unemployment effect.

The consistency condition for the MPE is given by equation (21). This is not a necessary condition to the control problem. It is a restriction on the unknown function $Q(L)$ to insure that when the government chooses its optimal policy rule $\tau^M(L)$, taking as given $Q(L)$, agents’ expectations are confirmed in equilibrium. (The superscript $M$ indicates the equilibrium policy rule in a MPE.) In terms of the notation introduced above, (21) must hold in order that $\tau^M(L) \equiv \tau^r(L,0)$.

Unless we introduce an additional restriction, the MPE is not unique. However, because this issue is tangential to our chief concern, and because in our view there is a reasonable restriction to select a unique equilibrium, we relegate this discussion to Appendix B. Here we merely note that $L = L^*$ and $\tau^M(L^*) = 0$ is a steady state MPE; $L^*$ is the steady state under laissez faire, the wage subsidy, and the full commitment tariff. If the initial size of sector 2 is small, in the sense that migration would occur under laissez faire, then the steady state size of the sector is the same under all of the policy regimes we have considered.
In every case there is no government intervention in the steady state. Because we want to use this model to gain some insight into policy-making in reforming economies, the steady state is of limited intrinsic interest. It is, however, important because it provides a boundary condition which determines the transitional dynamics. In order to examine the equilibrium trajectories under the various policy regimes we use numerical methods.

4.3 A Numerical Comparison

Extensive simulations results establish the following properties of the equilibrium wage subsidy and tariff policies: (1) The time trajectories of these policies are all monotonic. (2) The first year OLE tariff is similar to, but slightly lower than the first year MPE tariff; from the second year on the OLE tariff trajectory is above the MPE’s.

Computational Method We used the algorithm introduced by Fair and Taylor (1983)\(^{10}\). Starting from an initial guess of the time paths of the jump variables, a sequence of temporary equilibria is computed, giving rise to a sequence of actual values of the jump variables. A linear combination of initial guesses and actual values is computed in order to improve the guesses of the jump variable. This procedure is iterated until a fixed point is reached. To implement this algorithm, we replaced the continuous time control programs with their discrete time analogues.

Calibration We choose units so that \( p=1 \) and \( L=1 \). We use constant elasticity for the production functions, the expenditure function and the probability function: \( \bar{E}(1+\tau)=(1+\tau)^{0.55} \), \( R^1(1+\tau,L)=(1+\tau)(1-L-U)^{0.75} \), \( R^2(L)=L^{0.75} \), and \( f(U)=0.5(1+U)^{-1} \). Using the discrete version of

\(^{10}\) This algorithm is now used to solve large-scale applied general equilibrium models [e.g. Goulder (1989) or Keuschnigg (1991)].
(5), the steady value of \( L \) is \( L^* = 0.401 \). The unit of time equals 1 year and the discount, \( r \), is set to 0.05. The time horizon is 20 years and the initial condition on \( L \) is \( L(1) = 0.1 \) so that 90\% of workers are employed in the dying sector before migration takes place. The boundary value \( L(20) \) is free and there is no scrap value.

We conducted sensitivity tests by changing (i) the initial condition \( L(1) \), (ii) the functional form of the probability \( f(U) \) and (iii) the parameter of the expenditure function. We also changed the unit of time from a year to a quarter, thus reducing the government’s period of commitment. In all cases the basic conclusions are the same.

**Results** We considered four policy scenarios: the private adjustment equilibrium (PAE), the first best equilibrium (FBE), the open loop equilibrium (OLE) and the Markov Perfect equilibrium (MPE). Figures 1 and 2 show the time paths of \( U \) and \( L \) in the four simulations. The initial level of unemployment is 35\% higher in the PAE relative to the FBE. However, the higher unemployment in the first years implies that workers are more rapidly employed in the growing sector. Full employment is reached in about eight years under PAE and twelve years under FBE.

The unemployment trajectories for the OLE and the MPE scenarios lie between the PAE and FBE paths. The most important qualitative result is that for this model of unemployment the requirement of Markov perfection does not substantially reduce the efficacy of government intervention. Unemployment in the MPE is first slightly higher and then slightly lower than in the OLE. As a result, the MPE engagement rate in the expanding sector is higher in the first years.

The time paths of the wage subsidy, the OLE tariff and the MPE tariff are shown in
figure 3. The two tariff trajectories are both monotonic, so protectionism is phased out under either assumption about government commitment. The trajectories are very similar, so a large change in the government's ability of making binding commitment does not affect the qualitative properties of tariff policy. Both of these conclusions differ from results obtained using a different model of migration, which we discuss below.

The fact that the initial MPE and OLE tariffs are similar could be due to two causes. The first explanation is that the announcement effect is very small compared to the unemployment effect. The open loop trajectory is then mostly the result of the interaction between the import effect and the unemployment effect and therefore is similar to the MPE tariff trajectory. The second explanation is that the consumption distortion is relatively unimportant and therefore the open loop tariff is not "very" time-inconsistent. However, if the second explanation were true, the three instruments (FBE subsidy, the OLE tariff and the MPE tariff) should have similar values in the first year when the state variable is the same for each. This is not the case. The initial open-loop tariff is about 9% lower than the initial subsidy, but only 1.2% lower than the MPE tariff. We therefore eliminate the second explanation and conclude that the announcement effect is small compared to the wage effect.

The first year OLE tariff is lower than the first year MPE tariff because the announcement effect, although small, is not negligible. In the MPE, the inability to use announcements of future tariffs to slow current migration, forces the government to increase slightly the initial tariff (relative to the OLE). Despite the fact that the initial MPE tariff is

\[11\] For given value of the state variable L, the wage subsidy is higher than the tariff, since the former entails no consumption distortion.
higher, we noted that initial unemployment is also higher. Therefore adjustment occurs more quickly in the MPE. This tends to make the second period MPE tariff lower than the OLE tariff. This tendency is reinforced by the fact that the second period OLE tariff is high partly because of the prior commitment. To summarize, we see that the government’s inability to make commitments causes it to begin with a higher degree of protection, but to reduce this more quickly (relative to the OLE). However, the tariff trajectories under either zero or perfect commitment have very similar profiles.

5. Comparison with an Alternative Migration Model

The previous section suggests that when the migration decisions of forward looking agents involve an externality, the qualitative aspects of the optimal tariff trajectory do not depend on the government’s ability to make commitments. This conclusion appears diametrically opposed to the results in Karp and Paul (1994). We explain the source of the difference in this section.

In the model in our previous paper, agents incur a one-time cost in moving from the dying to the growing sector. If \( L(t) \) workers migrate at time \( t \), the social cost is \( c(L) \) and the marginal social cost is \( c'(L) \). The market failure in this model is that agents’ private cost is a fraction \( \theta < 1 \) of social marginal cost. We assume that marginal costs are increasing, so that adjustment is not instantaneous.\(^\text{12}\) Equilibrium migration equates private benefits and costs:

\[ \theta c'(L) = q. \]

We invert this to write

\[ c'(L) = \frac{q}{\theta}. \]

\(^\text{12}\) This is a fairly standard model of convex adjustment costs. Krugman (1991) uses it in a model of intersectoral migration.
Equation (22) replaces equations (2) and (3), but in other respects the model is the same. To distinguish the two migration models, we refer to the one described by (2) and (3) as the "unemployment model", and the one described by (22) as the "cost of adjustment model".

In the cost of adjustment model, the current tariff has no effect on current migration: the tariff is not an argument of the function $g(\cdot)$. The first order condition (for the cost of adjustment model) corresponding to (12) does not have the third term, which we denoted the unemployment effect, although both the import effect and the announcement effect are present. The import effect is negative for nonzero tariffs and the announcement effect vanishes at time 0 (as is the case in the unemployment model), so the perfect commitment tariff is zero at time 0. For $t > 0$ the announcement effect is not identically 0, so it is optimal to use a tariff. Eventually, the tariff approaches zero as $L$ approaches the first-best steady state. In the cost of adjustment model then, protection is "phased in and phased out" in the full commitment equilibrium. In the zero commitment equilibrium, the announcement effect vanishes for exactly the same reason that was discussed in 4.2. Since the import effect remains negative (for non-zero tariffs), and since there is no offsetting unemployment effect, it is optimal to set the tariff identically equal to zero in the cost of adjustment model.\(^{13}\)

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\(^{13}\) The problem of non-uniqueness does not arise in the cost of adjustment model under zero commitment. The reason is that whatever are the agents expectations, and the resulting (differentiable) function $Q(L)$, it is optimal for the government to set the tariff identically equal to zero. Therefore the only function $Q(L)$ that is consistent with equilibrium is based on expectations of a zero tariff.
It is useful to consider the discrete stage version of these two models for the MPE.\textsuperscript{14} The length of each stage is $\varepsilon$, and in each stage the government sets the tariff before agents make their migration decision. In this case, the lower bound on the period of commitment is $\varepsilon$. The discrete stage version of (1) is $q_t = e^{-\varepsilon}Q(L_{t+\varepsilon}) + d(L_t, \tau)\varepsilon$, where (as above) $d(\cdot)$ is a flow variable, giving the wage differential. From this relation it is apparent that the effect of the current tariff on $q_t$ is proportional to $\varepsilon$. The discrete version of (22) is $L_{t+\varepsilon} = L_t + g(q_t/\theta)\varepsilon$, so the effect of $q_t$ on the stock of labor is also proportional to $\varepsilon$. Using the chain rule, or direct substitution, we see that the effect of the current tariff on $L_{t+\varepsilon}$ is proportional to $\varepsilon^2$ in the cost of adjustment model. We can write the flow of welfare in a given period as $W(t, L_t)$; because of the consumption distortion, the current tariff reduces this flow. Welfare for the period is simply $W(t, L_t)\varepsilon$. The welfare loss due to a non-zero tariff is therefore proportional to $\varepsilon$. Since the benefits (of affecting the state variable) are of a smaller order of magnitude than the costs (of distorting current consumption), the adjustment cost model implies that the MPE tariff must be small when $\varepsilon$ is small.

The discrete version of (3) is simply $L_{t+\varepsilon} = L_t + Uf(U)\varepsilon$. Unemployment is still given by equation (2), which is independent of $\varepsilon$. In the unemployment model, then, the effect of the current tariff on the future state variable is proportional to $\varepsilon$, as is the consumption cost of the current tariff. Since the costs and benefits of a tariff are of the same order of magnitude, optimality will in general require using a non-zero tariff even as $\varepsilon \to 0$.

\textsuperscript{14} Karp and Paul (1994) provide the discrete stage version of the cost of adjustment model. It is straightforward to write this down for the unemployment model, but in order to conserve space we merely provide a verbal sketch.
6. Conclusion

We studied the use of a second best policy to affect the reallocation of labor, following a shock such as that experienced by Central and East European countries. We considered two extreme cases, in which the government has either unlimited or negligible ability to commit to future actions. In the situation were the government has negligible ability to commit, we restricted attention to Markov Perfect Equilibria, thus ruling out reputational, or "grim trigger" strategies. Those types of strategies may be of little use if the public expects the current government’s tenure to be short.

We asked whether externalities in the labor adjustment process justify gradual liberalization, as opposed to the big bang. We investigated whether the answer to this question depends on the extent to which governments are able to make credible commitments about their future behavior. Numerical examples showed that the equilibrium paths under both unlimited and infinitesimal commitment ability are qualitatively the same, and in both cases involve gradual liberalization. Therefore, the inability to make commitments does not necessarily provide an argument against gradualism.

This result was obtained for a migration model in which the current tariff has a first order effect on current migration. We discussed another model in which current migration is affected chiefly by the expectation of future tariffs. For that model, the extent of commitment ability has a dramatic effect on the equilibrium tariff. However, that model supports the gradualist prescription for neither of the two extreme assumptions about commitment ability. The two models taken together therefore indicate that externalities in the labor adjustment process may justify gradual liberalization, but only if the current tariff affects current
migration. In that case, the inability to make commitments may be surprisingly unimportant.

It is well-known that "commitment matters", in the sense that the equilibrium changes with the degree of commitment. However, what is not known is whether commitment matters very much. We think that this is the more interesting question. Given the current state of data, economists can hardly provide precise recommendations concerning policy trajectories. For the purpose of advising governments, the comparative statics of optimal trajectories with respect to small changes in commitment ability, is probably not very useful. It is, however, useful to determine whether the shape of a policy trajectory can change for large changes in commitment ability. The framework we have used enables us to answer this question.

The relative merits of gradualism Vs. the big bang depend on many issues. The questionable ability of governments to make credible commitments is only one argument against gradualism. Our results suggest that this argument may have been overstated.
Appendix A: The Private Adjustment Equilibrium Without Government Intervention

By linearizing the dynamic system at the steady state, we can show that the steady state is a saddle point. This procedure assumes that $U \geq 0$ is not binding. The $L = 0$ isocline is the positively sloped curve in Figure A1. In the shaded region below this curve, the non-negativity constraint on $U$ is binding; all points on the segment $E^aE^a^*$ in Figure A1 are steady states. Therefore, any level of $L$ in the interval $[L^*, E^a^*]$ is a steady state. For initial values of $L$ below $L^*$, the size of the growing sector moves to $L^*$; for initial conditions in the interval $[L^*, E^a^*]$, the distribution of labor remains fixed; for initial conditions $L > E^a^*$, the economy moves immediately to $E^a^*$.

Comparative statics with respect to the relative output price, $p$, are shown in Figure A2. A decline in $p$ shifts downwards the $L = 0$ locus and shifts upwards the $q = 0$ locus, as shown by the dashed lines in the figure. This leads to an increase in the value of $L^*$, given by the solution to (7). The sign of the change in $q^*$ depends on the relative magnitudes of $1/r$ (a measure of agents' patience) and $1/f(0)$, the expected waiting time. Figure A2 is drawn with the assumption that agents are "very patient" [$1/r > 1/f(0)$], so that $q^*$ increases when $p$ falls. If the size of sector 2, prior to the price change, is less than $L^b$ in Figure A2 (e.g., at $L^a$), the sector grows. If the initial size is in the interval $[L^b, L^a^*]$, the price change does not alter the distribution of labor, although it does change the wage differential.

If the economy begins at a steady state such as $L^a$ in Figure A2, a fall in the relative price of sector 1 causes the endogenous value of $q$ to increase from $q^a$ to $q^o$. The price fall

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15 In the limiting case as $f(0) \to \infty$, the $L = 0$ isocline becomes the horizontal axis. In this case, wages in the two sectors are equal at the unique steady state, which is independent of the initial condition.
leads to an immediate discrete increase in unemployment and decrease in the size of sector 1. The equilibrium trajectory is shown as E^0E^b; on this trajectory, L (the size of sector 2) increases and q decreases. This, together with (3) and (4), implies that unemployment falls monotonically during the adjustment period: \( \dot{U} = U_l \dot{L} + U_q \dot{q} < 0 \). The size of sector 1 is \( L_1 = \bar{L} - L - U \), which implies, using equation (2), \( \dot{L}_1 = [f(U)q + f'(U)qU]/R_{22} \). The two terms in the numerator have opposite signs, so the direction of the trajectory for labor in sector 1 is ambiguous. However, in the limiting case where there is no congestion \( f'(U) = 0 \), \( L_1 \) is unambiguously positive. In this case, all quits occur immediately after the price fall; during the adjustment phase, some unemployed workers find jobs in sector 2, and some return to sector 1. With congestion, however, sector 1 may continue to shrink for a period after the price fall. Congestion in the labor market causes workers to leave the dying sector more slowly.
Appendix B: Non-uniqueness of the MPE

Here we explain why the MPE is not unique, and we relate this to similar results in other models. We also explain why we think that it is reasonable to select a particular MPE.

The necessary conditions which define the equilibrium constitute a system of differential and algebraic equations, and transversality conditions. In standard control problems we find the steady state values by setting the differential equations equal to zero and use these with the first order conditions. This leads to a system of unknowns and equations; if the Jacobian of this system, evaluated at a steady state, is of full rank, then the steady state is "locally unique". In other words, there does not exist an interval of state space such that each point in that interval is a steady state. Even if the Jacobian is of full rank at a steady state, there may be multiple steady states, but these are at isolated points. In that case the equilibrium depends on the initial condition.

Models that depend on agents' expectations exhibit another kind of non-uniqueness. There may be intervals of state space such that every point in that interval is a steady state for some equilibrium; then the equilibrium is not "locally unique". Even when the set of steady states are isolated points, there may be multiple equilibria that reach those states, as in Krugman (1991) In our model, there exists a continuum of steady states; to each of these there corresponds an equilibrium trajectory. The intuition for non-uniqueness can be obtained simply by counting unknowns and steady state equilibrium conditions. Using \( U = 0 \) in the steady state [from equation (3)] we have six remaining unknown steady state values, \( L, q, \lambda, \beta, \tau, \) and \( dQ/dL \). However, we have only five linearly independent equations: the steady state's of (1) and (20), and the algebraic relations (2), (18) and (19). Equation (3) and the
consistency relation, equation (21), are linearly dependent in the steady state. This means that (21) is vacuous in the steady state, although not, of course, outside it. Equation (21) does not help pin down the steady state. Tsutsui and Mino (1990) note that an analogous situation arises in a MPE to some noncooperative differential games; they refer to this as an incomplete transversality condition.

The method of counting unknowns and equilibrium conditions is suggestive, but does not provide a rigorous proof of non-uniqueness. The reason is the following: Equation (21) can be written as \( \frac{dQ}{dL} = \frac{[rQ - d]}{Uf(U)} \). In the steady state both the numerator and denominator vanish. Since \( \frac{dQ}{dL} \) exists (by assumption), we can evaluate the ratio using L'Hospital's Rule. Therefore, it is not obvious that equation (21) really provides no information in the steady state. However, the application of L'Hospital's rule merely allows us to write the unknown function \( \frac{dQ}{dL} \) in terms of higher derivatives which are also unknown; therefore (21) is indeed vacuous in the steady state.

In order to demonstrate this statement, it helps to rewrite the model using more concise notation. To this end, we use the constraints (2) and (9) to eliminate \( \tau \) and \( Y \), allowing us to write instantaneous social welfare as \( z(U,Q,L) \). In a MPE the government takes the function \( Q(L) \) as given and chooses \( \{U\} \) to maximize \( \int_{t}^{\infty} e^{-rs}z(U,Q,L)ds \) subject to \( L = g(U) \), where \( g(U) \equiv Uf(U) \). The dynamic programming equation for this problem is

\[
\max_{U} \left[ z(U,Q(L),L) + J'(L)g(U) \right] \tag{B1}
\]

where primes denote derivatives. The first order condition to (B1) can be inverted to obtain the control rule, which we write as \( U = B(Q,J',L) \). Differentiating (B1) with respect to \( L \) and using the envelope theorem gives
\[ rJ' = z_Q Q' + z_L + J''g(U). \] (B2)

In a steady state, denoted by \( \hat{\cdot} \), we have

\[ B(\hat{Q}, \hat{J}', \hat{L}) = 0 \] (B3)
\[ r\hat{J}' = z_Q \hat{Q}' + \hat{z}_L \] (B4)

If we knew the function \( Q(L) \), (B3) and (B4) would comprise two equations in the two unknowns \( \hat{L} \) and \( \hat{J}' \), allowing us to solve for the steady state.

Defining the function \( F \) as \( F = g(B(.)) \), and using the fact that the equilibrium tariff is a function of \( L, Q \) and \( U \), where \( U = B(Q,J',L) \), allows us to rewrite the consistency condition (21) as

\[ Q'F(Q,J',L) = rQ - d(Q,J',L) . \] (B5)

Equation (B5), evaluated at the steady state, implies

\[ r\hat{Q} = d(\hat{Q}, \hat{J}', \hat{L}) . \] (B6)

Equations (B3), (B4) and (B6) comprise three equations in the four unknowns: \( \hat{Q}, \hat{L}, \hat{J}', \hat{Q}' \).

The question is whether we are able to use the limit as \( L \to \hat{L} \) of (B5) to obtain the final condition needed to select a locally unique steady state. To this end, we differentiate (B5) with respect to \( L \). Evaluating the result at a steady state leads to an equation involving the four unknowns above, plus the higher order derivative \( \hat{J}'' \). If we differentiate (B2), and evaluate the result at the steady state, we obtain an expression for \( \hat{J}'' \). However, this contains the unknown value \( \hat{Q}'' \). It is clear that if we proceed in this manner, each additional equation obtained by differentiating again either (B2) or (B5), brings with it an additional unknown.

Therefore, the restriction to a differentiable MPE does not imply a locally unique steady state.
In our model there is a plausible additional requirement which enables us to select a locally unique steady state for initial conditions which satisfy \( L_0 \leq L^* \). Recall that \( L^* \) is the solution to equation (5) and thus the smallest steady state under laissez faire; see Figure A1. This gives us a "natural boundary condition" that can be used to select a steady state. We noted in Section 2.2 that absent government intervention there is a continuum of steady states: values of \( L \) in the interval \([L^* E^*]\) in Figure A1. If the size of sector 2 were in this interval, the government would have no incentive to intervene, even if it were able to use a first best wage subsidy. Therefore, it is reasonable that agents would expect the government not to intervene when sector 2 is in this region and the government has only second best policies. If agents do hold these expectations, the optimal policy for the government is to set a zero tariff for \( L \in [L^* E^*] \). Non-intervention is therefore obviously one equilibrium; for \( L \in [L^* E^*] \), it seems the most reasonable one. The natural boundary condition is that \( L^* \) is a steady state.

The requirement that \( L^* \) is a steady state allows us to solve (B3) and (B6) to obtain the steady state values \( Q^* \) and \( J^* \). Equation (B1) is then used to obtain \( J^* \). Given these boundary values, we can solve the pair of ordinary differential equations (B1) and (B5) to obtain equilibrium functions \( J(L) \) and \( Q(L) \). Given mild regularity assumptions on the exogenous functions \( z(\cdot) \) and \( g(\cdot) \), this solution exists and is unique, at least in the neighborhood of the steady state. (See Boyce and DiPrima, page 268 for sufficient conditions.) Once we have the functions \( J(L) \) and \( Q(L) \) we can write \( U \) as a function of \( L \) using the control rule \( U = B(\cdot) \equiv b(L) \). At this level of generality, we can not eliminate the possibility that \( b'(L) \equiv 0 \) in an interval of \( L \) which includes \( L^* \); this is analogous to the Jacobian, discussed in the second paragraph of the Appendix, being singular. This is not an
interesting special case. However, even if we ignore this possibility, and moreover assume that there exists a unique solution to (B1) and (B5) for all $L \leq L^*$, it may still be the case that $b(L) = 0$ has a solution for $L < L^*$. In that case, there are multiple steady states, which are isolated points. Again, this possibility arises in standard control problems, and is not of special interest in our model.

Given the solution to (B1), $U = b(L)$, we can retrace the steps taken in constructing the function $z(\cdot)$, and retrieve the Markov tariff rule, $t^M(L)$. 
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