1. Introduction

It is well known that gauge coupling unification [1–5] looks very promising in the minimal supersymmetric extension of the standard model, the MSSM. A precise analysis reveals that, under the assumption of a ‘standard’ supersymmetry breaking scenario, gauge couplings do not meet precisely but in many cases the strong fine structure constant \( \alpha_3 = g_3^2/(4\pi) \) turns out to be about 3% smaller than \( \alpha_1 \) and \( \alpha_2 \) at \( M_{GUT} \), which is defined as the scale where \( \alpha_1 \) and \( \alpha_2 \) unify. This happens in scenarios in which

- gaugino masses unify at \( M_{GUT} \);
- scalar masses are universal.

One can correct for the discrepancy in many ways, most plausibly through threshold corrections at the TeV or at the GUT scale (see, e.g., [6]; see also [7, S. Raby, Grand unified theories]). Such thresholds may stem from exotics around the GUT scale (possibly from the GUT breaking sector) or, as in orbifold GUTs, from Kaluza–Klein modes between the compactification and cut-off scales (see e.g. [8–11]). In either case, such corrections will always exist. However, depending on the mechanism of GUT breaking they can be highly suppressed (see e.g. [12] for a discussion). In this Letter, we will focus on the question of how to obtain precision gauge coupling unification in the MSSM, i.e. with a subdominant contribution from extra states.

One may attribute the ‘non-unification’ to the fact that \( \alpha_3 \) runs too fast in scenarios that have the above two properties. In order to slow down the renormalization group (RG) evolution of \( \alpha_3 \) one might therefore lower the masses of colored states. Perhaps the simplest option for such a colored particle is the gluino, which transforms as an \( \mathbf{8} \)-plet under \( SU(3)_C \); lowering the masses of triplet–anti-triplet pairs often leads to problems with proton decay. Specifically, in scenarios in which the gaugino masses are non-universal at (or slightly below) the GUT scale, the deviation \( \epsilon_3 \) from the other two \( \alpha \) can vanish.

Let us note that, in general, the threshold effects at low energies seem to be more important than those at the high scale. That is, any correction to the fine-structure constants will be of the form

\[
\Delta \alpha_i = b'_i \alpha_1^2 \ln(m/M_\star),
\]

where \( b'_i \) is a ‘color factor’, \( m \) is the mass of the particle, \( M_\star \) is either \( M_{GUT} \) or \( M_{SUSY} \), and \( \alpha_i \) is the fine structure constant at \( M_\star \). Since \( \alpha_3 \) is much bigger at low energies, in order to get the same numerical correction one has to have a much larger ratio \( m/M_\star \) at high energies than at low energies. We will hence focus on settings in which the gluino is lighter than in the above-mentioned ‘standard’ scenarios.

It has been known for some time that a comparatively light gluino allows us to alleviate the tension between the predicted and measured values of \( \alpha_3 \) [13,14]; the main point of our analysis is that there are in fact well-motivated patterns of supersymmetry breaking which lead to precision gauge unification. It is worthwhile to stress that non-universal boundary conditions for the gaugino masses can be consistent with unification; they arise, for instance, in the scheme of mirage mediation [15], which has been first discussed in the context of type II B flux compactifica-
tions [16–18], but may also occur in heterotic string theory [19]. In particular, non-universal gaugino masses are something that one expects in settings in which dominant supersymmetry breaking comes from a field that does not enter the gauge kinetic function, e.g. a matter field [20,21]. There, the interplay between a suppressed tree-level term and quantum corrections render the gaugino masses non-universal slightly below the GUT scale. These corrections turn out to lower the gluino mass with respect to the gaugino masses non-universal slightly below the GUT scale. These suppressed tree-level term and quantum corrections render the non-universal gaugino masses. Non-universal gaugino masses may arise in settings in which dominant supersymmetry breaking occurs in settings in which dominant supersymmetry breaking by changing the running of the gauginos. In particular, the bino does not contribute to the running, only the squarks to the three gauge couplings (11/10, 3/2, 2).

Let us assume that scalar masses are universal and heavy at the GUT scale. If the gaugino masses are somewhat lighter, the scalar masses will also be nearly universal at the electroweak scale and will not induce differential running between the gauge couplings. Then the differential running is governed by the gauginos only. Hence whether or not we have precision gauge coupling unification crucially depends on the mass ratio of the gauginos. In particular, as the hino does not contribute to the running, only the masses of the wino and gluino will be relevant.

Both, a light gluino and a heavy wino can lead to precision gauge coupling unification by changing the running of $\alpha_3$ and/or $\alpha_2$ respectively. In the following we will study scenarios of super-symmetry breaking which result in a mass pattern allowing for precision gauge coupling unification.

3. Scenarios

As we have seen in the previous section the ratios of gaugino masses are essential ingredients for achieving precision gauge coupling unification. Therefore let us now consider models which give non-universal gaugino masses. Non-universal gaugino masses may arise if the gauge kinetic function $f_{\alpha \beta}$ has some non-trivial gauge structure. There are several known mechanisms for obtaining such structures even in grand unified theories or in string theory. In the following we shall discuss three examples.

3.1. Mirage mediation and matter domination

One scenario in which precision gauge coupling unification can be realized is ‘mirage mediation’. Here the moduli entering the gauge kinetic function have suppressed $F$-terms such that the direct contribution to the gaugino masses and quantum corrections are of the same order. The boundary conditions for the gaugino masses at the GUT scale read [18]

$$M_i = \frac{m^2 \phi}{16 \pi^2} (\alpha + b_i g_i^2),$$

where the $b_i$ denote the one-loop $\beta$-function coefficients and $\alpha$ is a continuous positive parameter. While the mirage mass relations for gauginos are rather robust, the pattern of masses for the scalars is more model dependent (cf. [23]). In fact, in situations in which supersymmetry is broken by a matter field, the so-called ‘matter domination’ scheme, the mirage pattern of the gaugino masses (5) is preserved (up to Kähler corrections [24]) while the scalars attain large positive masses.

Our ansatz for the soft parameters at the GUT scale is (cf. the general formulae in [20,21])

$$M_i = \frac{m^2 \phi}{16 \pi^2} (\alpha + b_i g_i^2).$$

1. We neglect here the small contributions from the Higgs fields.
Fig. 1. In panel (a) and (b) we show contours of different $\epsilon_3$ in the case of 'matter domination' for $\tan \beta = 10$ and $m_0 = m_{3/2}$ (a) and $m_0 = 0.01 m_{3/2}$ (b). The white region is excluded. The yellow and pink stars denote points in parameter space for which we show the spectra in Tables 1 and 2. In panel (c) and (d) the corresponding LSPs are shown with $\chi_1^0$ the lightest neutralino, $\tilde{g}$ the gluino and $\tilde{f}_1$ the lighter stau. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this Letter.)

\[ M_1 = \frac{m_{3/2}}{16\pi^2} \left( \alpha + b \, g_1^2 \right), \] 
\[ m_0^{16} = m_{3/2}, \] 
\[ A = m_0^{\text{Higgs}} = 0. \]

The continuous parameters in this scheme are $m_{3/2}$, $\alpha$ and $\tan \beta$;

we fix sign $\mu$ to be $+1$. We conducted a scan over the continuous parameters $m_{3/2}$, $\alpha$ and $\tan \beta$. It turns out that the deviation $\epsilon_3$ is almost independent of $\tan \beta$. In Fig. 1 we therefore show $\epsilon_3$ for varying $m_{3/2}$ and $\alpha$ with fixed $\tan \beta = 10$. The white regions are excluded as follows: we take into account all points which are invalid according to SOFTSUSY [25] as well as collider bounds on sparticle masses.\(^2\) This implies a minimal value of $m_{3/2} \gtrsim 7$ TeV in order to be consistent with precision gauge coupling unification.

At smaller values of $m_{3/2}$ the lighter chargino is below its experimental limit. Also shown in Fig. 1 are the corresponding LSPs.

To see the impact of smaller $m_0$ let us also consider the case where not only the gaugino masses but also the scalar masses are suppressed against the gravitino mass, e.g. $m_0^{16} = \frac{1}{100} m_{3/2}$. The corresponding plots are also shown in Fig. 1. Sample spectra for the two cases can be found in Table 1.

While this Letter is mainly concerned with precision gauge coupling unification, there are circumstances under which positive values of $\epsilon_3$ might also be interesting. Let us briefly discuss an example. If one is to accommodate negative $\epsilon_3$ in a concrete model, as it occurs in the 'usual' scenarios, one needs to have colored states lighter than the GUT scale. The most straightforward possibility for such states are the Higgs triplet partners (see e.g. [26]). However,\(^2\) Here we require that the lightest Higgs mass be above 112 GeV rather than above 114.4 GeV, since the theoretical error on the Higgs mass is about 2–3 GeV. Further, we are using version 3.0.9 of SOFTSUSY with a top quark mass of 173.1 GeV and $\alpha_s = 0.1176$. 
there are tight constraints on the masses of such states coming from proton decay as such triplets induce dangerous dimension five operators, which disfavor this possibility. On the other hand, a positive $\epsilon_3$ would allow for color triplets which are substantially heavier than $M_{\text{GUT}}$, thus ameliorating the problems with dimension five operators. For example in SU(5) the contribution to $\epsilon_3$ of the color triplet Higgs is given by \[\epsilon_3^{\text{Higgs}} = \frac{3\alpha_{\text{GUT}}}{5\pi} \log \left( \frac{M_{\tau}}{M_{\text{GUT}}} \right)\] (7) with $M_{\tau}$ the effective mass of the Higgs triplet. Hence for positive $\epsilon_3$ the triplet Higgs can naturally be heavier than $M_{\text{GUT}}$ (see e.g. \[28\] for a discussion on proton stability and an upper bound on $M_{\tau}$). In scenarios with positive $\epsilon_3$ the gluino becomes even lighter. Sample spectra with $\epsilon_3 = 0.02$ are shown in Table 2.

### 3.2. Non-singlet F-terms

Another possible source for non-universal gaugino masses is to have non-singlet chiral fields which break supersymmetry by F-term vacuum expectation values. These fields have to transform as the symmetric product of two adjoint representations of the GUT group, but not necessarily as singlets. For concreteness we focus here on SU(5) GUTs. In the following we concentrate on the two smallest possible representations for the supersymmetry breaking fields, which are simply the singlet and the 24-plet. Including also the 75 and 200 would of course widen the available parameter space. Similar relations hold for SO(10).

The high-scale mass patterns of the gauginos of SU(3)$_C$, SU(2)$_L$ and U(1)$_Y$ turn out to be given as linear combinations of singlet $(m_{1/2})$ and 24-plet $(m_{24})$ contributions \[29\],

\[
M_1 = m_{1/2} - \frac{1}{2}m_{24},
\]

\[
M_2 = m_{1/2} - \frac{3}{2}m_{24},
\]

\[
M_3 = m_{1/2} + m_{24}. \tag{8}
\]
In Fig. 2 we show \( \epsilon_3 \) as a function of the two gaugino mass parameters \( m_{1/2} \) and \( m_{24} \) for fixed \( m_0, m_0^{Higgs}, A_0 \) and \( \tan \beta \). We find that in the allowed regions \(-0.04 \leq \epsilon_3 \leq 0.03\). The white regions are again excluded. The excluded regions are mainly due to the Higgs LEP bound. A sample spectrum is shown in Table 1.

### 3.3. GMSB with Higgs messenger mixing

As a last example of a supersymmetry breaking mechanism which can lead to precision gauge coupling unification let us look at a particular version of gauge mediation [30–32]. The boundary conditions at the messenger scale \( M \sim M_{GUT} \) are determined by five SUSY breaking parameters, \( \Lambda \sim 10^5 \text{ GeV} \), \( a \sim b \sim 0.01–0.1 \), \( \mu \) and \( B_H \). In addition a \( D \)-term contribution is needed in order to obtain a phenomenologically acceptable theory [32]. The boundary conditions for the gauginos and scalars at the scale \( M \) read

\[
M_1 = \frac{3}{5} \frac{\alpha_1(M)}{4\pi} \Lambda \left( 1 + \frac{20}{3} b^2 \right), \\
M_2 = \frac{\alpha_2(M)}{4\pi} \Lambda (1 + 4b^2), \\
M_3 = \frac{\alpha_3(M)}{\pi} A b^2, \\
m^2 = 2A^2 \left[ C_1 \left( \frac{\alpha_1(M)}{4\pi} \right)^2 \left( \frac{3}{5} + \frac{2}{5} a^2 + 4b^2 \right) + C_2 \left( \frac{\alpha_2(M)}{4\pi} \right)^2 (1 + 4b^2) + C_3 \left( \frac{\alpha_3(M)}{4\pi} \right)^2 (a^2 + 4b^2) \right]
\]

with \( C_1 = \frac{3}{8} y^2 \), \( C_2 = \frac{3}{4} \) for weak doublets and zero otherwise and \( C_3 = \frac{3}{4} \) for color triplets and zero otherwise. The additional \( D \)-term contributions to the scalar masses are parameterized as

\[
\delta_D m^2 = d Q_a^X M_2^2
\]

with \( d \) an arbitrary parameter of order one and \( Q_a^X \) the \( U(1)_X \) charge of the field with label \( a \). In the following we will assume that \( Q_a^X \) is (1) for the matter fields and (2) for the Higgs fields [32].

In Fig. 3 we show again the contours of different \( \epsilon_3 \) for the boundary conditions (9) at \( M = M_{GUT} \) and \( a = 0.01 \), \( A_0 = 0 \), \( \tan \beta = 10 \) and \( d = 0 \) or \( d = 1 \). In the region with small values of \( b \) the gluino is the NLSP with the gravitino being the LSP and \( \epsilon_3 \) is positive. A sample spectrum is shown in Table 2. Precision gauge coupling can be achieved for larger values of \( b \) where the neutralino is the NLSP. A sample spectrum is shown in Table 1.

### 4. Phenomenological implications

#### 4.1. Prospects for the LHC

Let us now briefly comment on possible phenomenological implications of the requirement of precision gauge coupling unification. Sample spectra for the considered models are shown in Table 1. One common feature of the models which admit precision gauge coupling unification seems to be that the gauginos are rather light while the scalars are typically somewhat heavier. It is therefore well possible that the gauginos are within the kinematical reach of the LHC while the scalars are not. At the LHC one would then expect that supersymmetric particles are mainly produced via gluino pair production, resulting in a signature with at least four jets plus missing energy. With an integrated luminosity of 100 fb\(^{-1}\) the LHC could probe gluino masses up to \( \sim 2 \) TeV [33]. As the squarks are heavier than the gluinos, the gluinos can either appear as three-body decay via a virtual squark or two-body loop decay (for branching ratios see e.g. [34,35]). The latter process is usually suppressed. As can be seen in Table 1 the lighter stop is the lightest among the squarks, implying that the gluino decays will produce a high multiplicity of top quarks [33], if kinematically accessible. To summarize, squarks may be so heavy that they cannot be directly produced, in which case their masses have to be reconstructed by investigating the gluino decay processes, where they appear as intermediate states.

For positive \( \epsilon_3 \) the gluino is likely to be the lightest MSSM superpartner, in which case it may be long-lived. Depending on the lifetime the gluino hadronizes, travels macroscopic distances and might even escape the detector (see e.g. [36]). Potential signatures of such a scenario could therefore be displaced vertices or metastable R-hadrons which could be stopped in- or outside the...
Fig. 3. Contours of different $\epsilon_3$ (upper panels) as well as the corresponding (N)LSPs (lower panels) for the GMSB with Higgs messenger mixing with $a = 0.01$, $A_0 = 0$ and $\tan \beta = 10$. The white region is excluded as before. The yellow and pink stars denote points in parameter space for which we show the spectra in Tables 1 and 2. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this Letter.)

4.2. Further comments on scenarios with a low gluino mass

As is well known, scenarios with low gluino masses can also be interesting in the context of the little hierarchy problem. This is because the gluino mass has the largest impact on the Higgs mass parameters (cf. [37]). Given a specific measure of “fine-tuning” one also finds that low gluino masses increase naturalness (cf. e.g. [38, 39] for recent discussions). On the other hand, one should also mention that a large gluino mass has the virtue that the supersymmetric flavor problem gets ameliorated by flavor-universal contributions from the gluino in the renormalization group [40].

5. Summary

We have shown that it is possible to have precision gauge coupling unification in the MSSM without invoking high-scale thresholds. This can be simply achieved in schemes in which the gaugino masses are non-universal at (or slightly below) the GUT scale. Such non-universal gaugino masses are consistent with unified theories. They occur, for instance, in settings in which the tree-level value of the gaugino masses are suppressed such that quantum corrections, such as 'anomaly mediated' contributions, become important. They can also be obtained in settings in which supersymmetry is broken by a field furnishing a non-trivial GUT representation as well as in GMSB with Higgs-messenger mixing. We also note that $\epsilon_3$ can be positive. Interestingly, there is a correlation between low fine-tuning and precision gauge coupling unification. Both features can be achieved by having a rather light gluino. The emerging schemes have the virtue that it will be relatively easy for the LHC to produce gluinos copiously. On the other hand, squarks may turn out to be too heavy to be directly produced such that one would have to reconstruct their masses by analyzing gluino decays. There is also the possibility of displaced vertices from gluino decay as well as meta-stable gluino bound states stopped in the detector.
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References