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ABSOLUTE DECAY RATE FROM $K^0 \rightarrow \pi^+ + \pi^- + \pi^0$
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Donald Stern, Thomas O. Binford, V. Gordon Lind,
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February 4, 1964
ABSOLUTE DECAY RATE FOR $K_2^0 \rightarrow \pi^+ + \pi^- + \pi^0$

AND THE $|\Delta I| = 1/2$ RULE

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February 4, 1964

In this Letter we describe a measurement of the absolute decay rate

$\Gamma_2(\pm 0) = \Gamma(K_2^0 \rightarrow \pi^+ \pi^- \pi^0)$. Our result is based on 16 events of the type $\pi^- p \rightarrow \Lambda K^0$ followed by $\Lambda \rightarrow p \pi^+$ and $K_2^0 \rightarrow \pi^+ \pi^- \pi^0$, and 2608 double-vee events $\pi^- p \rightarrow \Lambda K^0$ with $\Lambda \rightarrow p \pi^-$ and $K_4^0 \rightarrow \pi^+ \pi^-$. We find

$$\Gamma_2(\pm 0) = (2.90 \pm 0.72) \times 10^6 \text{ sec}^{-1}. \quad (1)$$

This result can be compared with the prediction of the $|\Delta I| = 1/2$ rule for nonleptonic decays, that $\Gamma_2(\pm 0)$ and $\Gamma(\pm 00) = \Gamma(K^+ \rightarrow \pi^+ \pi^0 \pi^0)$ are related by

$$\Gamma_2(\pm 0) = 2(1.032) \Gamma(\pm 00), \quad (2)$$

where the factor 1.032 corrects for small mass differences. Equation (2) holds for any linear combination of the three $I = 1$ three-pion states. Taking $\Gamma(\pm 00)$ from a compilation of $K^+$ branching ratios and lifetimes, one obtains the prediction of the $|\Delta I| = 1/2$ rule,

$$\Gamma_2(\pm 0) = (2.87 \pm 0.23) \times 10^6 \text{ sec}^{-1}. \quad (3)$$
The excellent agreement between our experimental result (1) and the prediction (3) shows that the $|\Delta I| = 1/2$ rule is well satisfied. The ratio $\Gamma_2(\rightarrow 0)/\Gamma(\rightarrow 00)$ affords a sensitive test of the $|\Delta I| = 1/2$ rule. To exhibit this sensitivity we parameterize the effect of a small $|\Delta I| = 3/2$ amplitude $A_{3/2}$ for $K \rightarrow 3\pi$ under the assumption that the dominant $|\Delta I| = 1/2$ amplitude leads to the symmetrical $I = 1$ three-pion state, but with no such restriction on the $|\Delta I| = 3/2$ amplitude. Then, if $|A_{3/2}/A_{1/2}|^2$ is neglected, our experimental result expressed in the notation of reference 2 becomes

$$\sqrt{2} \quad \text{Re} \left( \frac{A_{3/2}}{A_{1/2}} \right) = \frac{2(1.032)\Gamma(\rightarrow 00) - \Gamma_2(\rightarrow 00)}{4(1.032)\Gamma(\rightarrow 00) + \Gamma_2(\rightarrow 00)}$$

(4)

$$= 0.00 \pm 0.09.$$  

(5)

The remainder of this paper is concerned with experimental details.

The Alvarez 72-inch hydrogen bubble chamber was exposed to $\pi^-$ beams with momenta between 1035 and 1325 MeV/c. All film was scanned for single and double vees which were analyzed using the least-squares fitting program KICK. For the present experiment events of the type $\pi^- p \rightarrow \Lambda K^0$ were used; $\pi^- p \rightarrow \Sigma^0 K^0$ events were not used. All single-$\Lambda$ events were rescanned along the direction of the unobserved $K^0$, as predicted by the fitting program. We believe that the resulting overall efficiency for finding associated $K$ decays is nearly 100%.

Most of the double vees fit the hypothesis $\pi^- p \rightarrow \Lambda K^0$ with $\Lambda \rightarrow p\pi^-$ and $K^0 \rightarrow \pi^+ \pi^-$. Those within the fiducial volume that fail because the assumption $K^0 \rightarrow \pi^+ \pi^-$ fails are analyzed as follows. The fitted parameters for the decay $\Lambda \rightarrow p\pi^-$ are propagated to the production vertex. At the production vertex we perform a 3-constraint (3-C) fit using the decay-fitted $\Lambda$, the incident $\pi^-$, and...
and the measured direction of the $K^0$ as determined from the two-point neutral track. We accept events with production $\chi^2(3-C) < 25.6$. The $K^0$ momentum vector is then well known and is used as input information for the following hypotheses:

A. $K^0 \rightarrow \pi^+ \pi^- \pi^0 (1-C, \pi^0 \text{ unseen})$. For $\chi^2(1-C) < 10$, the decay is a $\tau^0$ candidate. About five events having an obvious $e^\pm$ or $\mu^\pm$ track are discarded. There are then 20 candidates.

B. $K_1^0 \rightarrow 2\pi^0 \rightarrow e^+ e^- \gamma \pi^0$. We assume that the charged tracks are electrons and calculate their invariant mass $m(e^+ e^-)$. For $m(e^+ e^-) < 85$ MeV, the event is to be discarded, unless we can prove by other means that this hypothesis is wrong. We expect a total of about 32 Dalitz decays in the entire experiment, and of these we expect 99% to have $m(e^+ e^-) < 85$ MeV. None of the candidates is discarded.

C. $K^0 \rightarrow \pi \nu, \pi \pi \nu, \text{ or } \pi \pi \gamma (1-C \text{ fit})$. Candidate 1720440 has $\chi^2(\pi^+ e^- \nu) = 2.6$ and $\chi^2(\pi^+ \pi^- \pi^0) = 2.9$; candidate 1739122 has $\chi^2(\pi^+ e^- \nu) = 0.3$, $\chi^2(\pi^+ \pi^- \pi^0) = 6.2$. Gap counting on the negative track unambiguously proves these events both are $\pi^+ e^- \nu$. Candidate 1492562 has $\chi^2(\mu^+ \pi^- \nu) = 0.0$ and $\chi^2(\pi^+ \pi^- \pi^0) = 3.8$; gap-counting does not resolve the ambiguity. From our 1-C $\chi^2$ distribution, (reference 8) we bet 14 to 2 against $\pi^+ \pi^- \pi^0$. In addition, any completely ambiguous 3-body decay would have a priori about 5/1 odds against $\pi^+ \pi^- \pi^0$ on the basis of known branching ratios. We should perhaps count this as 1/35 event; instead we discard the candidate. Candidate 1458048 is nearly unmeasurable and is completely ambiguous. It should perhaps count as 0.2 events, but we discard it. None of the remaining 16 $\tau^0$ candidates is ambiguous.

D. $K_4^0 \rightarrow \pi^+ \pi^-; \pi^+ (\text{or } \pi^-)$ suffers a small (unobserved) scatter. We delete the $\pi^+ (\text{or } \pi^-)$ and fit (1-C) to the hypothesis $K_4^0 \rightarrow \pi^+ (\text{unmeasured}) + \pi^- (\text{measured})$ (and also with the signs reversed). For $\chi^2(1-C) < 10$, the
event is a possible Coulomb scatter. However, we do not reject the event as a \( \tau^0 \) candidate unless the "scattered" pion satisfies 
\[
(p_\beta)_{\text{fitted}} \mid \theta_{\text{fitted}} - \theta_{\text{measured}} \mid \leq 2000 \text{ (MeV/c) deg.}
\]
This condition is chosen after considering the form and magnitude of the Rutherford-scattering cross section, so that out of 3000 normal double vees, only a calculated 0.3
\( K_4^0 \rightarrow \pi^+ \pi^- \) decays followed by a single Coulomb scatter will fail to be cut off. This cutoff also eliminates \( K_4^0 \rightarrow \pi^+ \pi^- \pi^0 \) decays followed by a small-angle decay \( \pi^\pm \rightarrow \mu^\pm \nu \) in flight. Small-angle nuclear scatters are also eliminated, but in any case they are estimated to be negligible. None of the 16 \( \tau^0 \) candidates are rejected by this cutoff.

We are left with 16 \( \tau^0 \) events. There are no correction factors for lost or cutoff events. The characteristics of the events are exhibited in Table I.

Their time distribution is shown in Fig. 1. There is no evidence for an enhanced \( \tau^0 \) decay rate within the first \( K_4^0 \) mean life. This agrees with the expectation that the rate for \( K_4^0 \rightarrow \pi^+ \pi^- \pi^0 \) is negligible compared to that for \( K_2^0 \rightarrow \pi^+ \pi^- \pi^0 \). We therefore use all of the \( \tau^0 \) decays, including those within a few \( K_4^0 \) mean lives.

The rate \( \Gamma_2(\pi^0) \) is equal to \( 1.014 N(\pi^0)/T_2 \), where \( T_2 \) is the total of the \( K_2^0 \) flight times in the fiducial volume, and \( N(\pi^0) = 16 \) is the number of observed \( \tau^0 \) decays. We obtain \( T_2 \) by using the 2608 acceptable normal double vees. The acceptance criteria for \( \Lambda \rightarrow p \pi^- \) are independent of the \( K \) decay mode. For a given acceptable \( \Lambda \), the probability of obtaining an acceptable \( K_4^0 \rightarrow \pi^+ \pi^- \) decay and hence an acceptable double vee is given by

\[
P = \{1/2\} B[\exp(-t_0/\tau_4) - \exp(-t_4/\tau_4)].
\]

Here the factor \( 1/2 \) arises from \( \left| \left| K^0 \right| \right|^2 = \{1/2\} \left| \left| K_4^0 \right| \right|^2 \), \( B \) is \( \Gamma(K_4^0 \rightarrow \pi^+ \pi^-)/\Gamma(K_4^0 \rightarrow \text{all } 2\pi) = 0.725 \), \( \tau_4 \) is \( 0.90 \times 10^{-10} \) sec, and \( t_0 \) and \( t_4 \) are the minimum and maximum acceptance times for.
$K_+^{0 \rightarrow \pi^+ \pi^-}$ Corresponding to an accepted normal double vee, the average number of acceptable $\Lambda$'s is $1/P$, and their expected contribution to $T_2$ is $(1/2)(1/P)t_4$.\(^{14}\) We then obtain $T_2$ by summing over all acceptable normal double veses. $T_2 = \sum (1/2)(1/P)t_4 = B_{-1} \sum [\exp(-t_0/\tau_{-1}) - \exp(-t_1/\tau_{-1})]^{-1} t_4$.\(^{15}\)

We find $BT_2 = 4.06 \times 10^{-6}$ sec. Our final result is:

$\Gamma_2^+ (\rightarrow 0) = (1.044) \frac{16}{[4.06 \times 10^{-6} / 0.725]} = (2.90 \pm 0.72) \times 10^6 \text{ sec}^{-1}$.\(^{11}\)
It is a pleasure to acknowledge the advice and encouragement of Luis W. Alvarez. We are indebted to Gideon Alexander and Silverio P. Almeida for their contributions to the experiment, and to our scanners and measurers for their excellent work.
FOOTNOTES AND REFERENCES

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4. Four of the present 16 decays $K^0 \to \pi^+ \pi^- \pi^0$ are included in the experiment of Alexander et al. Based on these four decays, and a correspondingly smaller number of associated production events $\pi^\pm p \to \Delta K^0$, Alexander et al. obtained the direct result

$$\Gamma_2(\pm 0) = (2.66 \pm 1.34) \times 10^6 \text{ sec}^{-1}$$

in agreement with the present result based on the complete sample of 16. These authors also obtained

$$\Gamma_2(\pm 0)$$

indirectly, using their own absolute decay rate for $K^0 \to$-leptons (based on 14 events) and the branching ratio $\Gamma_2(\pm 0)/\Gamma_2$ (all charged) of D. Luers, I. S. Mittra, W. J. Willis, and S. S. Yamamoto, Phys. Rev. Letters 7, 255 (1964). In this way they obtained

$$\Gamma_2(\pm 0) = (1.44 \pm 0.43) \times 10^6 \text{ sec}^{-1}$$

in poor agreement with their own direct result, with the result of the present complete sample, and with the value predicted by the $|\Delta I| = 1/2$ rule.

5. The decay rate $\Gamma_2(\pm 0)$ can also be obtained indirectly by combining the $K^0$ lifetime $\tau_2$ with the branching ratios $\nu = \Gamma_2(000)/\Gamma_2(ch)$ and
\[ \lambda = \frac{\Gamma_2(\pm 0)}{\Gamma_2(\text{ch})}, \] under the assumption that there are no additional unobserved neutral modes, like \( K_2^0 \rightarrow \gamma \gamma \). R. H. Dalitz, Proc. Brookhaven Conf. on Weak Interactions (Brookhaven National Laboratory, September 1963), has combined the available data on \( \tau_2, \nu, \) and \( \lambda \) from eight different experiments, to obtain \( \Gamma_2(\pm 0) = (1.93 \pm 0.35) \times 10^6 \text{ sec}^{-1} \). This result differs by 2.3 standard deviations from the prediction of Eq. (3). In the notation of Eq. (4), it corresponds to
\[ \sqrt{2} \text{ Re} \left( \frac{A_{3/2}}{A_{1/2}} \right) = 0.42 \pm 0.06. \]

6. The 3-C production \( \chi^2 \) distribution for the final 16 \( \tau^0 \) events is as follows:
- \( \chi^2 = 0 \) to 3.67, 8 events (we expect 11.20); 3.67 to 7.82, 5 events (we expect 4.00); 7.82 to 16.27, 3 events (we expect 0.78); > 16.2, zero events (we expect 0.02). Thus the expected and observed \( \chi^2 \) distributions are in excellent agreement. Since \( \Lambda \) production and decay occurs only about once in 30 pictures, there is only one chance in 900 of finding a 3-body \( K^0 \) decay with a possibly ambiguous origin. (We do not use single-\( \Lambda \) 3-body \( K^0 \) decays.)


8. The 1-C decay \( \chi^2 \) distribution for the 16 final \( \tau^0 \) events is as follows:
- for \( \chi^2 = 0 \) to 1.07, 7 events (we expect 11.2); 1.07 to 3.84, 7 events (expect 4.00); 3.84 to 6.63, 2 events (expect 0.64); > 6.63, zero events (expect 0.02).

9. The decay \( K_4^0 \rightarrow \pi^+ \pi^- \pi^0 \) is forbidden for totally symmetric 3\( \pi \) states, and the contribution from nonsymmetric states is expected to be small because of angular-momentum barrier effects. See for instance, S. Treiman and S. Weinberg, Phys. Rev. 116, 239 (1959).

10. The mean decay distance for \( K_2 \) is large compared to the bubble chamber. A small correction factor of 1.014 arises from the total attenuation by
decay of the $K_2$'s. The attenuation by interaction in the hydrogen is even less important and is neglected.


13. In the film analyzed at Wisconsin $t_0 = 0$ was used. At Berkeley $t_0$ corresponded to a cutoff at 0.8 cm. The time $t_1$ is the potential proper time corresponding to the decay fiducial volume. The production fiducial volume is slightly smaller than the decay fiducial volume, so that large values of $1/P$ are excluded.

14. We impose no $t_0$ cutoff for $\tau^0$ decays.

15. In reference 3, the procedure was to use all of the acceptable $\Lambda$ decays, irrespective of whether there is an acceptable $K_4^0$ decay, and sum over the calculated potential $K^0$ times. In that case one need not use the value of $B$. However, $B$ is extremely well known, so that the two methods are equivalent. This was verified by comparing the methods in the film analyzed at Berkeley (75% of the total).
Table I. Details of the decays. $\chi_p^2$ is for the production (3-C), $\chi_d^2$ is for the decay (1-C); $p_{K^0}(\text{lab})$ is obtained from the production fit; $t_{K^0}$ is the $K^0$ proper time from production to decay. $T_+$, $T_-$, and $T_0$ are the decay pion kinetic energies in the $K^0$ rest frame.

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<th>$\chi_d^2$</th>
<th>$p_{K^0}(\text{lab})$ (MeV/c)</th>
<th>$t_{K^0}$ (10^{-10} sec)</th>
<th>$T_+$ (MeV)</th>
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FIGURE LEGEND

Fig. 1. Proper time distribution of $K_2^0 \rightarrow \pi^+ \pi^- \pi^0$ events. The vertical lines give the individual times of the 16 events. The smooth curve is their expected time distribution; its shape is entirely determined by the fiducial volume and the $K^0$ momentum distribution, because attenuation of the $K_2^0$ by decay is almost negligible. The histogram of the 16 events has the same normalization as the smooth curve.
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