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THE RESISTANCE OF SUPERCONDUCTING-NORMAL METAL-SUPERCONDUCTING SANDWICHES

Michael Lynn Rappaport
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THE RESISTANCE OF
SUPERCONDUCTING-NORMAL METAL-SUPERCONDUCTING SANDWICHES

Michael Lynn Rappaport

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TO THE REFRIGERATOR

Down—toward the mystery, the shadow world. You travel
Through this tangled mesh of confused actions
And find your rest of separated fractions
Where heliums are distinct and amplitudes unravel.

As God once swept the seas from land, all things made clear
So do your gasses now condense
Heat's mad passion, anarchy, is stilled. Symmetry and sense
Shape space as Lord—and men revere.

We serve you, pour LN and seal each leak
We are the drones who ease your slide to ground
And when the final millidegree is found
You shall reward us with the trace we seek.

The upper world may swirl in rapture, fear, and hate
But you—stay cool, stay low, refrigerate.

Physics Today/May 1973

This ode was written to the author's He³-He⁴ dilution refrigerator by his friend, Robert L. Kovsky, during the time the measurements reported in this thesis were made.
THE RESISTANCE OF
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THE RESISTANCE OF
SUPERCONDUCTING-NORMAL METAL-SUPERCONDUCTING SANDWICHES

ABSTRACT

The resistance, \( R \), of superconducting-normal metal-superconducting (SNS) sandwiches was measured as a function of temperature, \( T \), and measuring current, \( I \), usually from 20 mK to \( T_{cS} \). A semi-phenomenological theory for the resistance of these sandwiches below \( T < 1 \) K which includes both temperature and current dependance was derived using the de-Gennes theory of the proximity effect and the idea of Andréé reflection. Comparing experimental and theoretical \( R \) vs \( T \) curves for \( T < 1 \) K allows us to estimate the magnitude of the induced pair potential in \( N \), \( \Delta_N \), and then to deduce the values of the BCS interaction parameter \( [N(O)V]_N \), where \( N(0) \) is the density of states of one spin at the Fermi surface and \( V \) is the net electron-electron coupling interaction. The values of \( [N(O)V]_N \) found for Rh, Cu, and Ag are 0.078 ± 0.010, 0.045 ± 0.010, and 0.015 ± 0.005, respectively.

The large current dependence of resistance at high currents and at low temperatures indicated the necessity of accounting for the effect of the current on \( \Delta_N \). However, comparison of experimental and theoretical \( R \) vs \( I \) curves showed that this effect is not well understood.

Near \( T_{cS} \) the theory of the resistance of SNS sandwiches is presented most clearly by Waldram. Comparing theoretical curves of \( R \) vs \( T \) based on his work to our experimental curves for \( T > 1 \) K allowed us to
estimate the relaxation time, \( \tau \), for the excitation spectrum branch imbalance in S due to the transmission into S of excitations from N. The magnitude and temperature dependence of \( \tau_Q \) agree well with the theory of Tinkham for Sn, where we find \( \tau_Q(0) = (6 - 9) \times 10^{-10} \) sec. Waldram's theory is not strictly correct for Pb, so the agreement between experiment and theory was not as convincing. Our estimate of \( \tau_Q \) in Pb is \( (5-9) \times 10^{-11} \) sec.

Finally, we also point out that branch imbalance relaxation also occurs within N if \( \Lambda_N \neq 0 \) (because of the proximity effect) and there is a current flowing normal to the interface. It is shown that Andrev reflection is a special case of elastic branch crossing due to energy gap anisotropy.
I. INTRODUCTION

The present work began as an effort to determine experimentally the interaction parameter \([N(0)V]_N\) (\(N(0)\) is the density of states of electrons of one spin at the Fermi surface and \(V\) is the net electron-electron coupling interaction) of the BCS\(^1\) theory of superconductivity in a number of "normal" metals whose superconducting transition temperatures, \(T_{cN}\), are too low to measure directly. Using the BCS expression

\[ T_c = 0.85 \theta_D \exp \left( -1/[N(0)V] \right) \]

\(\theta_D\) is the Debye temperature, we could then estimate when these metals would go superconducting.\(^2\)

The method chosen to determine \([N(0)V]_N\) takes advantage of the "proximity effect." This requires placing the normal metal, \(N\), in intimate contact (i.e. no boundary layer oxide) with a superconductor, \(S\), so that the Cooper pairs in \(S\) "spill over" into \(N\) and induce a pair potential, \(\Delta_N\), therein. (Conversely, "normal" electrons from \(N\) spread into \(S\) and depress its pair potential, \(\Delta_S\). From the values of \(\Delta_N(0)\) and \(\Delta_S(0)\), the pair potentials at the interface of \(N\) and \(S\), respectively, we can then calculate \([N(0)V]_N\) using the boundary condition.

\[
\frac{\Delta_N(0)}{[N(0)V]_N} = \frac{\Delta_S(0)}{[N(0)V]_S}
\]

\(\Delta_N(0)\) derived by de Gennes\(^5\) and the value of \(\Delta_S(0)\) calculated by McMillan.\(^6\)

The values of \(\Delta_N(0)\) and \(\Delta_S(0)\) are determined by measuring the resistance, \(R\), as a function of temperature of an \(SNS\) sandwich. Typical values of \(R\) are \(\sim 10^{-7}\). A current of 1 - 100 mA is passed through the specimens and the voltage is measured with a superconducting voltmeter.
employing a SLUG or SQUID. Maximum resolution is \( \approx 10^{-14} \) volts. Since
the voltage across the sandwich is due only to the excitation (i.e. normal)
current and not the supercurrent, the resistance should be proportional to
the length of \( N \) in which the excitation current exists. The point in \( N \)
where the current changes from "excitation" to "super" was shown by
Andreev\(^7\) to be at the distance, \( x \), from the interface where the energy of
the excitations, \( E \sim k_B T \), is equal to \( \Delta_N(x) \).

In the limit of zero measuring current, \( I \), the Ginzburg-Landau (G-L)
equation\(^8\) gives \( \Delta_N(x) \sim \Delta_N(0) \exp(\exp(-k_N x)) \), where \( k_N^{-1} \) is the temperature
dependent Cooper pair decay length in \( N \). \( k_N^{-1} \) is only weakly dependent on
\( [N(0)V]_N \) for \( T \gg T_{cN} \).

Measurements of \( R \) vs \( I \), however, showed that the effect of the mea-
suring current was not negligible at low temperatures. As in a bulk super-
conductor, \( I \) produces a magnetic field which suppresses the pair potential;
the percentage change in the pair potential depends on its magnitude.
Therefore there is relatively little effect on \( \Delta_N \) near the interface where-
as at some distance from the interface the suppression increases to 18%.
Further from the interface \( \Delta_N(x) = 0. \)

Combining these ideas, we arrive at an expression for \( R(T) \) with

\[
[N(0)V]_{Rh} = 0.089 \pm 0.010, \quad [N(0)V]_{Cu} = 0.045 \pm 0.010, \quad \text{and} \quad [N(0)V]_{Ag} = 0.015 \pm 0.005.
\]

If \( R \) is to be sensitive to \( [N(0)V]_N \), it is clear from the de Gennes
boundary condition and the theory of Andreev that the temperature of the
experiment must be below \( \frac{\Delta_N(0)}{k_B} \). For this reason a dilution
refrigerator, capable of attaining 18 mK, was used. It is also clear that
the superconductor which will allow the detection of the smallest \( [N(0)V]_N \)
is the one with the largest ratio of \( [N(0)V]_S \) to \( \Delta_S(0) \). Because of its
low diffusion rate with most of the normal metals of interest and because of the ease of deposition, lead was usually chosen. Since \( \frac{\Delta_S(0)}{k_B} \approx 9.9 \text{ K} \) for \( \text{Pb} \) and \( [N(0)V]_{\text{Pb}} \approx 0.56 \), we were capable of detecting \( N(0)V \) in Pb with a sensitivity of \( 0.001 \).

Unsuccessful attempts to determine \( [N(0)V] \) in Au, Mg and Pt were also made. The Au apparently alloyed with the Pb. The surface of the Mg was not successfully cleaned before deposition of the Pb and leaks in the dilution refrigerator did not enable us to cool the Pb-Pt-Pb sandwiches below \( \sim 0.1 \text{ K} \).

Two other normal metals were also made into SNS sandwiches. The first was Ir, a known superconductor. Pb-Ir-Pb and Sn-Ir-Sn sandwiches were used as a check on the theory. The second was Ni, a ferromagnet. Since the internal field of the Ni should destroy the Cooper pairs very close to the interface, no temperature dependence of the resistance at low temperatures should be observed. The Pb-Ni-Pb sandwiches were used as a check on the experimental procedure.

In addition to being able to estimate \( [N(0)V]_N \), it was later realized that these same SNS sandwiches could provide some interesting information about S. This is because the average excitation energy, \( k_BT \), becomes larger than the bulk pair potential in S, \( \Delta_S(\infty) \), when the sample temperature is above \( \sim 0.9 T_{cs} \). Consequently, as was first demonstrated by Pippard, Shephard, and Tindall \( ^{10} \) (PST) and explained by PST \( ^{10} \) and Waldram \( ^{11} \), the excitation current in N is able to flow into S and thereby produce an extra voltage across S. The value of this extra voltage is proportional to the measuring current and depends on the decay time for the non-equilibrium excitation distribution in S. We show that one relaxation time important in the decay is the branch imbalance relaxation
time, $\tau_Q$, first measured by Clarke$^{12}$ and evaluated theoretically by Tinkham and Clarke$^{13}$ and in more detail by Tinkham.$^{14}$ From the increase in sandwich resistance near $T_{cS}$ we find $\tau_Q(Sn) = (6-9) \times 10^{-10}$ sec and $\tau_Q(Pb) = (5-9) \times 10^{-11}$ sec.

Although it first appeared that analysis of $R$ versus $T$ in SNS sandwiches required that the behavior of the excitations be treated differently in $N$ and $S$, i.e., in terms of Andreev reflection and a decay time, respectively, we show that the latter describes their behavior at least qualitatively in both metals. In other words, whether in $N$ with an induced pair potential or in $S$ below $T_{cS}$, a non-equilibrium excitation distribution has a finite lifetime. This lifetime determines how far within $N$ or $S$ an excitation can travel. The dissipation, and thus the resistance, is proportional to the length of travel.
II. THEORY

A. Pair Potential

In this section we determine the variation of the pair potential, \( \Delta(x) \), with distance from an SN interface. This has been calculated by a number of authors. Unfortunately, the calculations have been done in a number of limits which do not apply exactly to our problem: i.e., it is assumed that the metals are clean (the coherence lengths, \( \xi_N(T) \) and \( \xi_S(T) \) of N and S, are much less than the mean free paths, \( l_N \) and \( l_S \), respectively) or dirty (\( \xi(T) \ll l \)), the layers are thin (thicknesses, \( d_N \) and \( d_S \), are much smaller than the pair decay length, \( k_N^{-1}(T) \) and \( k_S^{-1}(T) \)), \( [N(0)V]_N = 0, \) \( T = 0, \) or \( T \approx T_{cS} \).

Contrarily, in our SNS sandwiches, S changes from dirty to clean and N changes from clean to dirty as the temperature is lowered from \( T_{cS} \). In general, \( d_S \gg k_S^{-1} \) and \( d_N \gg k_N^{-1} \) and except in sandwiches with Ni as N, our results indicate that \( [N(0)V]_N \neq 0 \). Lastly \( T \approx T_{cS} \) is usually not a good approximation (although \( T = 0 \) often is). Our analysis of the behavior of \( \Delta(x) \) will, therefore, be rough at best.

We start with the work of de Gennes. He defines

\[
\Delta(r) = V(\mathbf{r})f(\mathbf{r})
\]

(1)

where \( V(\mathbf{r}) \) is the usual net electron-electron interaction parameter and \( f(\mathbf{r}) \), the so-called condensation amplitude, is the probability amplitude for finding a pair at \( \mathbf{r} \). \( \Delta(\mathbf{r}) \) obeys Gor'kov's self-consistency equation

\[
\Delta(\mathbf{r}) = V(\mathbf{r})f(\mathbf{r}) + \int d^3\mathbf{r}' H(\mathbf{r},\mathbf{r}') \Delta(\mathbf{r}'),
\]

(2)
where $\omega = \frac{n \hbar k_B T (2n+1)}{\hbar}, n = 0, \pm 1, \cdots$. Equation (2) is a linear approximation valid only for small values of $\Delta(r)$, usually requiring $T \approx T_C$.

Assuming that the metals are dirty, de Gennes derived the Fourier transform of $H_\omega(\mathbf{r}, \mathbf{r'})$,

$$H_\omega(q) = \frac{2\pi N(0)}{2|\omega| + Dq^2},$$

where $D = \frac{1}{3} v_F \ell$ is the diffusion constant ($v_F$ is the Fermi velocity).

The one dimensional Fourier transform of $H_\omega(q)$ for the case where $\Delta$ depends on only one space coordinate is

$$H_\omega(xx') = \frac{\pi N(0)}{2|\omega| \xi_\omega} e^{-|x-x'|/\xi_\omega}.$$  

(4)

$\xi_\omega \equiv (D/2|\omega|)^{1/2}$ gives the range of $H_\omega(xx')$. Its largest value corresponds to $\omega = \omega_o \equiv \frac{\pi k_B T}{\hbar}$:

$$\xi_{\omega_o} \equiv \xi_D(T) = \left(\frac{\hbar D}{2\pi k_B T}\right)^{1/2} = \left(\frac{\hbar v_F \ell}{6\pi k_B T}\right)^{1/2}.  

(5)

$\xi_D$ is the coherence length in the dirty limit.

There is a different coherence length in the clean limit, given by

$$\xi_C(T) = \frac{\hbar v_F}{2\pi k_B T}.$$  

(6)

When $\ell \sim \ell$, neither $\xi_D(T)$ nor $\xi_C(T)$ is appropriate. Silvert and Singh have found a generalized coherence length said to be valid for any mean free path:
\[ \xi(T) = \left( \frac{1}{3} \xi_c \xi \right)^{1/2} \left( 1 + \frac{q}{3 \xi_c} \right)^{-1/2} = \xi_D \left( 1 + \frac{q}{3 \xi_c} \right)^{-1/2}. \]  

Even though de Gennes' calculation is not valid in the clean limit, it seems that \( \xi(T) \) would be a better choice than \( \xi_D(T) \) when one tries to extend his results to regimes where \( \lambda \sim \xi_c \).

The boundary conditions for \( \Delta(x)/V(x) \) are the same as for \( H_\omega(xx') \). Since \( D(x) \frac{d}{dx} H_\omega(xx') \) and \( \frac{H_\omega(xx')}{V(x')} \) are continuous across an interface,

\[ \xi_D(T) >> \lambda \] 

\[ \left( \begin{array}{c} \Delta(0) \\ \frac{N(0)V}{N} \\ D \frac{d\Delta(x)}{dx} \\ \frac{V}{dx} \end{array} \right) \text{ continuous.} \]  

(8a)  

(8b)

We see that, in general, \( \Delta \) is not continuous across an interface.

In order to solve the integral equation, Eq. (2), in closed form, de Gennes resorts to the "one-frequency approximation" for the case where \( d_S \xi_S(T) \) and \( d_N \xi_N(T) \). In this approximation only the maximum range kernel, with the range of the coherence length, Eq. (5), is retained, and all of the others are replaced by a suitably normalized \( \delta \)-function. Thus the actual rapid variation of \( \Delta \) near the interface will appear to be smoothed out. This form of \( H_\omega(xx') \) leads to

\[ (1-CN(0)V)[2\pi k_BT \Delta(x) - HD^2 \Delta(x)] = 4\pi N(0)V \Delta(x) k_BT \]  

(9)

where \( C = \ln\left( \frac{0.85 \beta D}{T} \right)^{-2} \). The boundary conditions in the one-frequency approximation are
Thus the smoothing of $\Delta$ is compensated for by an extra discontinuity in $\Delta$.

We solve Eq. (9) for the case of semi-infinite $N (-\infty < x \ll 0)$ and semi-infinite $S (0 \ll x < \infty)$. Since $\frac{d\Delta}{dx} = 0$ when $\Delta = 0$ (at $x = -\infty$), we find in $N$

\begin{equation}
\Delta_N(x) = \Delta_N(0)e^{+K_N x},
\end{equation}

where $K_N^{-1}$ is the pair decay length in $N$:

\begin{equation}
K_N^{-1} = \xi_D(T) \left(1 - \frac{2[N(0)V]_N}{1-C[N(0)V]_N} \right)^{-1/2},
\end{equation}

\begin{equation}
= \xi_D(T)(1 + \frac{2}{\xi nT/T_{cS}})^{1/2}.
\end{equation}

It is also possible to derive $\Delta_N(x)$ from the generalized Ginzburg Landau (G-L) equation\textsuperscript{20} which is valid when $T \sim T_{cS}$. In the dirty limit, it has the form, found by Fulde and Moormann\textsuperscript{21},

\begin{equation}
\left(\xi_D(T) - \frac{\pi^2}{4}\frac{\xi_D^2(T)}{dx^2}\right)\Delta(x) + \frac{7}{8}\frac{\zeta(3)}{(\pi k_B T)^2} \Delta^3(x) = 0
\end{equation}

$\zeta(3) \approx 1.202$ ($\zeta(x)$ is the Riemann zeta function). Blackburn, Smith and Rowell\textsuperscript{16} showed that in $N$, Eq. (13) can be expressed as
(in \( N, \infty < x < 0 \)) \[
\frac{d^2 f}{dx^2} - \alpha^2 (\beta^2 f + \delta^2 f^3) = 0
\] (14)

where

\[
f = \left( \frac{\pi m^2}{\hbar^2} \right)^{1/2} \frac{\Delta}{\psi_{\alpha S}}
\] (15)

\[
\psi_{\alpha S}^2 = \frac{2n_S \pi^2 k_B T^2}{\zeta(3) \hbar v_F T_{cS} cS} \ln \frac{T_{cS}}{T}
\] (16)

\[
\alpha^2 = 4 \frac{\ln T_{cS}}{\pi^2 \xi_S^2 (T)}
\] (17)

\[
\beta^2 = \xi_S^2 (T) \frac{\ln (T/T_{cN})}{\xi_N^2 (T)} \frac{\ln (T_{cS}/T)}{\xi_N^2 (T_{cS}/T)}
\] (18)

\[
\delta^2 = \frac{n_{cN}}{n_{cS}} \left( \frac{\epsilon_S}{\epsilon_N} \right)^2
\]

\( n_S \) and \( n_N \) are the valence electron densities in \( S \) and \( N \). For semi-infinite \( N \), \( f \) is given implicitly by

\[
\ln \left[ \left( \frac{\beta^2 + \delta^2 f^2/2 + \beta}{f_0} \right) \left( \frac{f}{\beta^2 + \delta^2 f^2/2 + \beta} \right) \right] = \alpha \beta x
\] (19)

where \( f(x = 0) = f_0 \). If \( \frac{T_{cN}}{T_{cS}} \ll 1 \), then \( \delta^2 \ll 1 \). Thus Eq. (19) can be approximated by

\[
f(x) = f_0 e^{\alpha \beta x}
\]

or
where the pair decay length is $k_N^{-1} = (\alpha \beta)^{-1}$:

$$k_N^{-1} = \frac{\pi}{2} \xi_D(T) \left( \frac{1}{\ln(T/T_{cN})} \right)^{1/2}$$

$$= \frac{\pi}{2} \xi_D(T) \left( \frac{1}{N(0)V} - \ln \left( \frac{0.85D}{T} \right) \right).$$

We see that both the one-frequency approximation and the G-L equation give an exponential decay of $\Delta_N$. However, if $[N(0)V]_N = T_{cN} = 0$, the former gives a pair decay length in $N$ of $\xi_D(T)$ whereas the latter gives zero. This appears to invalidate the G-L result since the decay length of the condensation amplitude, $F$, is the same as $\Delta$ [see Eq. (1)], and a sharp cut-off of the pair density at the interface is an unphysical consequence. From the boundary conditions, though, it is seen that $\Delta_N(0) = 0$ when $[N(0)V]_N = 0$. Thus, as far as $\Delta_N(x)$ is concerned, either a finite or zero decay length in $N$ gives indistinguishable results for $T_{cN} = 0$. As shown in Fig. 1, the ratio of $k_N^{-1}$ to $k_N^{-1}$ is about one for "reasonable" values of $T_{cN}$.

In spite of its unphysical behavior when $T_{cN} = 0$, we believe that $k_N^{-1}$ is the more accurate decay length for the following reason. The one-frequency approximation should give a good estimate of $\Delta(x)$ far from the interface for the case of a thick but finite $N$. It does this by exaggerating the discontinuity of $\Delta$ at the interface, but having $\Delta_N(x)$ decay slowly from $\Delta_N(0)$. Since we will use the exact boundary conditions, Eqs. (8a) and (8b), $k_N^{-1}$ should, in general,
Fig. 1. Ratio of Ginzburg-Landau pair decay length in $N, k_N^{-1}$, to the de Gennes "one-frequency approximation" pair decay length in $N, k_N^{-1}$. 
make $\Delta_N(x)$ too small for small $x$. The G-L equation, on the other hand, is valid if $\Delta$ is small, a condition fairly well satisfied in $N$. Thus $k_N^{-1}$ seems to be the better choice; whether this is correct will be determined from the data.

Before we consider the behavior of $\Delta$ in $S$, we must pay attention to the special case where the normal metal, $M$, is magnetic. If $H_E$ is the exchange field, then the electrons in the Cooper pair have their degeneracy split by $2\mu_B H_E$, where $\mu_B = 0.927 \times 10^{-20}$ erg/gauss is the Bohr magneton. This is equivalent to having $M$ at a temperature of $T_M = \frac{2\mu_B H_E}{k_B}$. For example, in Ni $H_E = 1.2 \times 10^5$ gauss, so $T_M = 16$ K. $T_M$ should then be used in place of the actual temperature in determining the coherence length, $\xi(T)$. Since the exchange fields in other ferromagnets are on the order of $10^4$ gauss or more, their decay lengths in general will be very short or zero, depending on whether $k_M^{-1} \equiv k_N^{-1}(T_M)$ or $k_M^{-1} \equiv k_N^{-1}(T_M')$ is correct, respectively.

This "exchange energy" decay length was proposed by de Gennes and Sarma and verified by Hauser, Theuerer, and Werthamer. Hauser, et al. combined the analysis of de Gennes and Werthamer for the proximity effect with that of Abrikosov and Gor'kov for the role of dilute, randomly oriented, paramagnetic impurities in $N$. They found that

$$\ln \frac{T_{cN}}{T_c} = \chi \left[-\xi_N(T)\kappa_N^2 + \frac{h}{2\hbar \tau \kappa_{SB} T_c} \right],$$

where $T_c$ is the transition temperature of an SN bi-layer, $\tau_S$ is the collision time due to the exchange interaction between a conduction...
electron and a magnetic impurity, $\kappa_{N}^{-1}$ is a new pair decay length in $N$, and $\chi(Z) = \psi(\frac{1}{2} + \frac{1}{2}Z) - \psi(\frac{1}{2})$. $\psi(Z)$ is the digamma function.

Assuming $T_{CN} = 0$, and setting $\hbar/2\pi T_{SC} = \zeta$, Eq. (22) gives in the limit of large $\zeta$

$$\kappa_{N}^{-1} = \frac{\xi_{NS}}{\sqrt{\zeta}}.$$  \hspace{1cm} (23)

$\xi_{NS} \equiv \xi_{N}(T_{CS})$. Assuming that Eq. (22) (and thus Eq. (23)) is valid when $N$ is a ferromagnet ($N = M$), Hauser, et al found from their data of $T_{C}$ versus $d_{S}$ that $\zeta_{Fe} = 14$, $\zeta_{Ni} = 6$ and $\zeta_{Gd} = 338$ with Pb as S. Setting $T_{M} = \zeta T_{CS}$, we find for thick Pb-Ni bi-layers that $T_{M} = 43$ K. This is in fair agreement with our simple estimate for Ni using the exchange field. Thus $\kappa_{N}^{-1}$ is very short and differs from $\kappa_{M}^{-1}$ or $K_{N}^{-1}$ in that it is not temperature dependent.

We next wish to determine $\Delta(x)$ in $S$. The one-frequency approximation, Eq. (9), is not valid for semi-infinite $S$ ($0 < x < \infty$). The generalized G-L equation, Eq. (13), is applicable and has the form

$$(in\ S, \ 0 < x < \infty) \quad \frac{d^{2}f}{dx^{2}} + \alpha^{2}(f - f^{3}) = 0.$$ \hspace{1cm} (24)

The solution is

$$(T-T_{CS}) \quad f_{S}(x) = \tanh \frac{\alpha}{\sqrt{2}} (x + C'),$$ \hspace{1cm} (25)

where $C' = \frac{1}{\alpha \sqrt{2}} \ln \left( \frac{1 + f(0)}{1 - f(0)} \right)$. 

The pair decay length in \( S \) is defined as

\[
(T - T_{cS})^{-1/2} = \frac{k_S}{\xi_S(T) (\ln(T_{cS}/T))^{1/2}}.
\]  

The problem now is to determine \( \Delta_S(0) \). We follow the procedure of de Gennes. Dividing Eq. (8a) by Eq. (8b) we find

\[
\frac{\Delta}{DN_d^a} \text{ continuous.}
\]  

Using this new boundary condition and the values of \( f \) in \( N \) and \( S \), we find

\[
(T - T_{cS}) \frac{\Delta_S(0)}{\Delta_S(\infty)} = \left( \frac{D_N N_k S}{\sqrt{2D_N N_k S}} \right)^{1/2} \propto \tau_{cS}^{-1/2} T_{cS}^{1/2}.
\]  

Since \( \Delta_S(\infty) \propto \left( \frac{T_{cS} - T}{T_{cS}} \right)^{1/2} \) near \( T_{cS} \),

\[
(T - T_{cS}) \frac{\Delta_S(0)}{\Delta_S(\infty)} \propto \Delta_S(\infty).
\]  

Eq. (28) is not valid for \( T \ll T_{cS} \), which is the region of interest for determining \( [N(0)V]_N \). Out of necessity, therefore, we turn to a calculation by McMillan for \( \Delta(x) \) in a clean SN bi-layer for which \( [N(0)V]_N = 0 \). Starting with the G-L equation, Eq. (13), he derives an expression for clean \( S \) which is analogous to Eq. (23):

\[
(T - T_{cS}) \frac{\Delta_S(x)}{\Delta_S(\infty)} = \tanh \left( \frac{x + b_{cS}}{\xi_S(T)^{1/2}} \right),
\]  

\( x \) is the distance from the interface.
where \( \xi_{cs} = \frac{h v_F}{2 \pi k_B T_{cs}} = 0.88 \xi_0 \) and \( \xi_0(T) = \frac{0.74 \xi_0}{(1-T/T_{cs})^{1/2}} \).

\[ \xi_0 = \frac{h v_F}{\pi \Delta_{os}^2} = 0.18 \frac{h v_F}{k_B T_{cs}} \]

is the Pippard coherence length, and \( b \) is chosen so that \( \Delta(x) \) satisfies the self-consistency equation, Eq. (2).

McMillan finds \( b = 0.6 \). At the interface, Eq. (30) becomes

\[
\frac{\Delta_S(0)}{\Delta_S(\infty)} = \frac{\tan \theta \xi_{cs}}{\xi_0 \sqrt{2}} = \tan \left( 0.50 \sqrt{1 - \frac{T}{T_{cs}}} \right)
\]

(31)

If we assume this is valid for \( T \ll T_{cs} \), then \( \frac{\Delta_S(0)}{\Delta_S(\infty)} = \tanh 0.50 = 0.46 \).

McMillan also calculates \( \Delta_S(x) \) directly for \( T = 0 \) and finds

\[
\frac{\Delta_S(0)}{\Delta_{os}(\infty)} = \frac{1}{2} \left[ 1 + \frac{[N(0)V]_S}{2} \right],
\]

(32)

where \( \Delta_{os}(\infty) = \Delta_S(\infty) \) at \( T = 0 \). Using \( [N(0)V]_{Pb} = 0.56 \) and \( [N(0)V]_{Sn} = 0.26 \), Eq. (32) gives \( \Delta_S(0) = 0.64 \Delta_{os}(\infty) \) for Pb and \( \Delta_S(0) = 0.56 \Delta_{os} \) for Sn.

The shapes of \( F \) and \( \Delta \) as determined from the preceding calculations are shown in Fig. (2) for both non-magnetic, \( N \), and ferromagnetic, \( M \), metals. \( \Delta \) in \( N \) is shown for the three cases \( V_N = 0 \), \( V_N > 0 \) and \( V_N < 0 \). It should be noted that a metal with \( [N(0)V]_N = 0 \) is not equivalent to a ferromagnet. In an \( SN \) bi-layer, a metal with \( [N(0)V]_N = 0 \) is an example of gapless superconductivity, since there are pairs (and thus correlations) in a region where \( \Delta = 0 \). In a ferromagnet, on the other hand, both \( F \) and \( \Delta \) are essentially zero.
Fig. 2.
Fig. 2. Condensation amplitude, $F$, and pair potential, $\Lambda$, versus position, $x$, near an MS interface(a). $F$ versus $x$ in a NS bi-layer(b). $\Delta$ versus $x$ in NS bi-layers for the three cases $V_N = 0$ (c), $V_N > 0$ (d), and $V_N < 0$ (e).
$V_N > 0$ implies that $N$ may be superconducting at some temperature $T_{CN} > 0$. The case of $V_N < 0$ is confusing. It would occur if, for example, the Coulomb repulsion were greater than the phonon coupling. de Gennes$^5$ depicts $\Delta$ as being negative for $V_N < 0$, as shown in Fig. 2(e). How a negative pair potential would manifest itself in the resistance of SNS sandwiches or in experiments such as tunneling, ultrasonic attenuation, or infrared absorption is certainly not intuitive.

Although Eq. (10a) derived by de Gennes$^5$ implies that the pair decay length for $[N(0)V]_N = -A$ is shorter than for $[N(0)V]_N = +A$, Deutscher and de Gennes$^{28}$ report that preliminary studies suggest that the range of $F$ for $[N(0)V]_N < 0$ stays equal to $\xi_N(T)$, independent of the value of $V_N$.

We note also that even $[N(0)V]_N = -\infty$ is not equivalent to a ferromagnet (although they both apparently have in common a temperature independent pair decay length). This was shown by Hauser, et al$^{25}$ by the fact that their data of $T_c$ versus $d_S$ of Fe-Pb bi-layers was fit by the Abrikosov-Gor'kov spin scattering theory, but not by the de Gennes$^5$-Werthamer$^{15}$ theory with $[N(0)V]_N = -\infty$.

B. Excitation Current Relaxation Length

In the previous section, the shape of the position dependent pair potential for an SN bi-layer with semi-infinite $S$ and $N$ were formulated. The interface was chosen to be at $x = 0$, with $S$ in the right half plane and $N$ in the left (see Fig. 2). If a current, $I$, flows through the interface, it is known that deep inside $N$ the current
flows dissipatively with a constant value $I_N = I$ and deep inside $S$ the current flows with no dissipation, with a value $I = I$. In the region near the interface, the normal current is converted to supercurrent. Exactly where the conversion takes place determines the resistance in this region. In this section we examine the mechanisms for conversion and determine where the conversion takes place.

In discussing either the normal metal side or the superconducting side, we will find it convenient to use the "excitation representation" rather than the "independent particle representation". In the independent particle representation of a normal metal, all of the conduction electrons in the metal are represented on the dispersion curve. Their energies, $E_k$, are measured with respect to the vacuum level. The dispersion curves for three cases, described in the caption, are shown in Fig. 3. The black dots represent electrons. Also shown are inverted occupation probability distributions, $f_k = \left( e^{\left( E_k - \mu_e \right) / kT} + 1 \right)^{-1}$ is the Fermi distribution, where $\mu_e$ is the chemical potential. At $T = 0$, $\mu_e$ equals the Fermi energy, $E_F$. The electrochemical potential, $\mu$, is $\mu_e + e\phi$, where $\phi$ is the electrical potential.

In the excitation representation one displays only the difference between the excited state and the ground state ($T = 0, I = 0$) distributions. Excitations are therefore electrons above the Fermi surface and holes below. Their energies are measured with respect to the electrochemical potential and are taken as positive. The dispersion curves for the same cases described in Fig. 3 are shown.
Fig. 3. Dispersion curves for a normal metal in the independent particle representation when $T = 0, I = 0$ (a), $T = 0, I \neq 0$ (b), and $T \neq 0, I = 0$ (c).
in Fig. 4, but in the excitation representation. The black dots are now excitations and may have charge either \(-e\) or \(+e\). All excitations with \(|k| < k_F\) have positive charge and will be represented by \(k_<\). All excitations with \(|k| > k_F\) have negative charge and are represented by \(k_>\). Their velocities are \(h^{-1}v_{kE}\). The circled numbers in Fig. 4(b) label particular branches, and comparison with Fig. 3(b) should make clear the relationship between electrons, holes, and excitations.

The current carrying case at \(T = 0\) is shown in Figs. 3(b) and 4(b). As is implied by calling an excitation current a normal current, a constant total current in a normal metal has a constant number of excitations. However, the \(k_4\) excitations (i.e. excitations on branch 4) can be elastically scattered to \(k_1\) and the \(k_3\) excitations can be scattered to \(k_2\). If the electric field, \(\vec{E}\), stays constant, the excitations on branches 1 and 2 will decay dissipatively into the ground state while the voltage (or current) sources will supply the energy to create new excitations on branches 3 and 4. The average time a state on branches 3 or 4 is occupied is \(\tau_1 = \ell_1/v_F\) where \(\ell_1\) is the mean free path for elastic scattering and \(v_F\) is the Fermi velocity. If the field is turned off, the number of excitations at \(T = 0\) goes to zero in \(\sim \tau_1\).

Scattering from \(k_>\) to \(k_<\) is not allowed in a normal metal since this would change the charge by two. Annihilation between branches, where both a \(k_>(-e)\) excitation and \(k_<(+e)\) excitation are destroyed, is allowed. This latter process was described in the preceding paragraph when an excitation from \(k_4\) relaxed to a vacant state
Fig. 4. Dispersion curves for a normal metal in the excitation representation for the three cases shown in Fig. 3: \( T = 0, I = 0 \) (a), \( T = 0, I \neq 0 \) (b), and \( T \neq 0, I = 0 \) (c).
below the Fermi surface (a hole), thus annihilating the excitation on branch 3.

The case of a normal metal with an induced pair potential is a bit more complicated. The sources of the complication are the BCS parameters $v_k^2$ and $u_k^2$. $v_k^2 = \frac{1}{2}(1 - \frac{\varepsilon_k}{E_k})$ is the probability that a pair of states $(k, -k)$ is occupied in the ground state and $u_k^2 = 1 - v_k^2$ is the probability that $(k, -k)$ are empty. $\varepsilon_k = \pm \sqrt{E_k^2 - \Delta^2}$ is the kinetic energy of an excitation with total energy $E_k$, both being measured from the electrochemical potential. $u_k^2$ and $v_k^2$ are shown in Fig. 5. As $\Delta \rightarrow 0$, $v_k^2 \rightarrow f_0(T=0)$, where $f_0$ is the Fermi distribution. Dispersion curves for the excitations in a metal with $\Delta > 0$ (either a superconductor below $T_{cS}$ or a normal metal with an induced $\Delta$) are shown in Fig. 6 for the same three cases shown in Fig. 3 and Fig. 4. The occupation probability for ground state electrons (pairs), $n_k$, is shown inverted. The charge on an excitation is $e\varepsilon_k/E_k$ and changes sign at the Fermi surface.

If we are far from a point where $\Delta = 0$ and at $T = 0$, there will be no excitations and any current in the metal will be carried by pairs. This is true even if $\Delta$ is non-uniform but always greater than zero (e.g. two wires made of different superconductors soldered together). If we are far from a point where $\Delta = 0$ but $T > 0$, there will be excitations. The excitations will have an energy distribution identical to that for $I = 0$ and the total current will still be carried by the pairs.
Fig. 5. Ground state occupation probabilities. $v_k^2$ is the probability that the pair of states $(-k,k)$ is occupied and $u_k^2 = 1 - v_k^2$ is the probability that $(-k,k)$ are empty.
Fig. 6. Dispersion curves for a metal with $\Delta > 0$ in the excitation representation.

The three cases represented are the same as were shown in Figs. 4 and 5: $T = 0, I = 0$ (a), $T = 0, I \neq 0$ (b), $T \neq 0, I = 0$ (c).
In order for the excitations to carry an electric current, there must be a gradient in the electrochemical potential of the excitations, $\mu = \mu_e + e\phi$. For samples of uniform temperature, $\nabla \mu_e = 0$. Therefore the driving force for excitations must be an electric field $\mathbf{E} = -\nabla \phi$. However, the second London equation for superconductors,

$$\frac{d}{dt}(\mathbf{A}_S^\dagger) = \mathbf{E} = -\frac{A}{c} - \nabla \phi,$$

(33)

where $\Lambda(T) = \frac{4\pi \lambda^2_L}{c^2}$, $\lambda_L$ is the London penetration depth, and $J_S^\dagger$ is the supercurrent density, requires that in the steady state ($\dot{J}_S = \dot{A} = 0$), $\mathbf{E} = 0$. If Eq. (33) is correct, then one must conclude that there can be no steady state normal current flowing in a superconductor.

Now consider the SN interface shown in Fig. 7(a). We assume that the exponentially decaying pair potential in N goes to zero at some point $x_1$. We follow an excitation with energy $E_e < \Delta_N(0)$ which is at $x < x_1$, and traveling to the right [Fig. 7(a)]. The excitation could be either positive or negative depending on whether it is on branch 2 or 4.

When the excitation reaches $x_1$, it enters a region where $\Delta > 0$. As will be shown, it neither immediately stops moving nor immediately recombines with another excitation to form a Cooper pair. Therefore, if this excitation is part of a steady state flux of excitations from N traveling to the right, we will have a normal current in a superconductor (here, induced in N). Such a flux exists in a thick...
Fig. 7. Pair potential and dispersion curves for an excitation moving toward an SN interface.
SNS sandwich when driven by a current source. We are thus in contradiction with the conclusions based on Eq. (33).

A resolution to this difficulty has recently been proposed by Waldram. His work is a modification of the theoretical part of a paper by Pippard, Shepherd and Tindall (PST) on the resistance of SNS sandwiches near $T_c$. Waldram shows that the London equation, Eq. (33), should be more correctly written as

$$\frac{d(A\mathbf{j}_S)}{dt} = -\frac{\mathbf{A}}{C} + \nabla \mu_c,$$

(34)

where $\mu_c$ is the electrochemical potential of the condensate (pairs).

In the BCS theory,$^1$

$$\mu_c = \varepsilon_c + e\phi,$$

(35)

where $\varepsilon_c$ is a 'band energy for the condensate', measured from the vacuum level, and equal to the mean energy $\frac{1}{2}(\varepsilon_{k+q} + \varepsilon_{-k+q})$ of the pair at which $u_k^2 = v_k^2 = \frac{1}{2}$. We shall call $\varepsilon_c$ the chemical potential of the condensate in analogy with a normal metal in which the chemical potential is the energy at which the Fermi function equals $\frac{1}{2}$. If there is no current and $T = 0$, then $\varepsilon_c$ is equal to the Fermi energy, $\varepsilon_F$.

Combining Eq. (34) and Eq. (35) we see that steady state no longer requires $\mathbf{E} = 0$ in the superconductor, but rather that

$$\mathbf{E} = -\nabla \phi = \nabla \varepsilon_c.$$
We must conclude that $\varepsilon_c$ varies with position near an SN interface when there is a current. Rieger, Scalapino and Mercereau \textsuperscript{32} (RSM) reached the same conclusion, also from an analysis of current flow through an SN interface.

We now show that $\varepsilon_c \neq \varepsilon_F$ when there is a charge imbalance per unit volume, $Q^*e$, between $k_>$ and $k_<$ branches and that $Q^*$ varies with distance from an SN interface. We let $f_{k_>} (\varepsilon)(f_{k_<} (\varepsilon))$ be the (not necessarily equilibrium) distribution function for the excitations with kinetic energy $\varepsilon$ on the $k_>$ ($k_<$) branch and define $g(\varepsilon) = f_{k} - f_o$ as the difference between $f_{k}$ and the Fermi distribution, $f_o$. $\varepsilon$ is measured with respect to $\varepsilon_c$. Then

$$Q^* = 2N(0) \int_0^\infty (u_k^2 - v_k^2) q(\varepsilon) d\varepsilon ,$$

where $q(\varepsilon) = g_> - g_<$ is the difference in probability per unit energy of finding excitations with kinetic energy $\varepsilon$ on the $k_>$ and $k_<$ branches. $u_k^2 - v_k^2$ is the number of electrons added to the system when an excitation is added. This can be seen by the Bogoliubov-Valatin excitation creation operator, $\gamma_k^+ = u_k c_k^+ - v_k c_{-k}^-$, where $c_{-k}^+$ and $c_{-k}$ are electron creation and annihilation operators, respectively. An equivalent picture is to ascribe to an excitation the charge $u_k^2 - v_k^2$. $Q^*$ was first introduced by Tinkham and Clarke.\textsuperscript{13}

For an arbitrary value of $\varepsilon_c$, the charge excess in the metal per unit volume is

$$eN(0)(\varepsilon_c - \varepsilon_F) + eQ^* .$$
Hence

\[ \mathbf{E} = \nabla \varepsilon = - \frac{\nabla Q^*}{N(0)}. \] (40)

The voltage due to current flow in the interface region is therefore just \( \int \mathbf{E} \cdot d\mathbf{l} \).

Qualitatively, how does this charge imbalance between branches come about? When there is a current in a normal metal of uniform temperature there is no imbalance, as illustrated in Fig. 4(b). There are as many \( k_\leq \) excitations (holes) moving to the left as there are \( k_\geq \) excitations (electrons) moving to the right. Deep inside \( S \) there is no normal current. Hence, near the interface, \( g_\geq > 0 \) because of the \( k_\geq \) excitations which come from \( N \), but there is no flux of \( k_\leq \) excitations coming from \( S \) to make \( g_\leq < 0 \). There is, therefore, a charge difference between branches, \( Q^* \).

To evaluate the spatial extent of \( Q^* \) we follow Waldram \(^{11}\) by writing down the one dimensional Boltzmann transport equation for the excitations of energy \( \varepsilon \):

\[ -v_S \frac{\partial g_1}{\partial x} = \frac{\varepsilon_1 - \frac{1}{2}(\varepsilon_1 + \varepsilon_4)}{\tau_1} - \frac{\varepsilon_1 - \varepsilon_{10}}{\tau_2} - \frac{\varepsilon_1 - \varepsilon_{20}}{\tau_3} - \frac{\varepsilon_1}{\tau_4} \] (41a)

and

\[ -v_S \frac{\partial g_2}{\partial x} = \frac{\varepsilon_2 - \frac{1}{2}(\varepsilon_2 + \varepsilon_3)}{\tau_1} - \frac{\varepsilon_2 - \varepsilon_{20}}{\tau_2} - \frac{\varepsilon_2 - \varepsilon_{210}}{\tau_3} - \frac{\varepsilon_2}{\tau_4}. \] (41b)

\[ v_S = h \frac{-1}{k} \frac{E}{k} \frac{e_k}{E_k} = \frac{v_p}{E_k} \] is the velocity of the excitation at \( k \) and
\[ Q = 2N(0) \int_{0}^{\infty} q(\varepsilon) \, d\varepsilon \]  \hspace{1cm} (42)

is the excess number of excitations. \( \tau_1 \) is the elastic scattering time (for \( k_1 \approx k_4 \) and \( k_2 \approx k_3 \)), which is the same as when \( \Delta = 0 \). \( \tau_2 \) is the cooling time (i.e. inelastic, non-branch crossing). \( \tau_3 \) is the branch crossing time (for \( k_3 \approx k_4 \)) and \( \tau_4 \) is the recombination time to form pairs. \( g_{10}(g_{20}) \) is the thermal equilibrium value of \( g_1(g_2) \) when \( Q \neq 0 \), assuming only \( \tau_2 \) is not infinite.

Why is branch crossing allowed in a superconductor (\( \Delta > 0 \)) but not in a normal metal (\( \Delta = 0 \))? The reason is that in \( S \) the change in charge which would occur in a normal metal is carried away by the supercurrent.

The \( g \)'s subtracted from \( g_1(g_2) \) in Eq. (41a) (Eq. 41(b)) in each of the first three terms on the right are the values of \( g_1(g_2) \) which would be attained in thermal equilibrium if \( g \) were allowed to relax by only one mechanism which proceeds at a rate \( \tau_i^{-1} \). Since none of the processes \( i = 1-3 \) allow for recombination, the thermal equilibrium values of \( g \) could not equal zero. The fourth term, on the other hand, accounts for recombination, so that subject to this process at the rate \( \tau_4^{-1} \), \( g_1(g_2) \) does relax to zero. The fourth term was not included by Waldram.

What happened to the term \(-eE \nabla S \frac{df}{d\varepsilon} \) which usually appears on the left side of the Boltzmann equation? This term accounts for the increase in energy due to the field. It will be recalled, however, that the energy of the excitations is measured with respect to the chemical potential of the pairs, \( \varepsilon_c \). Since \( \nabla \varepsilon_c = \dot{\varepsilon}_c \), the real increase in excitation energy due to the field has been accounted for by
the increase of the "zero" of excitation energy.

Defining \( j = g_1 + g_2 \) and \( q = g_1 - g_2 \), adding and subtracting Eq. (41a) and Eq. (41b), and using the symmetry conditions \( g_1 = g_3 \) and \( g_2 = g_4 \), we get

\[
\frac{\partial j}{\partial x} = \frac{q - q_Q}{\tau_2} \frac{q - q_Q}{\tau_2} + \frac{q}{\tau_4} \tag{43}
\]

and

\[
\frac{\partial q}{\partial x} = \frac{q}{\tau_0} - \frac{q}{\tau_2} \frac{1}{\tau_2} + \frac{1}{\tau_3}, \tag{44}
\]

where

\[
\frac{1}{\tau_0} = \frac{1}{\tau_1} + \frac{1}{\tau_2} + \frac{1}{\tau_3} + \frac{1}{\tau_4} \tag{45}
\]

and \( j_Q = g_{1Q} + g_{2Q} \), \( q_Q = g_{1Q} - g_{2Q} \). Multiplying Eq. (43) and Eq. (44) through by \( 2N(0)(u - v)^2 = 2N(0) \frac{vS}{F} \) and integrating over \( \varepsilon \) from 0 to \( \infty \), we get, respectively,

\[
\frac{dJ^*}{dx} = \frac{Q}{v_F} \left( \frac{2}{\tau_3} + \frac{1}{\tau_4} \right) \tag{46}
\]

and

\[
\frac{dQ^*}{dx} = \frac{J}{v_F} \left( \frac{1}{\tau_1} + \frac{1}{\tau_4} \right), \tag{47}
\]

where \( eJ^* = e2N(0) \int_0^\infty (u^2 - v^2) d\varepsilon \) is the branch imbalance charge current density and \( J = 2N(0) \int_0^\infty j d\varepsilon \) is the flux difference in the number of excitations. If we substitute Eq. (40) in Eq. (47) we find
J = \left( \frac{1}{\tau_1} + \frac{1}{\tau_4} \right)^{-1} \nu_F N(0) E \quad (48)

in analogy with Ohm's law in a normal metal, where \( \tau_4 = \infty \). Equation (48) again expresses our contention that an excitation current in S (or in N with an induced \( \Delta \)) is driven by an electric field.

Since excitations do not carry a full charge, i.e., \( J \neq J^* \). When \( E_k \sim \Delta \), \( u_k^2 - v_k^2 \approx 0 \) and \( J^* \ll J \). However, for \( E_k \gg 2\Delta \), \( |u_k^2 - v_k^2| \approx 0.87 \) and \( Q^* = Q \), \( J^* \approx J \). Ignoring the difference between starred and unstarred quantities, Eq. (46) and Eq. (47) can be combined to give

\[
\frac{d^2J}{dx^2} = \frac{J}{\lambda_1^2} \quad (49)
\]

where the decay length of the excitation current in one dimension is

\[
(1-D) \quad \lambda_1 = \nu_F \sqrt{\left( \frac{2}{\tau_3} + \frac{1}{\tau_4} \right)^{-1} \left( \frac{1}{\tau_1} + \frac{1}{\tau_4} \right)^{-1}} \quad (50a)
\]

In N, far from the interface, there is neither branch crossing nor recombination, so \( \tau_3 = \tau_4 = \lambda_1 = \infty \). Hence the excitation current is constant where \( \Delta = 0 \), but decays exponentially to zero where \( \Delta = \text{const} > 0 \).

There results are shown in Fig. 8 for the case of \( [N(0)\nu]_N = 0 \) and in Fig. 9 for \( [N(0)\nu]_N > 0 \).

(PST)\( ^{10} \) and (RSM)\( ^{30} \) arrived at similar results, but in the former \( \lambda_1 \) was the diffusion length for inelastic processes only and in the latter \( \lambda_1 \) was a diffusion length in the Ginzburg–Landau relaxation time.

In the next section we determine approximate values for \( \tau_1, \tau_3 \) and \( \tau_4 \).
\[ Q^* = -e \phi N(0) \]

\[ \mu_p(T_C) \]

\[ \mu_p(T_2) \]

\[ \mu_p(T_1) \]

ACTUAL Q*

XBL 759-7107

Fig. 8
Fig. 8. (a) Pair potential for three temperatures
$0 < T_1 < T_2 < T_3 < T_{CS}$ with $[N(0)V]_N = 0$.

(b) Branch unbalance $Q^*$ in Waldram's scheme
where excitation energies are measured with
respect to $\varepsilon_c$. $\mu_p$ is the electrochemical
potential of the pairs.

(c) Actual $Q^*$ obtained by subtracting $\varepsilon_c$ from $Q^*$
in (b). No boundary resistance is assumed
(which would create a discontinuity in $\phi$).
Fig. 9
Fig. 9. (a) Pair potential for $T \ll T_{cS}$ where $\left[ N(0) \right]_N > 0$.

(b) Branch imbalance in Waldram's scheme.

(c) Actual $Q^*$, obtained by subtracting $\epsilon_c$ from $Q^*$ in (b). No boundary resistance is assumed (which could create a discontinuity in $\phi$).
C. Relaxation Time

We extend the results of the preceding section to three dimensions by approximating the length in $S$ in which excitation (normal) current flows as:

\[ \lambda_3 = \frac{\lambda_1}{\sqrt{3}} = \frac{v_F}{\sqrt{3}} \sqrt{\left( \frac{2}{\tau_3 + \frac{1}{\tau_4}} \right)^{-1} \left( \frac{1}{\tau_1} + \frac{1}{\tau_4} \right)^{-1}}. \] (50b)

In this section we give the temperature dependence of $\tau_1$, $\tau_3$, and $\tau_4$ and compare their relative magnitudes.

$\tau_1$ is the elastic scattering time. It is equal to $\ell/v_F$, where $\ell$ is the mean free path and $v_F$ is the Fermi velocity. $\ell$ can be estimated from the product $\rho \ell$, and from the measured value of $\rho$ at low temperatures. $v_F$ can then be estimated from the relation

\[ v_F = \frac{2 \pi k_B^2}{e^2 \gamma \rho \ell}, \] (51)

where $\gamma$ is the coefficient of the linear term in the specific heat.

A time not quite identical to the branch crossing time, $\tau_3$, has been calculated by Tinkham. It is the "relaxation time for the imbalance $Q"$, $\tau_Q$. It includes recombination as a mode of $Q$ relaxation and may be written as $\tau_Q^{-1} = \tau_3^{-1} + (G \tau_R)^{-1}$.

Tinkham has calculated $\tau_Q$ for both inelastic phonon processes, $\tau_Q^{\text{ph}}$, which are present in both isotropic and anisotropic superconductors, and for elastic processes, $\tau_Q^{\text{el}}$, which occur only in anisotropic or non-uniform superconductors. We examine first the inelastic processes.
1. **Inelastic Phonon Processes**

The calculation by Tinkham was inspired by an experiment by Ckarke,\(^{12}\) which involved the injection of excitations from a normal metal into a superconductor by tunneling through an oxide barrier. The barrier resistance was \(\sim 10^\Omega\) and the injection current was varied from 0.1 to 20 ma. The injected excitations therefore had energies in the range 1 to 200 meV. Since these energies were usually much greater than \(k_B T (k_B = 8.6 \times 10^{-5} \text{ eV/°K})\), Tinkham had to consider the time, \(t(T)\), for the excitations to cool to the temperature, \(T\), and the time, \(\tau(T)\), for inelastic scattering to bring the separate branches into internal equilibrium. If \(\tau_Q(T) > \tau(T)\) and \(\tau_Q^\text{ph}(T) > \tau(T)\) then near \(T_c\)

\[
\frac{1}{\tau_Q^\text{ph}(E)} = \frac{2\Delta E^2}{\tau_0 \theta_D^3} \coth \left( \frac{E}{2k_B T} \right),
\]

(52)

\(\theta_D\) is the Debye temperature and \(\tau_0 \equiv \tau(\theta_D) = \ell/\nu_F (-10^{-14} \text{ sec})\) is the scattering time extrapolated to \(T = \theta_D\) from the electrical conductivity, \(\sigma\), at high temperatures where \(\sigma \sim T^{-1}\). Since we assume thermal equilibrium on each branch, it is possible to define chemical potentials for each branch separately, namely \(\mu_\uparrow\) and \(\mu_\downarrow\). The Fermi functions for these branches differ by an amount

\[
\delta f(E) = \frac{\partial f}{\partial E}(\mu_\uparrow - \mu_\downarrow)
\]

(53)

We can therefore define a distribution function,

\[
p(E) = 2N(0)\eta(E)\delta f(E)
\]

(54)
such that \( Q = \int_{\Delta}^{\infty} p(E) dE \) [Eq. (42)]. \( \eta(E) = E / |E| \). The average value of Eq. (52) is given by

\[
\frac{Q}{\tau_Q(T)} = \int_{\Delta}^{\infty} \frac{p(E) dE}{\tau_Q(E)}.
\]  

Setting \( \eta(E) = 1 \), since \( \Delta = 0 \) near \( T_c \), we have

\[
(T - T_c) \quad \frac{\tau_{ph}^{(Q)}}{\tau_Q(T)} = \frac{\tau_{\theta}^{\Delta} \Delta(0)}{2 \Delta(T)}.
\]  

Noting that \( 2\Delta(0) = 3.52 \text{ bK} T_c \) and setting \( T = T_c \), Tinkham finds

\[
(T - T_c) \quad \frac{\tau_{ph}^{(Q)}}{\tau_Q(T)} = \frac{\tau_{\theta}^{\Delta} \Delta(0)}{2 \Delta(T)}.
\]  

where

\[
\tau_{ph}^{(Q)} = \frac{0.068 \tau_{\theta}^{\Delta} \tau_{D}^{3}}{b (T_c / T_c)}.
\]  

Since the typical voltages across our samples are \( 10^{-7} \text{ V} << k_B T_c / e \), the excitations transmitted into \( S \) are essentially in thermal equilibrium. We would therefore expect \( \tau_{Q}^{ph}(T) > \tau(T) \) and \( \tau_{Q}^{ph}(T) > \tau(T) \) to be valid over our whole temperature range and for Eqs. (57a) and (b) to apply. For Sn (\( \tau_{\theta} = 2 \times 10^{-14} \text{ sec}, \theta_{D} = 200 \text{ K}, T_c = 3.72 \text{ K}, \) and \( b = 1 \)), Eq. (57b) gives \( \tau_{Q}^{ph}(0) = 2 \times 10^{-10} \text{ sec} \). For Pb (\( \tau_{\theta} = 3 \times 10^{-14} \text{ sec}, \theta_{D} = 105 \text{ K}, T_c = 7.2 \text{ K}, \) and \( b = 1.23 \)), \( \tau_{Q}^{ph}(0) = 5 \times 10^{-12} \text{ sec} \).
At very low temperatures where the excitations in $S$ have $E \gg k_B T$ (since $\Delta \gg k_B T$), Tinkham finds

$$\frac{1}{\tau_Q(E)} = \frac{2}{\tau_0^\theta \Delta(E-\Delta)^2}$$

(58)

Again assuming thermal equilibrium, we can average Eq. (58) over $p(E)$. Noting that $e^{E/k_B T} \gg 1$ (in $S$), the average is

$$\frac{1}{\tau_Q(T)} = \frac{\tau_{ph}^T}{\tau_Q^T} \frac{\Delta(0)^2}{\Delta(T)}$$

(59a)

where

$$\tau_{ph}^T = 2 \tau_{ph}^0.$$  

(59b)

The density of states, $\eta(E)$, was again set equal to one so as to allow an analytic solution. This approximation is not really very good for $E-\Delta$, but from Tinkham 14 we estimate that it will not put us in error by much more than a factor of 2. The temperature dependent part of Eq. (59) is plotted in Fig. 10 using the values of $\Delta(T)/\Delta(0)$ given by Mühlschlegel 33.

For $\tau(T) > \tau_{ph}^T$, Tinkham showed that the low temperature value of $\tau_{ph}^T(T)$ should be temperature independent. Indeed, experiments by Clarke and Paterson 34 on tunnel junctions of geometry similar to those of Clarke 12 found that in Sn $\tau_{ph}^T(T) = 5.5 \times 10^{-10}$ sec, independent of temperature, at low temperatures. However, they found in Pb that $\tau_{ph}^T(T)$ is still temperature dependent below $4.2^\circ K (\sim 0.6 T_c)$, as shown by the data points and solid line on Fig. 11, which is reproduced from Ref. 34. The straight dashed line
Fig. 10. Temperature dependent part of the inelastic branch imbalance relaxation time, $\tau_Q^{ph}$, versus reduced temperature.
Fig. 11. Net branch imbalance relaxation time, $\tau_Q$ found by Clarke and Paterson$^{34}$ (data points and solid curve). Dot-dash curve is the temperature dependent part of $\tau_Q^{ph}$ (Fig. 10) normalized to $T/T_c = 0.6$. 

$eV_{\text{inj}} = 10 \Delta (0)$

- $12 \ (2490\AA)$
- $16 \ (2220\AA)$

$\tau_Q \ (8 \times 10^{-12}\text{sec})$

$T/T_c$

$XBL736-6363A$
Fig. 12. Coherence factor for branch crossing versus excitation energy relative to the pair potential for four values of $\Delta'/\Delta$ (solid curves). Dotted curve is Andreev reflection coefficient, $R(E) = 1 - T(E)$. 

$\log_{10}(uu' - vv')^2$ vs $E/\Delta$.
corresponds to $\tau = 4 \times 10^{-12}$ sec. Shown by the dot-dash line is

$$\tau_{Q}^{ph}(T) = \frac{\Delta(0)}{\Delta(T)} \left( \frac{T_{c}}{T} \right)_{0.5}^{2},$$

normalized to 0.5 at $\frac{T}{T_{c}} = 0.6$. This is obviously a better fit than $\tau_{Q}^{ph}(T) = \text{const}$. If their estimate for Pb of $T(T_{c}) \sim 1.8 \times 10^{-11}$ sec is too large or their estimate of $\tau_{Q}^{ph}$ is too small, then Eq. (59a) may apply.

2. **Gap Anisotropy**

Elastic processes in an isotropic superconductor have been ignored because the coherence factor for branch crossing is

$$\langle uu' - vv' \rangle^2 = \frac{1}{2} \left[ 1 - \frac{\Delta \Delta'}{E E'} - \left( 1 - \frac{\Delta}{E} \right)^{1/2} \left( 1 - \frac{\Delta'}{E'} \right)^{1/2} \right]^2,$$

which equals zero if $E = E'$ and $\Delta = \Delta'$. Equation (60) is plotted as the solid lines in Fig. 12 for elastic scattering ($E = E'$), but in an anisotropic superconductor with $\Delta'/\Delta < 1$. Since Sn and Pb both have some gap anisotropy, $\tau_{Q}$ for elastic processes must be considered. Again we follow Tinkham.14

Setting $\delta \Delta = \frac{1}{2} (\Delta - \Delta')$ and $\overline{\Delta} = \frac{1}{2} (\Delta + \Delta')$ and assuming $\delta \Delta \ll \overline{\Delta}$,

$$\langle uu' - vv' \rangle^2 \approx \frac{(\delta \Delta)^2}{E^2 - \overline{\Delta}^2} << 1$$

(61)

Since Eq. (61) is the probability that branch crossing will occur during each elastic process,

$$\frac{1}{\tau_{Q}^{el}(E)} \approx \frac{2}{\tau_{Q}} \frac{(\delta \Delta)^2}{E^2 - \overline{\Delta}^2}$$

(62)

where the 2 accounts for the change of $Q$ by two excitations and $\tau_{Q}$.
is the elastic scattering time. Estimating the average value of 
\( E^2 - \Delta^2 \) to be \( T(\Delta + T) \) and following Markowitz and Kadanoff\(^{35} \) by writing

\[
\langle (\delta \Delta)^2 \rangle = \frac{1}{2} \frac{\Delta^2 \langle a^2 \rangle}{\langle a^2 \rangle} = \frac{1}{24} (\Delta_{\text{max}} - \Delta_{\text{min}})^2, \tag{63}
\]

where \( \langle a^2 \rangle \) is the normalized mean square anisotropy, Tinkham finds

\[
\tau_{Q,\text{el}} = \frac{\tau_1}{\langle a^2 \rangle} \frac{k_B T}{\Delta} \left[ 1 + \frac{k_B T}{\Delta} \right]. \tag{64}
\]

Clarke and Paterson\(^{34} \) find that a tolerable approximation of \( \langle a^2 \rangle \) in a dirty superconductor is given in terms of the anisotropy of the pure metal, \( \langle a^2 \rangle_o \), by

\[
\frac{\langle a^2 \rangle}{\langle a^2 \rangle_o} \approx \left[ 1 + \frac{\hbar}{2T_1 \Lambda} \right].
\]

They also find that Eq. (64) then has a minimum value when \( \tau_1 = \hbar/2\Delta \):

\[
\tau_{Q,\text{el}}(\text{min}) = \frac{2\tau_1}{\langle a^2 \rangle_o} \frac{k_B T}{\Delta} \left( 1 + \frac{k_B T}{\Delta} \right). \tag{65}
\]

For Sn, \(^{36} \) \( \langle a^2 \rangle_o = 0.02 \) and for Pb, \(^{37} \) \( \langle a^2 \rangle_o = 6 \times 10^{-4} \). In general, for pure evaporated Sn and Pb, \( \tau_{Q,\text{el}} >> \tau_{Q,\text{ph}} \) except at very low temperatures.

When the pair potential is non-uniform, as near an SN interface, there can also be elastic branch crossing. If scattering occurs when \( \Delta \) has a slope \( \frac{d\Delta}{dx} \), the maximum change in \( \Delta \) is \( \delta \Delta(\text{max}) = \frac{d\Delta}{dx} \xi \) where

\[
\xi = v_F \tau_1. \]

If \( E = E' = \Delta' = \Delta - \delta \Delta(\text{max}) \), the coherence factor, Eq. (60), becomes

\[
(E \approx \Delta) \quad (uu' - vv')^2 = \frac{\delta \Delta(\text{max})}{2\Delta}. \tag{66}
\]
The corresponding scattering rate is

\[ \frac{1}{\tau_{Q}(\Delta)} = \frac{\nu_{F}}{2\Delta} \frac{d\Delta}{dx} \]  

(67)

In \( N \) near an \( SN \) interface, \( \frac{1}{\Delta} \frac{d\Delta}{dx} = k_{N}(T) \) [Eq. (20)], so Eq. (67) becomes

\[ (E \approx \Delta) \quad \tau_{Q}^{el}(\Delta) = \frac{k_{N}^{-1}(T)}{\nu_{F}} \]  

(68)

Except near \( T_{cN} \) when \( k_{N}^{-1}(T) \to \infty \), \( \tau_{Q}^{el} \) for excitations with \( E \approx \Delta \) is shorter in a non-uniform superconductor than either \( \tau_{Q}^{el} \) in an anisotropic superconductor or \( \tau_{Q}^{ph} \) for inelastic processes.

3. **Recombination**

\( \tau_{4} \) is the time it takes for excess excitations to recombine into pairs so as to create an equilibrium distribution. \( \tau_{4} \) is also known as \( \tau_{R}^{eff} \), the effective recombination time. \( \tau_{R}^{eff} \) is greater than \( \tau_{R}^{-1} \), where \( \tau_{R}^{-1} \) is the recombination rate, because the phonons with \( E \geq 2\Delta \) which are emitted during recombination can break up pairs. It is \( \tau_{R} \) which is included in \( \tau_{Q} \).

The temperature dependence of \( \tau_{R} \) has been derived by Rothwarf and Cohen \( T_{cN} \) for low temperatures \( \frac{T}{T_{c}} \leq 0.9 \) where the equilibrium number of excitations is much less than the number of pairs.

Assuming also that the excess number of excitations injected into \( S \) is small, they found for excitations near the gap edge \((E \approx \Delta_{\infty})\) that
For Pb, they calculated $\Gamma_R = 3 \times 10^{-13}$ sec $K^{1/2}$. $\tau_R$ has not been calculated for Sn.

$\tau_{\text{eff}}^R$ can be determined experimentally, but with difficulty. The normal procedure is to evaporate a double tunnel junction so that one can be used to inject excitations into a superconductor and the second junction is used to detect the resulting excess. Rothwarf and Taylor have shown, however, that the experimentally measured decay time also depends on another parameter, $\tau_Y$. $\tau_Y$ is the lifetime of phonons for other than pair breaking processes, mostly for propagation out of the superconductor, and it may be comparable to $\tau_{\text{eff}}^R$. Therefore, the uncertainty in the values of $\tau_{\text{eff}}^R$ cited below may be considerable.

For small excess populations (i.e. small compared to the thermal equilibrium number), $\tau_{\text{eff}}^R$ has the same temperature dependence as $\tau_R$ [Eq. (69)], but with $\Gamma_R$ replaced by $\Gamma_{\text{eff}}^R$. Parker and Williams have measured $\Gamma_{\text{eff}}^R$ by tunneling into photoexcited Pb and found $\Gamma_{\text{eff}}^R (\text{Pb}) = 4 \times 10^{-12}$ sec $K^{1/2}$. This is about an order of magnitude more than $\Gamma_R (\text{Pb})$. Parker has estimated $\Gamma_{\text{eff}}^R$ in Sn to be $2 \times 10^{-9}$ sec $K^{1/2}$. Peters and Meissner have determined $\tau_{\text{eff}}^R (\text{Sn})$ by measuring the RF impedance near $T_c$ where the assumption that the number of excitations is small compared to the number of pair is not valid. Over the range $0.95 < \frac{T}{T_c} < 0.995$ they found $\tau_{\text{eff}}^R (\text{Sn}) = 2.6 \times 10^{-10} (1-\frac{T}{T_c})^{-1/2}$. This equals $1.2 \times 10^{-9}$ sec at $\frac{T}{T_c} = 0.95$. If $\tau_{\text{eff}}^R$ is assumed
to have the temperature dependence of Eq. (60) up to $\frac{T}{T_c} = 0.95$, then Parker's value of $\tau_{eff}^{R} (Sn) = 2 \times 10^{-9} \text{ sec } k^{1/2}$ gives $\tau_{eff}^{C} (Sn) = 2 \times 10^{-9} \text{ sec } k^{1/2}$ at $\frac{T}{T_c} = 0.95$. The agreement within a factor of 2 for these results is remarkably good.

Recently, Sai-Halasz, et al.\textsuperscript{43} have estimated $\Gamma_{R} (Sn)$ from the microwave reflectivity of laser-irradiated Sn and found it to be $\leq 4 \times 10^{-11} \text{ sec } k^{1/2}$. This is about two orders of magnitude less than $\tau_{eff}^{R} (Sn)$.

Finally, we consider the relation $\tau_{Q}^{-1} = \tau_{3}^{-1} + (G\tau_{R})^{-1}$.

We combine Parker and William's value of $\tau_{R}^{eff}$ for low temperatures with the high temperature results of Peters and Meissner to write down an expression valid for all temperatures:

\[
\tau_{R}^{eff} (T) = \Gamma_{R}^{eff} \left[ T^{-1/2} e^{A/k_B T} + 0.13 \left( 1 - \frac{T}{T_c} \right)^{1/2} \right]. \tag{70}
\]

If we assume $\tau_{R}^{eff} (T)$ also has this form, $\tau_{R}^{eff} (T)$ has a minimum at $\sim 0.96 T_c$. For both Pb and Sn, $\tau_{R}^{eff} (T)$ is comparable to Clarke and Paterson's experimental values of $\tau_{Q} (0)$. However, recombination depopulates the $k_>$ and $k_<$ branches at a rate roughly proportional to their populations, $n_>$ and $n_<$, respectively. When $Q = n_> - n_<$ $\neq 0$, the rates are different, but the contribution of recombination to $\tau_{Q}$ is $G\tau_{R}$ rather than $\tau_{R}$, where $G = 2 \frac{n_> - n_<}{n_> + n_<} \ll 1$.

Thus, the use of $\tau_{Q}$ for $\tau_{3}$ should cause insignificant error.
D. Andreev Reflection

The decay of the pair potential from its bulk value to zero in the normal regions of a superconductor in the intermediate state is qualitatively similar to the behavior of the pair potential near an SN interface. Andreev\(^7\) has analyzed the transmission of electrons incident on the SN boundary and was able to account for the increase in thermal resistance in going from the superconducting to the intermediate state. His conclusions will also allow us to calculate the electrical resistance of our SNS sandwiches for \(T < \Delta_B^{(\infty)}/k_B T\).

Starting with the coupled pair of non-linear equations of motion of the excitations derived by Gor'kov,\(^17\) Andreev showed that the transmission coefficient, \(T(E)\), for excitations of energy \(E\) in \(N\) incident on an SN interface where the pair potential changes abruptly from 0 to \(\Delta_\infty\) is given by

\[
T(E) = \begin{cases} 
2 \sqrt{E^2 - \Delta_\infty^2}/(E + \sqrt{E^2 - \Delta_\infty^2}) & \text{for } E > \Delta_\infty \\
0 & \text{for } E < \Delta_\infty
\end{cases}
\]  

(71)

\(T(E)\) is plotted in Fig. 13. For comparison, the transmission coefficient for electrons obeying the Schrödinger equation incident on a potential step of height \(\Delta_\infty\) is also shown. That \(T(E) = 0\) for \(E < \Delta_\infty\) is a consequence of the density of states being equal to zero for \(E < \Delta_\infty\).
Fig. 13. Plot of the transmission probabilities for excitation subject to Andreev reflection and for particles obeying the Schrödinger equation when incident on the barrier shown inset.
Although it is very difficult to calculate $T(E)$ for the shape of $\Delta$ actually existing near the SN interface, we can get a feeling for how $T(E)$ for the real, smoothly varying (we ignore the discontinuity at $x=0$) $\Delta$ differs from that of a step function. We consider the solved problem of a particle of mass $m$ which obeys the Schrödinger equation and which is incident on a barrier of shape

$$\Delta(x) = \frac{\Delta_\infty}{1 + e^{-x/w}}$$  \hspace{1cm} (72)

(see inset, Fig. 14). For such a barrier, $T(E)$ is given by

$$T(E) = 1 - \frac{\sinh^2 \left[ D \left( \sqrt{\frac{E_\infty}{\Delta_\infty}} - \sqrt{\frac{E}{\Delta_\infty}} - 1 \right) \right]}{\sinh^2 \left[ D \left( \sqrt{\frac{E_\infty}{\Delta_\infty}} + \sqrt{\frac{E}{\Delta_\infty}} - 1 \right) \right]}$$  \hspace{1cm} (73)

where $D = \frac{\sqrt{2m\Delta_\infty}}{\hbar}$. Setting $w$ equal to the coherence length in $S$, $\frac{\hbar v_F}{\pi \Delta_\infty}$, and $m$ equal to the mass of an electron we find that $D(\text{Pb}) = 39.7$ and $D(\text{Sn}) = 91.9$. Equation (73) is plotted in Fig. 14 for Pb and Sn at low temperatures. Also shown are $T(E)$ for $D = 0.1$ and $D = 1$. For $T_1 < T_c S$, $D = 0.1$ corresponds to $w$ of $1.7\text{Å}$ in Pb and $2.6\text{Å}$ in Sn. Such short decays lengths are indistinguishable from a step change in $\Delta$, shown dotted in Fig. 14. It is thus seen that for $E \approx \Delta$ a smooth change of $\Delta$ differs markedly from the step function used by Andreev. Of course, $T(E) = 0$ for $E < \Delta_\infty$ for any shape $\Delta$. 
Fig. 14. Transmission probability for particles obeying the Schrödinger equation incident on the barrier shown inset. The width of the "step", w, was chosen as a coherence length in Pb (39.7) and in Sn (91.7). See text. The dotted curve is for $W = 0$. 
In Section IIB we considered elastic branch crossing in the presence of an inhomogeneous pair potential such that the excitation was scattered from a point where the pair potential was $\Delta$ to where the pair potential was $\Delta'$. The special case considered by Andreev of a step change in $\Delta$ is equivalent to $\Delta'/\Delta = 0$. The Andreev reflection coefficient, $R(E) = 1 - T(E)$ is plotted as the dotted curve in Fig. 12. It is seen from Eq. (60) and Eq. (71) that for $E > \Delta_\infty$, $(uu' - vv')^2$ and $R(E)$ have the identical form $\frac{1}{2} \left( 1 - \sqrt{1 - \frac{\Delta_\infty^2}{E^2}} \right)$. Andreev reflection, therefore, can be explained, at least in the limit where $E > \Delta_\infty$, as elastic branch crossing.

Let us follow an excitation as it approaches the SN interface, as shown in Fig. 7. In Fig. 7(a), the excitation is in a region of $N$ where $\Delta = 0$. It has energy $E$, charge $\pm e$, and is traveling to the right with velocity $v_F = v_S$. In Fig. 7(b), the excitation still has energy $E$, but its charge has decreased to $\pm \frac{e}{\sqrt{\frac{E^2 - \Delta^2}{E}}}$ and it has slowed to velocity $v_S = h^{-1} \sqrt{\frac{E}{k} E}$. However, the probability of it being reflected here is small. This can be seen in two ways. First, the Andreev transmission coefficient is almost unity, as shown in
Fig. 13. Second, we can calculate the branch crossing time $\tau_3 \equiv \tau_Q$.

Equation (52) shows that for inelastic processes, $\tau_Q \gg \tau_0$, since

$$\frac{\theta_0^3}{D} \geq 10^6.$$  

The maximum change in $\Delta$ by elastic scattering is $\delta\Delta(\text{max})$

$$\Delta \text{d} \Delta \text{d} x \frac{k}{N} \delta \Delta.$$  

Then, for $E > \Delta$, the minimum value of $\tau^{el}_Q$ is given

by Eq. (62) as

$$\tau^{el}_Q(t) \equiv \frac{1}{2} \left( E^2 (\frac{k}{N} - 1)^2 \right)$$

for $N$. The distance traveled

before elastic branch crossing occurs is

$$\nu^{el}_S \tau^{el}_Q(t) \approx \nu^{el}_F \tau^{el}_Q(t) \approx \frac{1}{2} \left( \frac{E^2}{\Delta} \right)$$

$$\frac{(k-1)^2}{x} \gg k^{-1}.$$  

Similarly, $\nu^{el}_S \tau^{el}_Q(t) \approx \nu^{el}_F \tau^{el}_Q(t) \gg 10^6 \theta \gg k^{-1}$.

Therefore, if there is no recombination, the excitations should reach

the point $x_2$ where $\Delta \approx E$ before reflection is probable.

The recombination time, $\tau_R(E)$, for excitations in $N$ with

$E > \Delta$ is not given by Eq. (69), because the assumptions leading
to Eq. (69) that the number of excitations is much less than the
number of pairs and that the excitation is near the gap edge are
not valid. Rather, in analogy with the derivation of Eq. (62),
we estimate $\tau_R$ using the coherence factor for annihilation

$$(v u' + u v')^2 = \frac{1}{2} \left[ 1 - \frac{E E'}{E E'} + \frac{\Delta^2}{E E'} \right].$$  

(74)

For $E \approx E' > \Delta$ (we neglect the non uniformity of the pair potential),

$$(v u' + u v')^2 \approx \frac{\Delta^2}{E^2} \ll 1.$$  

Thus

$$(E > \Delta) \quad \tau_R(E) \approx \tau(T) \frac{E^2}{\Delta^2},$$  

(75)

where $\tau(T)$ is the phonon scattering rate at the temperature $T$. 51

This is much longer than $\tau_R$ given by Eq. (69). For $\Delta = 0$, $\tau_R(E) = \infty$
as expected for a normal metal. The distance traveled by the
excitation before recombination is $\nu^{ph}_S \tau_R(E) \gg k^{-1}$. We
thus see that neither branch crossing nor recombination is probable until the excitation reaches $x_2$ where $E \approx \Delta$.

At $x_2$ the excitation has moved to the bottom of the dispersion curve where both its charge and velocity are zero. Because $T(E)$ is now zero, it should cross branches in a time given by Eq. (68). Since its velocity is nearly zero when $E \approx \Delta$, the branch crossing should occur very close to $x_2$. After branch crossing the excitation has charge $\mp e\frac{E}{E}$ and moves to the left with velocity $-\hbar^{-1}V_k$.

An equivalent description of the reflection process can be given in the independent particle representation as follows. If the excitation is an electron with energy $E$, momentum $k_F+q$ and spin up, at the point where $E = \Delta$ it finds another electron in $N$ of momentum $-k_F+q$ and spin down to form a Cooper pair with momentum $2q$ which propagates through $S$. This leaves a hole at $-k_F+q$ with spin up in $N$. In the excitation picture, this hole is a $k_<$ excitation of momentum $k_F-q$ and the same spin as the incident excitation.

Finally, we note that although total reflection for $E \ll \Delta$ appears at first to be the same as for an excitation incident on an insulator, the processes are distinctly different. When incident on a thick insulator, the excitation is reflected with a change of sign of only its component of velocity normal to the interface. In one dimension, this looks like the usual elastic scattering, taking the excitation from branch 1 to 4 or 2 to 3 or vice versa (see Fig. 4). Its charge is unchanged. In Andreev reflection, on the other hand, all three of its velocity components change sign.
In one dimension, it appears as a transition from $k_4$ to $k_3$ or $k_2$ to $k_1$, or vice versa, and its charge is reversed.
E. Resistance

In this section we derive expressions for the resistance of our SNS sandwiches as a function of T. It is convenient to separate the problem into two temperature ranges: low temperatures, for \( 0 < T \leq \frac{\Lambda_S(0)}{k_B} \), and high temperatures, for \( \frac{\Lambda_N(0)}{k_B} < T < T_{cs} \), where \( \Lambda_S(0) \) and \( \Lambda_N(0) \) are the values of the pair potentials at the interface in S and N, respectively.

1. Low Temperature Resistance

As shown by Rickayzen,\(^2\) the current carried by an excitation is proportional to its momentum, \( \hbar k \). Since \( k \approx k_F \) for all excitations, all excitations carry nearly the same fraction of the total current even though their energies, and thus their velocities, are different. It seems appropriate, therefore, to consider that each excitation dissipates the same energy per unit length of its travel within the same metal.

We make the approximation for \( T < \Lambda_S(0) \) that no excitations enter S; i.e., all excitations with \( E > \Lambda_N(0) k_B \) are reflected at the interface. For constant measuring current, the resistance of an SNS sandwich is proportional to the dissipation and is given by

\[
R(T) = \frac{\rho_N}{A} \left[ \int_0^{\Lambda_N(0)} \ell(E)f(E)dE + a_N \int_{\Lambda_N(0)}^{\infty} f(E)dE \right] \cdot \frac{1}{\int_0^{\infty} f(E)dE}.
\]

(76)
\( f(E) \) is the Fermi function, \( \rho_N \) and \( a_N \) are the resistivity and thickness of \( N \), respectively, \( \lambda(E) \) is the distance between points of Andreev reflection for excitations of energy \( E \), and \( A \) is the cross-sectional area.

It is implicit in Eq. (76) that \( \Delta = 0 \) for some distance about the center of the sandwich. If not, pairs would flow all the way through \( N \) and \( R(T) = 0 \). In other words, the sandwich would have a supercurrent. Since the exponential decay of \( \Delta \) never by itself gives \( \Delta = 0 \), we would assume that \( \Delta \) is suppressed by the measuring current. That is, the measuring current density, \( J \), exceeds the critical current density, \( J_c \), over the region for which \( \Delta(x) \) is less than some critical value \( \Delta_c \) at \( x_c \).

However, it has been shown theoretically by Langer and Ambegaokar\(^{45}\) that a superconductor with \( J < J_c \) can exhibit resistance caused by thermal fluctuations if it is within a few millikelvin of \( T_c \). \( J < J_c \) means that \( <\Delta> = 0 \), where \(<\> \) indicates a time average. This has been observed by Newbower, Beasley and Tinkham\(^{46}\) in one dimensional Sn whiskers whose diameters, \( d \), were much less than a coherence length. Can such thermal fluctuations in the center of an SNS sandwich give a measurable resistance, even though \( <\Delta> > 0 \) everywhere?

We examine this possibility by following the discussion of Newbower, et al.\(^{46}\) for one dimensional samples which also have \( d << \lambda \), where \( \lambda \) is the magnetic penetration depth. Our specimens, with \( d = 0.32 \) cm, are never one dimensional, even at our lowest temperature (18mK), although \( \lambda \), as will be shown below, can be less than \( d \) near the center. Therefore our calculation provides an upper limit on
fluctuation caused resistance.

The minimum free energy change of a superconductor caused by a fluctuation is

$$\delta F_0 = \frac{8\sqrt{2}}{3} \frac{H_c^2}{8\pi} A \xi. \quad (77)$$

To put this in terms of the pair potential, we use the relation

$$\frac{H_c^2}{8\pi} = \frac{1}{2} N(0) \Delta^2. \quad (78)$$

Here $H_c$ and $\Delta$ are the critical field and energy gap at $T = 0$ in a bulk superconductor, but we assume Eq. (77) to be valid in an SNS sandwich where $H_c$ is a function of distance from the interfaces. Thus

$$\delta F_0 = \frac{4\sqrt{2}}{3} N(0) \Delta^2 A \xi. \quad (79)$$

The mean-field critical current is

$$I_c = (\frac{2}{3})^{1/2} \pi \frac{\delta F_0 / \phi_0}{\phi_0}, \quad (80)$$

where $\phi_0 = \frac{h}{2e} = 6.3 \times 10^{-18}$ erg sec esu$^{-1}$ is the flux quantum. For measuring current, $I$, such that $I_c >> I >> I_1 \equiv \frac{k_B T}{\phi_0} = 2.5 \times 10^{-8} \Lambda$, the resistance due to fluctuations is
\[ R_F = \frac{\phi}{I} \exp \left[ -\frac{\delta F_o}{k_B T} - \left( \frac{2}{3} \right) \frac{1}{2} \frac{1}{3\pi I^2 I_c} + \frac{1}{2\pi I} \right] \] (81)

where \( \Omega \) is an attempt frequency for phase slippage. It has been found by McCumber and Halperin\(^47\) to have the form

\[ \Omega = \frac{L}{\xi} \left( \frac{\delta F_o}{k_B T} \right)^{1/2} \frac{1}{\tau_s}, \] (82)

where \( L \) is the length of the superconductor and \( \tau_s^{-1} = 8k_B (T_c - T)/\pi \hbar \) is the relaxation time in the time dependent G-L theory\(^48\). It will turn out that the choice of \( L \) and \( T \) for the SNS sandwich is not critical as long as \( \tau_s \) is "reasonably" long \((\approx 10^{-14} \text{ sec})\).

In order to get a quantitative feel for the magnitude of \( R_F \), we will evaluate \( R_F \) using the parameters of Cu at 20 mK. With \( \gamma_{\text{Cu}} = 0.691 \text{ mJ/K}^2 \text{ mole} = 3.80 \times 10^{-4} \text{ J cm}^{-3} \text{ K}^{-2} \)\(^49\), and \( \rho(4.2^\circ\text{K}) = 3.44 \times 10^{-8} \Omega \text{ cm} (R_{300}/R_{4.2} = 50) \), Eq. (51) gives \( \xi_{F}^{E_{\text{Cu}}} = 3.09 \times 10^4 \text{ cm}^2 \text{ sec}^{-1} \) at low temperatures. Then in the dirty limit, \( \xi_{\text{Cu}} = (\hbar v_F \xi/6\pi k_B T)^{1/2} = 3.9 \times 10^{-4} \text{ cm at 20 mK.} \) \( N(0) \) is given by\(^32\)

\[ N(0) = \frac{3}{2} \frac{\gamma}{\pi^2 k_B^2}. \] (83)

For Cu, \( N(0) = 3.0 \times 10^{34} \text{ erg}^{-1} \text{ cm}^{-3} \).

We have calculated the values of \( \Delta, \delta F_o \) and \( I_c \) in the center of Pb-Cu-Pb sandwiches at \( T = 20 \text{ mK} \) for various thicknesses, \( a_N \). The results of these calculations for a fluctuating volume, \( A \xi \), are given in Table 1(a). Also shown are the values of \( \delta F_o/k_B T \) and
Table IA. $V = A_\xi$, $N(0) = 7.92\, K$

<table>
<thead>
<tr>
<th>$a_N/2$</th>
<th>$\Delta(a_N/2)$</th>
<th>$\delta F_o$</th>
<th>$\delta F_o/k_B T$</th>
<th>$I_c$</th>
<th>$M$</th>
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Table I. Calculated values of parameters defined in the text for various sandwich thickness $a_N/2$, and for $N(0) = 7.92\, K$. In IA, the volume of fluctuating region is $A_\xi$ and in IB, $\xi^3$. 
Table IB. $V = \xi^3 \Delta_N(0) = 7.92$ K

<table>
<thead>
<tr>
<th>$a_N/2$</th>
<th>$\Delta(a_N/2)$</th>
<th>$\delta F_o$</th>
<th>$\delta F_o/k_B T$</th>
<th>$I_c$</th>
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</tbody>
</table>
M for $I = 1$ mA, where $M$ is the second term in the exponent of Eq. (81). It is seen that the restriction for Eq. (81) that $I < I_c$ is valid only for $a_N/2 \leq 8.9 \times 10^{-3}$ cm. Since $\frac{I}{2I_1} = 4 \times 10^4$, $R_F$ is dominated by the first two terms in the exponent and is immeasurably small for any "reasonable" value of $\tau_s$. $R_F$ is even smaller for larger $I$.

Now, it may be argued that the proper volume over which a fluctuation in $\Delta$ may occur is $\xi^3$. This is most likely correct. However, for our specimens, whose diameters are much greater than a coherence length, any increase in resistance over a fraction $\sim (\frac{2\xi}{d})^2$ of the cross section will be completely shorted out. If $d \approx \xi$, this would not be true. Calculated values of $\Delta, \delta F_0, I_c, \frac{\delta F_0}{k_B T}$ and $M$ for Pb-Cu-Pb sandwiches with parameters given above, but with fluctuating volumes of $\xi^3$, are given in Table 1(b). We note that the condition $I < I_c$ is now valid only for $a_N/2 \leq 3.8 \times 10^{-3}$ cm.

Comparing $\frac{I}{2I_1}$ with $\frac{\delta F_0}{k_B T}$ and $M$ in Table 1(b) we again see that $R_F$ is immeasurably small.

Since, as will be shown, we do measure a resistance in Pb-Cu-Pb sandwiches at 20 mK, thermal fluctuations with $I < I_c$ (and therefore $<\Delta> > 0$) cannot be the cause. We conclude that $I > I_c$ and that $<\Delta>$ must equal zero for some distance about the center of the sample.

How does the measuring current suppress $\Delta$? In a bulk superconductor at $T = 0$ the supercurrent density $J_s$ is proportional to the net pair velocity, $v_s$. A good approximation is that $\Delta$ is unchanged by the current until $v_s$ equals the "depairing velocity", $\frac{\Delta(0)}{k_B T}$. 


$\Delta$ then drops precipitously to zero.

In a bulk superconductor near $T_c$, the situation is more complicated. Starting from the G-L equations, Bardeen has shown that $J_s$ is not proportional to $v_s$, but rather that

$$J_s = 2e|\psi|^2 v_s,$$  \hspace{1cm} (84)

where

$$|\psi|^2 = \psi_\infty^2 \left[1 - \frac{m v_s^2}{\alpha}\right].$$  \hspace{1cm} (85)

Here $\alpha$ is the coefficient of the linear term in the G-L equation. $|\psi|^2$ is proportional to $\Delta^2$. Substituting Eq. (85) into Eq. (84) and differentiating with respect to $v_s$, a maximum in the supercurrent is found to occur when

$$\frac{|\psi|^2}{\psi_\infty^2} = \frac{\Delta^2}{\Delta_\infty^2} = \frac{2}{3}.$$  \hspace{1cm} (86)

$|\psi|^2 = 0$ for currents above this maximum.

In the center of our sandwiches $\Delta$ is very small, so the G-L equations should apply. We assume then that the measuring current suppresses $\Delta$ to $\sqrt{2}/3$ of its unperturbed value at some distance $|x_c|$ from each interface and that $\Delta = 0$ for $|x| > |x_c|$. De Gennes has shown that
\[ I_c = \frac{\pi e D_{NN} N N k N}{2 T c s} \Delta_N \left( \frac{a_N}{2} \right)^2 \] (87)

where \( \Delta_N \left( \frac{a_N}{2} \right) \) is the unperturbed pair potential in the center of the sandwich.

We have not yet determined how much \( \Delta \) is suppressed by the current less than \( |x_c| \) from each interface. In a bulk superconductor, the depression of \( \Delta \) as a function of current is given by combining Eqs. (84) and (85). This gives

\[ |\psi|^6 = \psi_0^2 \left\{ |\psi|^4 - \frac{m_J}{\alpha} \frac{J_s}{4e^2} \right\}. \] (88)

Unfortunately, how this relates to \( \Delta \) in \( N \) when \( I > I_c \) is not clear, since it is only \( J_s \) and not the total current which appears. However, the magnetic penetration depth in \( N \) with an induced gap, \( \lambda(\Lambda) \), can provide insight to the effect of the current. In the dirty limit and for small \( \Delta \), \( \lambda(\Lambda) \) has been shown to have the form

\[ \frac{1}{\lambda^2(\Delta)} = \frac{4}{\rho c^2} \frac{\Delta^2}{k_B T} \psi_2 \left( \frac{1}{2} - \frac{1}{2} \right) \left[ 1 - \left( \frac{E_{NF}}{T} \right)^2 \right], \] (89)

where \( \rho \) is the resistivity in statohm cm = sec and \( \psi_2(Z) = \sum_{n=0}^{\infty} (n+Z)^{-2} \). \( \kappa^{-2} \) is defined by

\[ \ln \left( \frac{T}{T_{c N}} \right) = \psi \left( \frac{1}{2} \right) - \psi \left( \frac{1}{2} - \frac{E_{NF}^2 K^2}{2} \right). \] (90)
where $\psi$ is the psi (di-gamma) function. Equation (90) is often written as

$$\ln \left( \frac{T}{T_{cN}} \right) = \sum_{n \geq 0} \left( \frac{1}{n+1} + \frac{1}{n + \frac{1}{2} - \frac{\xi N K^2}{2}} \right),$$

(91)

using $\psi(Z) = -\gamma - \sum_{n \geq 0} \left( \frac{1}{2+n} - \frac{1}{n+1} \right)$ ($\gamma = 0.577$ is Euler's constant).

If only the $n = 0$ term is used in the sum, then $K^{-1}$ is just the one-frequency approximation decay length $K_N^{-1}$, Eq. (12b). $\ln$$

$$\left( \frac{T}{T_{cN}} \right)$$

versus both $\xi N K$ and $\xi N K_\varphi$ are plotted in Fig. 15. It is seen that $0 < \arg \psi_2 < \frac{1}{2}$; $\psi_2 \left( \frac{1}{2} \right) = 4.9$ and for small $Z$, $\psi_2(Z) \approx \frac{1}{Z^2} + \frac{\pi}{6}$.

The largest value of $\lambda(\Delta)$ obtains when $T - T_{cN}$. Using the same parameters for Cu as were used in the calculation of $\delta F_0$ above, we find that $\lambda(\Delta) \leq 5.9 \times 10^{-5} \frac{\sqrt{T}}{\Delta}$ cm with $\Delta$ in Kelvins. We therefore expect a Meissner effect in our specimens since, except near the center, $\lambda \ll d$.

We are faced with a contradiction. In Eq. (76) for $R(T)$ we assumed uniform excitation flow through the cross-section, $A$, of the specimen. Near the center of the specimen where $\Delta \ll k_B T$, this should be true. But near the point where $\Delta = k_B T$ where most of the excitations are Andreev reflected, $\lambda \ll d$. Lacking a proper understanding of the actual current distribution, we will assume the following. First, the pair current in $N$ will be subject to the Meissner effect and will flow only on the surface. Second, we will ignore the small fraction of the excitation current which flows in the annulus $\pi d \lambda$ and assume a uniform flow throughout $A$. 
Fig. 15. $\ln(T/T_{CN})$ versus $\xi_N^K$ (solid curve) calculated from Eq. (89) and $\ln(T/T_{CN})$ versus $\xi_N^{KN}$ (dotted curve) for the one-frequency approximation, Eq. (12b).
There will then be a field \( H(x,r) = \frac{2\pi Y(x)I_0}{cA} \) at a radius \( r \) due to the excitation current. \( Y(x) \) is the fraction of the total current, \( I \), which still exists as excitations at the point \( x \). It is given by

\[
Y(x) = \frac{\int_{\Delta(x)}^\infty f(E)dE}{\int_{0}^\infty f(E)dE} = \frac{\ln(1+\frac{\Delta(x)/k_B T}{k_B T}) - \frac{\Delta(x)}{k_B T}}{\ln 2} \tag{92}
\]

where \( f(E) \) is the Fermi distribution. Equation (92) is plotted in Fig. 16.

In analogy with the behavior of \( \Delta \) in a film whose thickness is much less than \( \lambda \) so that the field inside is uniform and equal to the applied field,\(^5^5\) the pair potential in \( N \) should be suppressed by the field from its \( I = 0 \) value to \( \Delta(I) \) according to

\[
\left( \frac{\Delta(I)}{\Delta(I=0)} \right)^2 = 1 - \left( \frac{H(x,r)}{H_c(x)} \right)^2 \tag{93}
\]

where \( H_c(x) = \sqrt{4\pi N(0)\Delta(I=0)} \) [Eq. (78)]. For Cu, \( H_c(x) = 84\Delta(x) \) Gauss, with \( \Delta \) in Kelvins. In our specimens with \( d = 0.32 \) cm, \( H(x,r)/Y(x) = 8.0 \) Gauss/Amp so that at \( r = \frac{d}{2} \), \( H(x,\frac{d}{2})/Y(x) = 1.27 \) Gauss/Amp.

Since \( \Delta \) is dependent on \( r \), our assumption that the excitation current flow is uniform cannot be correct. We will neglect this second order effect by taking \( H \) as independent of \( r \) and equal to the average value over the cross-sectional area, \( H(x) = \frac{2}{3} H(x,\frac{d}{2}) \).

We can now determine \( \Delta_c \), the value of \( \Delta(I=0) \) at \( x_c \) where the pair potential is suppressed to zero. Combining Eqs. (86) and (93) we find...
Fig. 16. Fraction of total current which still exists as excitations at the point where the pair potential equals $\Delta(x)$. Note that 67% of reflection occurs within one decay length of $\Delta = k_BT$. 
Equation (94) can be written to show the implicit relation between the measuring current, \( I \), and \( \Delta_c \):

\[
\Delta_c = \frac{I d y(\Delta_c)}{30 k_B} \sqrt{\frac{\pi}{N(0)}},
\]

(95)

where \( I \) is in Amps.

Setting \( E = \Delta(I) \), we find that the distance from the interface where the excitations are Andreev reflected is given by

\[
x = -k_N^{-1} \ln \left( \frac{1}{\Delta_N(0)} \sqrt{E^2 + \frac{H^2}{4N(0)}} \right).
\]

(95)

\( H(E) \) is just \( H(x) \) evaluated with \( \Delta(x) = E \).

By definition, \( \xi(E) = a_N - 2x \).

Equation (76) can now be written

\[
\begin{align*}
R(T) &= \frac{\rho_N}{Ak_B T \ell n^2} \left[ \left( a_N - 2k_N^{-1} \ln \frac{\Delta_N(0)}{\Delta_c} \right) \sqrt{\frac{2}{3\Delta_c}} \int_0^{2\Delta_c} f(E) dE \\
&+ \int_{\Delta_N(0)}^{\infty} (a_N - 2x) f(E) dE + a_N \int_{\Delta_N(0)}^{\infty} f(E) dE \right].
\end{align*}
\]

(97a)
Here $\Delta_N(0)$ is the value of the pair potential in $N$ at the interface taking into account the effect of the current. After some algebra Eq. (97a) becomes

$$R(T) = \frac{\rho_N}{A\xi n^2} \left\{ a_N \xi n^2 - 2k^{-1} \Delta_N(0) \sqrt{\frac{2}{3}} \frac{\Delta_c}{k_B T} - \ln(1 + e^{\frac{2}{3}} \frac{\Delta_c}{3k_B T}) \right\}$$

$$+ \ln(1 + e^{\frac{2}{3}} \frac{\Delta_c}{3k_B T}) + 2k^{-1} \int \frac{\Delta_N(0)}{\sqrt{3} \Delta_c} \ln \left( \frac{1}{\Delta_N(0)} \sqrt{1 + \frac{H^2(E)}{4\pi N(0)}} \right) f(E) dE \left\} \right.$$  

(97b)

2. **High Temperature Resistance**

At temperatures above $\frac{\Delta_N(0)}{k_B T}$, almost all excitations reach the SN interface. As $T$ approaches $T_{cs}$, $\Delta_S(\infty)$ goes to zero and the excitations are able to enter $S$. They travel an average distance $\lambda_3$ [Eq. (50b)], dissipating energy until $Q = 0$. 
We will approximate the actual hyperbolic tangent shape of $\Delta$ in $S$ by a step change from 0 to $\Delta_\infty(T)$ at the interface. In the clean limit the coherence length in $S$ is $\xi(T) = \frac{h v_F}{\pi \Delta(T)}$. For Pb, $v_F = 0.43 \times 10^8$ cm sec$^{-1}$ and $\frac{\Lambda_0}{k_B} = 15.5$ K, so at $T << T_{cS}$ $\xi_S(Pb) = 6.4 \times 10^{-6}$ cm. For Sn, $v_F = 0.65 \times 10^8$ and $\frac{\Lambda_0}{k_B} = 6.6$ K. Therefore at $T << T_{cS}$, $\xi_S(Sn) = 2.3 \times 10^{-5}$ cm. Typical mean free paths at 4.2 K are $\sim 10^{-3}$ cm for both Pb and Sn. Therefore, the clean limit is appropriate for our evaporated Pb and Sn, and for estimates of $\lambda_3$ from Sec. IIC we find $\xi_S << \lambda_3$. Close to $T_{cS}$ both $\lambda_3$ and $k_B^{-1}$ increase as $\sim (\Delta_\infty(T))^{-1/2}$, so a step change in $\Delta$ should be good approximation for all temperatures.

The transmission probability for excitations incident on a smoothly varying $\Delta$ with an asymptotic value ($x \rightarrow \infty$) of $\Delta_\infty(T)$ is given approximately by Eq. (73). The fraction, $F$, of the total current entering $S$ as excitations at any temperature $T$ is given by

$$F(T) = \frac{\int_0^\infty f(E)T(E)dE}{\int_0^\infty f(E)dE} = \frac{\int_0^\infty f(E)T(E)dE}{\int_0^\infty f(E)dE} = \frac{\int_0^\infty f(E)T(E)dE}{k_B T \ln 2} . \quad (98)$$

As we saw in Fig. 14, however, $T(E) \approx 1$ for $\frac{E-\Delta_0}{\Lambda_\infty} \sim 10^{-3}$ in both Pb and Sn. We therefore set $T(E) = 1$ in Eq. (9). Then $F(T)$ has the identical form to Eq. (92) where we write $\Delta_\infty(T)$ in place of $\Delta(x)$. $F(T)$ is plotted in Fig. 17 using the values of $\frac{\Lambda(T)}{\Lambda(0)}$ given by Muhlschlegel. 33

The resistance at high temperatures is thus

$$R(T) = \frac{\rho_N a_N}{A} + \frac{2 F_p \lambda_3}{A} . \quad (99)$$
Fig. 17. Fraction of total current which enters $S$ as excitations.
III. EXPERIMENTAL METHODS

A. Specimen Preparation

The SNS sandwiches were prepared in most cases by cleaning the N material by DC sputter etching and then vapor depositing S.

The normal metals used were, except in the case of iridiium, the purest readily available from commercial suppliers. (We measured the $T_c$ of Ir so that $[N(O)V]_\text{Ir}$ was not needed from the literature.)

Iridium and platinum were purchased as foils. Bulk samples of copper, silver, magnesium and nickel were cold rolled between stainless steel rollers. Rhodium was melted from a powder and then rolled. The foils were cleaned by scrubbing with Labtone and a cotton tipped applicators and rinsed with distilled water.

All foils were clamped between water-cooled masks of 0.32 cm thick OFHC copper (diameter = 6.7 cm) or 6061-T6 aluminum (diameter = 8.3 cm) depending on whether sputterer I(SI) or sputterer II(SII) was used (see Appendix 1). The masks also acted as the sputtering cathode. The sputtering anodes were about the same diameter as the masks and the anode-cathode separation about 4 cm in both SI and SII. The sputtering voltage was always 5kV, the sputtering current was 10 mA to 30 mA, corresponding to current densities in SII of $(1-3) \times 10^{-5}$ A/cm$^2$ and bombardment rates of $(1-2) \times 10^{15}$ sec$^{-1}$. The sputtering pressure was 20-30 mT (measured with a $T_c$ gauge calibrated for air). Sputtering times varied from 15 to 125 min. depending on the sputtering rates for the normal metal foils. The foils were sputtered long enough so that their surfaces had an etched and uniformly cleaned appearance. It was because a macroscopic amount of metal
was removed that there was no effort made to remove roller contamination by chemical etching.

The first set of Cu masks was made by countersinking four 0.31 cm diameter holes with a 45° countersink. The holes were about 0.05 cm high. It was found that copper specimens sputtered with these masks were measurably thicker after sputtering. It is thought that glancing incidence of the sputtering gas on the walls of the countersunk holes knocked masks material onto the foils. Consequently, the masks were remade by milling a 1.3 cm wide slot above the holes so that only 0.025 cm of mask material remained. All subsequent specimens showed net removal of the foil metal.

A 10.2 cm I.D. glass pipe cross served as the vacuum vessel for both SI and SII. The flanges on SI were sealed to the cross by 163×0.635 cm Viton-A o-rings. SII was initially sealed with 163×0.31 cm Viton-A o-rings. It was consistently found that silver and magnesium foils sputtered with these o-rings came out black and reddish-brown, respectively. Auger analysis of the magnesium indicated surface contamination by sulfur, fluorine and carbides, all believed to be present in Viton-A (a fluorocarbon rubber). The colors of AgS and MgS₂ are known to be black and reddish-brown, respectively.

It was found that by switching to o-rings of 0.31 cm wide 4N grade indium, sputtered Ag looked clean and Mg looked yellow-green. Auger analysis (Table 2) of the yellow-green magnesium indicated Al and O as the only contaminants. Neither Al or Mg-Al alloy is thought to be colored. MgN₂, though is yellow-green. A plausible explanation for how a nitride could have formed is as follows:
Table 2. Auger analysis of Mg sputter-etched in vacuum vessel sealed with In o-rings. The units are arbitrary. Comparisons between different columns of the same element are good to ~50%. Comparisons between different elements are not valid.

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<td>Unsputtered</td>
<td>Sputtered</td>
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<td></td>
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<td>Spot</td>
<td>Surface</td>
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<td>9</td>
<td>8</td>
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The oxide on the Mg formed by exposure to air was removed by sputter-etching. The O$_2$ in any air leak into the vacuum chamber or outgassing would have been pumped much faster than the N$_2$ by the liquid N$_2$ pump (vapor pressure of O$_2$ at 77 K is $\sim$ 200 Torr). The remaining N$_2$ would then have been ionized and impelled into the cathode, thus forming MgN$_2$.

It is noted that copper sputtered with the same Al masks always came out looking clean; this had been considered an indication that mask material was not being deposited on the samples. The black appearance of the sputtered Ag foils was first considered as a light trapping effect caused by etch pits, as has been noted by others. This obviously happens to sputter etched lead because even with the In o-ring it appears flat black when viewed normal to its surface, but shiny at glancing incidence. However, due to the change in the appearance of Ag after switching to the In o-rings, we concluded that our initial judgement on Ag was in error.

The base pressure of SI was $\sim$ 6$\times$10$^{-7}$ T (it was never baked) and that of SII $\sim$5$\times$10$^{-9}$ T with a 24 hour bake at 120°C. The sputter gas was 99.99% Ar in SI and 99.999% Ar in SII. The supplier of the high purity Ar cited as contaminants $<$ 5ppm N$_2$, $<$ 2 ppm O$_2$, $<$ 0.5 ppm CO$_2$, and $<$0.5 ppm hydrocarbons.

After sputtering, the superconductor was vapor deposited on the sputter-etched areas. This required different procedures in SI and SII. In SI, the boat was pushed into the space between anode and cathode by means of its water-cooled high current feedthroughs. However, the rubber o-rings sealing these feedthroughs leaked badly during this movement. We therefore put $\sim$ 2 psig of 99.995% He gas in the glass cross before moving the boat so that the only leak would be He outward. Ar was not used since
it liquifies on contact with the cryopump (77 K) in the vacuum vessel when above \(-200\) Torr. This procedure was not adequate. The contaminants in the He were 50 ppm so that the partial pressure of the contaminants was \(-5\)\(\mu\) when the total pressure was 2 psig. Since a gas with unity sticking coefficient builds a monolayer per second at \(10^{-6}\) torr, our sputter-etched surface could have been badly contaminated during the \(-2\) minutes it took to pressurize the system, move the boat, and pump down to \(-10^{-6}\) torr at which evaporation was begun. It was necessary in SI to open the vacuum vessel in order to turn the masks.

SII avoided many of these problems. The boat was always in place for evaporation. The only dynamic seal was for rotation of the sputtering cathode, and this raised the pressure only \(-10^{-9}\) torr. The total time between the end of sputtering and the beginning of evaporation was \(10\) seconds. SII did not have to be opened in order to sputter the second side.

At the typical sputtering pressure \((-20\mu\)), the mean free path was \(-1\) cm. It is therefore likely that neutral atoms which had been removed from the masks were ionized before hitting the glass cross or cold anode. The ions would then have been impelled into the masks and foils. That this was not a gross effect was clear from the fact that the foils got thinner from being sputtered and that Cu specimens sputtered with Al masks always looked clean. However, it is not clear whether the Al on the Mg revealed by the Auger spectrum was due to Al\(^+\) or from the 0.025 cm high walls of the holes in the masks. If it were due to Al\(^+\), then making the masks smaller would reduce the problem.
It should be noted that the ion gun used for depth profiles in the Auger spectrometer was able to remove the yellow-green color from Mg. It used 99.99% Ar. The foil was in a vacuum of \( \sim 10^{-10} \) torr.

Heating of the specimens during evaporation or during sputtering of the second side would promote diffusion of one metal into the other. To be sure this was not a problem, the temperature of the specimens was measured. Since the specimens were 5kV below ground, it was difficult to bring thermometer leads out of the vacuum system. Instead, heat sensitive foils which changed color irreversibly when heated above certain temperatures were clamped between the masks in place of the specimens. These sensors have a resolution of 5.6°C. It was found that the maximum temperature of the sensor during both sputtering and evaporation was <43°C.

The completed SNS sandwiches were either quickly placed in the plastic containers of Polaroid print coaters and submerged in liquid nitrogen or mounted in the dilution refrigerator specimen frame. The plastic containers did not leak liquid nitrogen.

**B. Dilution Refrigerator**

The dilution refrigerator used in this experiment was designed by Timothy L. Thorp, based on a published design. Its cooling power and operating characteristics followed closely those of Wheatley.
The refrigerator has four discrete OFHC copper heat exchangers and an OFHC copper mixing chamber. Both the heat exchangers and the mixing chamber are partly filled with Feltmetal sintered copper. The Feltmetal is made of copper fibers and has a density of 1.8 gm/cm³. According to the quoted surface area of 260 cm²/gm it must have a surface to volume ratio of 460 cm²/cm³. The sintered copper is cut from sheets and turned on a lathe into cylinders, care being taken not to plug the flat surfaces. The cylinders are 20\(\mu\) per 1 cm diameter larger than the holes into which they are to fit. The sintered copper is cooled to 77 K before pressing it into the body of the heat exchanger which is at room temperature.

The mixing chamber contains 3.6 cm³ of Feltmetal and can hold about 3.6 cm³ of liquid. It has a blind 10-32 tapped in the bottom for attaching experiments. Two tubes (normally capped) which are connected to the inside of the mixing chamber also come out of the bottom and can be used to draw off part of the mixture for providing thermal contact (e.g. to the CMN thermometer discussed below).

The 1° pot has a 30Ω carbon resistance thermometer calibrated in situ by comparison with the vapor pressure of the He⁴ in the pot measured with a manometer filled with Dow-Corning DC702 silicone diffusion pump oil. The still's 10Ω Allen-Bradley thermometer was not calibrated. Each heat exchanger had a 230Ω Speer resistor whose calibration has been published.⁶¹

The mixing chamber has three thermometers. One is a 100Ω Speer resistor.⁶¹ It is wired 4-terminal and is mainly used as the
sensor for a "Rochlin bridge" temperature controller. The controller drives a heater made of twisted manganin wire of total resistance $1000\Omega$ which was wrapped around the body of the mixing chamber and epoxied to it with Stycast 2850 GT.

A 4-terminal germanium cryogenic thermometer was relied on over most of the operating range of the refrigerator. Its resistance was $4.2\Omega$ at $300\text{K}$, $6.8\Omega$ at $77\text{K}$, $44\Omega$ at $4.2\text{K}$ and $\sim20k\Omega$ at $20\text{mK}$. It was calibrated from $0.3<T<25\text{K}$ before it was mounted in the refrigerator and least squares fitted to the function

$$\frac{1}{T} = \sum_{N=0}^{5} a_N (\log R)^N. |T_{\text{calc}} - T_{\text{obs}}| \text{ is } \sim4\text{mK} \text{ at } 7\text{K} \text{ and } <1\text{mK} \text{ at } 0.3\text{K}.$$

From $0.3\text{K}$ to $\sim40\text{mK}$ it was calibrated by the paramagnetic susceptibility of CMN. Below $40\text{mK}$ it was calibrated by both CMN and a Co nuclear orientation thermometer.

Our first attempt to calibrate the Ge thermometer below $0.3\text{K}$ by CMN was not successful. At the time we were using a mixing chamber machined out of Epibond 100A epoxy. The CMN was in a chamber at the bottom of a $7.62\text{ cm}$ long by $1.27\text{ cm}$ O.D. by $0.32\text{ cm}$ I.D. Epibond 100A tube. The CMN was a fairly tightly packed powder in the form usually employed by Wheatley: a right circular cylinder with diameter ($1.9\text{ cm}$) equal to its high. The other end of the epoxy tube had a $\frac{1}{4}$-28 male thread which was screwed into the bottom of the mixing chamber and sealed with a glycerine and soap solution. A reliable procedure evolved in which we heated both the male and female threads with a heat gun before applying the melted soap.
solution with a brush. The two pieces were then quickly and tightly screwed together.

The problem with the above arrangement was that the thermal conductance to the CMN provided by the He$_3$-He$_4$ mixture in the tube was very small. The thermal conductivity of the Epibond 100A was negligible, and the time constant for the thermometer was on the order of hours.

The time constant was reduced to minutes by going to the copper mixing chamber and the following arrangement. The CMN was again in the form of a right circular cylinder with diameter (1.9 cm) equal to its height, inside of an Epibond 100A container. Now, however, the container was attached to a stalk of #40aWG Formvar insulated copper wires about 1.7 cm long and 1.27 cm in diameter. The copper wires had been impregnated with Epibond 121 before being drawn into a Teflon cylinder in which it was cured. The other end of the stalk was electroplated with copper which was subsequently machined flat. A 10-32 copper screw was then electroplated to the stalk. The stalk provided thermal contact with the mixing chamber and a capillary leading from the mixing chamber into the CMN chamber provided He for thermal contact with the stalk. The insulation on the wires in contact with the liquid He (~1 cm long) had been removed. The CMN chamber was sealed to the stalk with Epibond 121. The insulated wires rather than a solid Cu rod were used so as to avoid eddy current heating from the AC mutual inductance measurement.
The susceptibility of the CMN was measured as described by Abel, Anderson and Wheatley. Mutual inductance coils were mounted on the outside of a vacuum can with a long stainless steel tail with O.D. = 3.81 cm. The mutual inductance was measured with a Cryotronics mutual inductance bridge driven at 17 Hz whose output was monitored with a PAR lock-in amplifier, Model HR-8. The bridge required adjusting two potentiometers, one which was proportional to the mutual inductance, M, and the other the resistance, R. Balance was indicated by a null in both quadratures of the lock-in.

M was plotted versus 1/T, where T was determined by the Ge thermometer over the range 0.3<T<4.2 K. The high temperature straight line graph was then extrapolated to T more than an order of magnitude below 0.3K. Assuming a Curie-Weiss law, \( \chi \propto (T-T_c)^{-1} \) with \( T_c \) negligible in the range of calibration, the refrigerator was determined to reach 0.018K. The Ge thermometer was thereby calibrated and the C thermometer was found to agree with the published data, except at the lowest temperatures where the published data indicated there had been self-heating.

During the successful CMN calibration run described above, a Co\(^{60}\) thermometer was clamped to the inside of the copper mixing chamber with a brass screw and washer. The cobalt had been spark cut into a needle with dimensions 0.1x0.1x1 cm, with the needle axis parallel to the c-axis and to the magnetization. The
strength of the thermometer was \( -1 \mu \text{ Curie} = 3.7 \times 10^4 \) disintegrations/sec. With this mounting arrangement the heating of \( 1.8 \times 10^{-2} \) erg/sec due to internal capture of the 0.31 MeV \( \beta \)'s emitted during the decay of Ni\(^{60}\) caused the Co to read 40 mK when the CMN indicated 20 mK. Before the next run the Co needle was electroplated with copper to a 4-40 copper screw which was screwed into another hole in the bottom of the mixing chamber. The next calibration run then showed agreement between the CMN and the Co\(^{60}\).

The Co\(^{60}\) was measured with 7.6 cm x 7.6 cm NaI(T\(\beta\)) scintillation counter with its axis parallel to the axis of the needle and placed 14.5 cm from the center of the needle. The temperature was derived from the ratio, \( W(\theta, T) \), of low to high temperature counts (after subtracting the background) using formula given by Thorp, et al.: 68

\[
W(\theta, T) = 1 + U_2 F_2 Q_2 B_2(T) P_2(\cos \theta) + U_4 F_4 Q_4 B_4(T) P_4(\cos \theta) \quad (100)
\]

\[
U_2 F_2 = - \frac{2}{7} \sqrt{\frac{13}{6}} = -0.4206
\]

\[
U_4 F_4 = -\frac{\sqrt{26}}{21} = -0.2428
\]

\( \theta \) is the angle of the counter with respect to the magnetization of the Co. \( Q_2 \) and \( Q_4 \) are solid angle corrections, dependent on scintillator size and experimental geometry. In our experiments, \( Q_2 = 0.961 \) and \( Q_4 = 0.871 \). \( B_2 \) and \( B_4 \) are statistical tensors which contain all the temperature dependence. We placed the scintillator at \( \theta = 0 \), so
\( P_2 = P_4 = 1 \). \( W(0, T) \) for our geometry is shown in Fig. 18. It can be seen that the nuclear orientation thermometer is not useful until \( T \leq 40 \text{ mK} \). The counting rate was \( \sim 10^2 \text{ sec}^{-1} \). Counting for \( 10^3 \text{ sec} \) given a statistical uncertainty \( \sqrt{N}/N = 0.003 \). We found, however a steady drift per day at constant temperature greater than this. The drift was due to the well known long term decrease in the gain in the photomultiplier tube. A gain stabilizer was made that measured the count rate difference between the upper and lower halves of one of the Co\(^{60}\) gamma peaks and fed back to the photomultiplier tube power supply to keep the difference constant. The rate decreased in each half as the temperature was lowered, but the difference was kept constant. It is noted that the "high" temperature counts were taken with the LN and LHe in the dewar since it was found the the LN and LHe attenuated the rates by 10\% and 3\%, respectively.

At low temperatures the resistance thermometers were usually measured with a 4-terminal bridge\(^69\) and an HR-8 run at 27 Hz. The bridge is transformer coupled to both the reference and the input of the lock-in and one corner of the bridge circuit was grounded. It was found near the end of this work that one wire in the epoxy filled (Stycast 2850CT) lead tube connected to the Ge thermometer had a 200 k\(\Omega\) short to ground (probably due to scratching by the quartz crystals in the (Stycast). This short caused a 6 k\(\Omega\) (\(-1\) mK) error in the reading of the Ge thermometer at 20 mK. Part of this error was due to ground loop currents heating the thermometer since its apparent temperature was
Fig. 18. Counting rate as a function of temperature for our Co\textsuperscript{60} nuclear orientation thermometer.
very dependent on grounding. Ungrounding the bridge alleviated the problem. The apparent temperature of the Ge thermometer became independent of the reference level at the lowest temperatures when the dissipation in the thermometer was kept below $7 \times 10^{-15} \text{W}$.

At 2K the resistance of the Ge thermometer is $58 \Omega$ and $dR/dT = \frac{0.01 \Omega}{4 \text{mK}}$. At 7.2K, $R = 32.5 \Omega$ and $dR/dT = \frac{0.01 \Omega}{4 \text{mK}}$. Since the long term accuracy of the Dekastat in the AC resistance bridge is $\pm(0.02\% + 10 \text{m}\Omega)$, at 7.2K our accuracy is $\pm 20 \text{m}\Omega$ or 8 mK, which is about twice the RMS uncertainty in the thermometer's calibration. Above about 4K, therefore, we used a stable current source and a potentiometer to measure the resistance of the Ge thermometer. The current source consisted only of Hg cells, switches and wire wound resistors. The potentiometer was a Leeds and Northrup Type K-3 with a Keithley Model 149 Milli-microvoltmeter as the null detector. This combination had an accuracy of $\sim 3 \text{m}\Omega$.

The C and Ge thermometers were greased (Apiezon N) inside separate holes drilled through the body of the mixing chamber. However, the thick bakelite surrounding the graphite in the C thermometer provided negligible thermal contact. The Ge element was suspended by its leads inside an hermetically sealed can to avoid stresses which might change its calibration, so greasing it into the hole also provided insignificant thermal contact. Thermal contact was attained by wrapping 100 cm of 35AWG Formvar insulated Cu wire for each thermometer lead around the mixing chamber and epoxzying them with Stycast 2850GT.

Around the outside of the dewar were three $\mu$-metal shields. A flux-gate magnetometer indicated that they reduced the earth's field to $<5 \times 10^{-5}$ Oe. However, the welds used to form the inner can of the dewar out of
stainless steel sheet were ferromagnetic. Using the magnetometer in a superinsulated "hot finger" it was determined that the welds produced a field of 0.8±0.2 mOe at 4.2K on the samples. Once at 4.2K, the field on the samples was held constant by a lead foil cylinder inside the dewar with diameter of 10.8 cm and length of 37.5 cm. The foil was varnished to the inside of a Cu cylinder so that the large temperature gradients during the initial He transfer would not produce thermoelectrically induced currents in the low thermal conductivity Pb.

On the outside of the brass vacuum can of O.D. = 8.9 cm was wound a copper field coil 20.3 cm long. It was calibrated at room temperature to produce a field at its center of 19.2 Oe/A. At 4.2K, the coil produced a field of 7.9 Oe/A. This reduction in field can be understood by an elementary application of Ampere's circuital law for a long solenoid in an infinite superconducting shield. The ratio of the field inside the solenoid with the shield, \( H_{\text{shield}} \), to the field with no shield, \( H_0 \), is given by

\[
\frac{H_{\text{shield}}}{H_0} = \frac{A_2 - A_1}{A_2},
\]

where \( A_1 \) is the cross-sectional area of the solenoid and \( A_2 \) is the cross-sectional area of the shield. From the diameters of the shield and solenoid given above, Eq.(101) gives \( H_{\text{shield}}/H_0 = 0.32 \). The difference between the observed and calculated ratios can be accounted for by the fact that neither the shield nor the solenoid are long (length/diameter>>1).

C. Mounting of Specimens

The specimens were clamped to a frame of OFHC copper (see Fig. 19) by two brass bars which were screwed to the frame with brass screws. The
specimens were electrically isolated from the frame and the clamps with a single layer of cigarette paper. Thermal contact was at first provided by soaking the paper in GE7031 varnish diluted with a 50:50 toluene and alcohol mixture and clamping while still wet. It was difficult, however, to clamp the foils tightly without shorting through the cigarette paper, even with the edges of the foils and the bars carefully rounded. Sometimes shorts appeared after the varnish had dried. To avoid having to soak off the dried varnish, we switched to impregnating the paper with melted Apiezon N grease. This did not eliminate the occasional shorts, but it made it easier to correct them.

The thermal resistances of GE7031 varnish and Apiezon N grease as a bonding agents between Cu blocks has been measured by Anderson and Peterson. In general, the thermal bond resistance is given by

$$R = 2r_B + \frac{\theta}{KA}$$

(102)

where $r_B$ is the Kepitza resistance, $l$ is the thickness of the bonding agent, $A$ the contact area, and $K$ the thermal conductivity. A theory by Little has shown that $r_B$ should have a temperature dependence roughly proportional to the inverse of the lattice specific heat; i.e. $r_B \propto T^{-3}$. For GE7031 the data is fit by

$$R(7031) = 2\left(\frac{8.5 \times 10^{-7}}{T^3}\right) + \frac{l}{350 T^2}$$

(103a)

and for Apiezon N by

$$R(N) = 2\left(\frac{6.5 \times 10^{-7}}{T^3}\right) + \frac{l}{100 T^2}$$

(103b)
Fig. 19. Specimen frame showing mounting and pressure contacts to superconductor on specimens.
where R is in K sec erg\(^{-1}\) cm\(^{-1}\).

Cigarette paper is 2.0x10\(^{-3}\) thick and A was about 1 cm\(^2\). At 20 mK we find from Eqs. (103a) and (103b) that R(7031) = 0.23 \(\frac{K}{sec\ erg}\) and R(N) = 0.22 \(\frac{K}{sec\ erg}\). Our maximum specimen current was 0.1A and typical specimen resistance 10\(^{-7}\)\(\Omega\) so that the dissipation was 10\(^{-9}\)\(W = 10^{-2}\ erg/sec\). The temperature rise of the specimens at 20 mK is thus 2 mK.

Electrical contact to the evaporated superconductors on the foils was at first made with pressure contacts. As illustrated in Fig. 19, these consisted of a Be-Cu spring with a drop of Stycast 2805GT on the end which was screwed to one of the clamps. On the back side of the frame running diagonally across was a brass bar with a screw in the center. The tip of the screw also had a drop of Stycast 2850 GT on it. Lead strips 50 \(\mu\)m thick, about 1 cm long, and somewhat less than 0.31 cm wide were mechanically cleaned and placed between the evaporated superconductors and the Stycast on each side and soldered to terminals made of \#20 AWG Formvar insulated copper wires which were epoxied to the frame with Stycast 2850GT. The screw on the back side was then adjusted so that there was good contact between the specimens and the lead strips. If the resistance between the ends of the strips was \(\leq 20\ \Omega\) at room temperature, it was found that the pressure contacts would then have a critical current \(> 100\ mA\) at 4.2K.
Unfortunately, the resistance sometimes went above 20 mΩ after the frame was mounted in the refrigerator, and far worse, sometimes not until the refrigerator was cooled to 77 K. The problem was overcome by spot-welding the Pb strips to the specimens. (This technique was only used on specimens with Pb as the superconductor.) A Hughes VTA-66 parallel gap weld head with broad copper tips and a 900 μF capacitor charged to ~25 V were used. The capacitor was discharged through an SCR. The evaporated lead disks and the lead strips were polished with a cotton swab and acetone before welding. Both weld tips were placed on the strips and welds were made at ~10 places on each side. The specimen with both strips attached could be lifted by one strip and could be quite rudely handled without coming loose.

To determine whether the weld penetrated down to the SN interface, Pb ~25 μm thick was evaporated onto a microscope slide. This was thinner than the Pb on the specimens which were spotwelded. Even after many strong welds of the Pb foil to the evaporated film, no damage was visible when looking through the glass. It was important, however to keep the tips clean with fine emery paper; otherwise, damage did occur.

We considered lack of visible damage on the slide as adequate assurance that the SN interface would be effected by the welds. This was because the thermal conductivity of the glass at 300 K is about 100 times less than that of the metal foils.

The spotwelded specimens were mounted on the frame with the Pb strips bent normal to the foil for ~0.3 cm and then bent parallel to the frame and soldered to the wire terminals.
D. Mounting of Frame in Dilution Refrigerator

The frame, with specimens mounted as described in Sec. IIIC, was screwed into the bottom of the mixing chamber. The current leads were then soldered to the copper terminals with 60/40 Sn/Pb solder.

The current leads were made by tinning #36AWG manganin wire with 50/50 Sn/Pb solder, twisting a pair of these tinned wires together, retinning the twisted pair, and then applying many coats of GE7031 vanish diluted with a 50:50 mixture of toluene and alcohol. The tinning was done by first applying acid flux (either Lloyds 75 stainless steel soldering flux or SCS No. 76) to the wires with a cotton swab and then pulling them through a cup of solder kept molten on a hot plate. The wires were kept under the surface of the solder by a notched Teflon rod.

The varnish was applied with an acid brush with the wires hanging vertically and stretched by weighting with a C-clamp or Vise-grip. The varnish was cured at 125°C for 1-2 hours in an oven. Getting the varnish on without it beading was tricky. It was later found quite useful to heat the wires so that they were just too hot to hold while applying the diluted varnish. The varnish then dried almost instantly, eliminating the beading. The wires were conveniently heated by hanging them from one terminal of a low voltage power supply, connecting the other terminal to the C-clamp or Vise-grip, and passing ~2A through them. The critical currents of these leads exceeded 2.8A at 4.2 K.
Why were not Pb or Nb wires used for current leads? Pb wires with large enough diameter so that they could be handled have large thermal conductivity, even when superconducting. Before we became familiar with spotwelding, we only knew how to make connections to Nb by sticking the wires in fairly large solder blots. Besides being inconvenient, these connections were not metallic, but rather were Josephson junctions with critical currents less than the 100 mA we wanted to apply. If solder-Nb unions with critical currents >100 mA at 7.2K can be made in a manner similar to solder-Ta unions, then Nb wires would be most convenient.

The current leads were ~200 cm long. They were brought down from a terminal board above the 1° pot, being wrapped around the 1° pot, still, heat exchangers and mixing chamber and heat sunk with Apiezon N grease. After being soldered to the copper terminals (one wire between each of the 4 or 5 specimens which were connected in series and 2 on each end), they were tied to the frame with waxed Dentalfloss.

A copper radiation shield was then secured to the frame by screwing it to two 0.63 cm diameter Lucite posts 3.2 cm long which were themselves attached to the frame by nylon studs. A copper tab silver-soldered to the radiation shield was clamped to the fourth heat exchanger which ran at ~25 mK when the mixing chamber was at 20 mK.
As shown in Sec. IIIC, self-heating of the specimens at the maximum current (~10^{-9} W) should not have produced a significant temperature rise. All wires to the frame were carefully heat sunk to the mixing chamber and the radiation shield should have reduced 4.2 K radiation to a negligible amount. Since the thermometers measured the temperature of the outside of the mixing chamber and the frame was in excellent thermal contact with it, it is felt that the mixing chamber thermometers gave the correct specimen temperature.

E. Superconducting Voltmeter Circuits

In each refrigerator run, 4 or 5 specimens were wired in series as described in the previous section. At the ends of the series were two solder blobs, into which were plunged at least 0.4 cm of #36 AWG Nb wires which had their Formvar insulation removed and which had just been cleaned with fine emery paper. If the Nb wire slipped out of the blob before it solidified, the wire was recleaned before reinserting. This procedure never failed to produce superconducting connections.

The Nb wires were ~200 cm long and tightly twisted. The wires were untwisted for ~10 cm at the bottom so that they could reach the blobs at opposite ends of the frame. They were woven between the specimens and the frame so as to decrease the pick-up area in a magnetic field. The twisted pair was wrapped around the mixing chamber, heat exchangers, still and 1° pot and heat sunk with Apiezon N.
Above the 1° pot the pair was connected to the superconducting voltmeter. The voltmeter initially utilized a SLUG \(^{78}\) and later an RF SQUID. Using either voltmeter, enough current was usually applied to the specimens so that their resistances could be measured to a part in \(10^4\).

1. **SLUG Voltmeter**

The Nb signal leads of three SLUGs \(^{78}\) were connected in series (see Fig. 20) using solder blobs (this was before we knew about spotwelding Nb) and with a standard resistor made of 1.1 cm of #12 AWG Cu wire. The resistance of the standard was \(5.38 \times 10^{-7}\) Ω. The heavy nichrome wires of the SLUGs and the ends of the resistor were soldered to heavy Formwar insulated Cu wires which were epoxied into holes in a 0.1 cm thick Cu plate. The plate was put inside a solder covered Cu can whose open end was then covered with Pb tape. Number 26 AWG Formwar insulated Cu wires were connected to the SLUGs and resistor for applying current to them and sensing the SLUG's voltage. There were also two Pb wires connected to the ends of SQUID-resistor series. The wires were brought out of the can through a hole in the tape. All the Cu wires were connected to pins on a terminal board which had been hard soldered to the outside of the Cu can. The Pb wires were connected to two solder blobs. The terminal board and pins were made similarly to that on which the SLUGs and resistor were mounted. The can was then clamped to the underside of the vacuum can top flange, with Apiezon N grease providing thermal contact. The twisted Nb leads
Fig. 20. Voltmeter circuit employing SLUGS.
connected to the specimens, were cleaned and plunged into the molten blobs as described for the specimen frame end. The SLUGs, resistor, and specimen current leads were connected at room temperature to a box with 52 banana sockets via a Stycast 2850 GT-filled lead tube in the refrigerator with #36 AWG Formwar insulated Cu wires.

Great pains were taken to have the entire specimen circuit electrically floating. After many thermal cyclings, current leads would occasionally short to the still or heat exchangers. This could have been avoided by having coated these parts with varnish before heat sinking the wires with grease.

The SLUGs were driven and monitored at 1kHz with a PAR HR-8 lock-in with a Type A pre amp. The SLUGs were coupled to the reference of the HR-8 through a 1:1 isolation transformer. Input A of the pre amp was coupled through a Triad G-10 transformer wired to match a 17.5Ω source to 712 kΩ. Connected to input B of the preamp was a PAR Model 123 AC zero offset driven by the HR-8.

The DC output of the HR-8 ("monitor out") went to a Dymec Model 2460A DC amplifier with a Model 2461A-M2 gain unit. The output of the Dymec went to the standard resistor, thus providing the cryogenic circuit ground. In series with the Dymec output was an ESI 70 1kΩ, 0.01% resistor with a temperature coefficient of ±0±15ppm/°C. The voltage across this resistor was read on a Data Technology Model 350 4 1/2 digit DVM filtered with a single stage RC filter with RC = 1 sec.

The SLUG dc bias, zero adjust, and the specimen currents were provided by battery operated current sources with long term
stability better than 100 ppm. The specimen current was read on a Digitec Model 275A 4 1/2 digit panel meter with a simple RC filter added to reduce the noise put out by the meter.

Typical current resolution of the SLUGs was ~1µA. The voltage resolution was therefore $5 \times 10^{-13} V$. The open loop gain of the circuit was $\sim 10^5$.

2. RF SQUID

In order to improve our voltage resolution by an order of magnitude, a Develco Model 8103 RF SQUID was installed in the refrigerator in place of the SLUGs. Its current resolution was $2 \times 10^{-9} A$. Our voltage resolution was therefore limited by Johnson noise from the standard resistor rather than by current resolution, as was the case with SLUGs.

The SQUID sensor is mounted at the end of a 95 cm long, 0.31 cm O.D. stainless steel tube which acts as the center conductor of a coaxial cable. The outer conductor is a thin wall 0.95 cm O.D. Be-Cu tube. The field coil leads were Pb strips 0.2 cm wide. They came out of the bottom of the Be-Cu tube.

The field coil leads were brought into the vacuum can of the refrigerator through an epoxy seal. The seal was made in a Teflon mold, as shown in Fig. 21. The wires used were a Pb alloy 3 amp fuse wire. Since the expansion coefficient of the epoxy (Stycast 2850 GT) did not match stainless steel, the tube walls were tapered so they were paper thin on the epoxy end. They could therefore easily deform without straining the epoxy. This is the practice in making copper-to-glass seals, commonly known as Housekeeper seals.
Fig. 21. Cross-section showing Teflon mold and superconducting feed-through.
The epoxy was filled to the same level inside and outside the tube. The tube was made long enough so that the epoxy was not significantly heated while soldering it into the top flange. This and other cryogenic vacuum seals made with this procedure have had a 100% success rate and perfect reliability.

Two standard resistors were heat sunk to the 1° pot. One of resistance $1.93 \times 10^{-7} \Omega$ was made of manganin foil $5 \times 10^{-3}$ cm thick and 1 cm square. The edges were varnished with GE 7031 and the rest was covered with Sn-Pb solder. The solder was then coated with a solution of rosin in toluene. Lead tape 1 cm wide was attached simultaneously on each side by heating the three pieces between sheets of Al foil. The other standard of resistance $3.3 \times 10^{-9} \Omega$ was made with the same procedure but out 1 mm thick Cu. It was also 1 cm square.

The standard resistors were made out of sheet rather than wire so as to eliminate thermal EMFs. The Cu wire resistor used with the SLUGs (see Sec. IIE2) could have conceivably had a temperature difference across it of $10^{-6}$ K. Since the thermopower of Cu is $\approx 10^{-7}$ V/K, this would produce voltage noise fluctuations of $\approx 10^{-13}$ V. This is two orders of magnitude greater than the rms Johnson noise voltage in a 1/4 sec$^{-1}$ bandwidth (a 1 sec RC time constant) due to the $10^{-7}$ standard at 1 K.

The standard resistors were tightly clamped between one piece of Cu sheet which was folded around them. Cigarette paper saturated with Apiezon N grease provided electrical isolation. The Cu
sheet was silver soldered to a Cu cylinder which was greased with Apiezon N over a 1.4 cm O.D. post on the 1° pot. The contact area between the resistors and the Cu support was thus about 1 cm² on each side, apparently keeping the temperature gradient across the standards small enough so that no excess noise (above Johnson noise) was attributable to thermoelectric effects.

The resistors were each wired in series with a thermal switch of normal state resistance ~10⁻² Ω. The combinations were then connected in parallel. Connections between Nb wires were made by spotwelding them to Ta foil and between Nb and Pb wires with Pb to Nb unions. The complete circuit is shown in Fig. 22. The open loop gain was 10⁷.

Another change from the SLUG circuit (Fig. 20) was the addition of a precision ramp-hold (schematic given in App. 2). The ramp-hold was connected to the current source in place of its Hg-cell reference. It allowed the current sources to be smoothly ramped up or down from any one of 12 preset currents to any other, from 0 to 100% of full scale. The linear ramp was followed by a 1 sec RC time constant. This had the effect of rounding the corners of the ramp, thus avoiding transients. The 12 preset levels were reproducible and stable to 100 ppm over many minutes.

The ramp-hold eliminated the terrible tedium of manually increasing the specimen current, as was done with the SLUG circuit. It became essential in the 0-100 mA range of specimen currents because the SQUID's high current sensitivity and low slewing rate made it easy for the SQUID to jump a flux quantum. A linear ramp was also
Fig. 22. Voltmeter circuit employing an RF SQUID.
useful at times in sorting out inductive pick-up in the specimen loop from currents produced by the voltage across the specimens.

Unfortunately, the SQUID would not operate reliably when first installed in the refrigerator. It was observed that the SQUID would only lock onto its AF modulation when the helium in the 4° space of the refrigerator had dropped to within ~25 cm of the top of the refrigerator's vacuum can. This left very little running time before the SQUID sensor became uncovered. It was concluded that the source of SQUID noise was Taconis oscillations in the SQUID co-ax. The problem was solved by putting the SQUID in its own vacuum can. The SQUID and its vacuum can are shown in Fig. 23.
Fig. 23. RF SQUID inside of its own vacuum can showing heater and thermometer on epoxy-coated copper. The pump out line and thermocouple vacuum gauge are not illustrated.
IV. EXPERIMENTAL RESULTS

A. High Temperature Resistance

In this section we will determine for Sn and Pb the branch crossing time, \( T_3 \approx T_Q \), and a lower limit on the recombination time, \( T_4 \). These times can be derived from the measured increase in resistance near \( T_c \) due to excitation current flow in \( S \), \( R_S(T) \).

\[
R_S(T) = \frac{2F(T)\rho_S(T)\lambda_3(T)}{A}
\]

where \( F(T) \) is the fraction of excitation current entering \( S \), \( \rho_S \) is the normal state resistivity of the superconductor, \( \lambda_3(T) \) is the excitation current decay length and \( A \) is the cross-sectional area of the specimen.

Equation (99) predicts that the total resistance of the specimen should increase monotonically with increasing temperature. However, for most of the specimens with Pb as the superconductor, there was a peak in resistance, \( R_{\text{max}} \), around \( T_{\text{max}} = 1.5 \) K and a relative minimum, \( R_{\text{min}} \), at \( T_{\text{min}} = 5.5 \) K. Harding, Pippard and Tomlinson\(^{80}\) have recently suggested that this structure may be due to a thin oxide layer covering a fraction, \( 1-\beta \), of the interface. The oxide areas form superconducting-insulator-normal metal (SIN) tunnel junctions whose properties have been reviewed by Douglass and Falicov.\(^{81}\) That \( \beta > 0 \) is clear from the fact that the resistance of the specimens does not continue to increase below \( \sim 1 \) K.

The measured resistance, \( R(T) \), can thus be considered as due to an ideal SNS sandwich of area \( \beta A \) and with resistance given by Eq. (99) in parallel with an SIN junction of resistance \( R_{\text{SIN}}(T) \):
\[
\frac{1}{R(T)} = \frac{1}{R_N(T) + R_S(T)} + \frac{1}{R_{\text{SIN}}(T)}. \tag{104}
\]

Since the voltage, \(\sim 10^{-7}V\), across the junction is \(< k_B T\), the current through the SIN junction is given by \(81\)

\[
I = \frac{2V}{R_{\text{SIN}}(T_c)} \sum_{m=1}^{\infty} (-1)^{m+1} \frac{m\Delta}{k_B T_c} K_1 \left( \frac{m\Delta}{k_B T_c} \right), \tag{105}
\]

where \(R_{\text{SIN}}(T_c)\) is the resistance of the SIN junction when \(S\) is normal and \(K_1\) is the modified Bessel function of the second kind.

Because the resistance is linear, the ratio of the differential conductance at a temperature \(T\), \(\sigma\left(\frac{\Delta}{k_B T}\right)\), to the normal state conductance, \(\sigma(0)\), is equal to ratio of \(R(T)\) to the static resistance \(R = \frac{V}{I}\). Bermon \(82\) has calculated values of \(\frac{\sigma\left(\frac{\Delta}{k_B T}\right)}{\sigma(0)}\) for \(0.4 \leq \frac{\Delta}{k_B T} \leq 14.6\) and for \(0 \leq \frac{eV}{\Delta} \leq 2.00\). His results for \(V=0\) are plotted in Fig. 24(a) as a function of \(\frac{\Delta}{k_B T}\), \(\sigma(T)/\sigma(T_c)\) has been replotted in Fig. 24(b) as a function of \(\frac{T}{T_c}\) for Pb using \(\frac{\Delta(0)}{k_B T_c} = 2.15\) and the values of \(\frac{\Delta(T)}{\Delta(0)}\) given by Muhlschlegel. \(33\) Also shown in Fig. 24(b) is a straight line approximation for \(\frac{\sigma(T)}{\sigma(T_c)}\) good to 4% for \(0.72 \leq \frac{T}{T_c} \leq 1\). In terms of resistance, this approximation becomes

\[
R_{\text{SIN}}(T) = 1.265R_{\text{SIN}}(T_c)(1.759 \frac{T}{T_c} - 1). \tag{106}
\]

The use of Eq. (104) in unraveling the data on specimens made with Pb is fraught with difficulties. One is knowing what value of \(\Delta\)
Fig. 24. (a) Conductance of an SIN junction, $\sigma(\Delta/k_B T)$, for $eV \ll \Delta$, normalized to the conductance when $S$ is normal, $\sigma(0)$ (from Bermon, Ref. 82). (b) Normalized conductance plotted versus $T$ for Pb, taking $\Delta(0)/k_B T_C = 2.15$. The straight line is an approximation for $\sigma(T)/\sigma(T_C)$ used to "subtract" the resistance of the SIN junctions from the data.
to use in calculating the temperature dependence of the resistance of the SIN junction. We have shown in Sec. IIA that the pair potential at the interface of an SN bilayer is reduced to approximately one half of the bulk value. Whether one uses $\Delta_{\infty}$ or $\sim 5\Delta_{\infty}$ in calculating the tunneling resistance then depends on the size of the oxide areas. If the smallest dimension of the oxide area is small compared to the coherence length in $S$, $\xi_S$, then $\sim 0.5\Delta_{\infty}$ should be used. If, on the other hand, the oxide area has dimensions large compared to $\xi_S$, then the pair potential behind the oxide should be largely uneffected by the proximity effect and $\Delta_{\infty}$ should be used.

We have also neglected the possibility in the case where the dimensions of the oxide are $\ll \xi_N$ that the oxide areas are actually SIS junctions, since $N$ has an induced pair potential. It might be interesting to compute $R_{SIS}(T)$ where the gaps on both sides are related by the de Gennes boundary condition.

Another difficulty associated with applying Eq. (104) is determining what areas to use in calculating $R_N(T)$ and $R_S(T)$. Although the clean fraction of the interface is $\beta$, the current in both $S$ and $N$ should spread out in a few mean free paths. Therefore, if the dimensions of the oxide areas are small compared to $\lambda_3$, then the total area rather than $\beta A$ should be used in calculating $R_S(T)$ and similarly, if the dimensions of the oxide areas are small compared to the thickness of $N$, $A$ rather than $\beta A$ should be used in calculating $R_N(T)$. 
Besides having an oxide, there are two other ways the interface can be contaminated, both of which can cause the specimen resistance to differ from that calculated by Eq. (104). The first is if mask material is sputter-deposited on the specimens, as exemplified by the Cu specimens made with the Cu masks with beveled holes mentioned in Sec. IIIA. With these masks, more Cu was deposited on the specimens than was removed and their resistance was typically twice what was expected. However, there was no relative minimum in the resistance.

The second way the specimen resistance can differ from Eq (104) is if the vapor deposited S either alloys with or diffuses into N. It is probably alloying which gave the peculiar behavior of the Pb-Au specimens 8A and 8B shown in Fig. 25. Indeed, the transition temperature of AuPb$_2$ at 4.42 K is near the kink in their R(T) curves at ~$5$ K. Their resistance is close to the calculated value and there is no relative minimum. If there is diffusion, the resistance of the specimen will increase by a temperature independent (near T$_c$) amount.

One would prefer that the measured resistance rises smoothly as T approaches T$_c$ and be equal at low temperature to the calculated value. One can then assume that there is no interface contamination. If there is contamination, analysis is very difficult and the values of $\tau_3$ and $\tau_4$ derived from it unreliable.

We now outline a procedure for determining $\tau_3$ and $\tau_4$: 
Fig. 25. Data for two Pb-Au-Pb sandwiches. The huge temperature dependence above 100 K and the kink at ~ 5 K are probably indicative of superconducting Pb-Au alloys at the interface.
1. Take $R_{N}^{\text{exp}}(0)$ as the resistance at the relative maximum, $R_{\text{max}}$. For Pb the maximum occurs at $1<T<2K$ where $R_{S}$ is negligible. $R_{\text{max}}-R_{\text{min}}$ can also be neglected since the fact that $R_{\text{min}}(T)\sim 25(R_{S}+R_{N})$, and $R_{\text{SIN}}(2K)\sim 130 R_{\text{SIN}}(T_{\text{min}})$.

2. Calculate $R_{N}^{\text{calc}}(0)$ using the measured thickness and resistivity ratio and compare it to $R_{\text{max}}$. If $R_{\text{max}}<R_{N}^{\text{calc}}(0)$, assume that the resistivity ratio measured along the foil was lower than would be found through the foil and take $\beta=1$. If $R_{\text{max}}>R_{N}^{\text{calc}}(0)$, take $\beta = \frac{R_{N}^{\text{calc}}(0)}{R_{\text{max}}}$. 

3. Neglect the resistance of the SIN areas and plot on transparent paper $\beta(R_{\text{exp}}(T)) - R_{\text{max}}$ for $0.5<\frac{T}{T_{c}}<1$. 

4. Using the values of $\frac{\Delta}{k_{B}T}$ calculated by Muhlschlegel and $\Delta(0) = 2.15 k_{B}T_{c}$ for Pb and $\Delta(0) = 1.76 k_{B}T_{c}$ for Sn, the measured resistivity ratios and Handbook values of the resistivity at room temperature of Sn and Pb, Fermi velocities of $v_{F}(\text{Pb})=0.48 \times 10^{8}$ cm/sec and $v_{F}(\text{Sn})=0.65 \times 10^{8}$ cm/sec and mean free paths at room temperature of $\ell(\text{Pb}) = 48\text{Å}$ and $\ell(\text{Sn}) = 91\text{Å}$, plot $R_{S}(T)$ for various values of $\tau_{3}$ and $\tau_{4}$ for $0.5<\frac{T}{T_{c}}<1$ and on the same scale as the data. $\tau_{3}$ and $\tau_{4}$ are chosen so that $R_{S}(T)$ and $\beta(R_{\text{exp}}(T) - R_{\text{max}})$ have the same order of magnitude.

5. Slide the data as plotted in Step 3 up and down over the calculated curves until the best fit is obtained over the largest temperature range from $T_{c}$ down.

6. Reading $R_{S}(T_{\text{min}})$ off the curve which gave the best fit to the data in Step 5. add this to $R_{N}^{\text{exp}}(T_{\text{min}})$. $R_{N}^{\text{exp}}(T_{\text{min}}) > R_{\text{max}}$ if $\rho_{N}$ is temperature dependent. Then find $R_{\text{SIN}}(T_{\text{min}})$ from Eq. (104).
7. Using Eq. (106) and $R_{\text{SIN}}(T_{\text{min}})$ find $R_{\text{SIN}}(T_c)$. Then replot the data on transparent paper using Eqs. (104) and (106) to "subtract off" $R_{N}(T)$ and $R_{\text{SIN}}(T)$.

8. The replotted data is again slid over the theoretical curves of $R_{S}(T)$. The $\tau_3$ and $\tau_4$ giving the best fit must be the same as were chosen in Step 6. If not, one goes back to Step 6 using the new value of $R_{S}(T_{\text{min}})$ and Steps 6, 7, and 8 are repeated until self-consistency is attained.

The merit of curve fitting in the above manner is that an error in determining $R_{N}^{\exp}(0)$ has only a second order effect on values of $\tau_3$ and $\tau_4$ which give the best fit. The difference in position of the ordinates is a measure of the error in determining $R_{N}^{\exp}(0)$.

Although it may seem that the curve fitting procedure would allow the theoretical curves to fit the data for almost any value of $\tau_3$ as long as $\tau_4$ was varied accordingly, this is not the case. It is found that one curve in the family plotted as in Step 4 always gives a "best fit".

1. Tin

Only two specimens were made with Sn, both with Ir as N. There was no relative minimum in their $R(T)$ curves and $R_{N}^{\exp}<R_{N}^{\text{calc}}$, so $\beta$ was taken as unity.

The resistivity ratio ($R_{300}/R_{4.2}$) of a $\sim3\mu$m thick film of Sn evaporated at a rate of $\sim100\AA$/sec onto a glass slide was 105. We take $\rho_{300} = 11.0 \mu\Omega\text{cm}$, $v_F = 0.65 \times 10^8$ cm/sec and $\tau_4 = 1.5 \times 10^{-12}$ sec.

In Figs. 26(a) and (b) are shown the theoretical curves for $R_{S}(T)$, which were compared to the data. Figure 26(a) is plotted for...
Fig. 26. Calculated values of $R_s(T)$ for Sn for various values of $\tau_3$ taking (a) $\tau_3 = \tau_3 \Delta(0) \left( \frac{T}{T_c} \right)^2$ and (b) $\tau_3 = \tau_3 \Delta(0)$. 
\[ \tau_3 \propto \frac{\Delta}{\tau_0} T^{-2} \] [Eqs. (59a and b)] and Fig. 26(b) is plotted for \[ \tau_3 \propto \frac{\Delta}{\tau_0} T^{-2} \] [Eqs. (57a and b)]. In both figures, \( 4 \times 10^{-10} < \tau_3 < 11 \times 10^{-10} \) sec and \( \tau_4 = 1 \) sec.

In Figs. 27(a) and (b) are shown the data for specimens 9E and 9F and the curves from Figs. 26(a) and (b) which gave the best fit. Also shown in Figs. 27(a) and (b) are theoretical curves for \( R_S(T) \) with \( \tau_3 \) independent of \( \Delta \) (see Fig. 28), a possibility suggested by Tinkham.\(^83\) It is apparent that \( \tau_3(T) \propto \frac{\Delta^{-1}}{T} \) is in much better agreement with the data than \( \tau_3 = \text{const} \). All three theoretical curves shown for each specimen were calculated with \( \tau_4 = 1 \) sec. Best fits were always obtained with \( \tau_4 > 10 \tau_3 \). Since \( R_S^{\text{calc}}(T) \) is not sensitive to \( \tau_4 > 10 \tau_3 \), the value of \( \tau_4 \) in Sn cannot be determined quantitatively from this experiment.

The short lines near the abscissas labeled A, B, and C correspond to the origins of the ordinates of the theoretical plots for \( \tau_3(T) \) with the three different temperature dependences. For both specimens, the ordinates indicate that the values of \( R(T) \) chosen as \( R_N^{\text{exp}}(0) \) were too large for \( \tau_3 \propto \Delta^{-1} T^{-2} \) and \( \tau_3 \propto \Delta^{-1} \).

The results for Sn are summarized in Table 2. \( T_N \) is the temperature at which \( R_N^{\text{exp}}(T) \) is equal to the value of \( R_N(0) \) indicated by line A.
Fig. 27. (a) Data for spec. 9E (circled points) and theoretical curves for $R(T)$ which provide the best fit to the data. (b) Data for spec. 9F and theoretical curves providing the best fit.
Fig. 28. Calculated values of $R_s(T)$ for Sn assuming $\tau_3(T) = \tau_3$ for various values of $\tau_3$. 

$\tau_3 = 4 \times 10^{-10}$ sec
Table 2. Summary of results for Sn-Ir specimens, also giving their thicknesses and resistivities at 4.2K.

<table>
<thead>
<tr>
<th>SPEC</th>
<th>N</th>
<th>$\tau^*_{3} \Delta^{-1} T^{-2}$</th>
<th>$\tau^{0}_{3} \Delta^{-1}$</th>
<th>$\tau^{*}_{3}$ = const</th>
<th>Thickness of N</th>
<th>$\rho(4.2^\circ K)$</th>
<th>$R_N^{calc}$</th>
<th>$\sigma_{4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\tau^*_{3} (\sec)$</td>
<td>$T_N (K)$</td>
<td>$R_N (\Omega)$</td>
<td>$\tau^{0}_{3} (K)$</td>
<td>$R_N (\Omega)$</td>
<td>$\tau^{*}_{3} (sec)$</td>
<td>$T_N (K)$</td>
</tr>
<tr>
<td>9E</td>
<td>Ir</td>
<td>$9 \times 10^{-10}$</td>
<td>1.4</td>
<td>$4.37 \times 10^{-8}$</td>
<td>$9 \times 10^{-10}$</td>
<td>4.48 $\times 10^{-8}$</td>
<td>$2.5 \times 10^{-9}$</td>
<td>1.4</td>
</tr>
<tr>
<td>9F</td>
<td>Ir</td>
<td>$6 \times 10^{-10}$</td>
<td>1.4</td>
<td>$4.56 \times 10^{-8}$</td>
<td>$6 \times 10^{-10}$</td>
<td>4.57 $\times 10^{-8}$</td>
<td>$1.5 \times 10^{-9}$</td>
<td>1.6</td>
</tr>
</tbody>
</table>
2. Lead

The resistivity ratio of Pb was measured on a 125µm thick strip evaporated in 2 min. in SII onto an anodized Al plate. The thickness of the strip and the evaporation rate (10^4Å/sec) were typical of the Pb evaporated in SII. Anodized Al rather than glass was used so that the lead would be deposited on a cooled substrate similar to the surface of a specimen. The resistivity ratio was 1000 and constant to within 2% from 4 to 7.2 K. In contrast, Pb evaporated onto a room temperature glass substrate at 600Å/sec had a resistivity ratio of 3000 and a temperature dependence described by R(0)(1+9.2×10^{-3}T^3). The room temperature resistivity was taken as 21µΩcm, v_F=0.48×10^8 cm/sec, and τ_l=1.1×10^{-11} sec.

The Pb data was analyzed as outlined in Sec. IVB, except that the iteration suggested in Step 8 to attain self-consistency was not done. This was because it was found that the final values of τ_3 and τ_4 were only weakly dependent on our initial choices for these times.

We will discuss each set of similar specimens separately and then summarize the results.

a. Pb-Cu-Pb, Specimen 4C. This specimen was sputtered in SI. The Pb was evaporated very slowly, taking 125 min. on the first side and 62 min. on the second.

The resistance of spec. 4C relative to R_{max} is plotted in Fig. 29(a). In Fig. 29(b) is shown the same data multiplied by β.
Fig. 29. (a) Resistance of spec. 4C plotted relative to $R_{\text{max}}$. Also shown is $\tau_3(T)$ which fits the data at the two highest temperatures. (b) Figure (a) multiplied by the fraction of the interface which is clean, $\beta$. 

\[ \tau_3 = 3 \times 10^{-10} \frac{\Delta(0)}{\Delta(T)} \left( \frac{T_c}{T} \right)^2 \]

\[ \tau_3 = 2 \times 10^{-10} \frac{\Delta(0)}{\Delta(T)} \left( \frac{T_c}{T} \right)^2 \]

$\beta = 0.743$
The resistance of a slab of Cu made from the same stock as spec. 4C was measured as a function of temperature. The increase in resistance above its constant low temperature value is shown in Fig. 30(a). The dimensions of the slab are given in the inset. The superconducting leads to the slab were "soldered" with 99.999% Pb. A polynomial of the form \( R(0)(1+C_1 T^5) \) gave the best fit to the data. The \( T^5 \) dependence is in agreement with the theory for a free electron gas at low temperatures. The value of \( C_1 \) was estimated; it is not the result of a least squares fit.

If one calculates the room temperature resistance of the slab assuming a cross section of 0.25×2.2 cm\(^2\), one finds for the slab \( R_{300}=2.47\times10^{-6} \Omega \). From Fig. 30(a) we see that \( R_{4.2}=95.1\times10^{-10} \Omega \) so that \( R_{300}/R_{4.2}=260 \). The Cu in spec. 4C which was strained more by rolling had \( R_{300}/R_{4.2}=132 \). We assume that the ideal (phonon) resistivity is unaffected by rolling. Thus \( R_N(T) \) for spec. 4C has the form \( R(0)(\frac{260}{132} + C_T^5) \).

In Fig. 31(a) the data is plotted after being corrected for parallel SIN junctions and in Fig. 31(b) the data is plotted after both parallel SIN junctions and \( R_N(T) \) have been "subtracted".

b. Pb-Ir-Pb, Specimens 8C and 8D. The Ir in these specimens was later used to make the Ir-Sn sandwiches discussed in Sec. IVA. Specimen 8C became 9E and 8D became 9F.

Specimens 8C and 8D were not sputtered. They were instead cleaned with acetic acid in \( H_2O_2 \), then HCl, and finally acetone and dried with dry \( N_2 \). They were in air <5 minutes before being mounted in SI and pumped down. The deposition of Pb took 30 minutes on the first side and 40 minutes on the second.
The data for spec. 8C and 8D are shown in Figs. 32(a) and 32(b), respectively. Since we assumed $\beta=1$ for spec. 9E and 9F, we will take $R_N^{\text{exp}}(0)$ from these specimens as the correct values for $R_N(0)$. Then spec. 8C has $\beta=0.866 \beta(R(T)-R_{\text{max}})$ for spec. 8C is plotted in Fig. 33(a) and the data corrected for parallel SIN junctions is shown in Fig. 33(b). Figures 34(a) and 34(b) show $\beta(R(T)-R_{\text{max}})$ and the data corrected for parallel SIN junctions, respectively.

c. Pb-Rh=Pb, Specimen 10D. This specimen was sputtered in SI. The Pb on each side was deposited in 8 min. The data for this specimen are plotted in Fig. 35(a). $\beta(R(T)-R_{\text{min}})$ is plotted in Fig. 35(b), with $\beta=0.621$.

The resistivity ratio of a strip of Rh, spec. 10E, cut from the same piece as spec. 10D was measured as a function of temperature. The increase in resistance of this 53$\mu$m thick strip is shown in Fig. 30(b) The dimensions of spec. 10E are shown inset. The current and voltage leads were Pb wires and were attached with Sn-Pb solder. The smooth curve has the form $C_2R(0)T^2$. $C_2$ was estimated and was not the result of a least squares fit. The $T^2$ dependence is not understood.

In Fig. 36(a) is shown the data for spec. 10D with the resistance of the SIN junctions "subtracted" and in Fig. 36(b) the data corrected for parallel SIN junctions is replotted assuming $R_N(T) = R(0)(1+9.6\times10^{-4}T^2)$. 
Fig. 30. (a) Increase of the resistance of a Cu slab near $T_c = 7.2K$. The slab was rolled from the same Cu as were all Cu specimens. The smooth curve is an approximation.
(b) Increase in resistance of a 53 μm thick strip of Rh near $T_c$. Strip was annealed after rolling. Smooth curve is an approximation.
Fig. 31. (a) Measured increase of resistance of spec. 4C near $T_c$ after correcting for parallel SIN junctions. (b) Data corrected for both parallel SIN junctions and $R_N(T)$. 

\begin{align*}
\text{Pb - Cu - Pb} \\
\text{4C} \\
\text{A. } \tau_3 &= 1.3 \times 10^{-9} \frac{\Delta(0)}{\Delta(T)} \left( \frac{T_c}{T} \right)^2 \\
\beta &= 0.743 \\
\text{B. } \tau_3 &= 3 \times 10^{-9} \\
\end{align*}
Fig. 32. Resistance relative to $R_{\text{max}}$ for spec. 8C (a) and spec. 8D (b).
Fig. 33. (a) Data of Fig. 32(a) multiplied by $\beta = 0.866$. Instead of using the calculated value of $R_N(4.2K)$ to determine $\beta$, $R_N^{\exp}(0)$ from spec. 9E was used. (b) Data corrected for parallel SIN junctions.
Fig. 34. (a) Data of Fig. 32(b) multiplied by $\beta = 0.874$. $R_N^{\text{exp}}(0)$ from spec. 9F rather than $R_N^{\text{calc}}(4.2)$ was used in determining $\beta$. (b) Data corrected for parallel SIN junctions.
Fig. 35. (a) Resistance of spec. 10D relative to $R_{\text{max}}$. (b) The data multiplied by $\beta = 0.622$. 
Fig. 36. The resistance of spec. 10D (a) corrected for parallel SIN junctions and (b) corrected for both parallel SIN junctions and $R_N(T)$. 

(a) 

$\text{Pb - Rh - Pb}$

10 D

A. $\tau_3 = 6 \times 10^{-11} \frac{\Delta(0)}{\Delta(T)} \left( \frac{T_c}{T} \right)^2$

$\beta = 0.621$

B. $\tau_3 = 6 \times 10^{-10}$

(b) 

$\text{Pb - Rh - Pb}$

10 D

A. $\tau_3 = 9 \times 10^{-11} \frac{\Delta(0)}{\Delta(T)} \left( \frac{T_c}{T} \right)^2$

$\beta = 0.612$

B. $\tau_3 = 8 \times 10^{-10}$
d. Pb-Cu-Pb, Specimen CuC. Specimen CuC was sputtered in SII with indium o-rings. The Pb was deposited in 3 min. on the first side and 2 min. 20 sec. on the second.

The data for this specimen is shown in Fig. 37(a). In Fig. 37(b) the data is replotted after being multiplied by $\beta = 0.892$.

In Fig. 38(a) the data for spec. CuC is shown corrected for parallel SIN junctions and in Fig. 38(b) the data is plotted with the resistance of the SIN junctions and the temperature dependence of the resistance of N "subtracted". $R_N(T)$ was taken to have the same temperature dependence as spec. 8E [Fig. 40(a)].

e. Pb-Cu-Pb, Specimen CuF. In order to get a better understanding of the cause of the relative minimum in the resistance of almost all our SNS sandwiches made with Pb, we sputtered CuF using air rather than argon. The Pb was deposited on the first side in 1 min. 15 sec. and on the second in 2.5 min.

$R(T) - R_{\text{max}}$ is plotted in Fig. 39(a). The same data multiplied by $= 0.169$ is plotted in Fig. 39(b). In Fig. 40(a) we have accounted for the parallel SIN junctions and in Fig. 40(b) we have corrected the data for the parallel SIN junctions and $R_N(T)$.

f. Pb-Ag-Pb, Specimen AgC. This specimen was made at the same time as spec. CuC. The Pb on these specimens should therefore be identical. The data for spec. AgC is plotted in Fig. 41(a).

In Fig. 41(b) the data corrected for parallel junctions is shown. The resistivity ratio of the Ag was only measured at 4.2 K where its value was 193. Because the Debye temperature of Ag is less than that of Cu (227 K versus 344 K), the fairly high resistivity
Fig. 37. (a) Resistance of spec. CuC relative to $R_{\text{max}}$. (b) Data multiplied by $\beta = 0.892$. 

\[ \tau_3 = 3 \times 10^{-11} \frac{\Delta(0)}{\Delta(T)} \left( \frac{T_c}{T} \right)^2 \]

\[ \tau_3 = 2.5 \times 10^{-11} \frac{\Delta(0)}{\Delta(T)} \left( \frac{T_c}{T} \right)^2 \]
Fig. 38. Resistance of spec. CuC corrected for (a) parallel SIN junctions and (b) corrected for both parallel SIN junctions and $R_N(T)$. 
Fig. 39. (a) Resistance of spec. CuF relative to $R_{\text{max}}$. CuF was sputtered in air and had $\beta = 0.169$. (b) Data multiplied by $\beta$. 
Fig. 40. Data for spec. CuF (a) corrected for parallel SIN junctions and (b) corrected for both parallel SIN junctions and $R_N(T)$. 

$a$: 

\[ R_s(T) = 10^{10} \alpha \left( T/T_c \right)^2 \]

$\beta = 0.169$

$A. \ \tau_3 = 2.5 \times 10^{-11} \frac{\Delta(0)}{\Delta(T)} \left( \frac{T_c}{T} \right)^2$

$B. \ \tau_3 = 2 \times 10^{-10}$

$b$: 

\[ R_s(T) = 10^{10} \alpha \left( T/T_c \right)^2 \]

$\beta = 0.164$

$A. \ \tau_3 = 2 \times 10^{-11} \frac{\Delta(0)}{\Delta(T)} \left( \frac{T_c}{T} \right)^2$

$B. \ \tau_3 = 1 \times 10^{-10}$
Fig. 41. (a) Resistance of spec. AgC relative to $R_{\text{max}}$. $R_{N}^{\text{calc}}(4.2K) > R_{N}^{\text{exp}}(4.2K)$ so $\beta$ was taken as unity. (b) Resistance of spec. AgC corrected for parallel SIN junctions. Although the correction for $R_{N}(T)$ of Ag should be significant, $R_{N}(T)$ is not known.
ratio would indicate significant temperature dependence of $R_N(T)$ for Ag. Correcting the data for $R_N(T)$ would then give $T_3$ considerably less than $2 \times 10^{-10} \frac{\Delta(O)}{\Delta(T)} \left( \frac{c}{T} \right)^2$.

f. Summary of results. Some parameters and results for the above described SNS sandwiches made with Pb are tabulated in Table 3.

A number of conclusions can be drawn:

1. Although not shown in the figures, values of $\tau_4 \geq 10 \tau_3$ gave the best fit to the data. As mentioned in regard to Sn, $R_{S}^{\text{calc}}(T)$ is not sensitive to $\tau_4$ for $\tau_4 > 10 \tau_3$.

2. Just as good a fit to the data is possible with $T_3(T) = \frac{\Delta(O)}{\Delta(T)}$ as with $T_3(T) = \tau_3 \frac{\Delta(O)}{\Delta(T)} \left( \frac{c}{T} \right)^2$ and the values of $\tau_3$ are nearly the same.

3. In general, a better fit to the data is possible with $T_3(T) = \tau_3 \frac{\Delta(O)}{\Delta(T)} \left( \frac{c}{T} \right)^2$ than with $T_3(T) = \tau_3$.

In addition, the value of $R_N(O)$ determined by the position of the origin of the ordinate of the theoretical curve is closer to $R_{\text{max}}$ for $T_3(T) = \tau_3 \frac{\Delta(O)}{\Delta(T)} \left( \frac{c}{T} \right)^2$ (line A in the figures) than for $T_3(T) = \tau_3$ (line B).

4. The Pb on the specimens made in SI was thinner and was deposited over a period as long as 30 times the deposition time for specimens made in SII. This and the higher base pressure would cause one to expect that the Pb went down highly oxidized. If that were the case, its resistivity ratio might be considerably smaller than the value of 1000 measured for Pb deposited in SII at the typical rate. Setting $\tau_4 = \infty$, we see that $R_S \propto \rho_S^{\frac{1}{3}} T_3 \propto \rho_S T_3$. 
Table 3. Values of $\tau_3$ in Pb as derived from $R(T)$ data for SNS sandwiches with various normal metals. The different columns indicate which corrections were made to the data before comparing the corrected data to the theoretical curves.

<table>
<thead>
<tr>
<th>SPEC</th>
<th>N</th>
<th>$\tau_3(T) = \tau_3 \frac{\Delta(0)}{\Delta(T)} (\frac{T_c}{T})^2$</th>
<th>$\tau_3 = \text{const}$</th>
<th>$R_{300}/R_{4.2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$R(T) - R_{\text{max}}$</td>
<td>$\beta(R(T) - R_{\text{max}})$</td>
<td>SIN</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$10^{-11}$ sec</td>
<td>$10^{-11}$ sec</td>
<td>$10^{-11}$ sec</td>
</tr>
<tr>
<td>4C</td>
<td>Cu</td>
<td>40</td>
<td>25</td>
<td>70</td>
</tr>
<tr>
<td>8C</td>
<td>Ir</td>
<td>20</td>
<td>15</td>
<td>30</td>
</tr>
<tr>
<td>8D</td>
<td>Ir</td>
<td>30</td>
<td>20</td>
<td>40</td>
</tr>
<tr>
<td>10D</td>
<td>Rh</td>
<td>9</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>CuC</td>
<td>Cu</td>
<td>3</td>
<td>2.5</td>
<td>9</td>
</tr>
<tr>
<td>CuF</td>
<td>Cu</td>
<td>40</td>
<td>1</td>
<td>2.5</td>
</tr>
<tr>
<td>AgC</td>
<td>Ag</td>
<td>4</td>
<td>$\beta=1$</td>
<td>20</td>
</tr>
</tbody>
</table>
Thus, underestimating $\rho_S$ by some factor gives an apparent $\tau_3$ too large by the same factor.

5. At $0.99T_c$, $\lambda_3 = 1.1 \times 10^{-3}$ cm for Pb with $R_{300}/R_{4.2} = 1000$. This decay length is comparable and possibly larger than the thickness of the Pb evaporated in SI. One might then expect additional scattering (and hence resistance) from the oxide near the outer surface.

6. For $\tau_3 = 5 \times 10^{-11}$ sec, $\lambda_3 < 10^{-3}$ cm for $T < 0.99 T_c$. Thus $\lambda_3$ is only about twice the mean free path, $\ell$. For the Boltzmann equation to be strictly valid, $\lambda_3 \gg \ell$. $\lambda_3 \propto V_F \tau_3$ might therefore have been a more appropriate choice for the excitation current decay length in Pb than $\lambda_3 \propto \sqrt{V_F \tau_3 \ell}$.

Although we have shown that SIN junction in parallel with the SN interface could produce the relative minima in $R(T)$ for the above described sandwiches made with Pb, there are four pieces of evidence which still leave us in doubt.

First, there was no relative minimum in the resistance of Sn-Ir specimens (see Fig. 27). This may be because the Sn is able to penetrate any oxide which may be on the Ir.

Second, three Pb-Pt specimens (see Fig. 42) made simultaneously in SI II with In o-rings had no relative minimum even though $\beta < 0.93$. As well be discussed in the next section, these specimens were the only ones made with a non-ferromagnetic normal metal whose resistance did not continue to decrease below 0.6 K. Although the fact that Pt does not oxidize in air may imply that SIN junctions were not formed, it is also possible that the lack of a relative minimum and the absence of a decrease in resistance below 0.6 K may be related.
Fig. 42. Resistance of three nominally identical Pb-Pt-Pb sandwiches made simultaneously. The rapid change in resistance near $T/T_c = 0.12$ is probably due to Al from the masks being sputtered onto the Pt.
Third Pb-Ni-Pb sandwiches sputtered in SII with In o-rings (Fig. 43) had relative minima the same order of magnitude \( R_{\max} - R(T_{\min}) \approx 10^{-11} \Omega \) as all other specimens with non-magnetic normal metals, except Pt as cited above. This is quite surprising. One would expect that the magnetic field from the Ni would depress the pair potential in S almost to zero at the interface. Then one would expect no temperature dependence due to SIN junctions until \( T < \Delta_S(0) \ll T_c \). As can be seen in Fig. 42, however, \( T_{\min} \) for Pb-Ni specimens is about the same as for non-magnetic normal metals.

Finally, the copper specimens which had copper deposited on them from the masks with beveled holes showed no relative minimum.

**B. Low Temperature Resistance**

In the previous section we assumed that the relative minima in the resistances of most of the SNS sandwiches with Pb as S was due to part of the interface being an SIN junction. Below \( T_{\max} \approx 1.5K \) at which the relative maxima occur, the resistance of SIN junctions made with Pb should be too large to be significant when added in parallel to the clean SN interface. Any change in specimen resistance below \( T_{\max} \) should therefore not be due to the presence of SIN junctions. The only effect the insulating barrier has is to decrease the effective area of the SNS sandwich by the factor \( \beta \) and therefore to increase its resistance by \( \beta^{-1} \).

We can eliminate the need to know the area by comparing theoretical and experimental values of \( R(T)/R(T_o) \), where \( T_o \) is some temperature \( \leq T_{\max} \). It is advantageous to choose \( T_o = T_{\max} \) so that the comparison
Fig. 43. Resistance of three Pb-Ni-Pb sandwiches. Specimen 12C was made with Viton A o-rings and specs. NiA and NiB were made simultaneously with In o-rings. Both were sputtered in SII.
can be done over the maximum temperature range. However, in a number of specimens sputtered with Al masks, there appears to be an anomaly in the resistance near 1K e.g., (see Fig. 42). For these specimens $T_o$ was chosen to be below the anomalous region.

Comparing the normalized measured resistance to theory has a further advantage when $\rho_N(T) = \text{const.}$: $\rho_N$ is determined by making a 4-terminal measurement of the resistivity ratio $(R_{300}/R_{4.2})$ of a long, narrow strip cut from the rolled foil. One would expect that the rolling would cause crystal planes to slide over each other, thus producing grain boundaries in the form of sheets parallel to the surface of the foil. The resistivity along the strip might then be significantly less than through it.

In order to test the validity of our model for $R(T)$ at low temperatures, Eqs. (97a) and (97b), we needed to use a superconductor as N whose $T_c$ we could measure directly. The optimum metal was one with a $T_c$ near the minimum attainable temperature of the refrigerator and which did not alloy with Sn or Pb. Our choice was iridium.

1. Iridium

Although the critical temperature of pure Ir is $0.1125$K, the approximate midpoint of the resistive transition of a strip of our 99.9% Ir was only 58 mK. The resistive transition of the strip is shown in Fig. 43A. Note that the resistance increased by 10% going from 70 mK to 108 mK. Inverting the expression $T_o = 0.85\theta_D \exp(-1/[N(O)V])$, we find for our Ir that $[N(O)V] = 0.115$, using $\theta_D = 420$K.
Fig. 43A. Resistance transition of Iridium.
In Fig. 44 are shown the experimentally determined dependences of resistance on measuring current, \( R(I)/R_N \), for a Pb-Ir-Pb specimen, 8D, and a Sn-Ir-Sn specimen, 9F. \( R_N \) is the experimentally determined value of the resistance of the normal metal. \( R_N = R_{\text{max}} = R(2.0K) \) at 100 mA for spec. 8D and \( R_N = R(T_N) \) for Sn at 100 mA. \( T_N \) was defined in the preceding section; it equals 1.4K for spec. 9F. Two things should be noted: (1) the resistance depends on current even for \( I = 1 \) mA and (2) \( d^2R/dI^2 < 0 \).

Since most excitations are Andreev reflected at \( \Delta \sim k_B T \), the current must depress the pair potential near \( \Delta = k_B T \) in order to significantly effect the resistance. However, Eq. (95) gives a cut-off, \( \Delta_c \), if only 0.08 mK for \( I = 100 \) mA at \( T = 70 \) mK. There is no significant effect of the current on the pair potential for \( \Delta > 10\Delta_c \).

It is possible that the problem with Eq. (95) lies in our assumption that the critical field of a normal metal with an induced \( \Delta \) is the same as for a bulk superconductor with \( \Delta_0 = \Delta \) given by Eq. (78). To examine this possibility, we have calculated \( R(I)/R_N \) at 70 mK using the parameter of spec. 8D with \( \Delta_N(0) = 2.015K \) but with the critical field reduced by a factor \( c \): i.e. \( H_c(x) = c\sqrt{\Delta N(0)} \Delta(x) \). The results are shown in Fig. 45. Each calculated point is for a particular value of \( D = \log_{10} \frac{\Delta}{\Delta_c} \). The values of \( D \) are given next to their corresponding points.

We determine \( c \) by finding which two orders of magnitude of current in Fig. 45 give best agreement with Fig. 44(a). It is seen in Fig. 45 that there is a point of inflection for \( D \approx 1.0 \) and that \( d^2R/dI^2 \) is negative for higher currents. If we choose the inflection point as \( I = 1 \) mA, then \( c = 10^{-4} \).
Fig. 44. Normalized resistance as a function of measuring current for the same piece of Ir with different superconductors. (a) Pb-Ir-Pb, (b) Sn-Ir-Sn.
Fig. 45. Calculated values of resistance as a function of current for Pb-Ir-Pb spec. 8D, normalized to the measured resistance at 100 mA. The numbers next to several of the calculated points are $D = \log_{10} \Delta_c$, where $\Delta_c$ is the cut-off in Kelvins.
A value of $c < 1$ is reasonable since we have treated $H_c$ in $N$ as a local property, whereas we know that properties depending on $\Delta(x)$ must be integrated over $x$, as was done in Eq. (2) to find $\Delta(x)$ itself. A value of $c = 10^{-4}$ is, however, surprisingly small.

We note that for $c = 10^{-4}$, $\Delta_c = 10^{-0.4} = 0.40K$ at 100 mA. It should also be noted that although the shape of Fig. 45 is in agreement with the data for $D \gtrsim 1.0$ its magnitude is not. We calculate only a 7% change in resistance when $I$ is increased from 1 to 100 mA, whereas experimentally we found a change of 12%.

We have calculated $R(T)/R_0$ using the parameters of spec. 8D for $c = 1.8 \times 10^{-3}$ and for two values of $\Delta_s(0)$. The calculated curves and the data are shown in Fig. 46. The fits are reasonable. However, it can be seen that the calculated resistance is too large at high temperatures ($> 100 \text{ mK}$) if it agrees at low temperatures. If on the other hand, the calculated resistance agrees at high temperatures, it falls too rapidly with decreasing temperature below $\sim 100 \text{ mK}$. This is true whether the one-frequency approximation decay length, $K_N^{-1}$, or the Ginzburg-Landau decay length, $K_N^{-1}$, in $N$ is used, although agreement is somewhat better with $K_N^{-1}$.

The resistance is expected to decrease with decreasing temperature because the point where Andreev reflection occurs moves further from the interface. The current dependent cut-off makes the calculated resistance fall even faster. This is because $\Delta_c(x_c)$ is proportional to $Y(\Delta_c)$ which, it will be recalled, is the fraction of total current which still exists as excitations just closer than $x_c$ from the interface. $Y(\Delta_c)$ is temperature dependent.
Fig. 46. Normalized resistance data for Pb-Ir-Pb, spec. 8D (dots). The smooth curves are the resistance of Pb-Ir spec. 8D calculated from Eq. (97a). A constant value of $\rho_N$ was assumed over the temperature range shown.
The factor of $Y(\Delta_c)$ would be eliminated if the total current and not the excitation current produced the field which depressed the pair potential. There is some justification for believing that this may indeed be the case.

Heretofore we have claimed that the Meissner effect in N forced the pair current to the outside of the specimen so that it would not effect the pair potential inside. We assumed that this movement occurred instantly at the point of Andreev reflection. In fact we would expect the pairs to move to the outside over a distance of the order of the decay length. Then both the uniformly flowing excitations and the spreading pairs would produce a field at $x_c'$, and $\Delta_c$ would no longer be temperature dependent.

Using different values of temperature independent $\Delta_c$ near the value of 0.4K found above, we have recalculated $R(T)/R_{\text{max}}$ using the parameters for spec. 8D. The results along with the data are shown in Fig. 47(a). The fit is excellent. In Fig. 47(b) we have plotted the calculated curve of $R(T)/R_{\text{max}}$ using the parameters for spec. 9F. Here, also, $\Delta_c$ was made to be independent of temperature. Again the agreement between the calculated value of $R(T)/R_{\text{max}}$ and the data (also shown) is excellent. This is encouraging since spec. 9F was made from the same foil as spec. 8D after the Pb was removed with acetic acid and $H_2O_2$.

However, a temperature independent cut-off does not give the correct dependence. This is seen in Fig. 48. The values of $D = \log_{10}\Delta_c$ are shown next to the calculated points. Values of CI run over five orders of magnitude, but nowhere is $d^2R/dI^2 < 0$. 
Fig. 47. Normalized resistance versus temperature for sandwiches with Ir as N. (a) Pb-Ir-Pb with two values of the cut-off $\Delta_c$, (b) Sn-Ir-Sn.
It is thus apparent that we do not know how to properly handle the current dependence. It is also clear that the $R(T)/R(T_0)$ calculations are quite sensitive to the current dependence used.

2. Magnesium

Magnesium is a particularly interesting metal to study by the proximity effect because calculations have predicted a probable $T_c$ in the range of $0.4 \text{ mK} \lesssim T_c \lesssim 200 \text{ mK}^{85}$ and direct measurements down to $2 \text{ mK}^{86}$ have shown no transition. Furthermore, Mota, Brewster, and Wang$^{87}$ have alloyed Mg with different percentages of Cd and by extrapolating (about one order of magnitude in temperature) to $100\%$ Mg have predicted $T_c = 0.5$ mK. If $T_c$ for Mg is within an order of magnitude of this temperature, $[\text{NV}]_{\text{Mg}}$ and thus $\Delta_N(0)$ will be large and there should be a relatively large change in resistance with temperature for an SNS sandwich with Mg as N. However, our best efforts were not able to produce a clean Mg surface, as was described in Sec. IIIA. This was not known until after a number of specimens were measured in the dilution refrigerator. The data for four samples which had been sputtered simultaneously in SII Veton o-rings are presented in Fig. 49. Note that the ordinate is logarithmic.

The samples were made from one piece of rolled Mg foil which was spark cut from a single crystal, 99.99% pure. The crystal was supplied
Pb-Ir-Pb
8 D
T = 70 mK
c = 1
ΔN(0) = 2.015 K

\[ \frac{R(I)}{R(0.1)} \]

\[ 10^{-3} \quad 10^{-2} \quad 10^{-1} \quad 1 \quad 10 \quad 10^2 \]

\[ cI (\text{Amphs}) \]

XBL 768-7320

Fig. 48
Fig. 48. Theoretical current dependence of the normalized resistance of Pb-Ir-Pb spec. 8 D for temperature independent cut-off, $\Delta_c$. The upper curve was calculated using $K_N^{-1}$ and the lower using $k_N^{-1}$. The numbers next to the calculated points are values of $D = \log_{10} \Delta_c$. $R(0.1)$ is the calculated resistance at $I = 100$ mA and $T = 2.0\, K$. 
by Research Organic/Research Inorganic. The resistivity ratio of the single crystal, measured by the eddy current decay method by Thorp, et al., was 330. After rolling, the resistivity ratio of a strip cut from one of the samples was 101.

The data have many peculiarities. The resistances of the four specimens were nearly the same at 26 mK but were about two orders of magnitude higher than the calculated resistance for the Mg alone. This would seem to indicate a very dirty interface, but if so, then one would expect the resistance to increase rather than decrease below ~5K, as discussed in Sec. IV-A1.

Although the resistances at 26 mK are about equal, the resistances range over an order of magnitude near the $T_c$ of Pb. The temperature dependence is huge.

The flattening of the curves at the lowest temperatures is also unexpected. We have two reasons to believe that this is not due to self-heating. First, although the specimen resistances were high, the measuring current was only 0.1 mA. The dissipation in the sample was therefore $10^3$ times less than in some other specimens whose resistances were still falling at 20 mK. Secondly, specimens B, C, and D had essentially no current dependence at 26 mK. $R(I)$ for Mg A and Mg B is shown in Fig. 50. Note that the resistance depends more strongly on current at 60 mK and at 100 mK than at 26 mK.

Neither the behavior of $R(T)$ nor of $R(I)$ is compatible with our theory. There seem to be two possible explanations. One is that the interface is mostly covered with an insulating barrier, that about 1% of the surface is a Mg-Al alloy formed from mask material sputtered onto the
Fig. 49. Resistance versus temperature of four Pb-Mg-Pb sandwiches cut from the same piece of Mg and sputtered simultaneously.
Fig. 50. Resistance as a function of measuring current for two of the Pb-Mg-Pb sandwiches shown in Fig. 49. The resistance of spec. MgA at 500 mK (dashed line) is ten times greater than shown.
The current density through the conducting parts of the interface is high enough to suppress the induced pair potential in Mg, or \( [N(0)V]_{Mg} \ll 0.01 \). The large current dependence, in the temperature range over which \( dR/dT \) is also large, is due to the effect of the current on the transition temperature of the Mg-Al alloy.

A second interpretation of the Mg data is that the flattening of the \( R \) vs \( T \) curves at \( \sim 0.1K \) is due to magnetic impurities in the Mg. Because the total impurity concentration is \( <100 \) ppm, there certainly is no ferromagnetic transition at \( \sim 0.1K \). However, the flattening could be due to the temperature independent decay length associated with the magnetic impurities. The net decay length is then

\[
k_{\text{net}} = k_N + k_M = \sqrt{6\pi k_B T} + \sqrt{3 \frac{3}{\sqrt{V_F^* B^*}}} \]

where \( \tau_S \) is the spin scattering time due to the magnetic impurities.

Setting \( k_N = k_M \), we see that the \( R(T) \) curve should begin to flatten near

\[
\frac{\hbar}{2\pi k_B \tau_S}.
\]

Hauser, et al.\(^{89}\) studied iron impurities in Mo in proximity with pure lead. They found that for 1\% Fe, \( z \) [Eq.(23)] equals 10.7. For our Mg we found \( z = \frac{T_M}{T_{CS}} = 0.1 \), or 0.013. Assuming that \( \tau_S \) is independent of the host metal and that \( \tau_S^{-1} \) is proportional to the concentration of magnetic impurities, we see that if all the magnetic impurities were Fe
their concentration would be \( \frac{0.013}{10.7} \) (1\%) \( \approx \) 10 ppm. This is in reasonable agreement with the impurities listed by the supplier.

Magnetic impurities also account qualitatively for the current dependence. The field from the current is \( <1 \text{ Gauss} \) and thus produces no significant ordering of the impurity spins. We therefore assume that the current does not effect \( k_M \) but increases \( k_N \) in the same manner as when magnetic impurities are absent. For \( T < T_M \), \( k_M > k_N \) and the increase of \( k_N \) makes a small change in \( k_{\text{net}} \). Above \( T_M \) when \( k_N \) dominates \( k_{\text{net}} \), the effect of the current on the resistance increases.

3. Rhodium

Rhodium has also been of great interest for us to test for a superconducting transition by the proximity effect ever since Mota, et al.\(^8\) had estimated \( (T_{c})_{\text{Rh}} = 0.2 \text{ mK} \). They measured the \( T_{c}'s \) of alloys of Rh-Ir and Rh-O\(_5\) and extrapolated to 100\% Rh.

The Rh used in our Pb-Rh-Pb sandwiches was received from United Mineral and Chemical Corp as 5N sponge. It was melted first in an argon arc furnace and then remelted with an electron beam gun. Slices were cut with a spark cutter. The slices were rolled between stainless steel rollers, reducing the thickness from its initial value of 0.4 mm in steps of \( \sim 50 \mu\text{m} \). After each rolling operation the slices were cleaned in \( \text{H}_2\text{SO}_4 \) in order to remove any contamination from the rollers and were then annealed with the e-gun. The final thickness was 53 \( \mu\text{m} \) and after annealing, its resistivity ratio was 170.

In Fig. 51(a) is shown the \( R(T)/R_{\text{max}} \) data for spec. 10c. The curves are calculated values of \( R(T)/R_{\text{max}} \) using the temperature independent cut-off
Fig. 51. Calculated and experimental values (points) of the normalized resistance versus temperature for Pb-Rh-Pb spec. 10 C. Figure 51(a) shows that $k_N^{-1}$ is the better choice for the decay length. Figure 51(b) shows the dependence on the cut-off, $\Delta_c$. 
of the pair potential with both the one-frequency approximation decay length $K_{N}^{-1}$, and the G-L decay length, $K_{N}^{-1}$. As was the case for Ir, $K_{N}^{-1}$ clearly gives the better fit.

In Fig. 51(b) are shown calculated curves for three different values of the cut-off and $T_{c} = 0.2$ mK. In Fig. 52(a) the same data are shown along with calculated curves for three values of $T_{c}$. Figure 52(b) again is for spec. 10C. By comparing Figs. 52(a) and 52(b) one can see the effect of changing the value of the cut-off.

It is apparent that the theory is quite sensitive to the choices of cut-off, $T_{c}$, and $\Delta_{S}(0)$. Although it may appear that for any value of $D$ one can choose a $T_{c}$ which gives a good fit to the data, this is not correct. The fitting procedure is not without order. One must choose a value of $\Delta_{S}(0)$ from theory or by curve fitting to a known superconductor, such as Ir. In the end we chose $\Delta_{S}(0) = 9.9K = 0.66 \Delta_{S}(\infty)$.

The high temperature end of the calculated curve is not affected by the cut-off. One can therefore fit the data for $T$ greater than the expected cut-off and then find the value of $D$ which gives the best fit at low temperatures.

Although we have no calculations for spec. 10C for $\Delta_{S}(0) = 9.9$ K which show the effect of varying $T_{c}$, it can be seen by comparing curves B and C in Fig. 52(a) that it is easy to distinguish the best value of $T_{c}$ within a factor of 2. Assuming the same resolution would be obtained for $\Delta_{S}(0) = 9.9$ K, we conclude that for $\theta_{D} = 480K$, $T_{c} = (1.4)$mK and, therefore, that $0.064 \leq [N(0)V]_{Rh} \leq 0.071$. 
Fig. 52. Experimental values (points) of normalized resistance of Pb-Rh-Pb spec. 10C.

The calculated curves show the effect of the assumed $T_c$ for (a) $\Delta_c = 0.392\,K$ and (b) $\Delta_c = 0.115\,K$. 

For Pb-Rh-Pb 10C:

- $A. T_c = 0.2 \times 10^{-3}\,K$
- $B. T_c = 3 \times 10^{-3}\,K$
- $C. T_c = 5 \times 10^{-3}\,K$

For Pb-Rh-Pb 10C:

- $A. T_c = 1 \times 10^{-10}\,K$
- $B. T_c = 1 \times 10^{-8}\,K$
- $C. T_c = 0.2 \times 10^{-3}\,K$
- $D. T_c = 3 \times 10^{-3}\,K$
4. **Copper**

Copper is a particularly interesting metal to examine for superconductivity because of its common usage and because its normal state properties have been extensively measured. As one might have predicted from its high room temperature conductivity and therefore small electron-phonon interaction, its superconducting transition has never been observed. Indeed, previous proximity effect measurements (see Table 4) have even been uncertain of the sign of \([N(0)V]_{\text{Cu}}\). Our data indicates, however, that \([N(0)V]_{\text{Cu}} = 0.045 \pm 0.010\).

In Figs. 53 and 54 are shown the normalized data and calculated curves for Pb-Cu-Pb spec. 5D. The constant cut-off model was used. This specimen was \(15 \times 10^{-3}\) cm thick and had a resistivity ratio of 52. By comparing Figs. 53 and 54 we see that here the G-L decay length gives a better fit than the one-frequency approximation decay length. The problem with \(k_{N}^{-1}\) is that it does not give a resistance which decreases fast enough with decreasing temperatures. As was pointed out in the previous section on Rh, the independence of D and \([N(0)V]\) in determining the best fit is apparent.

Figures 55 and 56 are for spec. Cu C. It was \(1.9 \times 10^{-2}\) cm thick and had a resistivity ratio of 237. Again \(k_{N}^{-1}\) gives the better fit. However, comparison of Figs. 54(b) and 56(b) gives different cut-offs for the same current. Why this might be true is not understood.

At very low temperatures, the theoretical curves do not fall fast enough with decreasing temperature, even using \(k_{N}^{-1}\). It should be remembered that one of the problems with the original temperature
Table IV. Partial list of estimated values of \([N(O)V]\) (unless otherwise noted) since 1962 of metals studied in this work. References are given in approximately chronological order. Numbered references correspond to those in the text. Lettered references are given below. (T) indicates theoretical value.

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<th></th>
<th>Magnesium</th>
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<th>Silver</th>
<th>Gold</th>
<th>Platinum</th>
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<td>0.08(T)</td>
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<td>Gor'kov-Eliashberg eq. using retarded electron interaction</td>
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<td>T_c of Pb-Cu bi-layers evaporated onto low temp. substrates</td>
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<td>c T_0 of Pb-Cu and Pb-Pt bi-layers sputtered onto substrate</td>
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<td>Magnetic transition of prox effect bi-layers of Mg–Ti, Ag–In, and Au–In</td>
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<td>Magnetic transition of prox. effect bi-layers of Mg-Ti, Ag-In, and Au-In.</td>
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<td>0.085</td>
<td>0.15</td>
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<td>t Tunneling into Al-AlO₃-N-Pb where N = Cu or Ag.</td>
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<td>0.078±</td>
<td>0.045±</td>
<td>0.015±</td>
<td>&lt;0.01</td>
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<td>(0.010) Resistance of Pb-N-Pb work sandwiches where N = Rh, Cu, Ag or Pt</td>
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</table>
References for Table 4:

Fig. 53. Normalized resistance of Pb-Cu-Pb specimen with $P_{\text{Cu}}(4.2) = 3.3 \times 10^{-8} \Omega \text{ cm}$ and thickness $d_N = 1.5 \times 10^{-2} \text{ cm}$. The solid curves were calculated using the one-frequency decay length with (a) $[N(0)V]_{\text{Cu}} = 0.025$ and (b) $[N(0)V]_{\text{Cu}} = 0.037$. $\Delta_c$ is the value of the pair potential at cut-off.
Fig. 54. Same data as Fig. 53 but with the theoretical curves calculated with the G-L decay length. The theoretical curves are: (a) for \([N(O)V]_{Cu} = 0.025\) for three values of the cut-off \(\Delta_c\), and (b) for \([N(O)V]_{Cu} = 0.037\) for three other values of \(\Delta_c\).
Fig. 55. Data (points) and theoretical curves of the normalized resistance of Pb-Cu-Pb spec.Cu calculated using the decay length $K_N^{-1}$. The theory is for (a) $[N(O)V]_{Cu} = 0.03$. Note that no choice of the cut-off, $\Delta_c$, in (b) fits the data as well as curve A in (a).
Fig. 56. Same data as in Fig. 55 but with theoretical curves calculated using the G-1 decay length, $k_N^{-1}$. The curves in (a) for $[N(O)V]_{Cu} = 0.05$ and (b) for $[N(O)V]_{Cu} = 0.04$. 

The curves in (a) show $A. \Delta_c = 0.32 \text{K}$ and $B. \Delta_c = 0.30 \text{K}$.

The curves in (b) show $A. \Delta_c = 0.25 \text{K}$ and $B. \Delta_c = 0.20 \text{K}$. 

The resistivity ratio $R(T)/R(1 \text{K})$ is plotted against temperature $T (\text{K})$.
dependent cut-off was that the theoretical curves fell too rapidly when trying to fit the Ir data. Although calculations were not performed, it may be that the Cu data could be better fit with the temperature dependent cut-off.

5. Silver

The data for Pb-Ag-Pb spec. Ag D are shown in Figs. 57 and 58 along with calculated curves (using \( k_N^{-1} \)) for four values of \([N(0)V]_{\text{Ag}}\). This specimen was \(2.2 \times 10^{-2}\) cm thick and a strip cut from this sample had a resistivity ratio of 193. This sample was one of four cut from the same Ag foil and sputtered at the same time. Though the absolute values of their resistances at 0.7K were spread over a range of 15%, the shapes of their R vs T curves were almost identical. Specimen Ag D is therefore typical.

Our best estimate of \([N(0)V]_{\text{Ag}}\) for this sample is \(0.015 \pm 0.005\). This is considerably smaller than what most other researchers have found (see Table 4). It also differs by being smaller than \([N(0)V]_{\text{Cu}}\).

As can be seen most clearly in Fig. 58(b), the calculated resistance does not fall fast enough with decreasing temperature. We pointed out in the previous section on Cu that the temperature dependent cut-off may not have this problem. However, as was true of the Cu data, no curve fitting with the temperature dependent cut-off was performed.

6. Gold

The measured resistance of two Pb-Au-Pb sandwiches was shown in Fig. 25. From the anomalous behavior near 5K and the very large temperature dependence above 1K it appeared that at least the superconducting
Fig. 57. Data (points) and calculated values of the normalized resistance of Pb-Ag-Pb spec. AgD. The curves were calculated using (a) \([N(0)V]_{Ag} = 0.015\) and (b) \([N(0)V]_{Ag} = 0.010\).
Fig. 58. The same data as in Fig. 57 but with curves calculated using (a) \([N(0)V]_{Ag} = 0.030\) and (b) \([N(0)V]_{Ag} = 0.020\).
alloy AuPb₂ was formed. Because the percentage of this alloy at the interface is not known, the value of \( \Delta S(0) \) is also unknown. For this reason no attempt was made to fit the data to the theory. However, it is clear from the qualitative features of the low temperature data that 
\[ [N(0)\nu]_{Au} > 0. \]

7. **Platinum**

Platinum is an especially nice metal to study by the proximity effect because of its resistance to oxidation. Clean SN interfaces can thus be relatively easily prepared.

The well-known properties of Pt led us to believe that Pb-Pt-Pb sandwiches would behave similarly to those of Cu and Ag. It is a metal and sits next to Ir in the periodic table. It is not magnetic. It has quite high room temperature resistivity (10.4 \( \mu \Omega\)-cm) indicating strong electron-phonon interaction. Its electronic specific heat constant, \( \gamma \), is large (6.8 mJ mol\(^{-1}\)K\(^{-2}\)) so that \( N(0) \) is large. Its Debye temperature is 240 K.

However, the R vs T measurements of three Pb-Pt-Pb samples, Fig. 42, show no decrease in resistance from \( \sim 0.6 \) K to \( \sim 0.1 \) K, the lowest temperature attainable at the time. Platinum is the only non-magnetic metal we tested which had this property. Thus the possibility that \([N(0)\nu]_{Pt}\) may be zero or even negative, also found by Hauser, Theuer, and Werthamer, make it worthwhile to continue these measurements at lower temperatures.

The Pt was purchased as sheet, 254 \( \mu \)m thick and 99.99\% pure, from Electronic Space Products, Inc. The certificate of analysis for this Pt lists the following magnetic impurities (typical): Fe, 25 ppm; Ni, <1 ppm;
and Mn < 1 ppm. It was not rolled after receipt.

8. **Nickel**

Because Ni is ferromagnetic, the decay length of Cooper pairs in it is very short and independent of temperature, as discussed in Sec. IIA. SNS were therefore made with Ni as the normal metal with the expectation of seeing no temperature dependence of resistance at low temperatures. This was in fact the case as seen in Fig. 43.

We therefore are confident that the decrease of resistance with decreasing temperature that we observed in all the other specimens with the exception of Pt was a real effect which can be explained by the semi-phenomenological theory developed in Sec. II.

Finally, we present in Table 4 a summary of the estimated values of $[N(O)V]$ found by other researchers for the same metals which were studied in this work. It is interesting to note the lack of agreement, especially in the columns for Cu and Ag.
V. CONCLUSIONS

From the work of Waldram and Tinkham we have derived an expression for the resistance of an SNS sandwich near $T_{cS}$. This expression contains three characteristic times of the superconductor: $\tau_1 = \frac{L}{v_F}$, assumed to be much smaller than $\tau_3 \approx \tau_Q$, and $\tau_4 = \tau_R$. $\tau_1$ we determined from the low temperature resistivity and the Fermi velocity. For tin we found $\tau_3 = (6-9) \times 10^{-10}$ sec and for lead $\tau_3 = (5-9) \times 10^{-11}$ sec. For both metals $\tau_R > 10 \tau_3$.

The uncertainty given for $\tau_3$ represents only the spread of the data and the lack of agreement between theory and experiment. It does not include uncertainties in the theory.

We feel that two approximations in the theory may cause our estimates of $\tau_3$ in Sn and Pb to be too large by as much as a factor of 6. Both approximations come from trying to apply the one-dimensional Boltzmann equation to a three-dimensional experiment. One factor of 2 is the coefficient of $\tau_3^{-1}$ which one would not expect if the excitations were undergoing a three-dimensional random walk during a time $\tau_3$. The other factor of three comes from the $\sqrt{3}$ in Eq. 50(b). In addition, as has already been mentioned, the Boltzmann equation does not really apply to the case of lead since $\tau_1$ is not much shorter than $\tau_3$.

The specimens used in these experiments were not ideal for measuring $\tau_Q$. The normal metal was thick so that the usual measuring current of 10-100 mA would be far above the critical current at 20 mK. Unfortunately, near $T_{cS}$ where supercurrents would not be a problem, the thick normal
metal decreased our sensitivity to small changes in resistance due to excitation current in S.

Another problem was that the Sn on the Sn-Ir-Sn specimens was not several excitation current decay lengths thick very close to $T_{cSn}$. This should have caused reflections of excitations at the Sn-vacuum interface, whose effect we did not consider.

The relative minimum in resistance near 5K was attributed to part of the interface being covered by a very thin oxide, but this explanation is not given with complete confidence. One would like to have specimens with SN interfaces believed to be really without oxide and without impurities not contained in S or N. If the relative minimum disappeared, one would be more certain of the given explanation. The best way to achieve clean SN interfaces is probably to evaporate both S and N in an ultrahigh vacuum system. This would eliminate impurities introduced by the sputtering gas and possible contamination due to sputtering of the masks. Pb-Ni-Pb sandwiches should be the easiest to prepare because the short decay length obviates the need for a thick normal section.

Also, any lingering doubt about the $\Delta(T)^{-1}$ dependence of $\tau_Q$ should be eliminated by measuring the resistance as a function of magnetic field. Because $\tau_I$ and $\tau_R$ are very weakly dependent on $\Delta_S$ near $T_{cS}$, one should see a significant increase in resistance attributable to $\tau_Q$ as the magnetic field approaches the critical field of S.

The good agreement between our phenomenological theory of the low temperature resistance as a function of temperature and the Ir data gives strong support to the basic correctness of our model. However, it is clear that the effect of the measuring current on the resistance is not
understood. Because the current dependence is significant, a proper calculation which treats self-consistently the current distribution must be done.

Even more important, perhaps, is a better understanding of the shape of the pair potentials in both S and N in the zero current limit. The de Gennes boundary conditions show that any uncertainty in the value of $\Delta_S(0)$ will be reflected in $\Delta_N(0)$ and thus in $[N(0)V]_N$. Furthermore, it is still not clear which is more correct, the one-frequency approximation or the G-L theory, or whether the exponential decay of the pair potential even applies close to the interface. The wrong choice may put $[N(0)V]_N$ in error by much more than the 30% uncertainty in $\Delta_S(0)$.

Our data on Rh are consistent with a $T_c$ of 0.2 mK, predicted by others. It should therefore be a prime candidate for direct observation of a superconducting transition when improved cryogenic techniques allow a reduction of the present minimum of ~500 $\mu$K. Superheating problems associated with its small critical field should be overcome by making a proximity effect sandwich of Rh with a superconductor of high $T_c$.

Magnesium still remains as the element with the highest predicted $T_c$ whose transition has not been seen directly. Although we were unsuccessful producing a clean Mg surface, it appears that other investigators can. In particular, the ion gun in the Auger spectrometer used to analyze the surface of our Mg was able to remove the colored contamination which our sputterer created.
The discussion of the cause of the flattening in Mg at low temperature pointed out the importance of having N metals with magnetic impurities of no more than a few ppm.

Copper and silver have positive values of \([N(O)V]\) with \([N(O)V]_{Cu} > [N(O)V]_{Ag}\). It should again be pointed out that our Cu and Ag might, nonetheless, never go superconducting. These measurements of \([N(O)]\) were made at a temperature much greater than the \(T_c\)'s given by inserting \([N(O)V]\) into the BCS expression. Magnetic impurities of less than 1 ppm would not have been observed, but could still prevent a superconducting transition. Also, there may exist unexpected effects at such low temperatures which would prohibit superconductivity.

Because of the Au-Pb superconducting compounds apparently formed at the interfaces of our Au sandwiches, we did not attempt to extract \([N(O)V]_{Au}\) from the data. The low temperature behavior of R(T) for these specimens, however, was qualitatively similar to that of Cu and Ag and thus we conclude \([N(O)V]_{Au} > 0\).

Platinum turned out to be the most interesting element studied. There were two reasons. First, it had no relative minimum in resistance near 5K. This may be due only to a lack of interface contamination. Platinum may thus be the ideal normal metal to use in the study of the decay of excitation currents in superconductors. Second, the resistance was constant over the temperature range 0.1K to 0.6K. One can conclude that \([N(O)V]_{Pt}\) is smaller than \([N(O)V]_{Ag}\) and may even be less than or equal to zero. It would therefore be exciting to continue the measurements to much lower temperatures in order to see if curvature in R(T) will appear.
Finally, from measurements of Pb-Ni-Pb sandwiches which showed no change in resistance from ~1K to 20mK, we are confident that the decreasing resistance with decreasing temperature that we saw at low temperatures in the other elements was indeed due to the proximity effect.
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APPENDIX I

Sputterer I (SI) is shown in Fig. Al. Part A is the water-cooled copper sputter cathode, shown with masks, B, attached. Under the relieved area of the masks is a foil to be sputter-etched. Around the cathode is the cathode guard, C, at ground potential and insulated from the cathode by a Teflon spacer, D. The outer water-cooled tube, E, is isolated from the Swagelok connector, F, by a Teflon sleeve, G, and Teflon ferrules. H is the Ar gas inlet. The high voltage lead (-5kV) is clamped to E.

The boat, I, for evaporation of the superconductor is shown clamped to copper water-cooled tubes, J. The boat is moved to the right during sputtering.

K is the sputter anode at ground potential. It is a copper disk which is silver soldered to and supported by its own water-cooled tube, L. The anode is not movable.

In order to sputter-etch the second side of the specimen it was necessary to open the vacuum vessel and turn over the masks. The superconductor, usually lead, which had gotten evaporated all over the glass cross as well as on the first side of the specimens would adsorb air during this time and make it difficult to pump down. Baking was avoided in order to prevent diffusion of the superconductor into the foil. As mentioned in the text, the movement of the boat to the left after sputtering tended to allow air to leak through the o-rings which sealed around the tubes, J.
Fig. A-1. Sputterer I. Arrows indicate water flow.
Fig. A-2. Sputterer II.
Sputter II (SII) is shown in Fig. A2. The sputter anode, a, is cooled by liquid nitrogen and is at ground potential. One mask, b, is welded to the water-cooled tube. The other mask is screwed to the first. The complete water-cooled, rotating, sputter cathode of which the masks are one part is described elsewhere. The shutter-guard, c, prevents the discharge from extending to the baseplate and also allows the evaporant in the boat, d, to be outgassed without being deposited on the specimens. The shutter-guard is pulled to the right with a magnet from outside the vacuum vessel before the cathode is rotated.
APPENDIX II

The schematic of the precision ramp-hold is shown in Fig. A3. Switch S4 selects one of twelve present levels, obtained by dividing the Hg-cell reference. The 5 kΩ trimpots in the divider allow the voltage levels to be set to 100 ppm.

Switching to a higher (lower) reference voltage causes the integrator to integrate up (down) at a rate determined by the 10 kΩ logarithmic pot. When the output of the integrator reaches the reference voltage, the discriminator (μA741) changes state, thus energizing the relay.

The energized relay (indicated by the LED) converts the integrator into a unity gain amplifier with a 1.2 sec time constant.

The knee created by the abrupt end of the integration is smoothed by the output amplifier which has a 1 sec time constant.

The small amount of positive feedback in the discriminator produces hysteresis which keeps the circuit from oscillating.

The benefit of converting the integrator into a unity gain amplifier is that the stability of the output voltage in the "holding" condition depends only on the stability of the Hg-cells and the offset voltages of the follower and 725 op-amps. If one just had the relay ground the input of the integrator when it reached the reference level, the integrator would have to have extremely low drift to remain at the reference level to within 100 ppm for several minutes.
Fig. A-3. Precision ramp-hold.
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3. M. L. Cohen has pointed out (private communication) that although
   a measured value of \( [N(0)V]_N > 0 \) at \( T \) allows one to predict
   \( T_cN < T \), ferromagnetic ordering or other unknown effects below
   \( T \) may prevent \( N \) from going superconducting.

4. The energy gap, \( E_o \), (a quantity independent of \( \hat{r} \)) is defined as
   the lowest excitation energy of Fermions in the self-consistent
   field, \( \Delta(\hat{r}) \) is dependent on only one space coordinate, \( x \), \( E_o \) is
   equal to the minimum value of \( |\Delta(x)| \) in the sample.


   see N. R. Werthamer in Superconductivity, R. D. Parks, ed.


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20. L. P. Gor'kov, JEPT 9, 1364 (1959).
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32. A straight-forward derivation is given by J. L. Paterson, Ph.D. thesis, Univ. of Cal., July 1973 (unpublished) Appendix II. There is an algebraic error in Eq. (5).


41. Private communication to Clarke and Paterson, Ref. 36.


51. See Eq. (18) in Ref. 14.
55. See, for example, Ref. 48, p. 124.
56. Performed by Thomas J. Kaminska, IMRD, Lawrence Berkeley Laboratory.
58. Tempilabels, Omega Engineering Inc., Stamford, Conn. 06907.
60. Product No. FM-1006, Huyck Metals Co., Melford, Conn.
64. Cryocal Corp., Reviera Beach, Florida.
65. We thank Prof. N. E. Phillips and co-workers in the Dept. of Chemistry, Univ. of California, for this careful calibration.
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69. Model 857262, Lawrence Berkeley Laboratory, Berkeley, California.
72. We thank M. B. Ketchen for pointing this out.
82. Stuart Berman, Research report, University of Illinois, Urbana, unpublished.

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