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INTERPRETATION OF THE PONDEROMOTIVE POTENTIAL FOR A MAGNETIZED PARTICLE IN A LOW FREQUENCY WAVE*

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ABSTRACT

One term in the expression for the ponderomotive potential of a low-frequency magnetoplasma wave has a simple interpretation: it is the parallel-electric-field potential produced directly by the wave.

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The average motion of a particle (charge \(e\), mass \(m\)) in a uniform magnetic field plus an electromagnetic oscillation,

\[
\mathbf{B}(x, t) = \hat{z}B_0 + \left(\mathbf{B}(x)e^{-i\omega t} + \text{c.c.}\right)
\]

\[
\mathbf{E}(x, t) = \mathbf{E}(x)e^{-i\omega t} + \text{c.c.},
\]

can be analyzed by means of the ponderomotive potential,\(^1\text{-}^3\)

\[
\psi(x) = \frac{e^2|\mathbf{E}_z|^2}{m\omega^2} + \frac{e^2(|\mathbf{E}_x|^2 + |\mathbf{E}_y|^2)}{m(\omega^2 - \Omega^2)} + \frac{i\omega(\mathbf{E}_x^* \mathbf{E}_y - \mathbf{E}_y^* \mathbf{E}_x)}{m\omega(\omega^2 - \Omega^2)},
\]

if the displacement of the particle in one oscillation period is small compared to the scale length of the wave amplitude. In the low frequency \((\omega \ll \Omega \equiv eB_0/mc)\) limit the ponderomotive potential (2) reduces to

\[
\psi(x) = \frac{e^2|\mathbf{E}_z|^2}{m\omega^2} - \frac{m(\omega^2 - \Omega^2)}{B_0^2} \mathbf{E}_z^2 - \frac{iec(\mathbf{E}_x^* \mathbf{E}_y - \mathbf{E}_y^* \mathbf{E}_x)}{\omega B_0}.
\]

The first term in this expression is the familiar parallel oscillation energy \(1/2 \frac{mv^2}{z}\). The second term also has a simple interpretation:\(^4\text{-}^6\) it is the negative of the perpendicular electric-drift energy \(1/2 \frac{m\omega^2}{B}\). Here we offer an interpretation for the last term of (3), for a low frequency \((\omega \ll \Omega)\) wave.

Consider the parallel electric field in the presence of this wave:

\[
E_\parallel \equiv \mathbf{E} \cdot \mathbf{B} / |\mathbf{B}|
\]

To second order in \(\mathbf{E}\) and \(\delta \mathbf{B} \equiv \mathbf{B} - \hat{z}B_0\) we find

\[
E_\parallel = E_z + (\mathbf{E} \cdot \delta \mathbf{B} - E_z \delta B_z)/B_0 + 0(E^3)
\]
Averaging this expression over one oscillation and using Faraday's law,\[ \mathcal{B} = -ic \nabla \times \mathbf{E}/\omega, \] we obtain
\[
E_{\parallel} = -\frac{\partial}{\partial z} \left[ -\frac{ic(\mathcal{E}_x^* \mathcal{E}_y - \mathcal{E}_y^* \mathcal{E}_x)}{\omega B_0} \right]
+ \frac{ic}{\omega B_0} \left[ \mathcal{E}_x^* \frac{\partial \mathcal{E}_z}{\partial y} + \mathcal{E}_y^* \frac{\partial \mathcal{E}_z}{\partial x} - \mathcal{E}_x \frac{\partial \mathcal{E}_z^*}{\partial y} - \mathcal{E}_y \frac{\partial \mathcal{E}_z^*}{\partial x} \right].
\] (6)

In order of magnitude, the ratio of the last term of this expression to the first term is \( L_z \mathcal{E}_z/L_\perp \mathcal{E}_\perp \), where \( L_z \) and \( L_\perp \) are the parallel and perpendicular scale lengths of the wave amplitude.

Now the low frequency \( (\omega \ll \Omega_\perp) \) modes of a plasma with \( T_e \) and \( T_\perp \) comparable are characterized by \( |\mathcal{E}_z| \ll |\mathcal{E}_\perp| \). (For example, Hasegawa and Chen, and Ott, Wehringer, and Bonoli show that kinetic Alfven waves and magnetosonic waves satisfy this inequality). Therefore, assuming parallel and perpendicular scale lengths to be comparable, we can neglect the last term of (6), and we find \( E_{\parallel} \equiv -\partial \psi/\partial z \), with the parallel-electric-field potential \( \psi \) given by
\[
\psi(z) = -\frac{ic(\mathcal{E}_x^* \mathcal{E}_y - \mathcal{E}_y^* \mathcal{E}_x)}{\omega B_0}. \] (7)

Comparing (7) and (3) we see that the last term of \( \psi(z) \) is simply the potential energy \( e\psi(z) \).

Thus we see that, in the low frequency limit, one term of the ponderomotive potential can be interpreted as the second-order parallel-electric-field potential produced directly by the wave. Of course an additional potential also arises in a self-consistent analysis of the effects of the ponderomotive potential.6,9,10
REFERENCES


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