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Examining the Representation and Understanding of Large Magnitudes Using the Hierarchical Alignment model of Analogical Reasoning

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Abstract
Understanding scale is fundamental in science education, but scale comprehension is difficult. One reason difficulties may arise is a disconnect between the linear scale of magnitude and how scale information is cognitively represented. An intervention was designed to foster a linear representation of magnitude, based on the theory that people represent magnitude information in a hierarchically organized structure. The intervention extends principles from the progressive alignment model of analogical reasoning to include hierarchical alignment. Half the students in an undergraduate introductory-level geology class were given multiple opportunities to progressively align time to a constant spatial scale in a linear representation, and locate all previous scales relative to the current scale. The other half of the class received the same content and practice aligning time to space. The intervention group demonstrated a more accurate sense of the relative durations of geological events and a reduction in the magnitude of temporal location errors relative to the control group. These findings suggest that the hierarchical and progressive alignment of geologic time is an effective way to reduce magnitude-based errors in understanding geologic time. These findings are consistent with the category adjustment model, and suggest commonalities between number and time magnitude representation. Educational implications are discussed.

Keywords: Scale; Hierarchical alignment; Progressive alignment; analogy

Introduction
Having a strong conceptual understanding of scale and the relationships between scales is essential for scientific literacy (Tretter, et al., 2006). Fundamental concepts in many disciplines require understanding scales outside those familiar to human experience. For example, research on geologic time, the atom, the size of the universe, and nanotechnology is all based on phenomena occurring at scales that cannot be directly perceived. Being able to understand important current social issues, such as the U.S. deficit, population growth, and global warming also require an understanding of magnitudes outside of direct human experience. Given the importance of understanding scale, it should be no surprise that the new NRC Framework for K-12 Science Education (NRC, 2011) and the Benchmarks for Science Literacy (AAAS, 1993) have both identified “size and scale” as fundamental and a unifying theme in science education. “Size and scale” was also identified as one of the “big ideas” at recent nanoscience and education national workshops (Swarat, et al., 2010).

Unfortunately, people consistently have trouble understanding and comparing values of very small or large magnitudes (e.g. Jones, et al., 2008; Libarkin, et al., 2005; Tretter, et al., 2006; Swarat, et al., 2010). Undergraduate students, even those in STEM majors, have difficulty mastering concepts of size and scale (Drane et al., 2008). Size and scale has been described as a critical barrier to learning and higher-level understanding (Hawkins, 1978). While people are more accurate at ranking relative sizes, they struggle assigning, comprehending, and comparing absolute sizes, especially at extreme scales (Jones, et al., 2008; Tretter, et al., 2006). For example, while most students are able to place major geologic events in the correct order, they fail to demonstrate an understanding of the magnitude of time between these events (Libarkin, Kurdziel, & Anderson, 2007).
Difficulties processing extreme sizes and scales may stem from how magnitude information is cognitively represented. Magnitudes at extreme scales are unfamiliar. Activation of representations of unfamiliar magnitudes is less automatic than of familiar values (Kadosh & Walsh, 2009). For example, people possess a weaker association between magnitude and number words for larger quantities than for smaller more familiar quantities (Sullivan & Barner, 2010).

Unfamiliarity with the magnitude and content information associated with extreme scales may lead to the large conceptual categories held by novices (Trend, 2000; Trettter, et al., 2006). While experts working with extreme scales are characterized as having a “detailed, secure, sophisticated, and well developed” mental framework, novices’ mental frameworks are found to be “scant, insecure, and nebulous” (Trend, 2000). For example, even in-service science teachers who teach geologic time represent the roughly 14 billion years of geologic events as only three conceptual categories: extremely ancient, moderately ancient, and less ancient (Trend, 2000). Conceptual boundaries are defined by consistent estimations of events near each other, creating the boundary, and increased variation of estimations of events within each conceptual category across participants.

Huttenlocher and colleagues’ (1988) category adjustment model offers an account for this pattern of estimations. The category adjustment model applies to both objects and events. It suggests that 1D, 2D, and 3D magnitudes, such as location, distance, and duration, are stored as a hierarchical combination of metric and categorical information. A person retrieves needed information at the level required by a question, as well as the boundaries of any associated higher-level units (Huttenlocher, et al., 1988). For example, remembering that dinosaurs first appeared in the Triassic Period implicitly contains information that dinosaurs also first appeared during the Mesozoic Era, which is a larger division that includes the Triassic period.

However, in the absence of exact information, people use boundaries of other objects/events to help make estimations. Variation in estimation, therefore, occurs because of imprecision of boundaries (Shipley & Zacks, 2008; Zacks & Tversky, 2001). As people use object/event boundaries to help make estimations, the more imprecise or the larger the boundaries, the more variation one could expect to find (Huttenlocher, et al., 1988; Shipley & Zacks, 2008). With no information at a lower level, estimations must default to a higher level. Thus, if a student cannot recall which period dinosaurs first appeared, but can recall it happened in the Mesozoic Era, their estimation will range 180 million years, spanning all of the periods that comprise the Mesozoic Era.

The placement of object/event boundaries will systematically distort estimations in predictable ways. Subjective experience of magnitude is influenced by the number of boundaries the person can recall; the more boundaries a person can recall the greater the subjective magnitude, and the converse for recollection of a smaller number of boundaries (Block, 1990). In line with the category adjustment model, when people hold relatively few conceptual categories, such as at extreme scales, they should underestimate magnitudes. For example, elementary to graduate-level students estimated objects as too small at large scales and as too large at small scales (Trettter, Jones, & Minogue, 2006).

Additionally, because change is usually perceptually salient, and thus plays a role in object/event comprehension and memory, at points of unpredictability humans are more likely to attend to information to permit more accurate future predictions (Shipley & Zacks, 2008). Subsequently, people tend to remember objects/event boundaries by attending to them (Speer, Zacks, & Reynolds, 2007), and recall those objects/events at boundaries more clearly than those in between (Zacks & Tversky, 2001). Therefore, regions sparsely populated with objects/events will tend to elicit more variation in estimation of location and an underestimation of magnitude.

The more organizational structure a person has for the material in memory, the better their recall (Mandler, 1967). Where people have more conceptual categories, perhaps arising from personal experience with the scale, they are more accurate when making judgments. For example, most adults (e.g. Dehaene & Marques, 2002; Dehaene, et al., 2008) and children (Booth & Seigler, 2008) are able to use a proportional linear number line to make estimations for smaller, more familiar numbers, but they fail to do so with larger or unfamiliar numbers. While there currently is a debate about the nature of people’s mental representation of size and scale (e.g. logarithmic, power, scalar variability, or segmented linear model), it is clear that there are compressive effects on people’s estimation of size and scale as magnitude increases to unfamiliar scales. The variation of people’s estimations of quantity increases as a function of the magnitude of the judgment (Dehaene, 2003). A consequence of a compressed number line is that, as magnitudes become less familiar, values will become less discriminable (the distance effect). The distance effect can be seen in slower response times when people make judgments about larger numbers compared to making judgments about smaller numbers (Dehaene, et al., 2008).

If the representation of scale information drives student difficulties in learning about size and scale, then learning interventions designed to address scale representation more directly should improve learning. Effectively teaching reasoning about unfamiliar scale magnitudes should require an intervention that fosters a linear representation of magnitude, populated with boundary information at that scale. An intervention was designed based on the theory that people represent magnitude information in a hierarchically organized structure.

The intervention tested in this study is based on the progressive alignment model of analogical reasoning (Kotovsky & Gentner, 1996). The progressive alignment model has been shown to foster a linear representation of number magnitude (Thompson & Opfer, 2010). The progressive alignment model advocates the comparison of two similar items. The more commonalities that exist...
between these items, and the more these commonalities are highlighted, the more salient corresponding relations will be. Comparing two similar items then helps extend the analogy to unfamiliar items (Gentner & Namy, 2006). Furthermore, the act of performing comparisons may change the original mental representations, increasing uniformity between the two representations. Thus, the process of alignment may make higher-order relational similarities more salient. Recognition of higher-order relational commonalities may promote making similar higher-order connections with subsequent unfamiliar items (Kotovsky and Gentner, 1996). The progressive alignment of scales may alleviate the conceptual dissimilarity between human scales and extreme scales by providing greater structural alignment across changes of scale.

The current study uses the Geologic Time Scale, extending from present day back 4.6 billion years. Novices have trouble understanding geologic time, demonstrating a pattern of errors consistent with a hierarchically organized representation of temporal magnitude (e.g., Libarkin, et al., 2005; Trend, 2000). Novices' estimations of when geologic events occurred may differ from the correct magnitude by as much as five orders of magnitude (Catley & Novick, 2008).

A commonly employed classroom exercise to teach students about the magnitude of geological time is to have them align time to a spatial representation. The current study builds on the use of space as an analogy for time. We note that using space to represent time is particularly important in geoscience education because geologically relevant temporal information is often stored in spatial arrays (e.g., as sequences of layers in a sedimentary deposit). In line with the progressive alignment model, the current intervention gives students multiple opportunities to align time to space in a linear representation, progressing from small familiar scales to geological scales. While the amount of time varies, the amount of space remains constant: students align increasing amounts of time to one meter.

Importantly, the current intervention extends the principles of the progressive alignment model to include the hierarchical organization of all previous scales. Every time students align a new temporal scale to space, they locate all previous scales relative to the current scale. This hierarchical organization highlights how each temporal scale is related to the others, helping to populate each scale with boundary information by providing internal structure of magnitude relations within event boundaries.

Methods

Participants

Participants consisted of 58 (control group) and 49 (experimental group) students enrolled in an undergraduate introductory-level geoscience course at a major university located in an urban setting. The demographics of participants were consistent with those of a large urban American university.

The geoscience course consisted of twice weekly lectures and a laboratory period. All lectures were given by the same faculty member; the students were divided into different sections for the laboratory period. One TA covered four sections and two TAs covered two sections each. The intervention was conducted by the first author (as a guest lecturer) in the laboratory sections as part of the standard stratigraphy lab. Experimental (intervention) and control conditions were evenly distributed across the TAs to control for instructor-based differences.

Intervention Design

In the hierarchical alignment intervention, students aligned time to space beginning with a familiar personal time scale, then worked through different historic and geologic timelines, up to the full Geologic Time Scale. For each timeline, students were required to indicate the timeline’s length, locate specific events, and locate where all previous timelines would begin on the current timeline (see Fig. 1).

![Examples of timelines](image)

Figure 1: Example of three time lines in the hierarchical alignment intervention. Note that for each time line, all previous time lines are located.

Procedure

In a two-hour laboratory session, the experimental group participated in the hierarchical alignment intervention (1.5 hr) after a shortened stratigraphy lab (30 min). The control group completed the full stratigraphy lab (2 hr). During the stratigraphy lab, students learned about the age and distribution of rock types and the types of environments in which those rocks are formed by making and examining stratigraphy columns. Importantly, stratigraphy columns involve aligning geologic temporal information to space. Thus, the intervention and control groups both received practice aligning geologic time to space and exposure to magnitude information. The only difference between the intervention and control groups is the way in which the magnitude information was presented (hierarchically or conventionally). Both the intervention and control groups received further instruction on the Geologic Time Scale and concepts explicitly related to geologic time.
(i.e. two fossil labs) prior to completing the outcome measures. The fossil labs include identifying fossils from different divisions in time.

**Measures** All students completed outcome measures one month after the stratigraphy lab as part of a laboratory exam. There were two items that assessed understanding of geologic time magnitude. One item came from the Geoscience Concept Inventory, which is a valid and reliable instrument measuring a range of geoscience concept knowledge (Libarkin et al., 2005). For this item, students were presented with five time lines that had the same geologic events in different locations. Four of the time lines represented common misconceptions students have (response option A. life occurred when Earth formed, B. humans and dinosaurs coexisted, C. dinosaurs appeared much earlier than they did, E. all life formed at the beginning of Earth’s history), and one time line showed the events in the correct relative locations (D). Students were asked to choose the most correct time line. Two of the incorrect response options (A/B) reflected relatively small magnitude errors (i.e. they are wrong on the scale of millions of years) and the other two incorrect response options (C/E) reflected relatively large magnitude errors (i.e. they are wrong on the scale of billions of years).

The second item is a new measure of geologic time developed for use with middle school students (Barghaus & Porter, 2010). This item is a multiple-choice item that required students to identify which duration-based statement was true using a conventional diagram of the Geologic Time Scale. The correct choice is the statement: The Proterozoic Eon lasted much longer than the Phanerozoic Eon. While numerical information is provided in the diagram, the correct choice may not be obvious to novices in the standard diagram because the spatial intervals of the eons do not proportionally correspond to their temporal lengths. This type of compressed representation is how the Geologic Time Scale is typically depicted. In past work the most commonly chosen incorrect response was a statement that is consistent with the visible spatial intervals (The Phanerozoic lasted much longer than the Proterozoic).

A third test item served as a control for other potential group differences (e.g. motivation). This item is a knowledge-based question, asking when mammals were the dominant land animal. This item did not require an understanding of magnitude.

**Results** On the Geoscience Concept Inventory item the intervention group was significantly less likely to make large-magnitude errors than the control group ($\chi^2(1) = 6.08, \ p = .01$), although both groups were just as likely to choose D, the correct option ($p>.05$). The intervention group was significantly less likely than the control group to choose C, the most common error ($\chi^2(1) = 7.35, \ p = .01$).

The intervention group was more accurate than the control group on the Geologic Time Scale Diagram item ($\chi^2(1) = 3.99, \ p = .05$). The groups did not differ significantly on the knowledge-based test item, which did not require an understanding of magnitude ($p>.05$).

**Discussion**

The current study found that the hierarchical and progressive alignment of geologic time is an effective way to reduce magnitude-based errors in understanding geologic time. The intervention group demonstrated a more accurate sense of the relative durations of geological events and a reduction in the magnitude of temporal location errors relative to the control group. Importantly, the intervention and control groups did not differ significantly on the knowledge-based item, indicating that the intervention affected understanding the magnitude of geologic time, and did not provide additional content or increase effort or motivation in the intervention group. These findings were attained one month later, suggesting a durable effect.
That the intervention, aimed at fostering a linear representation of geologic time, was successful at reducing magnitude errors suggests that mental representation of magnitude influences understanding of the Geologic Time Scale. Specifically, the increased accuracy on the Geologic Time Scale diagram item and the pattern of errors on the Geoscience Concept Inventory item are consistent with developing a more linear scale of time at large magnitudes. One limitation of the current study is the limited number of measurements assessing geologic time understanding. We are developing new assessments of scale representation that evaluate magnitude understanding in abstract numerical domains as well as spatial and temporal content domains. Such assessments will be important for further development of our understanding the nature of the role of analogical mapping in representing magnitude and scale.

The category adjustment model can be applied to any type of magnitude (e.g. space and time). Currently, there are competing accounts of the nature of the representation of magnitude information. Some researchers advocate a generalized mapping of more/less relations across dimensions (Walsh, 2003), while other researchers maintain separate or asymmetrical representations (e.g., Agrillo, Ranpura, & Butterworth, 2010). Our finding that progressive alignment helps foster a linear representation of magnitude for time, and Thompson & Opfer’s (2010) work on numbers indicates that there may be commonalities between number and time magnitude representation. Hierarchical alignment could serve as a valuable technique in future research on the nature of representations of different types of magnitude information.

There are other factors besides those studied here that may contribute to the representation and comprehension of size and scale. Measurement, estimation, perspective, and proportional reasoning may make a role in understanding size and scale (Jones & Taylor, 2009). For example, proportional reasoning is correlated with students’ ability to order objects and assign correct sizes to objects (Jones, et al., 2007). Being able to conceptualize a new unit from existing units (unitizing), and then use that new unit to make comparisons or calculations, are particularly important aspects of proportional reasoning related to understanding size and scale (Lamon, 1994). Further research is needed to examine how these other factors contribute to the understanding of size and scale.

The finding from the current study has clear educational implications. While analogy is one of the most commonly used pedagogical practices (Libarkin, et al., 2007), students still continue to demonstrate difficulties in understanding size and scale information (e.g. Jones, et al., 2008; Libarkin, et al., 2005; Tretter, et al., 2006; Swarat, et al., 2010). It is possible for analogies to fail to bring about conceptual change (Brown & Salter, 2010; Duit, 1991), and even mislead students’ understanding of a concept, making misconceptions hard to identify and resolve (Brown & Salter, 2010; Duit, 1991). Two obstacles faced when using analogy in representing scale information include failure of alignment and unrelated salient features (Gentner, 1983). The hierarchical alignment intervention is specifically designed to control for these issues by keeping everything aligned except for magnitude information. Thus, the hierarchical alignment intervention may be more effective teaching tool than current practices employing single analogical mapping exercises.

Thus far the research discussed has described representations of large unfamiliar whole numbers compared with relatively smaller more familiar whole numbers. While there has been little research examining scaling and ordering of numbers less than the integer one, it is hypothesized that as magnitudes become unfamiliar at both large and small scales, the representation of those magnitudes will become “fuzzy and indistinct” (Tretter, et al., 2006). For example, people also hold a limited number of conceptual categories of extremely small scales (e.g. things we can see versus things we cannot) (Jones, et al., 2008; Tretter, et al., 2006). The increased variation within categories and little to no variation across categories should result in difficulty discriminating among objects/events that are very small. However, in the case of small scales, there is an added complication with the hierarchical alignment intervention: familiar scales are not able to be hierarchically organized within the unfamiliar scales as one progresses towards smaller and smaller scales, as was the case when progressing towards larger and larger scales. A solution may be cycling back up to larger level (or levels) for each smaller scale. In any event, the implications of this complication in cognitive representations and pedagogical practices should be examined in future research.

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References


Hawkins, D. (1978), Critical barriers to science learning, Outlook, 29