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ANNIHILATION

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Publication Date
2014-04-21
Submitted to Physical Review D

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December 1980
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On Search for Heavy Hadron through Jet Invariant Mass

in

Electron-Positron Annihilation

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Abstract

On the basis of theoretical arguments that heavy hadrons carry almost all of the energy of heavy quark jets, we investigate a possible recipe to isolate or to concentrate events of heavy hadron production in electron-positron annihilation.
1. Introduction.

The electron-positron annihilation experiments at PETRA have examined the data to test the validity of quantum chromodynamics through the quantities such as sphericity, thrust, oblateness, and so on.¹ Because of nonperturbative effects still sizable even at the highest energy region, the test can be made only through comparison with results of Monte Carlo simulation. Production of heavy flavors is treated as a contamination folded into the Monte Carlo simulation. As statistics improve and when experiments operate in full swing at PEP² more conscious efforts will be made to separate the production of the bottom flavor from two-jet and three-jet events from light quarks and gluons and to identify hadrons of heavy flavors (simply "heavy hadrons", hereafter) by invariant mass plots. Even with the momenta of all the final particles measured accurately, however, the sheer size of the particle multiplicity will make such an attempt very difficult unless some prescription is given to reduce the combinatorial background in the invariant mass plots.

One obvious way to detect the B mesons is to do measurements at the \( T'' \) (10.57 GeV) peak and to search for B decay modes into smaller multiplicity. As an alternative to this, we study here a systematic method to isolate the signature of relativistic heavy hadrons. It is based on the theoretical argument that heavy hadrons carry almost all of the energy of heavy quark jets, nearly the beam energy. Defining first the meaning of the fragmentation function for our purpose in Section 2, we summarize three different models supporting such a behavior of heavy hadrons in jets. Then, methods are proposed to isolate or to concentrate events of heavy hadron production in \( e^+e^- \) annihilation. Relevant numerical estimates are given for \( \bar{b}b \) production at the highest PETRA-PEP energy region through Monte Carlo Calculation.
The most promising method seems to be to plot invariant masses of whole jets excluding soft hadrons whose energies are low in the overall center-of-mass frame. Separating $\bar{b}b$ events effectively is still not as easy as one might first think, but it may not be impossible with a sufficient number of data, depending on some details in nonperturbative QCD effects and weak decay mechanisms. For $tt$ production at LEP with $m_t = 20$ GeV and $\sqrt{s} = 200$ GeV, separation looks less difficult, but again it will be affected sensitively by the dynamics of strong interactions in the transitional region between perturbative and nonperturbative QCD for light quarks.

We will start with theoretical discussions in the following. If one is primarily interested in conclusions drawn for experimental physics, however, one may immediately proceed to Section 4 without losing any essence of the underlying theoretical background.
2. Fragmentation function; short distance and long distance QCD.

The concept of fragmentation was originally introduced in the phenomenological parton model. It includes all the effects due to low energy strong interactions that happen to quarks after they are created. In field theory, a fragmentation function is defined only when one includes perturbative interactions at short distances as well as non-perturbative interactions at long distances.

Our method of analysis depends entirely on the nature of fragmentation of heavy hadrons from heavy quark jets. It is necessary, therefore, to make its theoretical justification as firm as possible. At extremely high energies, quark fragmentation can be treated rigorously in QCD in the leading logarithmic approximation. The renormalization group analysis or equivalently the ladder diagram summation, which has been successfully applied to light quark jets, can be used for the fragmentation of heavy hadrons as long as the energy-momentum scale $Q$ is much larger than the heavy quark mass $M$ on a logarithmic scale, $\log Q \gg \log M$. Repeating the derivation for the light quark fragmentation, we obtain for the heavy quark of mass $M$

$$\int_0^1 z^{n-1} D(z, Q) \, dz = c_n \left( \frac{\log Q}{\log M} \right)^{-\gamma_n} + \ldots \quad (2.1)$$

in the limit of $\log Q/\log M \to \infty$ with $\log M/\log \Lambda \gg 1$, where $z$ is the usual fragmentation variable and $\Lambda$ is the scale of strong interactions, $\Lambda^2 = 0.1 \sim 0.5 \text{ GeV}^2$. The exponent $\gamma_n$ characterizes short-distance behaviors and is the same as in the case of light quark fragmentation. The coefficient $c_n$ incorporates all the nonperturbative QCD
effects at long distances, which are not calculable for light quarks. The moments of $D(z,Q)$ in (2.1) show an interesting $M$ dependence; $D(z,Q)$ tends to shift more to the large $z$ region for a heavier quark when $Q$ is fixed. The origin of this $\log M$ dependence is traced back to the running coupling of a heavy quark which is smaller by $(\log M)^{-1}$ than that of a light quark when transverse momenta of daughter jets are of $O(\Lambda)$; a heavy quark can emit a daughter jet of $k_T = O(\Lambda)$ but with a probability smaller by $(\log M)^{-1}$.

In spite of this clean perturbative derivation of heavy quark fragmentation, it may not be of more than theoretical interest in the foreseeable future. Even for the $b$ quark ($M = 5.3$ GeV) at LEP energies ($Q = 200$ GeV), the parameter of expansion $(\log Q/\log M)^{-1}$ is no smaller than $1/3$. What is more relevant to physics of our energies is the long-distance or nonperturbative behavior of heavy quark fragmentation. This corresponds to the small $k_T$ region in the perturbative picture of diagram summation (Fig.1). We define our fragmentation function as the one which includes only the small $k_T$ region. It is this fragmentation function that we used to call the fragmentation function in the parton model. Since we expect no more than one or two daughter jets to accompany each heavy quark jet in $e^+e^-$ annihilation even at the highest energies, we can adequately describe all jet phenomena by treating the light daughter jets of $k_T > \Lambda$ as separate jets using the usual parton model fragmentation function and then by applying our fragmentation function to produce well-collimated, low $k_T$ jets from heavy quarks. Since we have no method to calculate the long-distance behavior from the first principles of QCD, we have to resort to models with the help of our knowledge of low transverse momentum physics of strong interactions.
3. Why heavy hadrons carry most of the energy of heavy quark jets.

Two simple pictures were recently put forth for explaining this behavior. One may be called the fireball model, and the other can be called the universal hadronization model. They are consistent with each other and somewhat complementary to each other, too. We will interpret them and give another model based on diagram summation.

**Fireball model**

In deep inelastic processes a heavy quark is produced in an excited state with a cloud of light hadrons around it. It keeps fragmenting off the light hadrons as it moves away. The parton model (restricted to low $k_T$ according to the remark of Section 2) implies that the excited heavy quark states are not far off the mass shell. The invariant mass of such an object, a heavy quark plus light hadron cloud, is larger than the heavy quark mass only by a small amount, which we assume to be independent of the heavy quark mass, (Fig. 2)

$$M_{\text{fireball}} = M + m_{\text{cloud}},$$

$$\frac{m_{\text{cloud}}}{M} \ll 1,$$

$$m_{\text{cloud}} = \text{constant independent of } M = O(\Lambda).$$

It is one of the basic assumptions of QCD that the long-distance dynamics do not depend on quark flavors up to the logarithmic rescaling of the running coupling. Therefore, it is quite natural to assume that $m_{\text{cloud}}$ is independent of quark mass and and therefore that it is given by the order of the low energy strong interaction scale, $\Lambda$.

With this picture given, it is a matter of a simple Lorentz trans-
formation to obtain the $z$ distribution of heavy hadrons. The heavy hadrons carry the fraction of energy

$$z = \frac{\gamma m\sqrt{Q}}{\frac{1}{2}Q},$$

while the light hadrons making up the cloud carry only $z = \frac{\gamma m_{\text{cloud}}}{\frac{1}{2}Q} = O(\Lambda/M)$ though they are moving just as fast as the heavy quark.

Universal hadronization model

We know how to characterize low transverse momentum physics in hadron-hadron collisions. Hadrons are produced with small finite $k_T$ and with a universal and uniform density in the rapidity gap between two leading particles (a heavy quark pair in the present case) going back to back. The rapidity distance of two leading heavy quarks produced in $e^+e^-$ annihilation is given by

$$y = 2 \ln \left( \frac{Q}{M} \right).$$

The rapidity distance is shorter by $2 \ln(M/m_T)$ for a heavy quark pair than for a light quark pair ($m_T = \sqrt{k_T^2 + m_{\text{light}}^2}$). Therefore, light hadrons accompanying heavy quark jets are not only less copious by an amount proportional to $\ln(M/m_T)$, but also less energetic by a factor of $O(m_T/M)$. The hadronized light particles carry the fraction $O(\Lambda/M)$ of the total energy $Q$, thus leaving most of the energy to the heavy hadrons at the ends of the rapidity plot. (Fig. 3)

Ladder approximation in QCD

The summation of an infinite series of uncrossed ladders in the axial gauge is justified only when the entire kinematical region is
included for the transverse momentum $k_T$ of emitted daughter jets.

No proof can be made for the validity of the same approximation in calculating our fragmentation, since it includes only the small $k_T$ region. We use that approximation here, however, in order to compare its result with the conclusion of the preceding models.

With $k_T \leq O(\Lambda)$, heavy quark pairs can not be produced in the middle of the ladders, but they must be produced by the initial impulse of the deep inelastic collision and propagate all the way down to the final heavy hadrons. (See Fig.1a.) The problem becomes very similar to the nonsinglet channels of the light quark fragmentation: the light quark propagators are to be replaced by the heavy quark propagators. They are expressed as (see Fig. 1 for kinematics)

$$\frac{p + \not{k} + M}{(p + k)^2 - M^2} \approx \frac{p + \not{k} + M}{1 - z \frac{M^2}{p^2} + z \frac{k^2}{1 - z} k_T^2}, \quad (3.3)$$

where $(1 - z)$ is the fraction of energy transferred from a heavy quark to a gluon (daughter jet). This causes a strong damping because of the large mass $M$ unless $1 - z = O(k_T/M)$. That is to say, emission of a gluon is allowed only when it carries away a small fraction $O(k_T/M)$ of the heavy quark energy. To be more precise, the kernel of the nonsinglet channel problem for the light quark fragmentation

$$\propto \frac{g^2}{k_T^2 \ln k_T^2} \int_{z_{\text{min}}}^{1} \frac{dz}{z_{\text{min}}} \frac{1 + z^2}{(1 - z)} \frac{dz}{z} \quad (3.4)$$

is replaced by

$$\frac{g^2}{\int_{z_{\text{min}}}^{1} \frac{dz}{z_{\text{min}}} \frac{z^3(1 + z^2) \ln k_T^2 + (1 - z)^4 M^2}{[1 - z) M^2 + z^2 \ln k_T^2 + z^{-2} (1 - z)^2 M^2} \quad (3.5)$$
where \( m_T = \sqrt{k_T^2 + m^2} \) with \( m = O(A) \). Only the region between 1 and \( 1 - 0(k_T/M) \) can contribute to the integral over \( z \) significantly. Then, doing the \( k_T \) integral up to \( A \) (\( A \ll M \)), we obtain after a little algebra

\[
\int_0^1 z^{n-1} D(z, Q) \, dz = 1 - O(n A/M) \tag{3.6}
\]

for \( n \) sufficiently large, but smaller than \( O(M/A) \). This implies

\[
D(z, Q) \approx \delta(z - 1 + O(A/M)). \tag{3.7}
\]

Crossed ladders are of the same order in magnitude as the uncrossed ladders that are thus summed up. It is still true that even in the crossed ladders the value of \( 1 - z \) is restricted to \( O(k_T/M) \) in order to avoid the strong propagator damping. Therefore, the conclusion (3.7) is not affected by the crossed ladders. This seems to be a simple and general conclusion of a kinematical nature.

We have thus seen that three models, all consistent with existing experimental and theoretical knowledge, lead to the conclusion that the fragmentation functions of heavy hadrons must be peaked like a \( \delta \)-function near \( z = 1 \). This result is almost kinematical with a slight amount of dynamics. What would we have to assume if we want a fragmentation function in which heavy hadrons do not appear at the high \( z \) end?

In the fireball model, the invariant mass of the light hadron cloud must be large proportionally to the heavy quark mass at the center. This is clearly in contradiction with QCD and the unified gauge theory in which a heavy quark is heavy not because of its hadronic interactions, but because of its coupling to the Higgs particles. In the universal
hadronization model, it would have to happen that when a heavy quark at rest is struck by another quark, it either emits very energetic light hadrons in the backward direction, opposite to the direction of the incident quark, or else leaves a huge number of light particles ($\propto M$) in contradiction to the tested notion of universal pionization. In the QCD calculation, it seems that there is no diagram which allows production of energetic light particles without causing a large damping by heavy quark propagators.
4. Rapidity, sphericity and thrust of heavy quark jets

The conclusion of the preceding Section should apply to heavy quarks of mass $M$ much larger than $\Lambda$, the scale of strong interactions. The charmed quark mass ($1.5 \sim 2$ GeV) may be a little too small for this, but we expect that the fragmentation function of the charmed quark has an average value of $z$ larger than that for light quarks. It should peak broadly at some value of $z$ larger than $1/2$. For the $b$ quark, our reasoning should apply with better accuracy. Therefore, we explore $b$ quark and $t$ quark production in $e^+e^-$ annihilation.

For the purpose of improving momentum resolution of the final hadrons, it is advantageous to do spectroscopy with hadrons at low energies. For heavy quark pair production, however, decay products of two heavy hadrons are entangled at low energies and the large hadron multiplicity, characteristic of heavy hadron decays, could easily swamp the signature in combinatorial background. If the fragmentation function of heavy hadrons were similar to that of light hadrons, two heavy hadrons would come out with a small relative momentum even at high energies and one could never separate final hadrons into two groups of decay products. The situation is quite different if fragmentation occurs as argued in Section 3. By going to higher energies, where two heavy hadrons have a larger relative momentum, we will be able to separate final hadrons and leptons into two groups belonging to jets moving in opposite directions. In this way, we may have a chance to detect the bottom-flavored and top-flavored particles at the highest PETRA-PEP energies. In the following, we have in mind calorimeter type experiments which measure most of the neutral particles as
well as the charged ones.

**Rapidity distribution**

The fireball model concerns the fragmentation of leading groups of hadrons, the hadrons in the "fragmentation region" in the language of hadron-hadron collisions, while the universal hadronization model discusses the fragmentation through hadronization between the two leading particles. In the QCD ladder summation, there is no distinction between the "fragmentation region" and the "pionization region" of the rapidity plot.

A consistent picture is presumably that in each jet a fireball moves away with the highest velocity and breaks up into hadrons in the fragmentation region, leaving a tail of vacuum polarization that results in soft hadrons. We assume therefore that a jet consists of a fireball and a hadronization tail. Assuming the universality of hadronization, we know from hadron-hadron collisions that the average hadron multiplicity is given approximately by

\[ \langle n \rangle \approx 2y \quad (4.1) \]

in the central hadronization region. This leads to about 7 light hadrons being produced in the central plateau of \(\bar{b}b\) production at \(Q = 36\) GeV and 14 light hadrons for \(tt\) production at \(Q = 200\) GeV for \(m_t = 20\) GeV.

Heavy hadrons decay eventually through weak interactions and splash hadrons with momenta larger than the typical transverse momentum \(k_T\) of hadronization. Since the Lorentz factor along the direction perpendicular to the jet axis is negligibly small (\(\gamma_T = 0(k_T/M)\)), the transverse momentum distribution of weak decay products is determined only by the energy and angular distributions of the weak decay.
We thus expect that the rapidity distribution of final particles should look schematically like (a) for light quark jets and (b) for heavy quark jets in Fig. 4. For comparison, we have given in Fig. 5 the results of Monte Carlo generation of events for light and heavy quark production at the PETRA-PEP energies. The trend depicted in Fig. 4 clearly shows up in the Monte Carlo calculation. Our Monte Carlo events include gluon emission and uses the fragmentation algorithm of Feynman and Field for secondary light quarks in weak decays as well as primary light quarks and gluons. The former has not yet been tested with experiment. It is quite possible that the real distribution is different from the Monte Carlo result because of the uncertainty in the weak decay.

**Sphericity and thrust**

One can derive a relation between the average sphericity and the Lorentz factor $\gamma$ of a heavy hadron

$$<s> = \frac{3}{(4 \gamma^2 - 1)}$$

(4.2)

in the approximation of ignoring the hadronized light particles in the tail and the masses of final particles. This relation holds whatever the energy distribution of weak decays is, as long as the inclusive angular distribution is isotropic in the rest frame of the decaying hadron. This is subject to a statistical spread due to the finiteness of decay hadron multiplicity and a smearing due to the soft hadrons in the tail.

It has been claimed that heavy quark production is characterized by its large sphericity value. It is true only when a heavy quark pair is produced at relatively low energies. At the highest PETRA-PEP energies, for instance, the typical sphericity of the $\bar{b}b$ jets is no larger than
that of relatively narrow three-jet events of light quarks. In Fig. 6, the sphericity distribution of the events generated by the same Monte Carlo method as before is shown to confirm this fact. We hardly see any difference between the sphericity distributions for heavy and light quark jets. The reason is partly that a relatively large $k_T$ distribution of weak decay products is compensated by the large longitudinal momentum carried by the heavy hadron and partly that heavy quarks radiate gluons less frequently than light quarks. We therefore conclude that the sphericity can not be a good criterion to distinguish heavy quark production except at energies near its threshold. It will certainly not work for the $\bar{b}b$ production at the highest PETRA-PEP energies.

For the same dynamical reasons, thrust can not serve for our purpose either, unless it is combined with some other methods. In the limit of $k_T = 0$ for the soft hadrons in the central plateau and in the zero mass approximation to light hadrons and leptons, we obtain independently of the energy distribution of weak decay

$$<T> = \beta \left( = \sqrt{\beta^2 - 1/\gamma} \right).$$

The right-hand side is 0.94 for the $b$ quark at $Q = 36$ GeV ($\gamma = 3$), which is larger than typical values for wide angle three-jets. Though the finite $k_T$ correction reduces $<T>$, it still can not be a powerful means to separate highly relativistic heavy quark production.

5. Mass spectroscopy of jets

We propose to examine the invariant masses of jets in combination with other information. The invariant mass of a heavy jet is equal, up to
$O(\Lambda)$, to the invariant mass of the heavy quark or the weakly decaying heavy hadron, after one separates light hadrons in the hadronization tail. This was built in as a basic feature when the models were presented in Section 3.

The invariant mass of an entire jet is sensitive to the soft hadrons in the central hadronization region. When one includes them, the invariant mass increases substantially since the hadrons which are soft in the overall center-of-mass frame are very energetic in the rest frame of the fireball. If we make the approximation that the light hadrons produced in the central plateau are all relativistic along the jet axis ($k_T \ll k_\parallel$), the squared mass is given by

$$M^2_{\text{jet}} \approx \frac{1}{2} Q \left( \frac{1}{P} M^2_{\text{fireball}} + \sum_i \frac{m^2_{T1}}{k_{/i}} \right),$$

$$= \frac{1}{z} M^2_{\text{fireball}} + \sum_i \frac{m^2_{T1}}{z_i}, \quad (5.1)$$

where $P$ is the momentum of the fireball or the heavy quark, $z$ and $z_i$ are the fractions of energy partition in fragmentation, and $i$ is summed over hadronized light particles. The right-hand side of (5.1) diverges as $Q \to \infty$ because there are always hadrons with finite $k_{/i}$ independent of $Q$. At finite values of $Q$, however, it is a relevant quantity to distinguish between light quark jets and heavy quark jets, provided that one should exclude the soft hadrons in the sum.

For two-jets from light quarks, the invariant mass is given by

$$M^2_{\text{jet}} = \sum_i \frac{m^2_{T1}}{z_i} \quad . \quad (5.2)$$
Three-jets with small opening angles can simulate heavy quark jets. The mass distribution has been evaluated for three-jets with $k_T=0$ using perturbative QCD. In reality, nonperturbative small $k_T$ effects completely dominate over the calculable perturbative effects and enhance enormously the invariant jet mass at the PETRA-PEP energies. The precise distribution with the nonperturbative $k_T$ effects included depends on the transition between perturbative gluon emission and the nonperturbative dynamics of hadron formation in the nearly collinear quark-gluon system. One sensible way to distinguish heavy quark jets and light quark jets is to examine the invariant jet masses on both sides in each event. This seems to be an effective cut in our Monte Carlo events as will be shown below. After this cut, the light quark production that simulates heavy quark production is mostly four-jet events like $\bar{q}G + qG$ with both opening angles relatively narrow.

We generated events by the Monte Carlo method with the same inputs as in Section 4. In Fig. 7a, the invariant masses are plotted for the $\bar{b}b$ jets and light quark jets with gluons at $Q = 36$ GeV, including all soft particles. As was noted before, distinction between them is rather inconspicuous. We then excluded the soft particles with $p < 1$ GeV and replotted the invariant masses in Fig. 7b. The cut on the soft particles reduces the invariant masses of heavy quark jets and even more so for light quark jets. The correlation of invariant masses of both hemispheres can be seen clearly. It now looks feasible to skim out those events for which both invariant masses are larger than a certain value. They have a very high concentration of $\bar{b}b$ events as compared with all the data. The fact that many of the $b$ quark jet masses come out smaller than 5 GeV is due to the following reasons: One is obviously the cut of $p < 1$ GeV,
which occasionally excludes even genuine weak decay products of heavy hadrons. It can also happen with a small probability that a decay product emitted energetically in the rest frame of a decaying heavy hadron may come out as a relatively soft particle in the opposite hemisphere in the overall center-of-mass frame. The other reason is that the neutral $K_L$ and neutrinos are assumed to be undetected in our Monte Carlo. If $K_L$ is measured, the overlap in the invariant mass plot for heavy and light quark jets is reduced further.

The distinction between heavy and light quarks jets is not as clear as we wish. It may help to use the invariant mass plot in conjunction with sphericity or thrust distributions, though the latter alone may probably be useless. We have considered a few more criteria for heavy quark production which are commonly quoted. First of all, multiplicity of particles in the final state. From the recent experiment at CESR,$^9$ we know that the average charge multiplicity is $\approx 9$ for $\bar{b}b$ production. Adding the soft hadrons in the central plateau, we deduce the average charge multiplicity for $\bar{b}b$ production to be $\approx 14$, which is not much larger than the grand average.$^{10}$ The presence of strange particles is a signature of $\bar{b}b$ production, but equally of $\bar{c}c$ production, too. The difference between them is that the strange particles from $\bar{b}b$ tend to have larger transverse momenta than those from $\bar{c}c$. The same can be said for lepton signatures, but in this case one has to try reconstructing hadrons using the jet emitted into the opposite direction because of the missing neutrino. Combining these additional cuts with the invariant mass, it is fair to say that there is a reasonable chance to obtain a sample of events which consists largely of $\bar{b}b$ production.

The invariant mass plot was constructed for events generated by
Monte Carlo for $\bar{t}t$ production at LEP energies. We have used a running coupling constant of QCD which depends on emission angles of gluons and connects smoothly to $\alpha_s(q)$ in the wide angle region. The cut of soft particles is necessary in the invariant mass plot in order to distinguish the $\bar{t}t$ production from the rest. Though we feel that the uncertainty in the Monte Carlo may be greater at the LEP energy range, it looks promising to utilize the invariant mass for the search for $\bar{t}t$ production. The result is plotted in Fig. 8 with the cut of $p < 1$ GeV. A cut of $p$ larger than 1 GeV will probably be more effective.

6. B and T search

Our ultimate goal is to detect the bound states of a heavy quark and a light antiquark and their antiparticles far above their production thresholds. Once one succeeds in obtaining likely candidates of heavy particle production by the invariant mass plot, one should proceed to look into the invariant mass in each hemisphere, first including all particles and then subtracting, one by one, hadrons of lower energies and of small $k_T$. One will hopefully hit peaks of heavy hadrons in this way. Whether this is successful or not depends on how many data are left after the cut has been made on the events.

If one hits invariant mass peaks of hadrons, it is important to examine the angular distribution of the heavy hadrons. Because the heavy hadrons carry away most of the heavy quark energy and receive practically no recoil during fragmentation, the direction of a heavy hadron is the same as the jet axis of the event. Nonperturbative $k_T$ is less important here than for light quarks. The angular distribution of the
heavy hadrons is, therefore, given by

\[ \frac{d\sigma'}{d\Omega} = \frac{3\beta}{16\pi} \left( 1 + \cos^2\theta + \frac{1}{\gamma^2} \sin^2\theta \right) \sigma(Q), \quad (6.1) \]

where \( \theta \) is the polar angle of the heavy hadron momenta with respect to the beam direction, \( \gamma \) is the Lorentz factor of the heavy quark jets, and \( \sigma(Q) \) is the total cross section of a massless quark pair of the same charge at energy \( Q \). It shows a marked dependence of \( \approx 1 + \cos^2\theta \) even at \( \gamma = 3 \). This will serve as one of the consistency checks of the method.

7. Acknowledgment

We are grateful to I. Hinchliffe for useful comments and discussions. One of us (M.S.) owes to G. Gidal for communicating to him the Monte Carlo calculation for the MARK II experiment at PEP and S. Orito of the University of Tokyo/DESY for supplying him with useful informations. He would like to express a sincere gratitude to V. Barger and F. Halzen of the University of Wisconsin for stimulating conversations and suggestions, which eventually led to this work.

This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract No. W-7405-ENG-48.
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Figure captions

Figure 1. Ladder diagrams for summation.

Figure 2. Fireball model.

Figure 3. Universal fragmentation model. The peaks at the ends of the rapidity (y) plot and the z distribution denote the heavy hadrons.

Figure 4. Expected rapidity distributions of final particles from (a) light quark production and from (b) $\bar{b}b$ production.

Figure 5. Rapidity distributions of final particles in the events generated by Monte Carlo. $K_L$ and $\nu (\bar{\nu})$ are not included. The production rates are weighted with squared electric charges for $\bar{u}u + \bar{d}d + ss + cc$. The plot for $\bar{b}b$ is made with the same number of events as that for light quarks.

Figure 6. Sphericity distributions of the Monte Carlo events.

Figure 7. Invariant masses of the Monte Carlo events for $\bar{b}b$ and light particle productions (a) including all observable final particles and (b) excluding soft particles of $|p_t|<1$ GeV and requiring sphericity $>0.05$ for the events. $K_L$ and $\nu (\bar{\nu})$ are excluded. The two invariant masses of opposite hemispheres are plotted in the x and y axes in units of GeV. We have started with the same number of events for $(\bar{u}u + \bar{d}d + ss + cc)$ and for $\bar{b}b$.

Figure 8. Invariant mass distributions for $(\bar{u}u + \bar{d}d + ss + cc + \bar{b}b + \bar{t}t)$ and for $\bar{t}t$ with properly normalized production rates. $Q = 120$ GeV and $m_t = 20$ GeV.
Fig. 1

(a)

(b)

Fig. 2

heavy quark
Fig. 4
Fig. 5
Fig. 6
Fig. 7b
Fig. 8

The figure shows a histogram of the number of jets as a function of the jet mass, $M_{\text{jet}}$, in GeV. The data is categorized into three groups:

- **All events** indicated by a dashed line.
- **$t\bar{t}$ events** indicated by a solid line.
- **Events with $p > 1$ GeV** indicated by a dotted line.

The x-axis represents the jet mass in GeV, while the y-axis represents the number of jets.