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SOFT-GLUON EFFECTS IN NONLEPTONIC DECAYS OF CHARMED MESONS

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ABSTRACT

Soft-gluon effects in nonleptonic decays of D and F mesons are studied nonperturbatively by use of a QCD multipole expansion. For reasonable values of D-meson bound-state parameters, the soft-gluon effects lead to a significant difference in the lifetimes of the \( D^0 \) and \( D^+ \) mesons.

Recent experiments [1] have reported an appreciable difference in the lifetimes of the \( D^0 \) and \( D^+ \) mesons, with \( \tau(D^+)/\tau(D^0) = 3 \sim 10 \). A simple picture of charmed meson decays, based on the charm-quark decay process \( c \rightarrow \bar{u}d \) or \( s\bar{g} \) (fig. 1(a)), predicts equal lifetimes for \( D^0 \), \( D^+ \), and \( F^+ \) mesons. The observed lifetime difference suggests significant enhancements of nonleptonic \( D^0 \) decays by some dynamical mechanisms [2].

The \( W \)-exchange process ("quark-annihilation" process), as depicted in fig. 1(b), contributes solely to \( D^0 \) decays. A helicity factor \( (m_s/H_d)^2 \) and the small probability for quark-pair annihilation in the D meson combine to make its contribution negligibly small.

A number of authors [3-5], however, have pointed out that the presence of gluons may be crucial to the removal of helicity suppression which otherwise is inherent in quark-annihilation processes. Single-hard-gluon emission from the \( D^0 \) meson [3-5], as depicted in fig. 2(a), serves to remove helicity suppression of the accompanying weak decay and enhances the \( D^0 \) decay rate; however, such short-distance QCD effects alone seem to be too small to account for the \( D^0-D^+ \) lifetime difference. Soft gluons inside (or surrounding) the D meson are equally likely sources of enhancement for nonleptonic \( D^0 \) decays. Because of the nonperturbative nature of soft-gluon interactions, however, estimates of their effects have so far been purely phenomenological [4].

The purpose of this paper is to present a dynamical calculation of the soft-gluon effects on \( D^0 \)-decay enhancements. We study the
soft-gluon effects nonperturbatively by use of a QCD multipole expansion [6-8] and relate them to the vacuum expectation value

\[ \mathcal{Y} = \langle 0 | (\frac{e_g}{a}) F_{\mu\nu}[A]^2 | 0 \rangle > - 0.012 \text{ GeV}^4, \]  

(1)

whose magnitude is phenomenologically known from the charmonium sum rules of Shifman, Vainshtein and Zakharov [9]. This matrix element provides a measure of the nonperturbative soft-gluon fluctuations residing in QCD color-confinement mechanisms.

Figure 2(b) represents the process we consider. A hard gluon in fig. 2(a) is replaced by a collection of soft gluons here. (What is meant by a collection of soft gluons will be made clearer below.)

We treat the $D^0$ meson as a nonrelativistic bound state of $c$ and $\bar{u}$ constituent quarks of mass $m_c \approx 1.65 \text{ GeV}$ and $m_u \approx 0.34 \text{ GeV}$. Soft-gluon emission from the final quarks is not taken into account since it is not directly related to the removal of helicity suppression factors. Both the motion and the soft-gluon interactions of the $c$ quark are ignored in what follows; they are suppressed by a ratio $m_u/m_c$ in the decay amplitude as compared with those of the $\bar{u}$ quark.

Let us expand the soft-gluon field around the $c$ quark into multipoles. The multipole terms are cast into a manifestly gauge-invariant form by a suitable transformation [7,8]. The interaction of the $u$ quark with the gluon field is described by the Hamiltonian

\[ \mathcal{H}_{\text{QCD}} = g \bar{u}(x) \left[ \gamma^\mu \mathcal{E}^\mu(\vec{0}) + \frac{1}{2m_u} (\vec{\sigma} + \vec{1}) \cdot \mathcal{H}^{\mu\nu}(\vec{0}) \right] \frac{1}{2} \lambda^a u(x) + \ldots, \]  

(2)

where $u(x)$ is the $u$ quark field at position $x$, $E^{ka}(\vec{0})$ is the $k$th soft-gluon field defined at the $c$ quark position ($\vec{x} = \vec{0}$), and $\vec{L} = \vec{x} \times (p^k = \frac{i}{2} \sigma^k)$ is the angular momentum of the $u$-quark motion. Only the color-El and the color-M1 interactions are shown. The color-Coulomb interaction does not act on a color-singlet $cu$ system. Higher multipole interactions are not included. Note that the $L$ term in (2) does not contribute to an $S$-wave state.

The initial $D^0$ meson is a color-singlet $(1, 1S_0) (cu)$ bound state. The color-El interaction turns the $D^0$ meson into a color-octet $(8)$, $P$-wave $cu$ state while the color-M1 interaction changes both the color and spin of the $cu$ system; namely,

\[ (1, 1S_0) \rightarrow (8, 1P_1) + \mathcal{G}(\text{El}), \]  

\[ (1, 1S_0) \rightarrow (8, 2S_1) + \mathcal{G}(\text{M1}). \]  

(3)

Here $\mathcal{G}$ stands collectively for the color-octet states that are reached by either a color-El or a color-M1 soft-gluon interaction; we call $\mathcal{G}$ the "soft gluons".
The virtual color-octet $\bar{c}u$ systems have $J^P = 1^+$ or $J^P = 1^-$ so that their subsequent weak decays are free of helicity suppression. The $\Delta S = \Delta C = -1$ nonleptonic weak Hamiltonian is given by (we set the Cabbibo angle $\theta_C = 0$)

$$H^W = \frac{\Delta G}{\sqrt{2}} \left[ \frac{1}{N} f_1 + f_2 \right] (\bar{u}d)_L (\bar{u}d)_L + \frac{1}{2} f_1 (\bar{s}d)_L (\bar{s}d)_L + h.c.,$$

where Lorentz as well as color indices have been suppressed in usual fashion; $(\bar{u}d)_L \equiv \bar{u} \gamma^\mu (1 - \gamma_5) d$, etc. This is a Fierz-reordered form (suitable for $D^0$ decays) of the conventional weak Hamiltonian. In the above, $N = 3$ for color SU(3); we use SU(N) notation to keep track of color factors. Hard-gluon exchange corrections to the weak Hamiltonian [10] change the coefficients $f_1$ and $f_2$ from their zeroth-order values ($f_1 = 1$ and $f_2 = 0$) to $f_1 \sim 1.42$ and $f_2 \sim 0.74$.

Let us denote the $D^0$-meson wave function in momentum space by $\psi(p)$ normalized so that $(2\pi)^3 \int d^3 p |\psi(p)|^2 = 1$, where $p$ is the momentum of the $\bar{u}$ quark in the $D^0$ meson. Note that the S-wave function $\psi(p)$ is a function of $|\vec{p}|$. As usual, quark spinors are approximated by free quark spinors. The virtual $\bar{c}u$ states can be decomposed into spin-zero and spin-one components by use of appropriate spin-projection operators. The old-fashioned perturbation theory leads to the following amplitude for the process in fig. 2(b).

$$F = \sqrt{\frac{4}{N}} \int \frac{d^3 p}{(2\pi)^3} \frac{1}{M_{D^0} - \epsilon} \bar{u}^{(s)}(\vec{q}_1, \vec{q}_2) J^{\mu a} (\vec{p}, \vec{\omega}) \psi(p).$$

where $\epsilon$ is the energy of the intermediate $c\bar{u}_b + G$ state, $\vec{p}$ is the momentum of the "soft-gluon" state $|G>$ and $L^a_\mu(q_1, q_2)$ refers to the weak current of the light quarks

$$L^a_\mu(q_1, q_2) = \sqrt{2G(\frac{1}{2} f_1)} \sum_b (q_1)_\mu (1 - \gamma_5) v_\mu (q_2).$$

For the color-EL interaction $J^{\mu a}(p, \vec{\omega})$ is given by

$$J^{\mu a}_{EL}(p) = g <G|E^{ka}(\vec{\omega})|0> \left[ \frac{x^k}{p^\mu (2m_u)} \right] (\mu = 0) \quad (7)$$

while for the color-ML interaction

$$J^{\mu a}_{ML}(p) = \frac{1}{2} \frac{\bar{G}}{m_u} <G|H^{ka}(\vec{\omega})|0> \left[ \frac{p^\mu (2m_u)}{\delta^{kl} + i c_k l j / (2m_u)} \right] (\mu = 0) \quad (8)$$

where terms quadratic in $\vec{p}$ or higher as well as those $\vec{p}^2$ that vanish as $\vec{\omega} \to 0$ have been omitted.

Soft gluons are strongly interacting and need to be treated nonperturbatively. The decay rate, e.g. for the process with the color-ML interaction, involves a sum over the soft-gluon states $|G>$ of the form

$$\sum_0 |H^{1b}(\vec{\omega})|G> <G|H^{1b}(\vec{\omega})|0> = \frac{1}{(M_{D^0} - \epsilon)^2} \ldots \quad (9)$$
The energy denominator \( \epsilon - \gamma \) represents the energy difference between the initial \( D^0 \)-meson state and the virtual \((c\bar{u})_8 + G\) state. It will be reasonable to suppose that, owing to color confinement, the virtual color-octet \( c\bar{u} \) system has higher energy than the initial \( D^0 \)-meson state and that this energy difference does not vanish even for very soft gluons \((\omega \rightarrow 0)\). [For example, this is the case for mesons bound by a one-gluon-exchange potential, which is repulsive between a color-octet quark-antiquark pair.] This energy difference will be of the order of \( 100 \sim 200 \) MeV, a typical scale related to confinement; this point will be discussed later.

With these in mind, we approximate the soft-gluon sum in (9) as follows

\[
\sum \frac{\langle G \rangle}{\epsilon - M_{\gamma}} \sim (\Delta \epsilon)^{-1}. \tag{10}
\]

Namely, the energy denominator \( \epsilon - M_{\gamma} \) is replaced by a constant value \( \Delta \epsilon \) typical for the soft-gluon reaction we consider. It is understood that eq. (10) is inserted between soft-gluon states (i.e. excluding hard-gluon states); accordingly we set \( \omega \rightarrow 0 \) in (\(\cdots\)) of eq. (9).

(iii) The soft-gluon world would exhibit approximate Lorentz invariance relative to the wavelengths of color fluctuations in it. By making use of this invariance, eq. (9), which is now rewritten as

\[
\langle 0 | \phi_{\mu \nu}^1 | \phi_{\mu \nu}^0 > \sim (\Delta \epsilon)^{-2}(\cdots), \tag{11}
\]

is related to \( \langle 0 | \phi_{\mu \nu}^2 | \phi_{\mu \nu}^0 > \) in eq. (1).

An argument in favor of these approximations is made as follows: The phenomenological value for \( \mathcal{V} = \langle 0 | (u_{\gamma} / \pi)^2 | 0 > \) in eq. (1) represents nonperturbative (long-distance) QCD effects; namely, this matrix element is predominantly saturated by the long-distance color fluctuations of confinement physics. Each of the above approximations relies upon and could be justified by this dominance of long-distance dynamics in the phenomenological value of \( \mathcal{V} \).

The value of the unknown quantity \( \Delta \epsilon \) may be estimated in the following way. The annihilation of the virtual color-octet \( c\bar{u} \) system by the weak current occurs in a small domain of the size \( 1/m_c \), the Compton wavelength of the \( c \) quark. Correspondingly, \( \Delta \epsilon \) will be relatively sensitive to the short-distance structure (e.g. the spin-dependent part) of the \( c\bar{u} \) binding potential. Although the virtual \( c\bar{u} \) system is in color-octet, the \((c\bar{u})_8 + G\) state as a whole is a color-singlet and presumably is still in a bound state (of spatial spread of the order of \( 1/\Delta \epsilon \)). Namely, the virtual state may be pictured as a spin-one \( D \)-meson system with a (color-octet) gluon cloud; and the total energy of this meson system may not be drastically affected by the spatially small color concentration of the \( c\bar{u} \) system. If this picture is adequate, the \( \frac{3}{2} - \frac{1}{2} \) fine structure of the D-meson system provides a reasonable guess for \( \Delta \epsilon \) (for the color-M1 process):

\[
\Delta \epsilon \sim M(D^0) - M(D) \approx 140 \text{ MeV}. \tag{12}
\]
A naive estimate of the "binding energy" of the $D^0$ meson gives another measure of $\Delta \epsilon$:

$$\Delta \epsilon \sim m_c + m_\pi - M_D \approx 120 \text{ MeV}. \quad (13)$$

With the abovementioned approximations, the amplitude (9) simplifies. In particular, since $x' = i\delta p^1$ in $\vec{p}$ space,

$${\cal F}^3 \vec{p} \cdot \vec{x} \psi(\vec{p}) = -i\delta \epsilon \int \hat{d}^3 \psi(\vec{p}) \quad (14)$$

by an integration by parts. In the nonrelativistic approximation, the wave function at the origin $\vec{x} = 0$, $\psi(\vec{0}) = (2\pi)^3 \int \hat{d}^3 \psi(\vec{p})$, is related to the $D$-meson decay constant $f_D$ so that

$$\epsilon_D = 2\langle N/N_D \rangle^{1/2} |\psi(\vec{0})|. \quad (15)$$

Apart from an unobservable overall phase factor, the amplitude $\mathcal{F}$ is now rewritten as

$$\mathcal{F} = \left(4m_u \Delta \epsilon (N)\right)^{-1} \langle \psi_D, l_{1/2}^d | t_l \langle \psi_0, l_{1/2}^d | 0 \rangle, \quad (16)$$

where we have taken the same $\Delta \epsilon$ for the color-$M_l$ and color-$E_l$ transitions (although $\Delta \epsilon$ may well be different in the two cases). In the calculation of the decay rate, we use the relation [9]

$$\langle 0 | \varepsilon^{2 u}_D | 0 \rangle = -\langle 0 | \varepsilon^{2 u}_D | 0 \rangle - \frac{1}{12} \epsilon \langle 0 | \hat{p}^2_\mu | 0 \rangle, \quad (17)$$

in accordance with the approximations explained earlier. Note that $\varepsilon^2$ is antichromatic in the nonperturbative QCD vacuum. The contributions of the color-$E_l$ and the color-$M_l$ processes to the decay rate turn out to be of equal magnitude. The decay rate $\tau_{S_d} = \tau_{S_c}$ for the soft-gluon process is given by

$$\tau_{S_d} = \frac{G_F^2 \lambda}{8 \pi} \left( \frac{f_1}{N} \right)^2 \left( \frac{\tau}{4m_u \Delta \epsilon} \right)^2 \mathcal{V}, \quad (18)$$

where $\mathcal{V} = \langle 0 | \varepsilon^{2 u}_{d} | 0 \rangle$ (eq. (1)), and the light quark masses in the final state have been neglected. On the other hand, the $c$-quark decay process in fig. 1(a) leads to equal $D^0$ and $D^+$ decay rates

$$\tau_c = (2 + \lambda)G_F^2 m_c^2/(192\pi^3), \quad (19)$$

where the factor 2 is for leptons and the factor $\lambda = f_1^2 + f_2^2 + (2/N)f_1 f_2 = 1.8$ involves the hard-gluon exchange effect. As before, the final quark masses have been neglected. The relative importance of the soft-gluon process to the quark-decay process is seen from the ratio

$$R = \tau_{S_d}(E_l + M_l)/\tau_c$$

Substituting the numerical values quoted earlier for $m_u$, $m_c$ and $\mathcal{V}$ yields

$$R = 0.7 \times (f_D/\Delta \epsilon)^2. \quad (20)$$
A reasonable estimate for $f_D$, based on an empirical scaling law for the observed lepton-pair decay rates of vector mesons, gives \[5\]

$$f_D \sim 150 \text{ MeV}.$$  \hspace{1cm} (21)

With this value for $f_D$ and $\Delta \sim 140 \text{ MeV (120 MeV)}$, the present soft-gluon process leads to the lifetime difference

$$\tau(D^+)/\tau(D^0) = 1 + R \sim 2.5 \text{ (3.0)}.$$  \hspace{1cm} (22)

This result indicates that the soft-gluon effect by itself could account for a significant portion of the lifetime difference between the $D^+$ and $D^0$ mesons.

The decay rate $\tau_{\text{hg}}$ for the single-hard-gluon emission process in fig. 2(a), evaluated perturbatively [3,5], is smaller than the soft-gluon effect $\tau_{\text{sg}}$:

$$\tau_{\text{hg}} / \tau_{\text{sg}} = \frac{2}{3} \frac{(a_s/\pi)^3}{(M_0/c)^2} / V$$

$$\approx 0.02 a_s \times \langle \Delta c/60 \text{ MeV} \rangle^2,$$  \hspace{1cm} (23)

where the $a_s$ is the coupling constant characterizing the hard-gluon emission process. In $D^0$ decays, therefore, the hard-gluon emission effect is one order of magnitude smaller than the soft-gluon effect.

Quantitatively the present analysis is a crude estimate because of uncertainties involved in $\Delta c$ and $f_D$ and of the approximation scheme employed. It is conceivable that higher-multipole terms, especially those of soft gluons coupled to gluon exchanges between the constituent quarks in the $D$ meson [8], are non negligible for the $D$-meson system (although they are less important for heavy quarkonium systems). Qualitatively, however, the abovementioned conclusions of the present analysis will, we expect, survive a more detailed analysis.

Soft-gluon effects are expected to be important in other heavy-meson decays as well.

1) Charmed $F$-meson decays. The present analysis developed for the $D^0$ meson is carried over to the $F^+$ meson by interchanging $u$ quarks $\leftrightarrow s$ quarks and $f_1 \leftrightarrow f_2$. It is owing to the hard-gluon exchange corrections to the weak Hamiltonian that color-octet $c\bar{c}$ systems are annihilated into $u$ and $d$ quarks via the process in Fig. 2(b); with either a single color-$E_1$ or color-$M_1$ soft-gluon interaction, their annihilation into $\nu\bar{c}$ is still forbidden by color mismatch. The ratio of the soft-gluon corrections $\tau_{\text{sg}}$ for the $F^+$ and $D^0$ mesons is given by

$$\frac{\tau_{\text{sg}}(F^+)}{\tau_{\text{sg}}(D^0)} = \left( \frac{f_F}{f_D} \right)^2 \left( \frac{m_{c\bar{c}}}{m_{u\bar{d}}} \right)^2 \left( \frac{m_F}{M_D} \right)^3.$$  \hspace{1cm} (24)

With $M_F \approx 2.03 \text{ GeV}$, $m_s \approx 0.54 \text{ GeV}$ (constituent quark mass), $f_F/f_D \sim 1.4$ (an estimate based on the scaling law) and
\[ \Delta \mu_F - \mu(F^+) - \mu(F) \approx 110 \text{ MeV}, \] one obtains the estimate

\[ \frac{\tau_{F^+}}{\tau_{F^0}} \approx 0.4 . \] \hfill (25)

This indicates that the \( F \)-meson lifetime is somewhere between those of the \( D^0 \) and \( D^+ \) mesons:

\[ \tau(D^0) < \tau(F^+) < \tau(D^+) \] \hfill (26)

(2) For heavy mesons containing \( b \) or \( t \) quarks, the difference in the lifetimes of the neutral and charged members will not be as prominent as in the case of \( D \) decays. The enhancement of annihilation processes by the soft-gluon effect diminishes rapidly with increasing heavy-quark mass: The ratio \( R \) in eq. (19) decreases like \( 1/m_c^3 \) as \( m_c \to \infty \) (since \( \tau_D = m_c^{-1/2} \)). Correspondingly, in \( B \)-meson decays this ratio will presumably be more than one order-of-magnitude smaller than in \( D \) decays. As seen from eq. (23), the soft-gluon effect and the hard-gluon emission effect are comparable in \( B \)-meson decays. For sufficiently heavy mesons, nonleptonic enhancements will be dominated by short-distance mechanisms, hard-gluon emission from the initial and final quarks.

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Figure 1. $D^0$ decays. (a) charm-quark decay process. The $\bar{u}$ quark acts as a spectator. (b) quark-annihilation process. Shaded blobs represent $W$-boson exchanges with hard-gluon exchange corrections.

Figure 2. Quark-annihilation process with emission of gluons from the $D^0$ meson. (a) hard-gluon emission. (b) soft-gluon emission. The dashed line refers to a virtual state.

F1 The nonrelativistic description of the $D$-meson system, though not as reliable as that of $c\bar{c}$ charmonium systems, has certain phenomenological success; see, e.g., ref. [10]. The nonrelativistic picture improves somewhat for $c\bar{s}$ systems.

F2 Terms that depend on $\bar{u}$ correspond to higher-multipole terms (other than color-$E1$ and color-$M1$ terms).

F3 Similar approximations have previously been used for heavy-quarkonium systems by Voloshin [6].

F4 In standard perturbation theory, the contribution of very soft gluons to $\langle 0|p_{\mu}^2|0 \rangle$ is vanishingly small at least to lowest order because of derivative coupling. Hard gluons mainly contribute to the renormalization of the operator $\alpha_s p_{\mu}^2$ (which is a renormalization-group invariant in the absence of quarks). The nonvanishing value of $\langle 0|\alpha_s p_{\mu}^2|0 \rangle$, being a long-wavelength phenomenon, is considered to be a consequence of strong soft-gluon interactions.

F5 The surface term in eq. (14) vanishes for a large class of quark-binding potentials, including the Coulomb and the harmonic oscillator potentials.
REFERENCES


