CALCULATING OPTICAL CONSTANTS OF GLAZING MATERIALS

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ABSTRACT

An exact analytic solution for the optical constants of a homogeneous, dielectric material is derived in terms of the transmittance and reflectance at normal incidence of one parallel-sided layer.

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Calculating Optical Constants of Glazing Materials

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INTRODUCTION

This note describes a simplified photometric method for calculating the optical constants of homogeneous, dielectric materials. From these constants, the radiation properties of a window or solar-collector cover consisting of a series of parallel layers can be calculated for any angle of incidence [1]. Many solar energy applications do not require the accuracy of classical refractometric methods. Pettit [2] shows that "effective" optical indices can be used to predict the angular properties of coated or scattering glazing materials. The method described below can be used to find these effective indices as well as the indices of nonscattering bulk materials. Where high accuracy is needed, this method can be used for preliminary surveys of optical materials, especially plastics, which differ because of additives and density as well as chemical composition.

Photometric techniques usually involve measuring transmittance [3] or reflectance [4-6] of polarized radiation for at least three angles of incidence. Three methods use unpolarized light: one uses three or more measurements of reflectance at different angles [7]; another uses normal transmittance measurements on two samples of different thicknesses,
and the method described herein uses two normal-incidence measurements, one each of reflectance, $R$, and transmittance, $T$, on a single sample. Any of these sets of measurements uniquely determines the index of refraction, $n$, and the extinction coefficient, $k$.

The "two-transmittance" method is appealing because no specialized equipment is required, and because transmittance is easier to measure than is reflectance. However, samples of different thicknesses often are not available. Also, the "two-transmittance" method introduces an approximation that causes the method to fail for high $n$ and low $k$.

References 2, 10, and 11 state that, given $R$ and $T$, no analytical solution exists for $n$ and $k$; instead they offer numerical or graphical solutions. We derive the analytical solution as follows. We assume that the sample is specular and has little absorption $((n-1)^2 \gg k^2)$. Accuracy is limited only by the photometric measurements and the validity of the above assumptions.

**METHOD**

Let $r$ be the single interface reflectance at the boundary between the sample and the air, at normal incidence, as given by Fresnel's equation:

$$r = \left[\frac{1 - n}{1 + n}\right]^2. \quad (1)$$

If "$a$" is the absorptance on a single pass through a sheet of thickness $t$, then Lambert's law can be written as

$$x = 1-a = \exp(-\alpha t), \quad (2)$$

where $\alpha = 4\pi k/\lambda$ is the absorption coefficient at wavelength $\lambda$. Counting
all multiple reflections and summing the resulting infinite series gives

\[ T(r,x) = \frac{(1-r)^2 x}{1 - r^2 x^2} \]  \hspace{1cm} (3)

and

\[ R(r,x) = r + \frac{(1-r)^2 x}{1 - r^2 x^2}. \]  \hspace{1cm} (4)

Expanding (4) and solving for \( x \),

\[ x(r,T,R) = \left[ \frac{R - r}{R^2 - 2r^2 + r} \right]^{1/2}. \]  \hspace{1cm} (5)

Combining (3) and (4),

\[ x(r,T,R) = \frac{R - r}{rT}. \]  \hspace{1cm} (6)

Equating the right sides of (5) and (6) results in a quadratic equation in \( r \), the solution of which depends on only \( T \) and \( R \):

\[ r(T,R) = \frac{T^2 + 2 - (R-1)^2 - \left[ (T^2 + 2 - (R-1)^2)^2 - 4R(2-R) \right]^{1/2}}{2(2-R)}. \]  \hspace{1cm} (7)

The index of refraction can be obtained by reversing (1):

\[ n = \frac{1 + r^{1/2}}{1 - r^{1/2}}. \]  \hspace{1cm} (8)

The absorption coefficient is obtained by first substituting the value of \( r \) from (7) into (6). Then, from (2),

\[ \alpha(t) = -\ln(x). \]  \hspace{1cm} (9)

A sensitivity plot (Fig. 1) shows that the theoretical suitability of this method increases in regions of low to intermediate \( n \) and low \( \alpha(t) \). The optical constants of transparent glazing materials generally fall within these ranges (\( n \) from about 1 to 4 and \( \alpha(t) \) from 0 to 1) in the
solar and far infrared.

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REFERENCES


Figure 1. Sensitivity of $n$ and $k$ to measurements of normal reflectance and transmittance.