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UNUSUAL BEHAVIOR OF PROJECTILE FRAGMENTS PRODUCED
BY THE INTERACTIONS OF RELATIVISTIC AR IONS WITH COPPER

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ABSTRACT

Radiochemical activation techniques were used to study the behavior
of projectile fragments formed in the interaction of 0.9 A GeV and 1.8 A GeV
40Ar ions within thick Cu-targets. Two identical 1 cm Cu-disks were ir-
radiated with separations between the disks of 0, 10, and 20 cm, respectively.
The results show an enhancement in the formation of light mass (A<30)
radioactive residues in the second disk relative to the first in the contact
configuration for both projectile energies. This enhancement decreases
for increasing distance between the Cu-disks. While our results with
0.9 A GeV projectiles can be explained within the framework of conventional
nuclear physics, the data at 1.8 A GeV demand that either secondaries
of Z=1, neutrons and/or pions are emitted with unexpectedly large transverse
momenta, or that some projectile fragments have large interaction cross
sections but decay in flight (\tau \sim 10^{-10}s).

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1. INTRODUCTION

Observations reporting the formation of projectile fragments with anomalously short mean free paths in relativistic heavy ion collisions [1] have attracted considerable interest in recent years. This short mean free path of secondary fragments (often dubbed as "anomalons") could not be explained within the framework of conventional nuclear physics. However, the effect itself has not been definitely established experimentally [2-12]. A large number of different experiments have been presented at the 1984 GSI Heavy Ion Study and summarized by Heinrich et al. [13].

In some experiments using the nuclear emulsion technique, the anomalous behavior is observed while in others it is not. The same holds for experiments using CR-39 solid state track detectors. The results from large counter experiments, which detect interactions via changes in the charge Z of projectile fragments with \(8 \leq Z < Z_{\text{primary}}\), gave no evidence for anomalous projectile behavior. These data [9,10] had much smaller statistical uncertainties than those obtained by visual methods; the results, however, apply only to projectile fragments whose interactions lead to \(\Delta Z \geq 1\) (i.e. a charge change of at least one unit between an incident heavy ion and its secondary heavy projectile fragment) moving within a narrow forward cone of \(\pm 3^\circ\). Bubble chamber experiments with small statistical uncertainty performed at the JINR accelerator [4,12] show some striking evidence for anomalous behavior. In particular, Gasparian et al. [4] were able to show that for interactions of 4.1 A GeV \(^{12}\text{C}\) in a propane bubble chamber one could observe a very strong short mean free path effect for primary interactions, in which \(\Delta Z = 0\). In reactions with \(\Delta Z > 0\), no such anomalously short mean free path could be observed. This was shown convincingly for reactions with \(\Delta Z = 1\). Thus, from an experimental point of view, one
has to consider the question "Do anomalons exist?" as being open. Consequently, it is important that as many different techniques as possible be brought to bear on this problem.

We report here the results of experiments in which radiochemical activation techniques have been used to search for the possible formation, interaction and decay of anomalous projectile fragments. In these experiments, thick copper disks are exposed to a high fluence of relativistic heavy ions. Certain specific radioactive residues are produced in the interaction of the beam ions and of their secondaries with the disks; these are identified by off-line γ-ray spectroscopy. A typical example of such a radioactive nucleus is $^{24}\text{Na}$ ($t_{1/2} = 15.02$ h), which can easily be identified by its 1368.5 keV γ-ray. Given a sufficiently high beam intensity of primary heavy ions, we can measure the $^{24}\text{Na}$-activity in the disks quite precisely (±1%). Comparable precision is obtained for a few similar nuclides. The specific reasons for our special interest in this nuclide will be given in detail in Sec. 3.

Our radiochemical experimental technique is complementary to the other techniques used so far, in that we study the target-like fragments of the interactions rather than the projectile fragments themselves. It also differs from them in two respects namely:

- it is a global search for anomalous behavior of any fast secondary
- it looks at partial rather than at total cross-sections

These differences obviously entail advantages and disadvantages, which are given below:
Advantages

- Because of the high beam fluxes involved, the statistical uncertainties are considerably smaller than those of visual methods, and are comparable to those direct counter measurements on secondaries.
- The search encompasses both charged and neutral secondaries, and charged pions as well as nuclear fragments.
- It covers in angle the whole fragmentation cone, and the pion and "participating nucleon" cone out to 20°.
- Because of the specific energy dependence of the $^{24}\text{Na}$ cross-section, it selects only fast secondaries (and is hence insensitive to the relatively low energy target fragments.)
- Even if anomalous secondaries either do not exist, or do not contribute to the investigated partial cross-section, this method provides a new measure for secondary energy flow, and implicitly, may reveal interesting features of the reaction mechanism.

Disadvantages

- Any interesting effects may just happen to be absent in the partial cross-section.
- The sensitivity to all kinds of fast secondaries may drown out an existing effect if it happens to be localized in some special and rare subset.
- Hence, a null effect may be inconclusive.

The experimental techniques used are described in Section 2 and the results in Section 3. The interpretation of our experimental results is by necessity somewhat involved, since we do not directly observe the projectile fragments but only the products of their interactions. Partial results of this experiment were presented previously [14].
2. EXPERIMENT

We carried out the experiments at the Bevalac (Lawrence Berkeley Laboratory, University of California, Berkeley) using argon beams with 1.8 and 0.9 A GeV energy, respectively. The principle of our experiment is illustrated in Fig. 1. Two 1 cm-thick circular copper disks (r=4 cm) were irradiated by the Ar beam (Fig. 1). In a typical irradiation, \( \approx 10^{12} \) Ar ions passed through the Cu disks in a period of 2-4 hours. The beam was well focussed (nominal diameter \( \approx 1 \) cm), as can be seen on an autoradiographic picture of an irradiated Cu-disk (Fig. 1b). Ninety-nine percent of the beam was centered within an area of \( r \geq 2 \) cm, as measured via the induced \(^{24}\)Na-activity in the Cu-disks. These autoradiographic pictures showed no significant difference for Cu-disks numbered 1-4 (Fig. 1a). For a given energy, pairs of Cu-disks were irradiated together in a "contact" configuration (d=0 cm), with a separation of 10 cm and a separation of 20 cm. In the contact configuration and in the d = 20 cm configuration, the downstream Cu-disks were surrounded by a 1 cm thick Cu guard ring (\( r_{\text{out}} = 7 \) cm, \( r_{\text{in}} = 4 \) cm) (Fig. 1a). Both Cu-disks in a particular configuration serve as targets for the primary beam, as well as for secondaries interacting within the same disk in which they were produced (trajectories Ar-1 and Ar-2 on Fig. 1c). However, secondaries produced in the first disk (the "target" disk) and interacting in the second disk (the "detector" disk, trajectory Ar-3 in Fig. 1c) will enhance nuclide production in the "detector" disk. This enhancement may be especially strong if, among other reasons, such a projectile fragment has an unexpectedly high interaction cross section, i.e., a "too short" mean free path.
After the completion of the irradiation, short-lived activities were allowed to decay for approximately 12-24 hours. Afterwards, the radionuclides present in the irradiated Cu-disks and guard rings were assayed by off-line gamma ray spectroscopy. The guard rings were cut into sections and reassembled to simulate the disk geometry for counting. Measurements were made with Ge(Li) detectors (resolution ~1.8 keV). The analysis of the γ-ray spectra was based on standard radiochemical procedures [15]. Counting was carried out for approximately one week at LBL and was continued for several months at Marburg. Independent determinations of most of the radionuclides present in each disk were made at LBL, Purdue, and Marburg and the whole experiment was repeated. All the results agreed within experimental uncertainties and the values from the different laboratories were averaged to give the final results. The experimental details have been described elsewhere [16]. Possible differences in counting efficiencies due to sample geometry were evaluated using homogeneous 24Na sources produced by the 27Al (n,a) reaction. We compared the counting efficiency of a beam-size-source (area ~1.5 cm²) of 24Na in 27Al with a clearly exaggerated two-component source of 24Na in 27Al (70% within an area of 1.5 cm² and 30% homogeneously distributed over an area of 30 cm²) in our typical counting configuration (source-detector distance = 8 cm). This two-component source should simulate the slight beam blow-up in downstream Cu-disks (disk No. 2 and No. 4 in Fig. 1a). The difference in counting efficiency between these two sample geometries was determined to be (2±1)% [16,17]. As a matter of fact, our beam spot was considerably more concentrated than that.

We determined the ratio $R_d$ of the activity in the downstream disk to that in the upstream disk as a function of disk separation $d$, for specific nuclides within a pair of Cu-disks. Because each pair of Cu-disks was irradiated with the same particle beam simultaneously and assayed later
for its gamma activity in a fixed position with the same Ge(Li) gamma-ray detector, this activity ratio for a specific nuclide can be determined to a high degree of precision. All uncertainties due to particle fluxes, counting efficiencies, uncertainties in the decay scheme for a specific radioactive nuclide, etc., cancel out in the activity ratios $R$. Essentially, the only experimental uncertainty in this ratio comes from counting statistics. As the number of counts is typically $>10^4$ our activity ratio $R$ can be determined within $\pm 1\%$. Such a precision is comparable only to that of large counter experiments or of high-statistics bubble chamber experiments, as mentioned in the introduction.

3. RESULTS

We show (in Fig. 2) the dependence of $R_d$ on the product mass number for two different separations ($d=0$ cm, $d=20$ cm) of the disks for reactions induced by 0.9 A GeV $^{40}$Ar ions. The dependence of $R_d$ on $A$ is a reflection of the energy spectrum and angular distribution of the secondaries inducing reactions in the disks. The results for the 0.9 A GeV $^{40}$Ar projectiles show that when the two disks are in contact ($R_0$), the projectile fragments (PF) most likely to strike the detector disk lead to the formation (by target fragmentation) of products with $A \approx 55$ and substantial yields are seen for all products with $A > 40$. The products with $A < 30$ ($^7$Be, $^{22}$Na, $^{24}$Na, $^{28}$Mg) are formed only in high deposition-energy target fragmentation events [19]. They cannot be projectile fragments because these are much too energetic to stop in the copper disks.

When the disks are moved 20 cm apart, the "detector" disk samples a different subset of the PF created in the "target" disk, i.e., the more strongly forward-focused and thus higher energy fragments. As a result, the PF most likely to reach the second disk now lead to the formation
of products with \( A \geq 45 \) and the formation of heavier fragments is less likely. Not surprisingly, \( R_0 \) for these products is larger than \( R_{20} \) reflecting production by low-energy, wide-angle secondaries. The fragments with \( A < 30 \) are produced with about the same yields regardless of disk separation because they are only produced by highly forward-focused, energetic projectile fragments.

For the 1.8 A GeV \(^{40}\)Ar beam (Fig. 3) the dependence of \( R_0 \) on \( A \) for \( A > 40 \) is similar to that obtained with 0.9 A GeV \(^{40}\)Ar projectiles except for the shift of the most probable product mass number to a slightly lower value (\( A \approx 50 \)). The values of \( R_{20} \) (for \( A > 40 \)) are also similar to those obtained at 0.9 A GeV. However, the results for products with \( A < 30 \) are significantly different from those obtained with 0.9 A GeV \(^{40}\)Ar. The values of \( R_0 \) are substantially higher, ranging from 1.3 to 1.6. Furthermore, the ratios decrease with separation between the disks, to values of \( R_{20} \) between 1.1 and 1.2.

In order to emphasize the difference between measurements at \( d = 0 \) cm and \( d = 20 \) cm we show the ratio of cross-section ratios

\[
\rho \equiv R(1.8 \text{ A GeV})/R(0.9 \text{ A GeV})
\]

for \( d = 0 \) cm in Fig. 4 (\( \rho_c \)) and for \( d = 20 \) cm in Fig. 5 (\( \rho_{20} \)) for all the isotopes investigated. There is a sharp contrast between Fig. 4 and Fig. 5. It is interesting to note that \( \rho_0 \) has its maximum value (1.3) for low mass products. All other \( \rho \)-values are \( \leq 1.15 \). This is another way of stating our experimental finding that something surprising is happening for low mass fragments produced in \( d = 0 \) cm configuration by the 1.8 A GeV \(^{40}\)Ar beam. The lowered \( \rho_0 \) value for \(^{7}\)Be can be explained by noting this nuclide is presumably produced via two different reaction mechanisms. Indeed \(^{7}\)Be can be either the target residue of a very violent interaction
(then its $p$-value should be that of Na-isotopes) or it is an evaporation product. In the latter case one would conjecture a $p$-value close to unity, as observed for spallation products. As one would expect, a value intermediate between those extremes is observed.

We find that $R$-values for $^{24}\text{Na}$ can be obtained more precisely than the corresponding $R$-values for $^7\text{Be}$, $^{22}\text{Na}$ and $^{28}\text{Mg}$ under the conditions of our experiments. Accordingly, we focus our attention in the following discussion to $^{24}\text{Na}$, noting that all these light products ($^7\text{Be}$, $^{22}\text{Na}$, and $^{28}\text{Mg}$) behave similarly. We also note that the excitation function for $^{24}\text{Na}$ produced from Cu with high energy particles is rather well known (Fig. 6); its steep rise up to 2 GeV, and its subsequent saturation, turn our target-product combination into a "radiochemical threshold Cherenkov counter."

Numerical values of the ratios $R_d$ for $^{24}\text{Na}$ are given in Table 1.

The results for the ratio of the ring activity to that of the downstream disk $R_{\text{ring,d}}$ (No. 2, No. 4 in Fig. 1a) for specific radionuclides are given in Table 2. The results for $d=0$, $R_{\text{ring,0}}$, show that practically no high-energy particles hit this guard ring ($R_{\text{ring,0}} < 0.01$ for $^{24}\text{Na}$) for reactions induced by either 0.9 A GeV $^{40}\text{Ar}$ or 1.8 A GeV $^{40}\text{Ar}$. The results for $d=20$ cm, $R_{\text{ring,20}}$, show that only a small fraction of high-energy secondary particles strike the guard ring, as we observe only $R_{\text{ring,20}} = 0.04-0.07$ at both $^{40}\text{Ar}$ beam energies for light products (e.g. $^{24}\text{Na}$).

However, we observe larger values of $R_{\text{ring,20}}$ at both energies for spallation products close to the target (e.g. $R_{\text{ring,20}} = 0.17 \pm 0.01$ for $^{56}\text{Co}$).

The detailed discussion of the induced activity in the Cu-disk and its surrounding guard ring for a separation of 20 cm is given in the next section. For $d = 20$ cm the disk and the ring sample secondary fragments moving within a $11^\circ$ cone and between $11^\circ$ and $20^\circ$, respectively.
The results for $^{24}$Na (Table 1 and Fig. 7) show a few interesting features:

(a) A strong increase in $R_0$ when the Ar-energy increases.
(b) $R_d$ decreases slightly with increasing $d$ at 0.9 A GeV, but strongly at 1.8 A GeV.
(c) If we add the activity in the guard ring to that in the downstream Cu disk, the decrease in $R_d$ is canceled at 0.9 A GeV, but not at 1.8 A GeV.

These changes can be expressed quantitatively by comparing values of

$$X_{d,j}(\%) = 100 \left( \frac{R_d^j - 1}{R_d - 1} \right)$$

i.e. of the deviation from unity of "corrected" ratios $R_d^j$ defined analogously to the $R_d$ but with the activity of the guard rings added to that of the "detector" disks. The second subscript ($j$) refers to the projectile energy (i.e. $j=1$ to 0.9 A GeV and $j=2$ to 1.8 A GeV) (Note that for $d=0$ the ring activity is so low that $R_d^1$ and $R_d$ practically coincide!).

Then

$$X_{0,2} - X_{0,1} = (33 \pm 2)\%$$

while

$$X_{0,2} - X_{20,2} = (16 \pm 2)\%$$

This behavior will be examined in the next section, using the information available at present about the mechanism of relativistic heavy ion collisions.

4. DISCUSSION

The results described in the preceding section suggest striking differences in the production behavior of the light products such as $^{24}$Na produced by 0.9 A GeV and 1.8 A GeV, $^{40}$Ar projectiles, namely:
(a) the significant increase of $R_0$ with projectile energy for Cu-disks in contact (Table 1) and
(b) the decrease of $R_d$ with distance $d$ between the disks only at 1.8 A GeV (Fig. 7).

Both results could be understood in terms of a relatively abundant production at the higher beam energy of projectile fragments with unexpectedly high interaction cross-sections which "lose" this property ("decay") and revert to normal fragments after a flight path of the order of 20 cm. However, before drawing such a drastic and far-reaching conclusion, it is essential to investigate whether there exist other, possibly quite conventional, mechanisms capable of producing such a marked energy dependence.

We refer the reader again to Fig. 1c. Because of the strong energy dependence of $^{24}\text{Na}$ production (Fig. 6), $R_0$ is sensitive to the energy of the primary $^{40}\text{Ar}$ ion and to the multiplicities and the energy spectra of the secondaries, whereas the variation of $R_d$ with $d$ is also dependent on their angular distribution. Indeed the simplest interpretation for effect 2), above is that wide angle secondaries (illustrated by trajectory Ar-3 in Fig. 1c) miss the "detector" disk and its guard ring, with the proviso that their energy be sufficient (see Fig. 6) to ensure efficient $^{24}\text{Na}$ production. These two conditions can be fulfilled simultaneously only if these secondaries have very large transverse momenta.

In order to predict the $R_d$-value some would ideally need a detailed Monte-Carlo (MC) calculation of the extra-nuclear cascade induced by the secondaries in the two Cu-disks. For obvious reasons it is desirable that such a calculation be based as much as possible on experimental facts rather than on theoretical models. Unfortunately the MC procedure would require a wealth of experimental input (a complete set of fragmentation cross-sections in copper for all possible reaction products together with
their angular and momentum spectra) which is not available at present.

In the following we shall adopt a mixed approach which

- is based on experimental information about Ar interactions in nuclear emulsions, which simulate interactions in Cu rather well, and
- makes exaggerated assumptions regarding the subsequent fate of the secondaries, in order to derive conservative estimates of the effects under consideration.

4.1 The auxiliary emulsion experiment

A stack of Ilford G5 nuclear emulsions 10x20x0.06 cm each, exposed to the 1.8 A GeV argon beam of the Berkeley Bevalac, was scanned along the track for Ar interactions both at the maximum (beam) energy and at a residual range where the beam was slowed down to about 0.9 A GeV. The secondary particles were classified as usual into "heavy" ($N_h$) tracks (mainly slow, target-related fragments), fast ($E > 0.7$) Z=1 "shower-tracks" ($n_s$) and $Z > 1$ projectile fragments (PF). "Shower-tracks" can be pions ($E_p > 56$ MeV) or protons ($E_p > 375$ MeV). While among the PF's, most alpha particles were identified by their specific ionization, the charges of heavier fragments ($Z > 2$) were visually estimated and the fragments classified into "light" ($< Z < 4$), "medium" ($4 < Z < 8$), "heavy" ($8 < Z < 16$), and "beam-like" (Z=18) PF's. Emission angles of all fast secondaries (i.e. shower tracks and PF) were measured with the high accuracy characteristic of the nuclear emulsion detector (i.e. to better than 0.1 degrees). Fifty-six events were recorded at a projectile energy of 0.9 A GeV and ninety-five at 1.8 A GeV. These events gave rise to a total of 542 and 1178 fast secondaries, respectively. The data from these events were used as input for the subsequent MC calculation.

Table 3 shows the main characteristics of the events observed at both energies for the well-identified secondaries. The last two rows
of Table 3 present a few event characteristics based on the roughly estimated charges of PF's with \( Z > 2 \). The first two columns in Table 3 refer to the total sample of events. It is well known that, on the average, nuclear emulsion behaves with respect to nuclear reactions like copper (e.g. the proton mean free path for nuclear collisions is 134 g/cm², as compared to 135 g/cm² in Cu). However the emulsion target nuclei are a mixture of light (C,N,O) and heavy (Ag,Br) targets. To define the possible range of parameters used in the MC, the next four columns in Table 3 list the characteristics of events with \( N_h \leq 8 \) (corresponding mainly to collisions in CNO) and those with \( N_h \geq 9 \) (collisions with Ag and Br only, both of which are heavier targets than Cu). The mean number of target related fragments \( \langle N_h \rangle \) is approximately the same at the two energies as would be expected from limiting fragmentation. The mean number of shower tracks \( \langle n_s \rangle \) increases with beam energy by \( \approx 50\% \) as expected from increased pion production. The mean alpha multiplicity decreases slightly (if at all) from 0.9 to 1.8 A GeV indicating perhaps a more thorough break-up of \(^{40}\text{Ar}\)-projectiles at the higher energy. The angular distribution of the shower tracks (as reflected in the mean emission angle) appears to be roughly the same at both beam energies whereas the alphas are more bundled at the higher energy (the same is true for the heavier PF's). This can be understood qualitatively by considering the line labeled \( \langle n_s \rangle(0<\psi_C) \) in Table 3, which shows the mean multiplicity of shower tracks in the "fragmentation cone" of the projectile, taken to be \( \psi_C = 10^\circ \) at 0.9 A GeV and \( \psi_C = 6^\circ \) at 1.8 A GeV. The numerical value for the critical angle \( \psi_C \) has been derived empirically as described in Sec. 4.2.1. These numbers suggest that the angular distribution of shower tracks at 1.8 A GeV, which would be expected to be narrower than at 0.9 A GeV because of the higher Lorentz factors involved, is widened by the particles lying
beyond \( \Theta_C \), i.e., by the pions and the mid-rapidity protons. This is another hint of a somewhat more violent reaction mechanism at 1.8 A GeV.

4.2 Estimation of effects in \( R_0 \)

As indicated schematically in Fig. 1c, \( R_0 \) is determined by five factors. Each of these factors contributes an amount \( Q \) to the total \( ^{24}\text{Na} \) activity observed in any given disk. \( Q \) is expressed in terms of cross-sections, e.g., for one interaction \( Q_{\text{Ar}} = \sigma_{\text{Ar}} \) where \( \sigma_{\text{Ar}} \) is the primary production cross-section for the \( \text{Cu}(\text{Ar},X)^{24}\text{Na} \) reaction. The different contributions are:

(a) beam interacting in disk 1 (Ar-1), contributing \( Q_{\text{Ar}} \);
(b) beam surviving through disk 1 and interacting in disk 2 (Ar-2), contributing also \( Q_{\text{Ar}} \) (\( Q_{\text{Ar}} \) has been corrected for attenuation);
(c) secondaries from interactions in disk 1 (i.e., a, above) interacting in the same disk together with their progeny in the extra-nuclear cascade and contributing \( Q_{11} \);
(d) secondaries from interaction in disk 2 and interacting in the same disk, contributing \( Q_{22} \);
(e) secondaries emitted from interactions in disk 1 and interacting in disk 2 (Ar-3), contributing \( Q_{12} \).

The attenuation factor for \( Q_{\text{Ar}} \) in disk 2 is \( e^{-x/\lambda_{\text{Ar}}} \) where \( \lambda_{\text{Ar}} \) is the collision mean free path (mfp) of the beam nuclei in Cu and \( x=1 \) cm is the disk thickness. The same factor turns \( Q_{11} \) into \( Q_{22} \).

Then the activities \( Q_1 \) and \( Q_2 \) of \( ^{24}\text{Na} \) produced in the two disks are given by:

\[
Q_1 \sim Q_{\text{Ar}} + Q_{11},
\]

\[
Q_2 \sim Q_{1} e^{-x/\lambda_{\text{Ar}}} + Q_{12},
\]
Therefrom:
\[ R_0 \equiv \frac{Q_2}{Q_1} = e^{-x/\lambda_{Ar}} + \frac{Q_{12}}{Q_{Ar} + Q_{11}}, \]  
(6)

\[ x = 1 \text{ cm is the disk thickness.} \]

In the above equations the energy loss of both primaries and secondaries in the two copper disks have been deliberately ignored. Taking the energy loss into account could only lower (albeit slightly) the values of \( R_0 \).

Eq. (6) can be qualitatively interpreted as follows: in zero-order approximation \( Q_{12} \sim 2Q_{11} \) because secondaries of type (e) traverse on the average twice the thickness of Cu traversed by those of type (c) or (d); in this approximation \( R_0 \) is determined essentially by the ratio \( Q_{11}/Q_{Ar} \). Any increase in this ratio will increase \( R_0 \) up to a limiting value of \( 2 + e^{-x/\lambda_{Ar}} \) when \( Q_{11} \to \infty \). If \( Q_{11} \ll Q_{Ar} \), then \( R_0 \) should drop below unity.

Note that all mfp's (primary as well as secondary) are considerably larger than \( x \). Hence, to first order, one can approximate the (really exponential) distribution of beam interaction points in disk 1 by a uniform distribution. This places the average beam interaction at a depth \( \approx x/2 \) (a more rigorous calculation yields \( 0.48 - 0.49)x \). Thus secondaries traverse in disk 2 roughly twice the thickness they had available in disk 1. This leads to

\[ Q_{12} \approx 2Q_{11}, \]  
(7)

and hence

\[ R_0 \approx e^{-x/\lambda_{Ar}} \frac{2}{1 + \gamma}, \]  
(8)

where

\[ \gamma = \frac{Q_{Ar}}{Q_{11}}. \]  
(9)

In order to be independent of both the absolute value of \( \lambda_{Ar} \), (which affects only the absolute value of \( R_0 \)) and absolute values of beam projectile fragment cross-sections, we concentrate on the difference \( \Delta R_0 \) between
the $R_0$-values observed at 1.8 and 0.9 A GeV. (It is well known from counter experiments that both $\lambda_{AR}$ and projectile fragment cross-sections do not depend on energy in this interval.)

$$
\Delta R_0 = 2 \left( \frac{1}{1 + Y_{1.8}} - \frac{1}{1 + Y_{0.9}} \right)
$$

(10)

Experimentally (see the first row of Table 1):

$$
\Delta R_{\text{experimental}} = 0.334 \pm 0.014.
$$

(11)

The rest of this sub-section is devoted to a somewhat simplified calculation of $Q_{11}$ aimed at checking whether the predicted value of $\Delta R_0$ can be made to match the above-mentioned experimental value. A more sophisticated Monte-Carlo (MC) calculation, which leaves the main conclusion unchanged, is briefly described in Appendix. At this stage it is important to point out that a thorough look at the definition of $Q_{11}$, and at the form of Eq. (10) allows one to avoid a seemingly obvious misinterpretation of the raw emulsion data. Indeed, we see in Table 3 that between beam energies of 0.9 A GeV and 1.8 A GeV, $\langle n_s \rangle$ increases by $\% 43\%$. This might lead to the erroneous conclusion that this increase is sufficient to justify the observed $\Delta R_0$. However:

- $Q_{11}$ includes the energy dependence of cross-sections, and
- Eqs. (8) and (10) are non-linear in $Q_{11}$, so that its effects are weakened in $\Delta R_0$ (provided the $Y$-ratios do not assume unreasonably large values.)

This means that a direct comparison between $\Delta R_0$ (and, incidentally, also $R_0$) and numbers of secondaries is not as straight-forward as it seems.

In order to sort out the different components among the secondaries we note that there are two angular regions obeying different physics, i.e. the "fragmentation cone" ($\theta \leq \theta_C$) and the "mid-rapidity"
region (θ > θ_c) (The population of fast secondaries at very low rapidities (say, θ > 75°) is at these energies practically extinct.)

The former contains as a rule (more precisely in ~70% of the cases) one heavy fragment (Z ≥ 3), alphas and (p, d, t), with n_p >> n_d, n_t.

The latter angular region is populated by "midrapidity" protons and neutrons (coming mainly from the projectile and scattered outside the projectile fragmentation cone by interactions in the target) and by charged pions (these mainly at 1.8 A GeV). The delimiting angle θ_c separating these two angular regions comes from straightforward relativistic kinematics.

Consider particles of mass m evaporated from some system moving with velocity $\vec{v}_{\text{CMS}}$ (Lorentz factor $\gamma_{\text{CMS}}$) with respect to the laboratory frame. Denote laboratory values for total and kinetic energies, momenta, velocities and angles by $E$, $E_k$, $p$, $\beta$ and $\theta$, and corresponding quantities in the emitting (i.e. effective c.m.s.) system by primed symbols. The transverse and longitudinal components of $p$ are denoted by $p_T$ and $p_X$. The laboratory emission angle $\theta$ is given by:

$$\tan \theta = \frac{p_T}{p_X} = \frac{\sin \theta'}{\gamma_{\text{CMS}}(\mu + \cos \theta')}$$ (12)

where

$$\mu = \frac{\beta_{\text{CMS}}}{\beta}.$$ (13)

4.2.1 The fragmentation cone

For heavy particles which are slow in the emitting system $\mu >> 1$:

$$\gamma_{\text{CMS}} \tan \theta \approx \frac{\beta \sin \theta}{\beta_{\text{CMS}}}$$ (14)
This is indeed our case where alphas and nucleons are evaporated with temperatures between 5-20 MeV in the rest system of a residue of the argon projectile moving with beam velocity. Since the angles are small, \( \tan \theta \approx \theta \);

\[
\gamma_{\text{cms}}^2 \langle \theta^2 \rangle \approx \frac{\langle \theta^2 \rangle \langle \sin^2 \theta \rangle}{\beta_{\text{cms}}^2} \quad (15)
\]

Now for isotropic emission in the projectile frame:

\[
\langle \sin^2 \theta \rangle = \frac{2}{3} \quad (16)
\]

If thermal equilibrium prevails, then for two different kinds of particles of masses \( m_1 \) and \( m_2 \) the two mean kinetic energies are equal and the ratio of the two r.m.s. angles is

\[
\frac{\theta_{\text{rms,1}}}{\theta_{\text{rms,2}}} = \sqrt{\frac{m_2}{m_1}} \quad (17)
\]

In other words, the rms proton angle is twice the rms alpha angle.

For \( \mu > 1 \) (which is always our case) a maximum (or limiting) angle \( \theta_c \) exists in the laboratory system, such that

\[
\gamma_{\text{cms}} \tan \theta_c = \frac{1}{\sqrt{\mu^2 - 1}} \quad (18)
\]

Again, for \( \mu >> 1 \),

\[
\frac{\theta_{c,p}}{\theta_{c,\alpha}} = \frac{\mu_\alpha}{\mu_p} \approx 2 \quad (19)
\]

(An exact MC calculation, taking into account the Boltzmann spectra of the evaporated particles, shows that without any approximations and for temperatures up to 40 MeV, Eq. (19) is good to better than 5%.)

Hence the receipe for determining \( \theta_c \) empirically:

- Forget about the value of the temperature;
- Search the emulsion data for the largest observed alpha emission angle;
Double that value and obtain \( \theta_c \) i.e. the limit of the fragmentation cone (since the protons are the lightest fragments emitted).

4.2.2 Angular region covered by "participating nucleons."

Sorting out protons and pions outside the fragmentation cone does matter because at the same angle protons and pions will tend to have different energies, hence different cross-sections for producing \( ^{24}\text{Na} \). However, all fast particles (\( \beta > 0.7 \)) of \( Z=1 \) look alike in the emulsion. A rough delimitation, (which tends to exaggerate \( \Delta R_0 \)) can be based on charge conservation. The total charge emitted into \( \theta_c \) is given by

\[
Z_c = Z_F + 2n_\alpha + np,c
\]

(20)

where \( Z_F \) is the charge of the heavy fragment and \( np,c \) is that subset of the \( n_s \) ("fast" \( Z=1, \beta > 0.7 \) secondaries) which are emitted with \( \theta \) below \( \theta_c \). Since \( Z_{\text{incident}} = 18 \) we expect

\[
\Delta Z = 18 - Z_c
\]

(21)

protons (plus an equal number of neutrons) to be present beyond \( \theta_c \). The optimum conditions for \( ^{24}\text{Na} \) production (thus exaggerating our effect) obtain if we progress outward from \( \theta_c \) and accept everything as protons until their sum has reached \( \Delta Z \) (In fact, some pions reach into smaller angles as well, but this would only lower \( ^{24}\text{Na} \) production).

From the observed alpha angles we find that at 0.9 A GeV \( \theta_c \approx 10^\circ \) and at 1.8 A GeV \( \theta_c \approx 6^\circ \). The ratio of these angles lies close to the inverse ratio of the \( \gamma_{\text{CMS}} \) values (1.959 and 2.919, respectively).

We see in Table 3 that, as one would expect, there is no significant increase with energy of the multiplicity in the fragmentation cone, where (in view of the heavy objects and high energies which entail large cross-sections most of the \( ^{24}\text{Na} \) production must be concentrated. All the increase in
\( n_s \) is located outside \( \theta_c \) where the energies and hence the cross-sections are small. Obviously most if not all of this increase is due to pion production.

### 4.2.3 Cross-sections

Now we have to assign cross-sections to the different components.

- We assume that all projectile fragments with \( A \geq 4 \) have energy independent cross-sections \( \sigma_F \), and interpolate between the (saturation) proton (ref [18]), carbon (ref [19]) and argon (ref [16]) values by a power law:

\[
\sigma_F(A) \sim 3.6A^{0.387} \tag{22}
\]

- For protons and pions we approximate the curve in Fig. 6 by:

\[
\sigma_p(E_k) = e^{-0.17+2.50u-1.22u^2} \tag{23}
\]

where

\[
u = \ln E_k \tag{24}
\]

if \( E_k \leq 3 \) GeV, and

\[
\sigma_p(E_k) = 2.88E_k^{0.0496} \tag{25}
\]

otherwise.

The \( E_k \) for the different components are obtained as follows:

(a) For the protons and neutrons in the fragmentation cone we note that the Lorentz transform reads:

\[
E = \gamma_{CMS}(E' + \beta_{CMS}p' \cos \theta') \tag{26}
\]

and since for any symmetric angular distribution in the rest frame \( \langle \cos \theta' \rangle = 0 \),

\[
\langle E \rangle = \gamma_{CMS} E' \tag{27}
\]

which to first order reduces to

\[
\langle E \rangle \sim m\gamma_{CMS} \tag{28}
\]
In other words, one should assign to these particles "beam velocity"; this lead to \( \sigma_p = 0.64 \) mb at 0.9 A GeV and to \( \sigma_p = 2.24 \) mb at 1.8 A GeV.

This is, however, an upper limit, because the \((\sigma_p, E_k)\) relationship is highly non-linear and
\[
< f(x) > \neq f(< x >);
\] (29)
the inequality is always true; in our case substantially so.

In order to see how much Eq. (29) may be in error, we have also done an MC calculation taking into account both the spreads in \( E' \) and in \( \cos \theta' \); it turns out that the average cross-sections are always lower than for "beam velocity". Thus, Eq. (28) is a conservative estimate.

(b) For particles outside \( \theta_c \) we use the streamer chamber results of ref. [20]. These tell us that in Ar+KCl "central" collisions at 1.8 A GeV the transverse momentum distributions are of Boltzmann shape with "temperatures" \( T \) of 56 MeV for pions and 120 MeV for protons.

The shape of the transverse momentum spectrum has a quite dramatic effect on the cross-section for producing \(^{24}\text{Na}\) at a given angle. As an example, we illustrate in Fig. 8 the situation at an emission angle of 30° (secondaries which miss the downstream guard ring.) The transverse momentum distribution, as well as that of kinetic energy, dies out long before the cross-section has reached its saturation value.

So we have to average \( \sigma_p(E_k(p_T,0)) \) over the Boltzmann distribution between the limits
\[
P_{T \text{min}} = P_{\text{min}} \sin \theta
\] (30)
with $p_{\text{min}}$ computed for a given mass from $\beta \geq 0.7$ (the criterion for classifying the particle as "relativistic") and $p_{T,\text{max}}$ resulting from

$$E_{\text{beam}} = \sqrt{\frac{p_{T,\text{max}}^2}{\sin^2 \theta} + m^2}$$

(energy conservation):

$$\langle \sigma \rangle = \frac{\sum_{p_{T,\text{min}}}^{p_{T,\text{max}}} f(p_T) \sigma_p(E_k(p_T, \theta)) dp_T}{\int_{p_{T,\text{min}}}^{p_{T,\text{max}}} f(p_T) dp_T}$$

(32)

where

$$f(p_T) = \frac{p_T}{\omega^2} e^{-\frac{p_T^2}{2\omega^2}}$$

(33)

$$\omega = \sqrt{m_T},$$

(34)

$$E_k(p_T, \theta) = \sqrt{\frac{p_T^2}{\sin^2 \theta} + m^2 - m}$$

(35)

and $\sigma_p$ is given by Eqs. (23) through (25).

The integration has to be carried out numerically; it leads to Table 4 which gives, in mb, the average cross-section for $^{24}\text{Na}$ production by pions and protons at a given angle $\theta$ (the center of the intervals used in tables 5a and 5b) and at the two beam energies. The energy dependence comes entirely from the upper integration limit, whereas the particle mass appears in both limits.

4.2.4 Computing $Q_{11}$

What remains to be done now is to add all these contributions, properly weighted, into $Q_{11}$. The weights are products of the mean multiplicities $v_j$ of secondaries of a given kind and of the probability that they will interact in the residual disk thickness ($\% x/2$).
Thus

$$Q_{11} = \sum_j v_j (1 - e^{2\lambda_j}) \sigma_j$$

which reduces to

$$Q_{11} \approx \sum_j v_j \frac{x}{2\lambda_j} \sigma_j$$

where \( j \) goes through all kinds of particles and angular intervals and \( \lambda_j \) is the interaction m.f.p. of secondaries of type \( j \).

Since for any given target material

$$\lambda_j \sigma_{T,j} = \text{const} = \lambda_p \sigma_{T,p}$$

where the \( \sigma_{T} \) are total cross-sections and the subscript \( p \) refers to protons (for which the m.f.p. in Cu is known to be 15 cm), we get

$$Q_{11} \approx \sum_j v_j \frac{x}{2\lambda_p} \sigma_{T,j} \sigma_j$$

For the total cross-sections we use the geometrical (Bradt-Peters) formula

$$\sigma_{T,j} \sim (A_j^3 + \frac{1}{3} A_{Cu}^3 - b)^2$$

from which we get the ratio of total cross-sections:

$$\frac{\sigma_{T,j}}{\sigma_{T,p}} = \frac{A_j^3 + 4 - b)^2}{(5 - b)^2}$$

where the correction factor \( b \) lies close to unity (overlap \( \sim 1 \) fermi).

(We have done detailed calculations with \( b \) varying from 0.5 to 2 and they have brought no qualitative change in the data).

In order to visualize how the different components might add up in to \( Q_{11} \) we give in Table 5 a schematic display of the structure of \( Q_{11} \). Angular intervals have been compacted wherever necessary, in order to ensure reasonable statistics per interval (thus mean populations should be reduced correspondingly to yield a "readable" angular distribution.) The \( \pi \) vs. \( (p,n) \) assignments
are only symbolical, but the final results (the column labeled "Effective $^{24}\text{Na}$") are computed from the detailed data. Actually the calculations have been performed separately for the five groups of events with either
- No heavy fragments ($Z > 2$) in the fragmentation cone ("Explosions"), or
- with one fragment of mean (estimated) charge of either 4, 8, 16 or 18 (beam-like secondary fragment).

Obviously, in each group, the angular distributions are different, and thus the demarcation angle between "pions" and "nucleons" lies at a different value. All "nucleon" contributions to $^{24}\text{Na}$ production were doubled, in order to take approximate account of neutrons. Table 6 shows how the final result for $\Delta R_0$ is reached.

Since the "average emulsion target" cannot be expected to be identical to the homogenous $^{64}\text{Cu}$ target used in our experiments, we show in Table 7 the results of calculations using as input certain subsets of the emulsion events. They provide "bracketing" values meant to convey an idea of the extreme range of $\Delta R_0$-values:
- "Light" corresponds to events with mainly (C, N, O) target, i.e. $N_h \leq 8$; "Heavy" means (Ag, Br), i.e. $N_h \geq 9$
- $Z_{\text{frag}} = 0$ symbolizes "explosions," i.e. relatively central collisions, (in which the projectile is obliterated); $Z_{\text{frag}} \geq 2$ are correspondingly more peripheral events.
- The main values were computed for $b=1$; values in parentheses correspond to the exaggerated assumption of $b=2$ (which would increase the weight of large total cross-sections.) The values in parentheses are obtained by pushing the uncertain parameter $b$ up to the extreme value of 2.

All more realistic variations of parameters yield lower values of $\Delta R_0$. 
Thus, the only really meaningful (lower right-hand corner) element of the table drops to 0.092 if \( b = 0 \) and a realistic evaporation temperature of 10 MeV is chosen. In summary, we see that the computed \( \Delta R_0 \) does not to match the experimental value [Eq.(11)] as long as one believes at all that the errors assigned to the \( R_0 \) values are correct.

In order to see what increase in the experimental errors would be necessary to reconcile the computed and experimental values we assume the (clearly exaggerated) value of \( b=2 \), and subtract the extreme \( \Delta R_0 \) value (in parenthesis, i.e. \( \Delta R_0=0.141 \)) from the experimental one:

\[
\Delta R_{\text{exper}} - \Delta R_{\text{computed}} = 0.195
\]

(42)

To reduce this difference to \(~3\) standard deviations one would need an error in \( \Delta R_{\text{exper}} \) of \(~0.064\), i.e. larger by a factor of almost five than the value obtained from experimental statistics.

4.2.5 Further cascading?

* How often does further cascading occur? Denote by \( \varepsilon \) the mean number of collisions of a particle (supposed "non-degradable") over some length \( x \):

\[
\varepsilon = \frac{x}{\lambda}.
\]

(43)

This mean includes obviously all traversals of \( x \), including those with no collisions at all. The mean number \( \varepsilon_1 \) if at least one collision has occurred is then

\[
\varepsilon_1 = \frac{\varepsilon}{1 - e^{-\varepsilon}};
\]

(44)
with \(0.5 \geq x \geq 1\) cm and \(5 \geq \lambda \leq 15\) cm. Thus \(\varepsilon_1\) is at worst 1.103, at best 1.017. Hence tertiary, etc. collisions could account for something like a 5% increase in \(Q_{11}\). This will then translate, as we know, non-linearly into \(\Delta R_0\).

*How much does further cascading contribute to the effect?* The only way that a somewhat larger contribution of cascading could come about is if in a higher order generation some of the heavy fragments blow up completely into their \(\%\) constituent nucleons which then have a chance to create further \(^{24}\)Na. The extreme example for such a behavior is the first (i.e. left-hand) column in Table 7, which derives from the total breakup of argon nuclei. This kind of effect would then have to be weighted by the small probability of tertiary interactions in our 1 cm thick disks. The appendix explains how this effect was taken into account.

4.3 Estimation of the "angular loss" out-side the guard rings

Now we address the decrease of \(R_d\) at 1.8 A GeV as shown in Fig 7. We remember that all \(PF(Z \geq 2)\) are moving forward in a narrow cone, limited by \(\theta_C = 6^\circ\). The dependence of \(^{24}\)Na production by \(Z=1\) secondaries on their emission angles (outside the narrow fragmentation cone) in a layer of 1 cm Cu is given (in mb) in Fig. 9. The experimental setup is shown in Fig. 1: We have a 1 cm Cu disk as a generator of secondaries and 20 cm downstream a 1 cm extended Cu plate. As input data we use the experimental angular distribution for \(Z=1\) particles, as obtained from nuclear emulsions. The curves in Fig. 9 were obtained by averaging the cross-sections at each angle over the (Boltzmann) transverse momentum spectrum, as described in Sec. 4.2.3, Eqs. (30) through (35).
Extra-nuclear cascading was then taken into account as described in the Appendix. Curve a refers to pions while curves b through d refer to protons with temperatures of 120, 200, and 800 MeV, respectively. It is seen that in going from $\theta_c$ to 20° (the polar angle covered by the guard ring at d=20 cm) the effective pion cross-section drops by a factor of $\approx 20$; for protons at "reasonable" temperatures (100-200 MeV) the drop in cross-section is still of one order of magnitude. Only quite exaggerated "equivalent temperatures" would be capable of conserving any sizable $^{24}$Na production at and beyond 20°. In Fig. 10 the dashed curve a) shows the integral probability $F(>r)$ for a secondary particle irrespective of its energy to arrive at, or beyond a radial distance $r$ from the beam axis in a Cu-disk located at 20 cm from the source (at a beam energy of 1.8 A GeV) for the angular distribution seen in the emulsion data. However, taking into account the weights included by the energy distribution of these particles, the full curve b then shows the integral $^{24}$Na production by protons assuming 30% for the "blow-up" probability $P_b$ and a proton temperature $T_p = 120$ MeV. Numerical values for results integrated over angles are given in Table 8. The dotted curves c and d are obtained by assuming exaggerated temperatures of 400 and 800 MeV respectively. The vertical dashed lines show the edges of the main Cu-disk ($r=4$ cm) and of the guard ring ($r = 7$ cm). The horizontal arrow shows the integral probability needed to reproduce the observed decrease in $R_d$ beyond the guard ring which is one of the main puzzles resulting from this activation experiment. From Table 8 we see that it is possible to understand the "angular loss" outside the central ($r = 4$ cm) Cu "disk" at $d = 20$ cm for the 0.9 A GeV beam; experimentally we observe $(6\pm3)\%$ of the total $^{24}$Na activity outside the central "disk" and $(2\pm3)\%$ even outside the guard
rings. Assuming a reasonable source temperature within the fireball of $T_\text{p} = 120$ MeV and a 50% contribution of pions among the shower tracks (i.e. $P_\text{p} = 0.5$) we can easily fit the experimentally observed values. However, at 1.8 A GeV it seems to be impossible to obtain such a fit, except by assuming an absurdly high source temperature within the fireball ($T = 800$ MeV), and only 10% of pions among the shower tracks. Even the relatively large amount of $^{24}\text{Na}$ found in the guard ring itself cannot be accounted for.

4.4 Anomalous cross-sections?

In order to see what kind of cross-sections would be required for secondaries to reproduce the $R_0$-value at 1.8 A GeV we added to the MC procedure the possibility that a fraction $\alpha$ of secondary projectile fragments ($Z \geq 3$) interact with a total cross-section $f$ times larger than the normal one. It turns out that in order to raise $R_0$ to $\sim 1.5$, it is sufficient to keep the product of $\alpha$ and $f$ close to

$$\alpha f \sim 2,$$  (45)

(e.g. 10% secondaries with 20 times the normal cross-section). Lower values of the product $\alpha f$ would be needed if some secondaries with $Z < 2$ also had anomalously high cross-sections.

Then, in order to explain the loss at $d = 20$ cm (which, incidentally, reduces the value of $R_{20}$ to roughly that seen at 0.9 A GeV) the anomalous fragments would have to revert to the ground state ("decay") during the time required to reach the "detector" disk in the $d=20$ cm configuration. This would require

$$d \sim (2-3)\tau_Y$$  (46)
where $\tau$ is the assumed proper lifetime and $\gamma$ the Lorentz factor (which at 1.8 A GeV beam energy is about 3). For $d=20$ cm this leads to values of $\tau$ of the order of $10^{-10}$ s.

It is interesting to note that the emulsion results of ref. [1] required a similar value for the product $\alpha f$. Low values of $\alpha$ (a few %) require very large $f$, hence "highly anomalous" cross-sections, whereas, say, $\alpha \sim 1$ (for PF's of $Z \geq 3$) required the anomalous fragments to decay with $\tau \sim 10^{-11}$ s [21].

This latter possibility was excluded by an experiment, [8] in which the possible decay of "anomalons" produced in the interaction of 1.7 A GeV $^{56}$Fe ions with copper was investigated by means of a comparison of projectile fragment production in "dense" and in "diluted" target (with "decay" paths of $\sim 2$ cm). This experiment was sensitive to anomalous fragments with $Z \geq 5$ and $\tau \sim 10^{-11}$ s, emitted at angles $< 1^\circ$ to the beam direction. No evidence of decay was observed and it was concluded that the data were consistent both with no anomaly and with anomalous fragments decaying with a lifetime longer than $5.10^{-11}$ s. These conclusions are not inconsistent with the results of the present study where "decay" paths larger by an order of magnitude were used.

The emulsion experiments [1] (as well as those performed with plastic track detectors [2,3]) were, by their very nature, unable to resolve the $\alpha \leftrightarrow f$ ambiguity. It should be recalled that our experiment is sensitive to all PF's ($Z \geq 1$ as well as neutrons), while all other experiments were sensitive to charged secondaries only, with most of them requiring $Z \geq 3$. (One might even conjecture that anomalous cross-sections may occur among all PF's; then Z=1 secondaries might be relatively more long-lived and this could possibly account also for the rather large amount of $^{24}$Na found in the guard rings at 1.8 A GeV.)
Obviously, no anomalously high cross-sections need be invoked in order to account for the decrease of $R_d$ at the higher beam energy if large transverse momentum secondaries are responsible for the loss at $d = 20$ cm. It is hard to imagine that reactions in Cu, integrated over all impact parameters, should be so drastically different from those induced by the same $^{40}$Ar beam in KCl when "central" collisions are selected through the absence of heavy projectile fragments [20].

This problem can and should be addressed experimentally by repeating the experiment with a 2π geometry. Then, high energy secondaries, capable of producing $^{24}$Na in sizable amounts and emitted at angles sufficiently large to miss the guard ring of the present experiment would also be detected. Such measurements are in progress. However the puzzling large value of $R_0$ at 1.8 A GeV (as well as the slight excess of $R_0$ over the MC prediction at 0.9 A GeV) still remain unexplained. Anomalously large total and/or partial cross-sections would easily take care of this effect, but more detailed calculations of the effects of presumed anomalously large cross-sections for unstable PF's are needed in order to understand all features of our data.

5. Conclusion

We have observed an unusual behavior of the energetic secondaries produced in the interaction of 1.8 A GeV $^{40}$Ar with copper but not in an identical experiment at a lower (0.9 A GeV) energy beam. Two effects were observed: a large value of $R_0$ for the production of light target residues and a decrease in $R_d$ with increasing distance between the disks. Our calculations, which were based on conventional nuclear physics and the available high energy data, fail to explain either effect. There are at least two possible explanations based on new phenomena.
First, it is possible that energetic secondaries are abundantly emitted at wide angles. However, such secondaries were not seen in appreciable numbers in a related experiment (20) and hence our calculations based on this experiment failed to explain our observations.

Second, some unstable secondaries possess unusually large interaction cross-sections and "decay" in flight. Experiments are in progress to examine the first of these possibilities.

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References


APPENDIX

The results presented in Sec. 4 were obtained with compacted angular intervals and other approximations described therein. In order to estimate the influence of these approximations we also performed a MC calculation, which used as input the multiplicities and angles of the secondaries observed in individual events recorded in the emulsion experiment. These angles, by themselves, would be insufficient (because of the energy dependence of the $^{24}$Na production cross section) to predict $R_0$ and hence, the MC-calculation proceeded with the following assumptions regarding the energy spectra of the secondaries:

1. All secondaries lying within the projectile fragmentation cone ($\theta < \theta_C$) were assigned beam velocity. This ignores any admixture of mid-rapidity particles to the fragmentation cone and exaggerates $^{24}$Na production, hence $Q_{11}$, and consequently $R_0$.

2. Since all fast secondaries with $\theta > \theta_C$ have $Z=1$ they were assumed to consist of $P_p\%$ "mid-rapidity protons" emitted from a fireball with a temperature $T_p$, and of $(100-P_p)\%$ of pions emitted with a temperature $T_n$.

3. The cross-section values for $^{24}$Na production are those of Sec. 4.2.3.

4. Subsequent cascading of the secondaries in the two Cu-disks was taken into account as follows:
   - $Z=1$ secondaries were assumed to lose a fraction $\xi(\% \ 1/2)$ of their kinetic energy in each subsequent collision.
   - Heavier fragments were treated with two extreme assumptions, namely that they either suffer quasi-elastic collisions with negligible energy loss or break up (in $P_b\%$ of the cases) into $A_Z \% 2Z$ nucleons with beam velocity, which suffer then the fate described in the preceding paragraph.
With these assumptions the effect of extra-nuclear cascading was then computed by assigning to each particle of energy $E$ an equivalent cross-section:

$$\sigma_{\text{equiv}} = \sum_{j=1}^{\infty} \sigma_{p,\pi} (E_0 e^{-x/\lambda}) (1-e^{-x/\lambda}) \sum_{k=0}^{j-1} \frac{(x/\lambda)^k}{k!}, \quad (A-1)$$

where $\lambda$ is the secondary's collision mfp. Eq. (A-1) follows simply from the Poisson probabilities of successive interactions.

The energy $E$ for pions and protons with $\theta > \theta_c$ was derived from the observed angle and the MC-generated (Boltzmann) transverse momenta at temperatures $T_\pi$ and $T_p$, respectively. It turned out that varying $\xi$ between 1/2 and 1 made little difference to the result because the total contribution of $Z=1$ secondaries to $Q_{11}$ and $Q_{12}$ is small.

5. For secondaries of type $e$ (as defined in Sec. 4.2) their attenuation in the residual thickness of disk 1 was also taken into account.

6. Whenever a fast $Z=1$ track was assumed to be a proton (i.e. for all shower tracks with $\theta < \theta_c$ and for $P_{p/2}$ of the rest) its contribution to $Q_{11}$ and $Q_{12}$ was doubled in order to account for fast neutrons expected to be emitted with similar angular and energy distributions.

A few results of predictions for $R_0$ at the two beam energies are shown in Table 9 for different extreme assumptions about the characteristics of the cascade process.

The pion "temperature" $T_\pi$ which enters the calculation via the mean transverse momentum ($<p_T> = 2T_\pi$ for an exponential $P_T$ spectrum) was held at 58 MeV while proton temperatures $T_p$ of 120, 200 and 400 MeV were tried out in order to see the influence of exaggerated assumptions. As can be seen two main results emerge almost independently of the assumptions about the parameters of the calculation:
The $R_0$ value at 0.9 A GeV beam energy is close to the observed one, albeit slightly smaller (to reach agreement with the value of $1.16 \pm 0.01$ an absurdly high proton temperature would be needed).

The $R_0$ values at the higher beam energy (1.8 A GeV) are only slightly higher than at 0.9 A GeV and can in no way be raised to the value of $\sim 1.5$ observed experimentally (see Fig. 4).

In Table 4 the predictions were based on all events observed in the emulsion experiment; in other words they refer to the average target nucleus (thus including collisions with light, i.e. C, N or O nuclei). If only events occurring in (non-peripheral) collisions with (Ag, Br) (i.e. events with $N_h > 8$) are considered (this could be considered as an upper limit for a pure Cu target) the $R_0$ value at 1.8 A GeV beam energy can be raised (with "normal" temperatures) at most to 1.25.

Acknowledgements

This work was supported by the Director, Office of Energy Research, Division of Nuclear Physics of the Office of High Energy and Nuclear Physics of the U. S. Department of Energy under Contract No. DE-AC03-76SF00098, DE-AM06-76RL02227, DE-AC02-76ER01515, and the Swedish Natural Sciences Research Council.
Table 1
Rd-values and derived quantities observed at the two beam energies

<table>
<thead>
<tr>
<th></th>
<th>0.9 A GeV 40Ar</th>
<th>1.8 A GeV 40Ar</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_0^a$</td>
<td>1.167 ± 0.011</td>
<td>1.501 ± 0.008</td>
</tr>
<tr>
<td>$R_{10}$</td>
<td>1.118 ± 0.020</td>
<td>1.373 ± 0.034</td>
</tr>
<tr>
<td>$R_2^a$</td>
<td>1.102 ± 0.026</td>
<td>1.251 ± 0.020</td>
</tr>
<tr>
<td>$R_0/R_{10}$</td>
<td>1.044 ± 0.021</td>
<td>1.093 ± 0.028</td>
</tr>
<tr>
<td>$R_0/(R_{20} + R_{ring,20})$</td>
<td>1.019 ± 0.025</td>
<td>1.120 ± 0.019</td>
</tr>
<tr>
<td>$R_0 - (R_{20} + R_{ring,20})$</td>
<td>0.022 ± 0.028</td>
<td>0.161 ± 0.022</td>
</tr>
</tbody>
</table>

a) The experiments determining $R_0$ were performed independently in triplicate, those for $R_{20}$ in duplicate.
Table 2
Relative yields of different nuclides in the upstream and downstream guard rings at the two beam energies (for details see text)

<table>
<thead>
<tr>
<th>Product</th>
<th>( R_{\text{ring,}0} )</th>
<th>( R_{\text{ring,}20} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>7Be</td>
<td>&lt;0.05</td>
<td>0.046 ± 0.034</td>
</tr>
<tr>
<td>22Na</td>
<td>&lt;0.07</td>
<td>&lt;0.07</td>
</tr>
<tr>
<td>24Na</td>
<td>&lt;0.01</td>
<td>0.039 ± 0.002</td>
</tr>
<tr>
<td>28Mg</td>
<td>&lt;0.01</td>
<td>0.048 ± 0.005</td>
</tr>
<tr>
<td>44mSc</td>
<td>0.009 ± 0.002</td>
<td>0.124 ± 0.006</td>
</tr>
<tr>
<td>46Sc</td>
<td>&lt;0.02</td>
<td>0.140 ± 0.009</td>
</tr>
<tr>
<td>54Mn</td>
<td>0.029 ± 0.003</td>
<td>0.167 ± 0.007</td>
</tr>
<tr>
<td>56Co</td>
<td>0.027 ± 0.005</td>
<td>0.166 ± 0.008</td>
</tr>
<tr>
<td>58Co</td>
<td>0.054 ± 0.003</td>
<td>0.163 ± 0.006</td>
</tr>
</tbody>
</table>

b) 1.8 A GeV 40Ar:

<table>
<thead>
<tr>
<th>Product</th>
<th>( R_{\text{ring,}0} )</th>
<th>( R_{\text{ring,}20} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>7Be</td>
<td>&lt;0.02</td>
<td>0.072 ± 0.013</td>
</tr>
<tr>
<td>22Na</td>
<td>&lt;0.03</td>
<td>&lt;0.08</td>
</tr>
<tr>
<td>24Na</td>
<td>&lt;0.01</td>
<td>0.071 ± 0.005</td>
</tr>
<tr>
<td>28Mg</td>
<td>&lt;0.01</td>
<td>0.060 ± 0.010</td>
</tr>
<tr>
<td>44mSc</td>
<td>0.012 ± 0.001</td>
<td>0.134 ± 0.008</td>
</tr>
<tr>
<td>46Sc</td>
<td>0.015 ± 0.003</td>
<td>0.159 ± 0.007</td>
</tr>
<tr>
<td>54Mn</td>
<td>0.041 ± 0.002</td>
<td>0.174 ± 0.007</td>
</tr>
<tr>
<td>56Co</td>
<td>0.054 ± 0.003</td>
<td>0.169 ± 0.007</td>
</tr>
<tr>
<td>58Co</td>
<td>0.065 ± 0.003</td>
<td>0.168 ± 0.006</td>
</tr>
</tbody>
</table>
Table 3

Mean characteristics of events induced by 0.9 A GeV and 1.8 A GeV $^{40}$Ar projectiles in nuclear emulsion

<table>
<thead>
<tr>
<th></th>
<th>All events</th>
<th>$N_h \leq 8$</th>
<th>$N_h \geq 9$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.9 A GeV</td>
<td>1.8 A GeV</td>
<td>0.9 A GeV</td>
</tr>
<tr>
<td>$&lt;N_h&gt;$</td>
<td>10.8±1.5</td>
<td>9.8±1.5</td>
<td>3.8±0.5</td>
</tr>
<tr>
<td>$&lt;n_s&gt;$</td>
<td>7.3±0.7</td>
<td>10.4±1.0</td>
<td>4.6±0.7</td>
</tr>
<tr>
<td>$&lt;n_a&gt;$</td>
<td>1.6±0.2</td>
<td>1.2±0.2</td>
<td>1.8±0.3</td>
</tr>
<tr>
<td>$&lt;e_5&gt;$</td>
<td>16.8±0.8</td>
<td>16.7±0.5</td>
<td>12.5±1.0</td>
</tr>
<tr>
<td>$&lt;\theta_a&gt;$</td>
<td>2.5±0.2</td>
<td>1.9±0.1</td>
<td>2.2±0.2</td>
</tr>
<tr>
<td>$&lt;n_s&gt;\theta_\leq \theta_c$</td>
<td>3.2±0.3</td>
<td>3.3±0.2</td>
<td>2.6±0.4</td>
</tr>
<tr>
<td>$&lt;Z_{F,Z}&gt;^2$</td>
<td>13</td>
<td>12</td>
<td>14</td>
</tr>
<tr>
<td>$\Sigma Z_{F,\theta &lt; \theta_c}$</td>
<td>19</td>
<td>16</td>
<td>21</td>
</tr>
</tbody>
</table>
Table 4

Average cross-sections (in mb) for the production of $^{24}\text{Na}$ by fast singly-charged secondaries emitted at different angles (see text, Sec. 4.2.3).

<table>
<thead>
<tr>
<th>$6^\circ$</th>
<th>Pions</th>
<th>Protons</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.9 A GeV</td>
<td>1.8 A GeV</td>
</tr>
<tr>
<td>2.5</td>
<td>0.220</td>
<td>1.161</td>
</tr>
<tr>
<td>8.0</td>
<td>0.160</td>
<td>0.454</td>
</tr>
<tr>
<td>13.0</td>
<td>0.081</td>
<td>0.112</td>
</tr>
<tr>
<td>18.0</td>
<td>0.030</td>
<td>0.031</td>
</tr>
<tr>
<td>23.0</td>
<td>0.010</td>
<td>0.010</td>
</tr>
<tr>
<td>28.0</td>
<td>0.004</td>
<td>0.004</td>
</tr>
<tr>
<td>35.5</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>50.5</td>
<td>$\sim 0$</td>
<td>$\sim 0$</td>
</tr>
<tr>
<td>73.0</td>
<td>$\sim 0$</td>
<td>$\sim 0$</td>
</tr>
</tbody>
</table>
Table 5:

Schematic presentation of the factors contributing to Q_{11} (last column), using as input the "average emulsion event" a) at 0.9 A GeV, and b) at 1.8 A GeV.

(a)

<table>
<thead>
<tr>
<th>Ebeam=0.9 A GeV : 56 Events</th>
</tr>
</thead>
<tbody>
<tr>
<td>Secondary</td>
</tr>
<tr>
<td>Heavy Fragments</td>
</tr>
<tr>
<td>( Z &gt; 3 )</td>
</tr>
</tbody>
</table>

Z=1 with \( \theta^0 \)

| From | To | 0 | 5 | 97 | 1.73 | 0.500 | 0.640 | p,n | 124.4 |
| 6 | 10 | 82 | 1.46 | 0.505 | 0.640 | p,n | 104.4 |
| 11 | 15 | 64 | 1.14 | 0.513 | 0.214 | p,n | 24.3 |
| 16 | 20 | 49 | 0.88 | 0.526 | 0.170 | p,n | 16.9 |
| 21 | 25 | 27 | 0.48 | 0.543 | 0.127 | p,n | 5.3 |
| 26 | 30 | 23 | 0.41 | 0.566 | 0.094 | p,n | 3.4 |
| 31 | 40 | 31 | 0.55 | 0.614 | 0.001 | π | 5.5 |
| 41 | 60 | 26 | 0.46 | 0.786 | α \( 0 \) | π | 3.2 |
| 61 | 85 | 11 | 0.20 | 1.710 | α \( 0 \) | π | 2.1 |

Total Z=1 | 7.31 | 289.5 |

Grand total | 2109.4 |

\( Q_{11} \) per event
Table 5

(b)

<table>
<thead>
<tr>
<th>Secondary</th>
<th>Observed number</th>
<th>Mean number$^a$</th>
<th>Effective path</th>
<th>$\sigma(^{24}\text{Na})$ mb</th>
<th>Type</th>
<th>Effective $^{24}\text{Na}$(mb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heavy Fragments $Z &gt; 3$</td>
<td>73</td>
<td>0.77</td>
<td>0.500</td>
<td>12.3</td>
<td>Z$^a$12</td>
<td>2019.6</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>117</td>
<td>1.23</td>
<td>0.500</td>
<td>6.16</td>
<td>$\alpha$</td>
<td>945.1</td>
</tr>
</tbody>
</table>

$Z=1$ with $\phi^\circ$

<table>
<thead>
<tr>
<th>From</th>
<th>To</th>
<th>Observed number</th>
<th>Mean number$^a$</th>
<th>Effective path</th>
<th>$\sigma(^{24}\text{Na})$ mb</th>
<th>Type</th>
<th>Effective $^{24}\text{Na}$(mb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5</td>
<td>250</td>
<td>2.63</td>
<td>0.500</td>
<td>2.240</td>
<td>p,n</td>
<td>1118.0</td>
</tr>
<tr>
<td>6</td>
<td>10</td>
<td>196</td>
<td>2.06</td>
<td>0.505</td>
<td>0.918</td>
<td>p,n</td>
<td>357.4</td>
</tr>
<tr>
<td>11</td>
<td>15</td>
<td>132</td>
<td>1.39</td>
<td>0.513</td>
<td>0.529</td>
<td>p,n</td>
<td>133.8</td>
</tr>
<tr>
<td>16</td>
<td>20</td>
<td>114</td>
<td>1.20</td>
<td>0.526</td>
<td>0.031</td>
<td>$\pi$</td>
<td>59.9</td>
</tr>
<tr>
<td>21</td>
<td>25</td>
<td>69</td>
<td>0.73</td>
<td>0.543</td>
<td>0.010</td>
<td>$\pi$</td>
<td>13.2</td>
</tr>
<tr>
<td>26</td>
<td>30</td>
<td>50</td>
<td>0.53</td>
<td>0.556</td>
<td>0.004</td>
<td>$\pi$</td>
<td>8.8</td>
</tr>
<tr>
<td>31</td>
<td>40</td>
<td>82</td>
<td>0.86</td>
<td>0.614</td>
<td>0.001</td>
<td>$\pi$</td>
<td>$\sim0$</td>
</tr>
<tr>
<td>41</td>
<td>60</td>
<td>69</td>
<td>0.73</td>
<td>0.786</td>
<td>$\sim0$</td>
<td>$\pi$</td>
<td>$\sim0$</td>
</tr>
<tr>
<td>61</td>
<td>85</td>
<td>24</td>
<td>0.25</td>
<td>1.710</td>
<td>$\sim0$</td>
<td>$\pi$</td>
<td>$\sim0$</td>
</tr>
</tbody>
</table>

Total $Z=1$ | 10.38 | 1691.1 |

Grand total Q11 | $4655.8$ |

$^a$ per event
Table 6
Factors contributing to the computed value of $\Delta R_0$, normalized "per event"

<table>
<thead>
<tr>
<th></th>
<th>0.9 A Gev</th>
<th>1.8 A GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_{Ar}(mb)$</td>
<td>77.3</td>
<td>77.3</td>
</tr>
<tr>
<td>$Q_{11}(mb)$</td>
<td>37.7</td>
<td>49.0</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>2.05</td>
<td>1.58</td>
</tr>
<tr>
<td>$2/(1 + \gamma)$</td>
<td>0.655</td>
<td>0.776</td>
</tr>
<tr>
<td>$\Delta R_0$</td>
<td></td>
<td>0.121</td>
</tr>
</tbody>
</table>
Table 7

Computed values of \( \Delta R_0 \) for different subsets of emulsion events (described in the text), assuming \( b = 1 \) for the overlap parameter. Values in parentheses refer to an exaggerated value of \( b = 2 \).

<table>
<thead>
<tr>
<th>Target</th>
<th>( Z_{\text{frag}} = 0 )</th>
<th>( Z_{\text{frag}} &gt; 2 )</th>
<th>All ( Z_{\text{frag}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light</td>
<td>0.111 (0.127)</td>
<td>0.075 (0.089)</td>
<td>0.066 (0.077)</td>
</tr>
<tr>
<td>Heavy</td>
<td>0.236 (0.277)</td>
<td>0.123 (0.144)</td>
<td>0.179 (0.209)</td>
</tr>
<tr>
<td>Emulsion</td>
<td>0.178 (0.206)</td>
<td>0.084 (0.099)</td>
<td>0.121 (0.141)</td>
</tr>
</tbody>
</table>
Table 8
Fractional loss (in%) of $^{24}\text{Na}$ activity computed by the MC procedure for various values of the input parameters a) outside the disk b) outside the guard ring

<table>
<thead>
<tr>
<th>Pp</th>
<th>$T_p$(MeV)</th>
<th>0.9 A GeV</th>
<th>1.8 A GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>a</td>
<td>b</td>
</tr>
<tr>
<td>0.5</td>
<td>120</td>
<td>10.2</td>
<td>3.3</td>
</tr>
<tr>
<td>0.5</td>
<td>200</td>
<td>14.5</td>
<td>5.1</td>
</tr>
<tr>
<td>0.5</td>
<td>400</td>
<td>20.9</td>
<td>8.7</td>
</tr>
<tr>
<td>0.9</td>
<td>400</td>
<td>20.9</td>
<td>10.8</td>
</tr>
<tr>
<td>0.9</td>
<td>800</td>
<td>34.1</td>
<td>17.1</td>
</tr>
</tbody>
</table>
Table 9
Values of $R_0$ for $^{24}$Na computed by the MC procedure for various values of the input parameters

<table>
<thead>
<tr>
<th>$T_p$ (MeV)</th>
<th>$P_b$</th>
<th>$P_p$</th>
<th>$R_0$ (0.9 A GeV)</th>
<th>$R_0$ (1.8 A GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>120</td>
<td>0.05</td>
<td>0.50</td>
<td>1.09</td>
<td>1.10</td>
</tr>
<tr>
<td>120</td>
<td>0.30</td>
<td>0.50</td>
<td>1.10</td>
<td>1.12</td>
</tr>
<tr>
<td>120</td>
<td>0.75</td>
<td>0.50</td>
<td>1.12</td>
<td>1.13</td>
</tr>
<tr>
<td>120</td>
<td>0.30</td>
<td>0.10</td>
<td>1.08</td>
<td>1.09</td>
</tr>
<tr>
<td>120</td>
<td>0.30</td>
<td>0.90</td>
<td>1.13</td>
<td>1.15</td>
</tr>
<tr>
<td>200</td>
<td>0.30</td>
<td>0.50</td>
<td>1.12</td>
<td>1.18</td>
</tr>
<tr>
<td>400</td>
<td>0.30</td>
<td>0.50</td>
<td>1.14</td>
<td>1.24</td>
</tr>
</tbody>
</table>
Figure Captions

Fig. 1: (a) Schematic representation of the target setup using Cu disks and their surrounding guard rings. (b) Autoradiographic picture of a Cu disk after the irradiation. (c) Schematic representation of different reaction paths in the two Cu disks; details are given in the text.

Fig. 2: Dependence of \( R_d \) upon product mass number for the interaction of 0.9 A GeV \(^{40}\)Ar with Cu. Values for two disk separations, \( d=0 \text{ cm} \) and \( d=20 \text{ cm} \), are shown.

Fig. 3: Dependence of \( R_d \) upon product mass number for the interaction of 1.8 A GeV \(^{40}\)Ar with Cu.

Fig. 4: Dependence of \( \rho_0 = R_0 \times \frac{(1.8 \text{ A GeV})}{(0.9 \text{ A GeV})} \) upon product mass number.

Fig. 5: Dependence of \( \rho_{20} = R_{20} \times \frac{(1.8 \text{ A GeV})}{(0.9 \text{ A GeV})} \) upon product mass number.

Fig. 6: Excitation function for the production of \(^{24}\)Na from Cu using high energy projectiles [16, 18].

Fig. 7: Activity ratios \( R_d \) for \(^{24}\)Na produced in Cu disks with relativistic Ar ions (where \( d \) is the distance between a pair of Cu disks.)

Fig. 8: Comparison between the transverse momentum distribution \( f \) (open circles), the kinetic energy distribution \( f_1 \) (full circles), and the \(^{24}\)Na production cross-section in mb (open squares) for protons emitted at 30° to the beam. A Boltzmann spectrum with \( T=120 \text{ MeV} \) was assumed for \( p_T \).

Fig. 9: Dependence of effective \(^{24}\)Na production (in mb) from \( Z=1 \) particles on their emission angle in a 1 cm layer of Cu. Further details are given in the text. (a) refers to pions of \( T=80 \text{ MeV} \), (b) to (d) refer to protons with \( T=120, 400 \) and 800 MeV, respectively.

Fig. 10: Integral probability \( (F>r) \) for the production of \(^{24}\)Na by secondary particles beyond a radial distance \( r \) from the beam axis in a Cu-block located at 20 cm from the source. The experimental integral angular distribution for \( Z=1 \) particles observed at 1.8 A Gev \(^{40}\)Ar in nuclear emulsions is shown (a) and was used for calculations, as described in the text. (b), (c), and (d) refer to protons emitted from a source with temperatures of \( T=120, 400 \) and 800 MeV, respectively.
\[ d = 0 \text{ cm} \]

\[ d = 20 \text{ cm (or 10 cm)} \]

Fig. 1
Fig. 2
Fig. 3
Fig. 4
Fig. 5
Fig. 6

\[ \sigma (^{24}\text{Na}) \text{ from Cu} \]

- protons (pions)
- \(^{12}\text{C}\)
- \(^{40}\text{Ar}\)

E (GeV)\text{total}
Fig. 7
Fig. 8
Fig. 9

$\sigma$ (mb)

$\theta^0$

-54-
Fig. 10