Worked Examples and Tutored Problem Solving: Redundant or Synergistic Forms of Support?

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Abstract
The current research investigates a combination of two instructional approaches, tutored problem solving and worked-examples. Tutored problem solving with automated tutors has proven to be an effective instructional method. Worked-out examples have been shown to be an effective complement to untutored problem solving, but it is largely unknown whether they are an effective complement to tutored problem solving. Further, while computer-based learning environments offer the possibility of adaptively transitioning from examples to problems while tailoring to an individual learner, the effectiveness of such machine-adapted example fading is largely unstudied. To address these research questions, one lab and one classroom experiment were conducted. Both studies compared a standard Cognitive Tutor with two example-enhanced Cognitive Tutors, in which the fading of worked-out examples occurred either fixed or adaptively. Results indicate that the adaptive fading of worked-out examples leads to higher transfer performance on delayed post-tests than the other two methods.

Keywords: Cognitive Tutor, worked-out examples, adaptive fading

Introduction
Learning and cognitive skill acquisition can be supported effectively in a number of different ways. One very successful approach is the use of “tutored problem solving” by intelligent tutoring systems (Anderson, Corbett, Koedinger, & Pelletier, 1995; Beal, Walles, Arroyo, & Woolf, 2007; Koedinger & Aleven, 2007; Koedinger, Anderson, Hadley, & Mark, 1997; Mitrovic, 2003; Razzak et al., 2005; VanLehn et al., 2005). These systems provide individualized support for learning by doing (i.e., solving problems) by selecting appropriate problems to-be-solved, by providing feedback and problem solving hints, and by online assessment of the student’s learning progress. Cognitive Tutors are one particular form of intelligent tutoring systems, grounded in cognitive theory; they individualize instruction by selecting problems based on a model of the students’ knowledge state that is constantly being updated (Corbett & Anderson, 1995).

Although Cognitive Tutors have many advantages, they are not without limitations. As is the case with most tutoring systems, their main focus is on correct answers during problem solving, which may not be ideal for gaining a conceptual understanding of the domain principles in problem solving (cf. VanLehn et al., 2005).

One instructional idea to further improve the focus on principles in Cognitive Tutors, and thereby their effectiveness, is to reduce problem solving demands by providing worked-out solutions (e.g., Renkl & Atkinson, 2007) when the primary instructional goal is to gain understanding (cf. Sweller, van Merriënboer, & Paas, 1998). Thereby, more of the learners’ limited processing capacity (i.e., working memory capacity) can be devoted to understanding the domain principles and their application in problem solving, especially when worked-out examples are combined with self-explanation prompts (Roy & Chi, 2005). However, as learners progress through training, worked-out examples might not be as effective in later stages of the training, a phenomenon known as the “expertise reversal effect” (Kalyuga, Ayres, Chandler, & Sweller, 2003). Empirical results indicate that problem solving is more favorable in later stages of learning, whereas worked
examples are more favorable in earlier stages. The implications for instructional design are that (a) initially, worked-out steps should be presented together with self-explanation prompts, and (b) when the learner demonstrates understanding, the worked-out steps should gradually be ‘faded’ from worked-out examples (solution is presented to the learner) to problems (learner must find the solution) (Atkinson, Renkl, & Merrill, 2003; Renkl, Atkinson, & Große, 2004; Renkl, Atkinson, Maier, & Staley, 2002; for a detailed theoretical rationale see Renkl & Atkinson, 2007).

An important issue that has remained unaddressed until recently is whether “tutored problem solving” and worked-out examples are redundant or synergistic forms of support. On one hand, it might be that the guidance that Cognitive Tutors give to learners is so effective that embedding worked-out examples within a tutored problem solving setting would not improve learning. This kind of tutored problem solving represents a far tougher control condition than those that have been investigated in previous studies of the value of worked examples. On the other hand, it is conceivable that the two forms of instruction are synergistic: it is conceivable that early cognitive skill acquisition is better supported by examples than by tutored problem solving, because examples prevent potential pitfalls such as a performance orientation, combined with the use of shallow strategies or general heuristics instead of efforts to understand and apply domain principles in the course of problem solving (cf. also VanLehn et al., 2005).

This issue was addressed in two recent studies by Schwonke et al. (2007), which found that tutored problem solving combined with examples that are gradually faded has beneficial learning effects. In this approach, examples are added to tutored problem solving, and are faded gradually, according to a “fixed” fading scheme that is the same for all learners. Students self-explain the example steps, as well as problem steps, with feedback from the tutor, by identifying the geometry theorem that justifies the worked-out step (see Figure 1). The results indicated that tutored problem solving combined with example fading leads to better transfer than tutored problem solving alone. Furthermore, the combination was less time consuming.

As suggested by Schwonke et al. (2007) the fading of examples could be even more beneficial for learning if the rate at which the worked-out steps are faded would be adapted to the students’ individual learning progress. While studying and self-explaining worked-out solution steps prepares the learner to deal with subsequent problem solving demands in a principle-based way, a learner who has not yet gained a basic understanding of a principle and of the way in which it is applied to solve problems should not be exposed to the corresponding problem solving demands. Once the student shows a basic understanding of a principle and its application, s/he should go one step further and apply this knowledge to solve problem steps. An adaptive fading procedure will make it more likely that the student will be able to solve a faded step correctly. Such an adaptive fading method can take advantage of the fact that the Cognitive Tutor that we used in our research prompts students for menu-based self-explanations. Therefore, example steps can be faded adaptively based on the quality of students’ menu-based answers to self-explanation prompts, as a measure of their understanding of the underlying problem solving principles.

In order to investigate whether tutored problem solving and worked-out examples are synergistic when examples are adaptively faded, three experimental conditions were compared: 1) a problem solving condition that uses the standard Cognitive Tutor; 2) an example-enhanced Cognitive Tutor that fades worked-out steps in a fixed manner; and 3) an example-enhanced Cognitive Tutor that fades worked-out steps adaptively for each individual learner. The main hypothesis states that an adaptive fading procedure, combined with tutored problem solving, will lead to better learning and higher transfer than a pure tutored problem solving procedure and a fixed non-adaptive procedure for fading examples (also combined with tutored problem solving). Essentially, this hypothesis states that tutored problem solving and adaptively faded examples are synergistic forms of support.

We conducted two experiments, both comparing these three experimental conditions, a lab experiment (in Freiburg) and a classroom experiment setting (in Pittsburgh). Implementing and evaluating the same manipulations in both a lab and a classroom setting enables us to assess whether and how robust effects found in a lab setting transfer to a real-life environment. Such transfer cannot be taken for granted, given the many sources of variability in the classroom that are typically absent in the lab (e.g., distractions such as announcements over the intercom, students arriving late, off-task behavior, absenteeism, informal peer helping, not always in ineffective ways, etc.), and the fact that classroom studies often take place over longer periods of time. Thus, a more ecologically valid investigation of the experimental manipulations and a “clean” lab investigation complement each other, and possible effects will have stronger implications.

**Experiment 1: Freiburg Lab Study**

For this study 57 students (19 in 9th grade; 38 in 10th grade) were recruited from a German “Realschule” which is equivalent to an American high school. The participants (age $M = 15.63$, $SD = .84$) were randomly distributed across the three experimental conditions.

The experiment focused on a unit in the Geometry Cognitive Tutor that deals with the geometric properties of angles, covering four theorems: angle addition, separate complementary angles, vertical angles, and linear pair. Every aspect (interface, hints, and glossary) of the Cognitive Tutor was translated into German. In order to be able to implement a consistent fading procedure we created new Geometry problems covering the theorems of the selected unit. The problems were sequenced from simpler to more complex; with one-step problems presented first, followed by two-step problems, and eventually by three-step problems. In the Problem Solving condition all steps of all
problems were “pure problem solving,” meaning that the students needed to solve them. In the Fixed Fading condition, by contrast, as detailed in Table 1, students started out with fully worked-out examples, with example steps gradually being faded in subsequent problems, until in the last two problems, all steps were pure problem solving.

Table 1: The fading of worked-out steps in the “Fixed Fading” condition; in problems P1 to P11, steps involving theorems T1 to T4 were worked-out (W) initially, and were systematically faded later (S for solving).

<table>
<thead>
<tr>
<th>Problem solving</th>
<th>Examples</th>
</tr>
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<tbody>
<tr>
<td>T1 S T2 T3 T4 T1 T2 T3 T4</td>
<td></td>
</tr>
<tr>
<td>P1 S</td>
<td>W</td>
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<tr>
<td>P2 S</td>
<td>W</td>
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<tr>
<td>P3 S</td>
<td>W</td>
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<tr>
<td>P4 S</td>
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<td>P5 S S W W</td>
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<td>P6 S S S W W</td>
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<td>P9 S S S S S</td>
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<tr>
<td>P10 S S S S S</td>
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<td>P11 S S S S S</td>
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For the Adaptive Fading condition the presentation of worked-out steps was the same as the fixed fading condition up until the three-step problems (problems 7 to 11). Once students got to those problems any step could be presented as either pure problem solving or as worked-out, depending on the student’s performance explaining worked-out steps in earlier problems that involve the same geometry theorem (see Figure 1). As detailed below, in all conditions, students were required to provide menu-based explanations for all steps, whether presented as worked-out steps or pure problem solving. These explanations indicate which geometry theorem was used. In an earlier study (Aleven & Koedinger, 2002) involving the Geometry Tutor, these menu-based explanations were found to improve student learning. Thus, in the current study, they serve double duty.

Specifically, the fading decisions were based on the tutor’s estimates of each individual student’s ability to produce valid explanations on steps involving the relevant theorem. The tutor maintains these probability estimates (separately for each of the four theorems) using a Bayesian knowledge-tracing algorithm (Corbett & Anderson, 1995). The estimates are updated each time the student explains a step involving the giving geometry theorem; the direction of the update depends on whether the explanation was correct or not. The knowledge-tracing algorithm is a well-established method for student modeling in intelligent tutoring systems. In prior research, Cognitive Mastery Learning built on top of Bayesian Knowledge Tracing has been shown to significantly improve student learning (Corbett & Anderson, 1995). Further, the estimates of skill mastery based on the Bayesian knowledge tracing algorithm have been shown to accurately predict students’ post-test scores (Corbett & Anderson, 1995).

In the current project, in order to achieve effective fading of the worked-out steps, the estimates of an individual student’s mastery of each geometry theorem were compared against two thresholds, set at .7 and .5, respectively. The high threshold represents an estimate of the level of understanding at which a worked-out step is faded. However, even if a student attains this level of understanding, s/he may later fall below that level, due to errors on subsequent steps of that specific theorem. Once the estimate of skill mastery falls below the low threshold the Tutor will again present the student with a worked-out step for the given theorem, until s/he reaches the high threshold again. In this manner, the Adaptive Fading method adapts to each individual student’s evolving level of understanding.

An example of a worked-out step for the linear pair (“Lineares Paar”) theorem is shown in Figure 1. The value for the quantity sought in this step (of “Winkel 2”) is worked-out. The student still has to explain the step by indicating which theorem is used. To fill in this explanation (called “Grund”) the student can either type the name of the theorem, or select the theorem from the tutor’s online glossary of geometry knowledge. Figure 1 shows the “Glossar” hyperlink in the upper right corner which will open the glossary in which students can browse relevant theorems and definitions; each is described and illustrated with a simple example.

The experiment consisted of two lab sessions. Since the students were unfamiliar with the Cognitive Tutor they received paper instructions before using the Geometry Tutor during the first lab session. They then took a pre-test, administered by the tutoring software, though no tutoring was provided during this test. Next, the students completed the actual Cognitive Tutor training (11 problems plus 1 warm-up problem), a built-in untutored post-test, and a paper post-test. During the Cognitive Tutor training students
received correctness feedback from the tutoring software after each step they performed. Furthermore, they could request hints at any point in time. For each step, several hint levels were available, explaining which problem solving principle applies, and how. The final hint level stated the answer. The online pre- and post-test consisted of the same three tutor problems. During the online tests, tutoring was turned off, meaning that the students did not receive correctness feedback and could not request hints. These tests were created with the Cognitive Tutors Authoring Tools (CTAT, see Alevens, Sewall, McLaren, & Koedinger, 2006). The paper post-test consisted of three different tasks with the first task being word problems from different domains with different structures. In another task, participants had to decide whether a given problem was solvable and if so provide the principles. In a third task they had to generate real world examples for the to-be-learned principles and to illustrate that example in form of a drawing. In other words, the post-test contained both procedural and conceptual knowledge items.

During the second session, which occurred one week later, a delayed post-test on paper was administered which contained the same procedural and conceptual knowledge tasks as the immediate post-test. The students received 20 euro for their participation in the study.

Results

In line with the hypothesis a planned contrast comparing the adaptive fading condition \((M = .52, SD = .17)\) with the problem solving \((M = .41, SD = .19)\) + fixed fading \((M = .41, SD = .17)\) conditions revealed higher transfer performance for the adaptive fading condition on the regular post-test \((F(1, 54) = 5.05, p < .05, \eta^2 = .09)\). This effect was replicated on delayed post-test \((F(1, 54) = 4.42, p < .05, \eta^2 = .08)\) with adaptive fading: \(M = .49; SD = .18\); fixed fading: \(M = .38; SD = .13\); and problem solving: \(M = .38; SD = .20\). There were no differences in time spent on either of the post-tests \((Fs < 1)\).

Experiment 2: In Vivo Study

The study took place at a vocational school in the Pittsburgh area, where the Geometry Cognitive Tutor is used as part of the regular geometry instruction. The participants consisted of three 9th grade classes with 51 students led by one teacher. In order to assign the students to the conditions, the student list was sorted based on the students’ prior grade in the course. The first three students were then randomly assigned to one of the three conditions, followed by the second three students on the list, and so on.

Overall, the materials and procedure were very similar to the German lab study with a few differences. First, since the students were already familiar with the Cognitive Tutor, we did not provide instruction up front about how to use the tutor. Instead the teacher explained to the students what the differences were between the standard Cognitive Tutor and the two example-enhanced versions. Second, the Cognitive Tutor’s mastery learning mechanism was used during this study, in all three conditions. Thus, the tutor presented students with remedial problems for the theorems/skills they had not fully mastered yet, until all theorems/skills were mastered (according to the tutor’s estimate of the student’s mastery, described above). As a result, different students completed slightly different sets of problems. Third, since the school where the study took place uses the Geometry Cognitive Tutor as part of their regular geometry instruction, the study covered more material and had a longer duration than the first study.

The study comprised all five sections in the tutor curriculum that deal with the geometric properties of angles, including the unit that was used in the Freiburg study. New problems were developed for all units, as our fading procedure required problems that involve particular skill combinations. Over a period of three weeks, the students worked with the Cognitive Tutor for two hours per week, each according to the condition s/he was assigned to.

Furthermore, online pre- and post-tests were administered to the participants, which presented students with problems covering the same Angles theorems as they learned in the Cognitive Tutor. The pre-test and immediate post-test contained the same ten transfer problems of which eight problems were transfer problems (problem solving/procedural items) in which the students needed to indicate whether a step was solvable, and if so, to provide the value, the theorem that was used to find the value, and to the geometric objects (in the diagram) to which the theorem was applied. The remaining two problems were transfer problems (conceptual knowledge items), where students were presented with a diagram and given measures for a small number of angles. For each of the given angle measures, the students were asked to state which other angle measures could be derived in a single step (i.e., application of a single geometry theorem).

In addition to the immediate post-test, a delayed post-test was administered three weeks after the students finished working on the Cognitive Tutor. This test contained six transfer problems, four of which were procedural items and two of which were conceptual knowledge items. Since the Angles Unit is part of their regular curriculum participants were not paid for their participation.

Results

Considerable attrition occurred throughout the study, which explains the varying degrees of freedom in the analyses. Of the 51 students 20 completed all three tests. Furthermore, 28 students completed both the pre-test and the immediate post-test. In the analysis of the delayed post-test scores, we included those students who completed at least one other test \((N = 35)\), in addition to the delayed post-test.

Among students who completed both pre-test and regular post-test \((N = 28)\), significant learning occurred in all conditions from pre-test \((M = 15.46, SD = 14.01)\) to post-test \((M = 22.93, SD = 16.64)\; \(t(27) = 2.27, p < .05, d = .87\); cf. Cohen, 1988). The planned contrast of adaptive fading condition versus the problem solving + fixed fading conditions revealed no differences in performance either on
the pre-test or on the regular post-test ($F_s < 1$). Furthermore, while the planned contrast did not show an effect of the adaptive fading condition ($M = 12.80, SD = 5.61$) over the other two conditions ($M = 8.73, SD = 4.97$) on the delayed post-test ($F = 2.38, p = .11$), it did indicate a tendency in the expected direction ($t(32) = 2.10, p < .05, d = .74$). When excluding the fixed fading condition, the adaptive fading condition did attain higher transfer performance on the delayed post-test than the problem solving condition ($M = 8.08, SD = 4.68; t(20) = 2.15, p < .05, d = .91$). No differences in Cognitive Tutor time ($F < 1$) were found. Lastly, the overall number of worked-out example steps between the fixed fading and adaptive fading conditions was fairly close to each other ($F < 1$).

Discussion

Two studies were conducted comparing “standard” tutored problem solving with a Cognitive Tutor versus two conditions in which tutored problem solving was enriched with worked-out examples. The worked-out examples were faded in either a fixed or in an adaptive manner. These manipulations were tested both in a lab study and in an actual classroom setting as part of a regular vocational school curriculum. The results of the lab study show that adaptively fading worked-out examples leads to higher transfer performance on both regular post-test and delayed post-tests. While this effect was not fully replicated in the classroom study, a significant benefit in transfer performance for the adaptive fading condition over the problem solving condition was revealed on the delayed post-test.

A likely explanation for the lesser effect in the classroom study can be found in the larger amount of “noise” that inherently exists within a real life environment, as compared to the laboratory. Also, the classroom study took place over a longer period of time with a fairly high amount of attrition of students. More specifically, a considerable number of students missed one (some even missed two) of the three online tests that were given. Yet despite the general difficulty of replicating lab results in the classroom, the current study still shows a benefit of the adaptive fading condition over the standard Cognitive Tutor. The failure to find a significant difference between the adaptive and the fixed fading conditions might be explained by the possibility that the fixed fading procedure was already near optimal. This is supported by the non-significant result of the comparison in number of worked-out steps between both fading conditions.

It is also possible that any learning differences between the fixed and adaptive fading conditions might be offset by the use of the Cognitive Tutor’s mastery learning criterion, which, as mentioned, led students in the classroom study to receive remedial problems for the theorems they had not mastered fully yet (on an individual basis, after they completed the problem sequence described in Table 1). These remedial problems represent additional learning opportunities for students. It could be that the mastery learning mechanism caused the students’ knowledge level (upon completion of the tutor work) to be more equal than it was for the students in the lab experiment, who did not receive any remedial problems. The results indicate that a possible equalizing effect due to mastery learning did wear off over time, since the adaptive fading condition attained higher delayed post-test performance than the tutored problem solving condition. In other words, even with mastery learning on, a benefit of worked examples is seen.

The current findings confirm and extend the findings of Schonke et al. (2007), which indicated that tutored problem solving, combined with fixed fading of worked-out steps, leads to better transfer performance, as well as to more efficient learning. A tentative explanation might be that working with examples increases students’ procedural and conceptual knowledge compared to tutored problem solving without examples.

It is interesting to view the current findings in light of the Assistance Dilemma issue that was recently brought to the foreground by Koedinger and Aleven (2007). The Assistance Dilemma follows from old observation in the learning sciences, namely, (a) that the balance between giving assistance to students and withholding it (while letting students generate information by themselves, possibly with feedback) exerts a major influence on students’ learning, and (b) that current cognitive theory does not provide the criteria needed to decide when to give, and when to withhold information, in order to optimize learning.

The choice between worked examples and problems is a key manifestation of this dilemma: how should a tutor effectively switch from a “high assistance” form of instruction (i.e., worked examples) to a “low assistance” form of instruction (i.e., problem solving) in a manner that is adaptive to individual students’ needs? The present results show that an adaptive example fading method, in which the rate of fading is based on the quality of students’ self-explanations, is a promising way to make this determination, in a manner adaptive to students’ individual learning trajectories. A remaining open question is theory development aimed at being able to predict when examples will be more/less effective than problem solving (see also Koedinger, Pavlik, McLaren, & Aleven, in press).

In short, the results of both studies indicate that the implementation of an adaptive fading procedure of worked-out examples within a Cognitive Tutor can be useful in both lab and actual classroom environments. Tutored problem solving and worked examples, adaptively faded, are synergistic, not redundant, forms of support.

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References


