Similarity Measures for Network Time Prisms

Young Jaegal, Harvey J. Miller

Department of Geography, The Ohio State University, 1036 Derby Hall, 154 North Oval Mall, Columbus, OH 43210, U.S.A
Email: {jaegal.1; miller.81}@osu.edu

Abstract

Space-time paths and prisms are time geographic concepts delimiting the actual and potential mobility pattern of an object in space and time, respectively. While there is a range of similarity measures for space-time paths, it is only recently that researchers started to develop similarity measures for the space-time prism (STP). This paper proposes a new methodology for measuring the similarity between network time prisms (NTP), the extension of STP to a moving object on a transportation network. A temporal sweeping method reduces the dimensionality of NTPs to a temporal profile curve that summarizes properties of the NTP subnetwork at moments in time. Then, the existing path similarity measures can measure resemblance between the temporal curves for NTP. We demonstrate the method using selected network summary measures derived from graph theory.

1. Introduction

Time geography provides a powerful spatiotemporal framework for both observed and potential movement pattern of moving objects (Hägerstrand, 1970). For observed movement data, space-time paths capture the actual traces of an object in space with respect to time. For the potential mobility, space-time prisms (STPs) delimit the geographic limit of movement of an object with respect to time. Researchers have developed a variety of path similarity measures for quantifying the resemblance between two space-time paths (Long and Nelson, 2013; Ranacher and Tzavella, 2014; Yuan and Raubal, 2014). Previous studies have utilized these measures for various purposes including comparing, clustering and aggregating movement trajectories. However, it is only recently that researchers started paying attention to similarity measures for the STP.

Miller, Raubal and Jaegal (2016) develop a temporal sweep approach for measuring STP similarity. The basic idea is to measure selected properties of a STP at moments in time, reducing the dimensionality of a STP to a profile curve summarizing changes in the selected property over time. This paper extends the temporal sweep method for measuring STP similarity to the case of network time prisms (NTPs). We also provide a demonstration of the method using an empirical transportation network.

2. Background

2.1 Network time prism

The NTP is an extension of STP that models the potential mobility of an object moving within a transportation network. Since individual movement in the real world usually occurs within networks, the NTP can provide a more realistic model of accessibility than a planar STP. Analytically, the NTP between locations and times \( (x_i, y_i, t_i) \) and \( (x_j, y_j, t_j) \) is the collection of 3-tuples consists of space-time coordinates, \( (x, y, t) \), that satisfies the following three conditions (Kuijpers and Othman, 2009).
\[ \begin{cases} 
(x, y) \in RN \\
d_{RN}(x_i, y_i, (x, y)) + d_{RN}(x_j, y_j, (x, y)) \leq (t_i - t_j) \\
t_i + d_{RN}(x_i, y_i, (x, y)) \leq t \leq t_i - d_{RN}(x_j, y_j, (x, y)) 
\end{cases} \]  

(1)

Where \( RN \) is the set of locations on a road network; \( d_{RN}(p,u) \) is the shortest path travel time between two locations within \( RN \).

### 2.2 Path and Prism Similarity Measures

Path similarity measures provide a quantitative assessment of resemblance among paths based on geometric and sequential properties. Two major classes of path similarity measures are shape-based and time-based (Yuan and Raubal 2014). Shape-based measures consider only the geometric characteristics of a path. This includes classical Euclidean distance and Hausdorff distance. Time-based methods conceptualize a space-time path as multidimensional time series data. It includes synchronized Euclidean distance, Fréchet distance, dynamic time warping (DTW) and longest common sub-sequences (LCSS) (See Long and Nelson (2013) for the review of these measures). The applications of path similarity in transportation studies include clustering methods for movement trajectories, comparison of individual mobility patterns, and querying for paths in a database that are similar to a reference path.

STPs have been widely used in human and ecological studies as measure of individual accessibility and exposure to environments (Espeter and Raubal 2009; Long and Nelson 2012). They can also measure space-time path uncertainty based on sampled locations (Pföser and Jensen 1999). As with space-time paths, we also may wish to measure similarity between STPs for assessing differences in accessibility, exposure and mobile object location uncertainty. We also may wish to cluster, aggregate and search for similar STPs based on geometric and/or semantic properties. However, until recently, researchers have paid little attention to similarity measures for STP.

Miller, Raubal and Jaegal (2016) develop a methodology for measuring STP similarity based on dimensionality reduction. They temporally swept STPs to generate time series data of geometric and semantic properties of STPs. We can assess STP similarity by applying existing path similarity measures such as DTW to the resulting temporal profile curves. The next section of this paper describes the temporal sweep method and its extension to measure NTP similarity.

### 3. Methodology

#### 3.1 Temporal Sweeping Method

Figure 1 provides an illustration of the general strategy. The method records a selected property of the NTP at discrete moments in time. The left side of Figure 1 presents an example NTP constructed by network analysis tool available in ArcGIS 10.3. The right side of Figure 1 shows that we can reduce the dimensionally of NTP into a temporal profile curve. Since this sequence is a time series, existing path similarity measures discussed in the previous section can assess the resemblance between different temporal curves.

We can measure and record a wide range of relevant NTP properties to generate the temporal profile curves. These include geometric properties such as the size and shape of the spatial region covered by the NTP, as well as semantic properties such as the spatial distribution of activities, resources and people within that region. In this paper, we apply graph theoretic measure to assess the topology and connectivity of the NTP subnetwork at each moment in time. We will now discuss these measures.
3.2 Graph Theory

Since the temporal sweeping method for NTPs generates a series of subnetworks at each moment in time, we can use analytical tools available from graph theory to assess properties of NTP for similarity assessment. A graph is a symbolic representation of network using vertices, $v$, and edges, $e$. (Rodrigue, Comtois and Slack, 2013). Graph theory is a mathematical study of the encoding and measurement of the graph.

Two key aspects of a network affecting mobility are size and connectivity. The simple graph theoretic measures for network size include the number of vertices, $v_n$, and edges, $e_n$, but they provide no information about travel time between vertices. Other size measures considering transportation times on the network include cost, diameter and Pi index. Cost is the sum of network travel time of all edges on a network. Diameter is the shortest path between the two farthest apart vertices in the graph. The Pi index is the ratio of cost to diameter. The connectivity measures from graph theory include Beta index and Gamma index. The Beta index is the ratio of the number of vertices to edges. It ranges from zero for a network with no edges and increases in value with greater connectivity. It exceeds unity for more complex networks with more than one circuit or path that begins and ends at the same vertex (Bhaduri, 1992). The Gamma index uses the number of all possible edges instead of the number of vertices as the numerator.

We apply these measures to the example NTP in Figure 1 to demonstrate the concept. Figure 2 presents the temporal curves of selected graph theoretic measures. Figure 2a and 2b illustrates the diameter and cost capturing effectively the change in the size of NTP with respect to time. The Pi index curve in Figure 2c also captures the changes in network size, and it would be useful when comparing NTP from different networks since it is standardized by the diameter of the network. Lastly, Figure 2d shows that Beta index capturing the change of connectivity over time in the NTP.
4. Concluding remarks

This paper develops a methodology for measuring the similarity between NTPs based on a temporal sweeping method for reducing the dimensionality of NTP and demonstrates the approach using selected graph theoretical measures for assessing the size and connectivity of the NTP over time. We can compare other NTP properties in a similar manner, such as the geometric size and shape of the space-time regions, and the spatial distribution of activities, resources contained within the NTP at different moments in time.

The next steps for this research include assessing the performance of a wider range of graph theoretic measures for different types of networks and NTPs. After assessing the performance of different measures for carefully crafted experimental networks, the next step is to apply the methodology to empirical transportation networks and NTPs. Finally, we will assess the performance of the similarity measures in NTP clustering and aggregation.

References


