Bargaining and Network Structure: An Experiment

Gary Charness, Margarida Corominas-Bosch & Guillaume R. Frechette

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Abstract: We consider bargaining in a bipartite network of buyers and sellers, who can only trade with the limited number of people with whom they are connected. Such networks could arise due to proximity issues or restricted communication flows, as with information transmission of job openings, business opportunities, and transactions not easily regulated by external authorities. We perform an experimental test of a graph-theoretic model that allows us to decompose any two-sided network into simple networks of three types, with unique predictions about equilibrium prices for the networks in our sessions. We begin with two separate simple networks, which are then joined by an additional link. Participants appear to quickly grasp important characteristics of the networks. The results diverge sharply depending on how this connection is made, typically conforming to the theoretical directional predictions. Payoffs can be systematically affected even for agents who are not connected by the new link. We find strong evidence that shares (publicly) allocated in the past to others in one’s current position substantially and significantly affect what one is willing to accept.

Keywords: Bargaining, Experiment, Graph Theory, Network, Social Learning

JEL Classification: B49, C70, C78, C91, C92, D40

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1. Introduction

In institutions such as financial markets, people make independent and anonymous decisions that interact through a central clearing mechanism that matches buyer and seller. However, in other environments, individual agents are only in contact with a small number of other agents, and transactions can only take place if there is a direct ‘link’ (e.g., a social or business relationship) connecting two agents. In this sense one can talk about a network, which summarizes the structure of linkages among people. Intuitively, some connections may be better than others.

Network structure has economic implications for a wide range of situations that feature a limited number of agents and connections. A market may be inherently thin, as with exclusive dealers, airplane or arms sales, or international relations. Even where there are many players, useful information may be transmitted only through private channels, as is often the case for job openings, business opportunities, and confidential transactions. A network is a non-market institution, with important market-like characteristics. It can be seen to represent an intermediate case between bilateral bargaining and matching in a large centralized market.

Social networks may play an important role where there is little effective external regulation. For example, Lamoureaux (1986, p. 648) states that “the operation of New England banks [in the first half of the nineteenth century] was shaped primarily by kinship networks,” and attributes the successful industrialization of the region to these networks. Guseva and Rona-Tas (2001) find that social and business networks ameliorated problems in the emerging Russian

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1 Nevertheless, there are still human agents in this system. Professional traders and market makers may strongly prefer to transact with others of their cohort, so that links may still play a role.
2 Roth, Prasnikar, Okuno-Fujiwara & Zamir (1991) highlight the effect of competition, demonstrating that results are very different for an ultimatum game, with one-to-one matching, and a ‘market game’, where a single agent on one side can agree to a proposal from any of nine agents on the other side.
credit-card market in the early 1990s, which suffered from both difficulties in screening applicants and a lack of effective punishment or sanctions for violations.

While theoretical work on network structure has been progressing rapidly, there have been few empirical tests; however, the stylized theoretical environment lends itself to laboratory experiments. In this paper, we assume that the basic network structure is exogenously given. While this is clearly not the case in many environments, here the imposed network structure could be seen as representing social, legal, or cultural trading restrictions. This abstraction allows us to focus on the issue of the resulting bargaining allocations once the network has reached the form we consider.

We extend the infinite-horizon Corominas-Bosch (2004) graph-theoretic model of buyers and sellers to a finite-horizon experimental game. The model allows us to decompose any countable number of agents into relatively simple subgraphs (plus some extra links). This decomposition (and the associated equilibrium payoffs) is unique for the networks we study, (although it is not unique for general bipartite networks) and determines whether any particular link is relevant to the local equilibrium price. The process of price formation is central in economics, and there are many applications for which the traditional Walrasian approach seems inadequate; Rubinstein and Wolinsky (1985), Gale (1987), and others model price formation as an outcome of decentralized bilateral bargaining. Pairs of traders who reach agreements leave the market, and those remaining (and re-matched) continue to bargain, as in our design.

The crucial question that this paper investigates is how the network ‘architecture’ affects the outcomes and dynamics of bargaining. We perform a test on whether it matters how a simple link is added between two small groups of traders. One link is theoretically irrelevant, while the other should have dramatic effects. We find that the manner in which the link is added has a
major impact on the bargaining outcome, as the results diverge sharply across treatments and broadly conform to the theoretical predictions. Payoffs can be systematically affected even for agents who are not connected by the new link.

Most of the discrepancies between the data and theoretical predictions seem to be attributable to two behavioral phenomena: First, we find that people receive significantly less when they have only one connection, even where the theory predicts no difference in outcomes. Perhaps having only a single connection makes one nervous, or inspires perceived bargaining weakness. Second, we see strong evidence that shares (publicly) allocated in the past to others in one’s current position substantially and significantly affect what one is willing to accept, suggesting that a form of social learning (Ellison and Fudenberg 1993) is taking place.

2. Background

There are two strands to the network literature in economics. One branch of research examines the process of network formation; see for instance Jackson and Wolinsky (1996), Jackson and Watts (forthcoming), Bala and Goyal (2000) or Kranton and Minehart (2001). The second strand considers the impact of exogenously-specified network structures on outcomes; examples include Bala and Goyal (1998), Glaeser, Sacerdote & Sheinkman (1996), Morris (2000), Chwe (2000), and Calvó-Armengol (2001). Our study relates to the second strand, as we do not consider the issue of how networks were formed, or which networks we might expect to form if there are modest costs to forming (or severing) links. Our motivation for considering exogenously-specified networks is that we are primarily interested in isolating the effect of a small change in network structure on bargaining behavior and prices. We simply presume that the links are already in place due to some relationships that have (or had) value, and that the cost
of (endogenous) change is prohibitive. In this sense, the networks we use are effectively stable and have immediate economic application.

Although economics experiments on networks are recent, there are now several such papers, all with a very different focus than ours. Corbae and Duffy (2001) study 2x2 games, where participants play all of their ‘neighbors’ in the network. Cassar (2002) finds that play in a coordination game typically converges to the efficient Nash equilibrium less frequently with local interaction or a random network than with a ‘small-world’ network. Deck and Johnson (2002) examine the effect of cost-sharing institutions in endogenous networks, obtaining slightly better efficiency when players can bid between zero and the full cost of a direct link. Riedl and Ule (2002) find stable cooperation in a prisoner’s dilemma when people can choose their own partners. Falk and Kosfeld (2003) find that the Bala and Goyal (2000) model predicts outcomes fairly well with one-way flows, but does quite poorly with two-way flows. While all of these studies consider network-related issues, none directly address the asymmetrical nature of many networks, how network structure affects bargaining outcomes, or the value of different links.

Also related is the social-learning literature, which generally considers the effect of dynamic aggregation of social information on equilibrium outcomes. Ellison and Fudenberg (1993, p. 612) use the term social learning to describe contexts where “agents base their decisions, at least in part, on the experience of their neighbors,” listing (p. 613) three features for such learning environments, all of which are satisfied here. Ellison and Fudenberg (1995) find that word-of-mouth communication may lead to superior choices and socially-efficient

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3 Kosfeld (2003) offers an excellent survey of this embryonic field.
4 Kirchkamp and Nagel (forthcoming) study a prisoner’s dilemma game with local interaction. Berninghaus Ehrhart & Keser (1998) use a 3-person game; participants are either connected to neighbors on a circle or play within closed three-person groups.
5 1) Agents observe both their neighbors’ choices and the payoffs that these choices generate; 2) Agents periodically reevaluate their decisions, as opposed to making a once-and-for-all choice; 3) Players may be sufficiently
outcomes. Jackson and Kalai (1997) study recurring games, where a stage game is played repeatedly, but each stage is played by a new group of players. This set-up seems closest in spirit to our game, but involves signals that shed light on the distribution of types in the population.\footnote{See also Jackson and Kalai (1999) and Bala and Goyal (1998, 2001).}

In our environment, a link can have an important impact or no impact at all depending on which subnetworks it connects. In a sense, this result is reminiscent of the concept of ‘structural holes’ in the sociology literature. Burt (1992) contends that people have a competitive advantage if they are at bottlenecks in partially-connected networks. In terms of power, being adjacent to a link between two groups that are each internally well-connected but are largely separate is more valuable to the people at the nodes than are links within a fully-connected group. See Willer (1999) and Burt (2000) for discussions of the literature and issues in this field.

### 3. The Model

Corominas-Bosch (2004) provides a method for decomposing a network of buyers and sellers into relatively simple subgraphs, plus some extra links. Our adaptation to the laboratory is a two-sided market, with two types of agents (e.g., buyers and sellers) who engage in sequential bargaining – alternating offers over a shrinking pie, with multiple possible rounds.

Suppose there are $n$ sellers and $m$ buyers of a homogenous good for which all sellers have reservation value 0 and all buyers have reservation value 1. Each buyer desires only one unit of the good, and each seller can supply only one unit. Is the price dependent only on the relative sizes of $n$ and $m$, and will all trades take place at the same price? Here buyers (sellers) bargain heterogeneous that under full information they would not all make the same choice. In our case, one’s neighbors are in a sense temporal.
with a pre-assigned subset of all sellers (buyers); links are non-directed, which means that A is
linked to B if and only if B is linked to A. Any buyer may be connected to multiple sellers and
vice versa. The network structure is common information, as are all proposals and
acceptances.\textsuperscript{7,8}

Our analysis incorporates the following intuition: Some networks are ‘competitive’, so
that the short side of the market receives all surplus. On the other hand, other networks are
‘even’ (neither of the sides is stronger), and agents split the payoffs nearly evenly. This structure
generalizes to any network, and we can decompose any network into a union of smaller
networks, each one either a competitive or an even network, plus some extra links.

Consider three types of particular bipartite graphs ($G^S$, $G^E$, and $G^B$); in the graphs below
we arbitrarily have sellers reside at the top nodes and buyers reside at the bottom nodes. Let
graphs $G^S$ be those with more sellers than buyers, such that any set of sellers can be ‘jointly
matched’ with buyers if the number of sellers in this set does not exceed the number of buyers.\textsuperscript{9}
In the following figure, $G_1$ is of type $G^S$ since it has more sellers than buyers (3 versus 2), and
since we can find a joint matching involving any set of 1 or 2 sellers. Graphs $G^B$ are the
complement, substituting sellers for buyers and vice versa. Finally, graphs $G^E$ have as many
sellers and buyers and are such that there exists a joint matching involving all of them.

\textsuperscript{7} One might object that it is unrealistic for a seller to simultaneously advertise the availability of a good to many
parties when there is only one unit available. Indeed, we might expect somewhat different behavior if an agent
could only make an offer to one other specified agent in each round, even where the theoretical predictions are the
same. Nevertheless, there are markets where this practice occurs and where it is understood that there is only one
unit available. For example, when one wishes to publish a paper in law journals, one may simultaneously submit the
paper to all law journals, even though it can only be published in one of them. Similarly, stores (e.g., car
dealerships) may send out flyers to ‘linked’ buyers advertising a product of which there is only one unit available for
purchase. We feel that our game should be viewed as a reduced form that abstracts from many aspects of real-world
negotiations.

\textsuperscript{8} Of course, bargainers maintain personal anonymity in the laboratory.

\textsuperscript{9} Intuitively, a set of sellers can be jointly matched if there exists a collection of pairs of linked members such that
each agent belongs to at most one pair. See Appendix A for more detail and the formal definitions.
Not every graph is one of these three types, as is illustrated by the following graph:

Nevertheless, we can decompose this graph into two subgraphs, one of type $G^S$ and one of type $G^E$, plus an extra link.

Theorem 1 (shown in Appendix A) shows that any graph decomposes as a union of subgraphs which are of one of these three types, plus some extra links which will never connect a buyer in a subgraph $G^B$ with a seller in a subgraph $G^S$. As in Corominas-Bosch (2004), a simple iterative algorithm for decomposing countable bipartite networks is at the heart of the process.\footnote{An outline: We first remove the subgraphs that have a set of sellers of size $t$ collectively linked to less than $t$ buyers. We do so starting with the subgraphs in which multiple sellers are collectively linked to only one buyer. Then we remove the subgraphs in which more than 2 sellers are collectively linked to only 2 buyers. When we have exhausted all the possibilities we then remove the subgraphs that have a set of buyers of size $t$, collectively linked to less than $t$ sellers. The subgraphs removed in the first case, will be type $G^S_i$, the ones removed in the 2\textsuperscript{nd} case will be type $G^B_i$, and the remaining subgraphs will be type $G^E$.}

The graph below is an example of the decomposition process; it decomposes into two subgraphs of type $G^S$, one of type $G^E$ and one of type $G^B$. 

The decomposition we have defined above directly allows us to characterize equilibrium outcomes. The subgame-perfect equilibrium payoff will give all the surplus to the short side in the subgraphs that are $G^S$ or $G^B$ (competitive networks), while the surplus will be split relatively evenly (taking into account the first mover advantage) in $G^E$ subgraphs (even networks). We demonstrate below precisely how this works for the particular networks and procedures used in our experimental design.

**Implementation**

Before describing the details of the bargaining game, it may be helpful to show the particular networks that we use in our experiment:
Network (1) is analogous to the Roth et al. (1991) market game, but with only two proposers (instead of nine) for each responder. In both networks (2) and (3), there are two buyers and two sellers; in network (2) each buyer is connected to each seller, while in network (3) there is no link between \( b_2 \) and \( s_4 \). In these networks, nodes represent traders and arcs represent trading possibilities.

We begin our experimental sessions with one network (1) and one of either network (2) or (3). We later introduce a connection between the two networks, linking either the bottom of (1) to the top of the other network, or the top of (1) to the bottom of the other network. The discerning reader might also notice that the prediction of different behavior resulting from these different links would also hold with network (1) and a separate dyad, a simpler case. However, we chose the 7-person structure to provide a bit more richness in the environment; as it happens, the number of links in the 4-person network becomes behaviorally relevant, as we find some differences depending on whether this network is fully connected. To the best of our knowledge, we are the first to experimentally compare behavior in a \( N \) versus a \( |X| \) structure. In addition, one of the strong predictions of the model is that people who are not directly involved with the added link can nevertheless be affected by it. By using a 7-person network, we have more possibilities for testing this prediction without increasing the complexity too much.

In the first round of bargaining, sellers simultaneously make proposals to divide 2500 with any of their linked buyers. These offers are displayed, and buyers then simultaneously choose to accept at most one of the proposals made by linked sellers. If one or more responders accept the same share proposed by one or more proposers, the accepting player who is left

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\(^{11}\) Note that sellers and buyers are simply labels. We could have just as well called the first movers buyers and the results would be unchanged.
unmatched after a coin flip is then matched with the other proposer who made the same offer.\textsuperscript{12} This means that responders actually accept a proposal, but they do not care with which specific proposer they trade. If a buyer and seller trade, they and their links are removed from the network. If any linked buyers and sellers remain, the game proceeds to the 2\textsuperscript{nd} round, where now buyers propose divisions of 2400 to all of their linked sellers, who choose whether to accept. If a 3\textsuperscript{rd} round is necessary, sellers propose divisions of 2300, etc. There are at most 6 rounds of bargaining; a coin is flipped in front of the group after round 4 to determine whether the period ends after round 5 or round 6.\textsuperscript{13} All unmatched players receive 200.\textsuperscript{14}

Let us examine how to determine the subgame-perfect equilibrium payoffs (subsequently denoted by PEP). Our analysis begins with the study of the simplest possible cases: Networks with at most 2 sellers and 2 buyers. We start by considering triads:

\begin{center}
\begin{tabular}{c}

\begin{tikzpicture}

\node (buyer) at (0,0) {$200$};
\node (seller1) at (1,1) {$2300$};
\node (seller2) at (1,-1) {$200$};
\draw (buyer) -- (seller1);
\draw (buyer) -- (seller2);
\end{tikzpicture}

\end{tabular}
\end{center}

In the unique PEP of our game (see Proposition 1; all propositions and proofs are given in Appendix B), the buyer receives 2300, and the sellers receive 200. The result is intuitively clear: Competition is so strong that the agents on the long side are forced to yield all surplus to the agent alone on the short side. In this respect, it is worth noting that competition is much stronger

\textsuperscript{12} We thank Matt Jackson and Tom Palfrey for pointing out that in fact this process is necessary for the equilibrium predictions to hold.
\textsuperscript{13} We introduced this uncertainty in order to prevent unraveling effects, at least prior to round 5.
\textsuperscript{14} We chose a non-zero reservation payoff to avoid having to give individual participants no money other than the show-up fee. This may serve to reduce any ‘fairness’ considerations, which are not the research question for this paper. We shall see that the passage of time seems to reduce their impact in any case. While we could have chosen
than the ultimatum effect given by the last period. Even if it is the turn of \( s_1 \) and \( s_2 \) to propose in the last period, they are forced to yield all surplus to agent \( b_1 \).

A buyer and seller linked only to each other will split the surplus nearly evenly, with the initial proposer having a small advantage (Lemma 1), and we can extend this result to both networks feasible with two sellers and two buyers (Proposition 2):

\[
\begin{array}{cc}
1300 & 1300 \\
1200 & 1200 \\
\end{array}
\begin{array}{cc}
1300 & 1300 \\
1200 & 1200 \\
\end{array}
\]

According to this theory there is no difference between the predictions for networks (2) and (3). One might initially suppose that the equilibrium should favor the agents having more connections, but a closer look tells us that the extra connection in (2) is actually irrelevant. Suppose the seller with two links offers a small share to the buyers. Clearly the buyer with two links will reject such a proposal, since he has the other seller all to himself. If the other buyer also rejects the proposal, then the seller with two links will be forced to offer a larger share. This process continues until all buyers and sellers receive equal shares (subject to the slight inequality present from the asymmetric timing of offers).

When the number of players in each side is less than or equal to two, we have seen that either we have a network in which one side completely extracts the surplus from the other side (we will denote these networks as ‘competitive’ networks, like Network 1) or we have networks in which neither of the sides is stronger, with the surplus being split evenly with the proposer
having a slight advantage (denoted as ‘even’ networks, as Network 2 or 3). Equivalently, we can also state that the theoretical joint payoffs received by agents always falls into one of only two categories: 1) When agents are in a competitive network, the agent on the short side receives 2300 (all the surplus minus the reservation value), while the agent on the long side receives 200 (the reservation value). 2) In an even network, agents making initial proposals receive 1300 (slightly more than half of the pie) and agents responding to these proposals receive 1200 (slightly less than half of the pie).

Interestingly, this property can be generalized to any network. Theorem 2 (see Appendix B) tells us that in each network, no matter how complex it may be, there exists an equilibrium in which agents either extract all the surplus, receive only the reservation value, or split the pie nearly evenly. This result uses the existence of a decomposition (see Theorem 1) that allows us to split any network into subgraphs that are either competitive or even, plus some extra links. Competitive subgraphs reproduce situations in which, as in Network 1, there exists an equilibrium in which the long side extracts all surplus. Even networks (like Network 2 or 3) reproduce situations in which there exists an equilibrium in which agents split the surplus roughly evenly. This PEP is not unique in all cases, but it is unique for the networks used in this experiment. Our derivation closely follows Corominas-Bosch (2004), by adapting the model of the infinite-horizon game treated therein to our finite-horizon game.

Applying Theorem 2 to our networks, we get the following PEP for the 7-player networks, which we show is unique in Propositions 4 and 5.

\[ (4) \quad (5) \]

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statistical tests to show the effect on behavior of adding a link.
It may be helpful to provide some intuition for the theoretically-predicted differences between, for instance, networks (4) and (6), the 7-person groups that are formed by adding a link between the smaller networks. In network (6) the added link from $s_2$ to $b_2$ (hereafter, a top-to-bottom link, or $tb$) serves as a propagation mechanism. We know that one of the sellers must receive the reservation payoff, since there is no way to match all sellers with all buyers. Assume it is $s_1$ (we can start with $s_2$, $s_3$, or $s_4$ and get the same result), then $s_2$ must also receive the reservation payoff, or else $s_1$ could make a proposal that would yield him a slightly higher payoff than the reservation value, a proposal that should be accepted by $b_1$. Similarly, it must be the case that $s_3$ proposes receiving the reservation payoff, otherwise $s_2$ could deviate. This leaves $b_3$ in a position to also extract surplus. Essentially, the buyers are jointly able to exploit the sellers; $b_1$, $b_2$, and $b_3$ receive full shares and $s_1$, $s_2$, $s_3$, and $s_4$ receive only the reservation payoffs.

On the other hand, in network (4) there is no propagation across the added link (hereafter, a bottom-to-top link, or $bt$). Seller $s_3$ knows that either $s_1$ or $s_2$ must get 0, so that $b_1$ will expect to get a full share. This implies that $s_3$ eliminates $b_1$ from his bargaining plans, and the $(s_3, s_4, b_2, b_3)$ network can be considered in isolation.
Thus, an apparently minor change in the network (differing by only one connection) may strongly alter the situation. A new connection can affect players who are not directly involved. On the other hand, some new connections are theoretically irrelevant.

4. Experimental Design

This experiment was conducted at the Universitat Pompeu Fabra in Barcelona, Spain. Participants were recruited by posting notices at campus locations. A total of 105 people participated in our study (each person could only participate in one session). Most of these were students in economics or business, with a smaller percentage of students in the humanities. Session lasted about 100 minutes and average earnings were approximately 1600 Spanish pesetas (at the time, $1 = 140 pesetas), including a show-up fee of 500 pesetas.

Participants were given written instructions (an English translation of the instructions is presented in Appendix C) and these were read aloud.\textsuperscript{15} We used a three-person network and one of two types of four-person networks in the initial phase of our experimental sessions. Thus, there were seven agents in each experimental session. These are the networks we used in the initial phase:

\begin{center}
\begin{tikzpicture}
  \node (s1) at (0,0) {$s_1$};
  \node (s2) at (1,0) {$s_2$};
  \node (b1) at (0,-1) {$b_1$};

  \node (s3) at (2,0) {$s_3$};
  \node (s4) at (3,0) {$s_4$};
  \node (b2) at (2,-1) {$b_2$};
  \node (b3) at (3,-1) {$b_3$};

  \draw (s1) -- (s2);
  \draw (s1) -- (b1);
  \draw (s3) -- (s4);
  \draw (s3) -- (b2);
  \draw (s4) -- (b3);

  \node at (1.5,-0.5) {or};
\end{tikzpicture}
\end{center}

\textsuperscript{15} The instructions did not correctly reflect the specifications of the theoretical model when two agents simultaneously accept the same individual’s offer, while there was another identical offer available (see footnote...
We conducted 15 sessions. There were 4 sessions for each type of network, except for Treatment 1, as one session was canceled due to an insufficient number of participants. Each session consisted of 10 separate bargaining interactions or ‘periods’. Every participant received a sheet of paper that stated his or her letter assignment in the period.

The networks were drawn on the board and each proposal was written next to the player’s letter. In the first half of the first round, agents $s_1$, $s_2$, $s_3$, and $s_4$ made proposals on sheets of paper. Blank sheets were also collected from agents $b_1$, $b_2$, and $b_3$, so that anonymity with respect to role was preserved. In the second half of this bargaining round, agents $b_1$, $b_2$, and $b_3$ indicated which one (if any) of the outstanding proposals they wished to accept.$^{16}$ Acceptances and rejections were indicated on the board and an ellipse was drawn around links between those agents who had reached agreements, removing them and their links from the network. Bargaining rounds continued as needed. In the second bargaining round, agents $b_1$, $b_2$, and $b_3$ made proposals and agents $s_1$, $s_2$, $s_3$, and $s_4$ indicated which one (if any) of the outstanding proposals they wished to accept. This pattern continued in odd and even rounds. Each period was comprised of up to 5 or 6 bargaining rounds (decided by a coin flip after round 4), with the total amount to be divided shrinking after each unsuccessful bargaining round.

$^{12}$ However, this situation was rare in the laboratory and was in fact implemented in accordance with our model in three of the five cases where it occurred.

$^{16}$ In the 2nd half of the first round, we collected responses from $b_1$, $b_2$, and $b_3$, as well as blank sheets from $s_1$, $s_2$, $s_3$, and $s_4$. In all subsequent rounds (if necessary), we continued to collect sheets from every player.
The session then proceeded to the next period. We randomly changed positions for each individual in each period, subject to the constraint that each person remained in their original 3- or 4-person network. While it might seem more natural to keep the same letter assignments throughout the session and learning might well have been thereby accelerated, fixed roles in our multi-period design could lead to people deciding to invest in some form of reputation; in addition, role changing allows for ‘smoothing’ of the heterogeneity of individuals and minimizes arbitrary performance by a participant unhappy at being stuck in a disadvantageous role throughout the session. More importantly, as will be shown later, this does not lead to different results (for the basic networks in period 1 to 4) from comparable studies without role changes. Finally, one of our findings, that subjects are influenced by others’ behavior in their position, would not have been identifiable without this role rotation. 17 Nevertheless, it is possible

We played four periods before adding a link between the two networks and six periods after the link was added. The number of periods to be played either before or after the link was added was not divulged to the participants, although they were told that there would be a change in the network at some point in time. The participants only learned the nature of the new network at the time that the change was publicly introduced.

People were told that one of the 10 periods would be chosen at random for implementation of actual monetary payoffs. At the end of the experiment, a 10-sided die was rolled to determine the period chosen for payment. 18 Participants were then paid individually and privately. Table 1 below summarizes our treatments:

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17 Furthermore, Charness and Rabin (forthcoming) find no significant effects from role rotation in simple binary-choice games, and Heijden, Nelissen and Verbon (2002) show how role rotation is qualitatively irrelevant in a sequential exchange-game experiment (note that we have role randomization, not role rotation). In fact, Binmore, Shaked and Sutton (1985) find that role reversal brings results closer to equilibrium in a bargaining game.

18 This random-payment design avoids possible ‘income effects’ from participants having accumulated wealth in early periods.
Table 1 – Treatment Summary

<table>
<thead>
<tr>
<th>Treatment</th>
<th>7-person network</th>
<th>Sessions</th>
<th>Periods/Session</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>V /</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>V / N</td>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>V \</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>V \ N</td>
<td>4</td>
<td>10^19</td>
</tr>
</tbody>
</table>

5. Experimental Results

In this section, we present our results and the performance of the theoretical predictions. Overall, we find that the maximum number of matches (three) was achieved in 139 of 149 periods, with 98% (437 of 447) of all potential matches made. Table 2 shows the distribution of the number of rounds needed to reach agreement in each session:

Table 2 – Agreements Reached, by Round

<table>
<thead>
<tr>
<th>Round</th>
<th>Treatment</th>
<th>1 (71.1%)</th>
<th>2 (21.1%)</th>
<th>3 (3.3%)</th>
<th>4 (0.0%)</th>
<th>5 (2.2%)</th>
<th>6 (0.0%)</th>
<th>None (2.2%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>64</td>
<td>19</td>
<td>3</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>93</td>
<td>19</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>99</td>
<td>14</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>79</td>
<td>24</td>
<td>5</td>
<td>6</td>
<td>1</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>335</td>
<td>76</td>
<td>11</td>
<td>10</td>
<td>5</td>
<td>0</td>
<td>10^20</td>
</tr>
</tbody>
</table>

On average, participants received 96.2% of the possible surplus of 6300 for each 7-person group, and this was fairly consistent across treatments and pre- and post-link periods.\(^{21}\)

\(^{19}\) There were only nine periods in one session of Treatment 4.

\(^{20}\) Two disagreements in N networks were the result of players a\(_3\) and b\(_2\) reaching an agreement, and leaving players E and F isolated. Thus, only 8 disagreements reflect failed bilateral bargaining.
Three-quarters of all possible matches are made in the first round, while another 17% are made in the second round. Only 15 bargaining sessions go beyond round 4, and two-thirds of these never become agreements.\textsuperscript{22} Thus, of the total inefficiency of 3.8%, 2.2% can be attributed to failed negotiations while the rest is due to agreements reached after round 1.

Table 3 summarizes the average payoffs for first-round agreements in each treatment, both before and after the introduction of the additional link after period 4.\textsuperscript{23,24}

<table>
<thead>
<tr>
<th>Table 2 – Average Payoffs by Network Position (First-round Agreements)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Network Position</strong></td>
</tr>
<tr>
<td>----------------------</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

\textsuperscript{21} The proportions range from 94.7% (periods 1-4 of Treatment 3) to 97.4% (periods 5-10 of Treatment 2).
\textsuperscript{22} Although the numbers here are quite small, a similar result in time-decay bargaining is found in Charness (2000). This result shows that even a relatively large financial disincentive in the last period may not induce an agreement in a final dyad. However, these disagreements are rare, suggesting that even though there is some form of ‘fairness’ present in these dyads, it is not a major factor in our results.
\textsuperscript{23} We only use first-round agreement data since bargaining behavior in later rounds may also be confounded by changes in the remaining network structure. In theory, all bargaining should end in round 1; recall that this occurs 75% of the time. This selection criterion will be used in tests throughout the paper, unless otherwise noted. For completeness, the average payoffs for all cases are shown in Appendix D.
\textsuperscript{24} Data for periods 1 to 4 reveal two features. The V network has unequal divisions with the bottom extracting most of the pie. This is similar to the result observe in Roth, Prasnikar, Okuno-Fujiwara & Zamir’s (1991) market game. Second, in the “balanced” network |X|, both sides of the market obtain almost equal shares as in the standard 2 person pie shrinking game. We take both of these results as indication the role randomization did not have a substantial effect on our results.
A nonparametric Wilcoxon-Mann-Whitney rank-sum test (Siegel and Castellan 1988) on payoff changes, using fully-independent session-level data (15 observations), confirms that the type of link added (\(tb\) or \(bt\)) strongly affects bargaining behavior. The changes in average \((s_1, s_2)\) payoffs and the average difference in \((b_2, b_3)\) and \((s_3, s_4)\) payoffs are always highest when a \(tb\) link is added; this reverses for player \(b_1\)’s payoffs (see Table D1). Each of these comparisons indicates a difference significant at \(p = 0.002\).

Several implications of the theory can be tested; we first consider the point predictions, and then discuss the qualitative predictions. Table 4 breaks the average payoff per position down by link-type, and reports whether the payoff differs (at the 5% significance level) from the theoretical prediction. We pool all the (first-round agreement) data for \(s_1\), \(b_1\), and \(b_2\), as the type of link does not affect the predicted payoffs for these positions.\(^{25}\) For \(s_2\), \(s_3\), \(b_3\), and \(b_4\), the theory distinguishes between whether or not there is a \(tb\) link. The columns give the theoretical predictions, and in each cell the average amount is reported, accompanied by \(r\) if the theoretical prediction is rejected and \(nr\) otherwise. As can be seen, only two of the 11 predictions cannot be rejected. However, the relative magnitudes of the estimates seem to go in the direction suggested by theory.

\(^{25}\) We note that the apparent differences in A and B payoffs in periods 1-4 in some treatments vanish when these are (appropriately) aggregated across treatments.

---

### Table 4 – Tests of the Quantitative Theoretical Predictions

<table>
<thead>
<tr>
<th>Type</th>
<th>Link</th>
<th>200</th>
<th>1200</th>
<th>1300</th>
<th>2300</th>
</tr>
</thead>
<tbody>
<tr>
<td>(s_1)</td>
<td>All</td>
<td>641</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
To test the qualitative predictions, we first average the payoffs by subject (for each position and relevant link). Using these per-subject averages, we conduct sign tests. Each cell compares whether the column and row components are equal, with a one-sided test if there is a directional hypothesis:

<table>
<thead>
<tr>
<th>Type</th>
<th>Link</th>
<th>s₁</th>
<th>s₂</th>
<th>s₃</th>
<th>s₄</th>
<th>b₂</th>
<th>b₃</th>
<th>s₃</th>
<th>s₄</th>
<th>b₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>s₂</td>
<td>All</td>
<td>All</td>
<td>No tb</td>
<td>No tb</td>
<td>No tb</td>
<td>No tb</td>
<td>tb</td>
<td>tb</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b₁</td>
<td>All</td>
<td>r, t</td>
<td>r, t</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>s₃</td>
<td>No tb</td>
<td>r</td>
<td>nr, t</td>
<td>r, t</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>s₄</td>
<td>No tb</td>
<td>r</td>
<td>nr, t</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b₂</td>
<td>No tb</td>
<td>nr</td>
<td>nr, t</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b₃</td>
<td>No tb</td>
<td>nr, t</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>s₄</td>
<td>tb</td>
<td>nr, t</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b₂</td>
<td>tb</td>
<td>r, t</td>
<td>r, t</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b₃</td>
<td>tb</td>
<td>r, t</td>
<td>r, t</td>
<td>nr, t</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

|r (nr) means we can (cannot) reject (at \( p = 0.05 \)) the hypothesis that the payoffs are equal. t means the result is consistent with the theory.

All but two of the theoretical predictions find support. Even for the two exceptions, the difference between actual average payoffs and predicted payoffs is never more than 100.\(^{27}\) We will explore the causes for these deviations in our discussion. Generally the directional

\(^{26}\) All tests in this Table are sign tests, except for the (s₃, s₄) vs. (s₃, s₄) and the (b₂, b₃) vs. (b₂, b₃) comparisons, which are Wilcoxon-Mann-Whitney rank-sum tests.

\(^{27}\)
comparisons are very much in line with the theoretical predictions, at high levels of statistical significance.\textsuperscript{28}

Although the theoretical point predictions generally fail, the evolution of play suggests that this might be in part remedied over time. The theory seems to fare poorly when it predicts a very uneven split. In most of these cases however, payoffs either stabilize or move closer and closer to the predicted values. Figures 1-3 illustrate how payoffs change over time for the top and bottom of each small network.

Figure 1 shows that player b\textsubscript{1}’s earnings increase in all treatments prior to the addition of a link. When the new link connects b\textsubscript{1} and s\textsubscript{3}, b\textsubscript{1}’s earnings continue to increase steadily. However, when the new link connects s\textsubscript{2} and b\textsubscript{2}, b\textsubscript{1}’s earnings do not change as much after period 4.

\textsuperscript{27} The (s\textsubscript{3}, s\textsubscript{4}) average payoff with no tb link is only slightly larger than the (b\textsubscript{2}, b\textsubscript{3}) average payoff, (1248 vs. 1221), instead of being 100 larger, and the s\textsubscript{3} and s\textsubscript{4} payoffs with no tb link are different (1286 to 1211).

\textsuperscript{28} While we do not focus on the networks remaining after the first round of bargaining, some summary statistics may be useful. There were 65 cases where only 2 connected bargainers remained after the first round. These were always members of the original four-person network, except for one case where s\textsubscript{1} and b\textsubscript{1} remained unmatched.
Figure 2 shows that the average earnings for players $s_1$ and $s_2$ decline in all treatments prior to the addition of the new link. This trend continues when the new link connects $b_1$ and $s_3$; however, when the new link connects $s_2$ and $b_2$, the average earnings increase somewhat.

**Figure 2 - Average ($s_1$, $s_2$) Earnings Over Time**

![Graph showing earnings over time]

Figure 3 looks at the difference between the average payoffs for the top and the bottom of the 4-person network. In the first 4 periods, there is very little difference between the average payoffs for $b_2$ and $b_3$ and the average payoffs for $s_3$ and $s_4$. However, behavior after the new link is added is very sensitive to whether the link connects $b_1$ and $s_3$ or $b_2$ and $s_2$. In the first case there is no change, but in the second case $b_2$ and $b_3$ payoffs increase dramatically.

---

There were also 24 cases where a set of 3 connected bargainers remained after the first round, with 15 of these being $(s_1, s_2, b_1)$. All 8 bargaining failures occurred in the dyads.
Hence, all of these trends appear to bring the division of payoffs closer to the theoretical predictions, except in the case where a $tb$ link is added. A possible reason for the latter behavior will be explored later.

6. Discussion

Thus far it seems fair to say that the model predicts the directions of the split rather well, and that the $tb$ link has the expected effect. Yet questions remain concerning what makes behavior differ from the exact predictions of the theory and how to explain the few qualitative-prediction anomalies. Of course, to the extent that we find factors that contributed to these deviations, these may very well be due to our particular experimental environment.

The observed outcomes are the result of at least two decisions. On the one hand, people decide what they are willing to offer. On the other hand, people receiving these offers decide what they are willing to accept. We first attempt to identify the determinants of whether an offer is accepted or rejected. Table 6 presents probit estimates of the determinants of whether, in round 1, subjects accepted or rejected the best offer they received (when available offers
differed, the lower one(s) were never chosen).\textsuperscript{29} Two separate probit regressions are estimated for the responders, one for type $b_1$ and one for pooled types $b_2$ and $b_3$.\textsuperscript{30}

### Table 6 - Determinants for Accepting an Offer

<table>
<thead>
<tr>
<th></th>
<th>Type $b_1$</th>
<th>Types $b_2$ and $b_3$ (pooled)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Share offered (all link types for $b_1$ and no $tb$ link for $b_2$ and $b_3$)</td>
<td>12.95*** (3.95)</td>
<td>11.79*** (2.40)</td>
</tr>
<tr>
<td>Share offered with $tb$ link (for $b_2$ and $b_3$)</td>
<td>--</td>
<td>8.92*** (2.04)</td>
</tr>
<tr>
<td>Average share the position received in the past</td>
<td>-13.27*** (4.10)</td>
<td>-2.10*** (0.77)</td>
</tr>
<tr>
<td>2-link</td>
<td>--</td>
<td>0.54** (0.30)</td>
</tr>
<tr>
<td>3-link</td>
<td>0.05 (0.62)</td>
<td>1.00** (0.46)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.97 (1.39)</td>
<td>-2.76** (1.21)</td>
</tr>
<tr>
<td>Number of Observations</td>
<td>132</td>
<td>264</td>
</tr>
<tr>
<td>Log likelihood</td>
<td>-37.93</td>
<td>-107.67</td>
</tr>
</tbody>
</table>

Period one omitted. *, **, *** indicates statistical significance at $p = 0.10, 0.05, 0.01$.

2-link is a dummy that has a value of 1 if the offerer is linked to 2 people, and is 0 otherwise. 3-link is a dummy that has a value of 1 if the offerer is linked to 3 people, and is 0 otherwise.

As expected, the share offered is a significant factor with regard to acceptance or rejection – higher proposed shares are more likely to be accepted. However, note that $b_2$ and $b_3$ are generally willing to accept less money when there is a $tb$ link. This is a bit surprising, as we

\footnote{In this case, a likelihood ratio test strongly rejects the random-effects specification for $b_2$ and $b_3$ types. Since the results are not markedly different for $b_1$, we only include the probit in the text. The random-effects probit estimates are available in a longer working-paper version. There were no significant period effects, so these are also omitted in the regression. The results were unaffected.}

\footnote{Pooling data for positions $b_2$ and $b_3$ may seem incorrect since although they are predicted to be equivalent in equilibrium, they might not be in practice. The extent to which they differ is the number of connections. Thus we control for the number of links even if these are not predicted to have an impact theoretically. Another (separate) issue is that, as shall be seen later, the added link between the two (originally) separate networks, $tb$ and $bt$, may not be irrelevant in the laboratory. Hence we have also estimated the probit regressions interacting the share offered with a dummy for each type of link. We show that for type $b_1$, none of the three coefficient estimates differ statistically; for types $b_2$ and $b_3$, the cases with no link and with a $bt$ link do not differ statistically, but the coefficient for the $tb$ link differs from both of the others. This both confirms the theory and validates the specification choice for the probit regressions.}
have seen (Tables 3 and 4) that \( b_2 \) and \( b_3 \) receive a greater amount with a \( tb \) link. It would appear that this effect must be driven by higher \( tb \)-link offers being made. In fact, \((s_3,s_4)\) round 1 offers average 1366 with a \( tb \) link, compared to 1180 with a \( bt \) link (the Wilcoxon-Mann-Whitney test gives \( p < 0.001 \)). Since \( b_2 \) can reach an agreement with \( s_2 \) with a \( tb \) link, both \( s_3 \) and \( s_4 \) should be concerned with being left unmatched in this case, and so must compete.

We also observe the fact that the shares allocated in the past affect what one is willing to accept. Thus, people appear to be learning the ‘social norm’ for the group.\(^{31}\) In an unfamiliar situation (either in the laboratory or in the field), where people may be uncertain about appropriate behavior, it is natural to consider that individuals update their beliefs about social norms on the basis of other observed outcomes. Here we see that the higher the average share previously received by a position, the less likely it is that a person in that position will accept any given offer.

We find that both \( b_1 \) and \((b_2,b_3)\) decisions on whether to accept offers are sensitive to whether the subject is connected to only one person; this is not predicted by the theory. However, this factor seems quite plausible psychologically; perhaps people get more nervous when they only have one connection, and are correspondingly less aggressive. Also notice that the coefficient for a 3-link is nearly identical to that of the 2-link (it is not statistically different) for \( b_2 \) and \( b_3 \), and the coefficient for a 3-link is effectively zero for \( b_1 \) decisions.\(^{32,33}\)

\(^{31}\) Note that this is not the usual form of learning, where agents learn from the agents to whom they are connected. Instead, since agents change positions, they see what other people do in the same position. We find that their behavior is influenced by what they observe.
\(^{32}\) Note that \( b_1 \) is always connected to at least 2 people in the first round of bargaining, which is all we consider in our analysis.
\(^{33}\) The analysis of the marginal effects is available in a longer working-paper version.
We can also examine the determinants of the offers made in the first round by $s_1$, $s_2$, $s_3$, and $s_4$. Our results for first-round offers by $s_1$ and $s_2$ (pooled) and by $s_3$ and $s_4$ (pooled) are shown in Table 7:

<table>
<thead>
<tr>
<th>Independent variable</th>
<th>Coeff.</th>
<th>Std. Error</th>
<th>Coeff.</th>
<th>Std. Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period 2</td>
<td>0.030</td>
<td>0.023</td>
<td>0.030</td>
<td>0.019</td>
</tr>
<tr>
<td>Period 3</td>
<td>0.075***</td>
<td>0.022</td>
<td>0.034**</td>
<td>0.015</td>
</tr>
<tr>
<td>Period 4</td>
<td>0.106***</td>
<td>0.023</td>
<td>0.013</td>
<td>0.019</td>
</tr>
<tr>
<td>Period 5</td>
<td>0.176***</td>
<td>0.025</td>
<td>0.012</td>
<td>0.021</td>
</tr>
<tr>
<td>Period 6</td>
<td>0.204***</td>
<td>0.024</td>
<td>0.034*</td>
<td>0.020</td>
</tr>
<tr>
<td>Period 7</td>
<td>0.214***</td>
<td>0.025</td>
<td>0.034*</td>
<td>0.020</td>
</tr>
<tr>
<td>Period 8</td>
<td>0.215***</td>
<td>0.024</td>
<td>0.040**</td>
<td>0.019</td>
</tr>
<tr>
<td>Period 9</td>
<td>0.238***</td>
<td>0.025</td>
<td>0.068***</td>
<td>0.018</td>
</tr>
<tr>
<td>Period 10</td>
<td>0.227***</td>
<td>0.025</td>
<td>0.056***</td>
<td>0.020</td>
</tr>
<tr>
<td>$tb$ link</td>
<td>-0.101***</td>
<td>0.022</td>
<td>0.041***</td>
<td>0.016</td>
</tr>
<tr>
<td>2-link</td>
<td>-0.052***</td>
<td>0.017</td>
<td>-0.031***</td>
<td>0.010</td>
</tr>
<tr>
<td>3-link</td>
<td>--</td>
<td>--</td>
<td>-0.030**</td>
<td>0.014</td>
</tr>
</tbody>
</table>

# of observations     | 298     | 296

Period one omitted     

*,**,*** indicates statistical significance at $p = 0.10, 0.05, 0.01$. 

2-link is a dummy that has a value of 1 if the offerer is linked to 2 people, and is 0 otherwise. 

3-link is a dummy that has a value of 1 if the offerer is linked to 3 people, and is 0 otherwise.

Here we see positive time trends for ($s_1$, $s_2$) offers, with significant t-statistics almost from the beginning. Perhaps these offers would approach the extreme level predicted with more time. In conformance with the payoffs displayed in Figure 2, but in contrast with the predictions, it appears that $s_1$ and $s_2$ adjust their offers downwards when a $tb$ link is added. However, the negative coefficient for the $tb$ link dummy is overwhelmed by the coefficients for the later periods. The reason for this seems to be that after the $tb$ link is added, $s_3$ and $s_4$ do not increase.

---

34 These were estimated using fixed-effects regressions. We can reject the hypothesis that the fixed effects are not statistically significant.

35 Pooling is justified both theoretically and empirically (see Table 5).

36 Although pooling is justified theoretically, one might oppose this on the basis of the results in Table 4. Note, however, that the difference in payoffs between $s_1$ and $s_4$ only occurs if they are connected to a different number of buyers (see Table 8 below). By controlling for the number of links, this is taken into account in the regression.
their offers as much as is predicted by theory. This allows \( s_2 \) to undercut \( s_3 \) and also allows \( s_1 \) to decrease his offers.

Note that this also explains the anomalous jump in Figure 2 for Treatments 3 and 4. We should also expect \( s_3 \) and \( s_4 \) to adjust their offers, and this is exactly what is observed.

As with \( b_1 \) and \( (b_2,b_3) \) offer-acceptance decisions, we find that both \( (s_1,s_2) \) and \( (s_3,s_4) \) offers are sensitive to whether the subject is connected to only one person. This may explain the surprising result (Table 4) that \( s_3 \)'s and \( s_4 \)'s payoffs are not equal. To see this, we will show that results for networks 2 and 4 (where \( s_3 \) and \( b_3 \) have more connections than \( s_2 \) and \( b_2 \)) drive the payoff inequality. Also notice that for \( s_3 \) and \( s_4 \), the coefficient for a 3-link is nearly identical to that of the 2-link (it is not statistically different). Table 8 presents some sign test results:

**Table 8 – Sign Tests on \((s_3,s_4)\) and \((b_2,b_3)\) Payoffs**

| Link category, Network type | none, N | \( tb, N \) | \( bt, N \) | none, \( |X| \) | \( tb, |X| \) | \( bt, |X| \) |
|-----------------------------|---------|-------------|-------------|----------------|----------------|----------------|
| \( s_3 \) vs. \( s_4 \)    | \( r \)  | \( nr \)    | \( r \)      | \( nr \)       | \( nr \)       | \( nr \)       |
| \( b_2 \) vs. \( b_3 \)    | \( r^* \)| \( nr \)    | \( r \)      | \( nr \)       | \( nr \)       | \( nr \)       |

\( r (nr) \) means we can (cannot) reject the hypothesis that the payoffs are equal.

\( r^* \) means significant at \( p = 0.10 \), but not 0.05.

Our theory predicts that \( s_3 \) and \( s_4 \) payoffs should be the same, and \( b_2 \) and \( b_3 \) payoffs should be the same. While \( s_3 \) and \( s_4 \) payoffs are not different for the \( |X| \) network, they are significantly different in most conditions for the \( N \) network. As the only real difference between these networks is the number of connections for \( s_4 \), it appears that having only a single connection inspires perceived bargaining weakness.

---

37 A sign test indicates that \( s_1 \) and \( s_2 \) payoffs are significantly lower before a \( tb \) link than after one is made.

38 This is also consistent with the observation that \( b_1 \)'s payoffs are higher in treatments 1 and 2 than in treatment 3 and 4 (significant at the 1% significance level using a Mann-Whitney test).

39 Note that for \( s_1 \) and \( s_2 \), this simply identifies the different impacts of the link on types.
As in bilateral bargaining games, buyers and sellers payoffs are closer than predicted, a phenomenon often attributed to some form of social preferences. However, evidence from market games (e.g., Roth et al. 1991) indicates that bargaining agreements are quite lopsided when there are unequal numbers of buyers and sellers. Our evidence from the V network is consistent with this; for example, Figure 1 shows that b\textsubscript{1} earnings begin to approach the full 2300 predicted in the final periods. It is worth noting that some prominent social-preference models (e.g., Fehr and Schmidt 1999, Bolton and Ockenfels 2000) successfully predict (via heterogeneity) the lopsided payoffs in the market game, so that the presence of fairness is not inconsistent with lopsided payoffs in a multi-player environment. Finally, we note that our main result, the sharply divergent outcomes after bt versus tb links, occurs despite any existing social preferences, which might tend to diminish this divergence.

Some modeling and design concerns and issues

We have modeled bargaining as an alternating-offer process, with multi-lateral simultaneous bargaining. We begin with the long side of the market making offers, and we do not consider the simpler case of 5-person networks with a bt or tb link between the triad and the dyad. Finally, we provide full information about the network structure, offers, and acceptances to every experimental participant; these characteristics are not completely general to bargaining environments. In view of these issues, we must consider whether our results are artifacts of our specific design.

The first issue is that a more unstructured bargaining protocol might lead to different outcomes, as alternating-offers models may make idiosyncratic predictions about the effects of

\[40\] Indeed, the only s\textsubscript{3} vs. s\textsubscript{4} comparison that is not significantly different in N networks occurs with a tb link. This is consistent with our explanation, as here s\textsubscript{3} is being directly undercut by s\textsubscript{2}, and so needs to adjust more (in an immediate sense) than does s\textsubscript{4}. 28
outside options in bilateral bargaining and this could carry over to the network structure in the
design.\textsuperscript{41} While this is a reasonable concern, there is some evidence that the observed results are
robust to the bargaining protocol.\textsuperscript{42}

A second concern is whether our results are robust to which side of the market made the
first offer (in round 1), even though there is no theoretical difference. We do have indirect
evidence that allowing the short side to make the first proposal would not have affected the
results greatly. There were nine cases in periods 1-4 where no agreement was reached in the
unconnected V in the first round of bargaining, but an agreement was reached in the second
round, where the short side made the proposal to the long side of the market. In all of these
cases, the short side indeed received more than the matched agent from the long side. The
average amount received by the short side was 1683 (out of 2400), or 70\% of the total. This
compares with 59\% for agreements reached in the unconnected V in round 1; perhaps the
divisions would be more extreme in the unequal case if the short side could move first.\textsuperscript{43}

While we do not consider the simpler 5-person network, we also have some evidence
from those six instances where a connected 5-person network remained after the first round of
bargaining. In one of these cases, the remaining network was type $bt$, decomposing into a triad
and a dyad; unsurprisingly, the dyad settlement was fairly even, 54\% vs. 46\%, while the short
side of the V received 69\% of the total. More interesting are the five cases where a $tb$ 5-person
network remained; since this network cannot be decomposed, the theoretical predictions are for
each agent on the short side to receive the lion’s share. There were six agreements reached in

\textsuperscript{41} We thank Vince Crawford for this observation.
\textsuperscript{42} Four students at Universitat Pompeu Fabra, A. Alsina, R. Fuertes, E. Moret & M. Planell, replicated our network
and payoff structures in a couple of sessions for an undergraduate research project. They used a double-auction
design instead of alternating offers. All subjects could submit bids in each round. To maintain anonymity, bids
were written down on pieces of paper and an experimenter would copy them on the blackboard. Their results were
close to ours, both with respect to payoffs, treatment effects, and efficiency.
the second round of bargaining, with the agent on the short side receiving an average of 72% of the total payoff (the range was 63-79%). If a dyad remained after the second round, the split was 50% for each agent, while if a triad remained, the agent on the short side received 77%. These data suggest that results would not have been dramatically different if we had used 5-person networks in our design, although perhaps we would have observed more extreme splits in the $tb$.

Another issue is that it may be unrealistic to expect bargainers to have the full information that we provide, limiting the applicability of our results. One could reasonably argue that in practice one might not actually know details about transactions between parties with whom one cannot transact, so that the network must also determine the information flow in the group. We chose this information structure in part for practical reasons (public display in a hand-run experiment), but also to introduce the possibility of social learning. We recognize that eliminating imperfect information might well affect behavior and decision rules.

Nevertheless, in the small markets/networks mentioned earlier (airplanes, arms, etc.), the limited number of participants may well know the full information that we provide.$^{44}$ In addition, the Lovaglia et al. (1995) test of the effect of restricting this information showed no discernible difference in (late-session) outcomes, although it took substantially longer to arrive at these outcomes. In line with these results, we conjecture that the public display of all information helped to accelerate the learning process, and perhaps served to minimize disagreements. This additional information may facilitate a common perception of the pertinent social norm. Clearly, since this allowed subjects to see what others in their current position

$^{43}$ Of course, there is also a selection-bias issue here, since agents at the bottom node of the V who make proposals in round 2 must have declined proposals in round 1.

$^{44}$ Alternatively, consider the academic job market. While a lower-ranked school might not be able to hire (or even interview) top candidates, it is still able to learn hiring information that is relevant to its own actions.
obtained in the past, it might also have contributed to the deviations. Perhaps one useful direction for future work would be to investigate how changes in the information structure affect bargaining behavior and outcomes.

7. Conclusion

We conduct an experiment to study the effect of network structure on bargaining outcomes in a bipartite market of buyers and sellers of a homogenous and indivisible good. In many markets, a buyer is connected to only a small subset of all sellers, and *vice versa*. Such an interaction can be modeled as a network, which describes the feasible links (trading possibilities) between agents.

We observe a high degree of bargaining efficiency, in that the total payoffs received are 96% of the maximum possible. The public display of all bids and acceptances may accelerate learning with respect to both bargaining power and group norms about appropriate division.

While there are only 10 separate bargaining interactions for each individual in an experimental session, there is strong evidence of substantial changes in bargaining behavior even over this limited period of time.

In the laboratory, the payoffs often do not match the theoretical point predictions; this is the norm more than the exception in experimental economics (see Chapter 4 in Kagel and Roth

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45 Of course, the ultimate effect of providing this information is an open empirical question. However, Blume, DeJong, Kim, and Sprinkle (1998) find that providing a population history leads to an increase in the proportion of separating outcomes achieved in a sender-receiver game, and Duffy and Feltovich (1999) find that observation of other players' actions and payoffs appears to affect the evolution of play. Armantier (1998) finds that in a common-value auction, unless subjects learn the signals and bids of other players, their bidding behavior never approaches the behavior predicted by theory.

46 It is true that the high degree of efficiency may be partially an artifact of the modest decline in payoffs with successive bargaining rounds. Nevertheless, efficiency would still be quite high (92%) with the same bargaining behavior and a (for example) 20% discount from round to round, since 75% of the possible agreements are reached in the first round, and 17% in the 2nd round. A steeper discount rate could also induce more rapid agreement, partially compensating for the higher efficiency loss from round to round.
which is why the focus is often on whether a model predicts the correct comparative statics with respect to changes in the key variables in the model. Here the central contribution of the theory, the decomposition, seems to capture the fairly strong differences in bargaining behavior depending on how a link is added between two groups. Payoffs are typically asymmetric where predicted so, and in the expected direction; we find these effects in a short time in a moderately complex environment, perhaps in part because we provide public information about transactions. We also find that subjects not directly connected to an added link are affected by its presence where the theory so predicts.

To the extent that behavior deviates from the theory, we see some explanations outside of the bargaining model and probably related to the specifics of our experimental environment, such as a significant decrease in earnings when an agent is connected to only one other agent, even where an extra connection (in the \( N \) network) should not make any difference. This indicates that what makes the results from the \(|X|\) network look so similar to what has been observed before in multilateral bargaining game is not simply the fact that there is the same numbers of buyers and sellers connected together, but also that they all have the same number of connections. Also, there is strong evidence of a form of social learning: The shares that have been allocated in the past to others in one’s current position substantially and significantly affect what one is willing to accept. This phenomenon could be the result of people updating their beliefs about the success of different strategies, as in belief-based learning models (Fudenberg and Levine 1999); it could also be a more general form of social learning, where subjects are trying to learn the acceptable norm by observing the behavior of others (see Ellison 2002, for example).

Chatterjee and Dutta (1998) find that public offers in thin markets lead to a unique subgame-perfect equilibrium in pure strategies, whereas private offers do not permit any efficient equilibria in pure strategies.
We feel that the network framework is a useful metaphor for many market environments. Natural extensions of our work include changing bargaining timing and protocols, as well as considering endogenous link-formation with bipartite markets. For example, if we consider that it may be possible to add or subtract links in markets, we can predict the effect of such changes in the trading regime. It is plausible that the lower payoffs experienced by people with few links could lead to over-connectedness, from a social standpoint. As network theory is still evolving and general solutions are often unobtainable, experimental study seems a very natural complement in this area.

References


48 For example, suppose there is a \(tb\) link in existence and \(s_2\) can sever it. \(b_2\) and \(b_3\) would pay \(s_2\) to do so, but would be bidding against \(s_3\) and \(s_4\).


Appendix A – Graph Theory Notation and Results

We now start introducing basic concepts in graph theory. All concepts are standard (excepting the definition of \( G^S \), \( G^B \), and \( G^E \)) and can be found in any graph theory textbook, e.g., Gould (1988).

A non-directed bipartite graph \( G=<S\cup B, L> \) consists of a set of nodes, formed by \( n \) sellers \( S=\{s_1, ..., s_n\} \) and \( m \) buyers \( B=\{b_1, ..., b_m\} \), and a set of links \( L \), each link joining a seller with a buyer. An element of \( L \), say a link from \( s_i \) to \( b_j \) will be denoted as \( s_i : b_j \).

A subgraph \( G_0=<S_0\cup B_0, L_0> \) of \( G=<S\cup B, L> \) is a graph such that \( S_0 \subseteq S \), \( B_0 \subseteq B \), \( L_0 \subseteq L \), and such that each link in \( L_0 \) connects a seller of \( S_0 \) with a buyer in \( B_0 \). When we speak of the subgraph \( G_0 \) induced by the set of nodes \( S_0 \cup B_0 \) in \( G \) we mean the subgraph formed by the nodes \( S_0 \cup B_0 \) and all the links that connect a seller in \( S_0 \) and a buyer in \( B_0 \) in \( G \).

A matching in a bipartite graph \( G=<S\cup B, L> \) is a collection of pairs of linked members of \( B \) and \( S \) such that each agent in \( S \cup B \) belongs to at most one pair. We say that a subset of agents can be matched if there exists a matching involving these agents.

For instance, in the following graph there exists a matching involving agents \( s_1, s_3, b_1 \) and \( b_2 \) (depicted in the figure with the dotted lines), but there exists no matching involving \( s_1 \) and \( s_2 \).

We now define three different types of connected graphs, that we will denote by \( G^S \) (where ‘s’ stands for a surplus of sellers), \( G^B \) (with a surplus of buyers) and \( G^E \) (with an equal number of buyers and sellers). For each of these graphs, one can construct a matching with a set of nodes in the long side at most equal to the short side.

**Definition:** A graph \( G=<S\cup B, L> \) with more sellers than buyers (\( |S|=n > m = |B| \)) is a \( G^S \) graph if any subset of sellers of size smaller or equal than \( m \) can be matched. Symmetrically, a graph with more buyers than sellers (\( n < m \)) is a \( G^B \) graph if any subset of buyers up to size \( n \) can be matched. Finally, a graph with as many sellers and buyers (\( n = m \)) is a \( G^E \) graph if there exists a matching involving all its agents.\(^{49}\)

Theorem 1 shows that any graph decomposes as a union of subgraphs which are of one of these three types, plus some extra links which will never connect a buyer in a subgraph \( G^B \) with a seller in a subgraph \( G^S \). A simple iterative algorithm for decomposing bipartite networks is at the heart of the process.

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\(^{49}\) The task of checking whether a graph is of one of this three types is simplified by using Hall's Theorem (1935), a well-known result in graph theory that relates the existence of a matching to the property of a subset of nodes being collectively linked to a set of at least the same number of members.
Theorem 1

1) Every graph $G$ can be decomposed into a number of subgraphs $G_{S1}^{S},...,G_{SnS}^{S}$ (of the $G^S$ type), $G_{B1}^{B},...,G_{BnB}^{B}$ (of the $G^B$ type), $G_{E1}^{E},...,G_{EnE}^{E}$ (of the $G^E$ type), such that each node of $G$ belongs to one and only one of the subgraphs and any seller (buyer) in a $G_{Si}^{S}$ ($G_{Bi}^{B}$) is only linked to buyers (sellers) in a $G_{Sj}^{S}$ ($G_{Bj}^{B}$).

2) Moreover, a given node always belongs to the same type of subgraph for any such decomposition.

We will write $G = G_{S1}^{S} \cup ... \cup G_{SnS}^{S} \cup G_{B1}^{B} \cup ... \cup G_{BnB}^{B} \cup G_{E1}^{E} \cup ... \cup G_{EnE}^{E}$, with the union being disjoint.


We can adapt the theory to the particular networks used in our design: applying Theorem 1 to the networks used in the experiment, we get the following decompositions:

![Graphs](image-url)
Appendix B – Propositions and Proofs

For simplicity, in the propositions and theorems below, we denote by $\Pi_t$ the amount that can be split among any two agents at round $t$. That is, $\Pi_t = 2500 - (t-1) \times 100$, for $1 \leq t \leq 6$. Whenever we speak about a proposal $(p, 1-p)$, we mean that the proposer suggests a share of $p$ for himself and $1-p$ for the responder.

**Proposition 1:** Suppose that the initial network is given by:

![Diagram](image.png)

Then there is a unique PEP equilibrium payoff, which gives a payoff of 200 to each of agents $s_1$ and $s_2$, and 2300 to agent $b$.

**Proof:** First note that if two agents reach agreement the game is immediately finished, since there is no possibility of a second agreement among another pair of agents. Suppose now that we are in the 6th round, when agent $b$ has to propose a division of $\Pi_6$. If $s_1$ or $s_2$ reject, they do not reach an agreement and so they receive a payoff of 200. Therefore the game reduces to an ultimatum game, and the only equilibrium is the one in which $b$ proposes $(\Pi_6 - 200, 200)$ and the proposal is accepted.

Now, suppose that we are in the 5th round. Interestingly, whether this is the last round of the bargaining session or not, the only equilibrium is the one in which both $s_1$ and $s_2$ propose $(200, \Pi_5 - 200)$ and $b$ accepts. To see why, suppose w.l.o.g that agent $s_1$ proposes a share of $(p, \Pi_5 - p)$ with $p > 200$ and $b$ accepts. Then, either $s_2$ also proposed $(p, \Pi_5 - p)$, in which case $s_1$ and $s_2$ both get a payoff of $p/2$, or $s_2$ proposed a different share, in which case $s_2$ gets 200 and $s_1$ gets $p$. But then, $s_2$ could undercut and instead propose (in round 5) the division $(200+\varepsilon, \Pi_5 - 200 - \varepsilon)$ which $b$ should accept, since $\Pi_5 - 200 - \varepsilon > \Pi_5 - p$ and since the most $b$ can get in the next round is $\Pi_6 - 200$, which is smaller than $\Pi_5 - 200 - \varepsilon$. With similar arguments we can determine the strategies for the other rounds.

We now specify the set of strategies leading to the PEP equilibrium.

- **At round $t$, with $t \in \{1,3,5\}$**
  Agents $s_1$ and $s_2$ both propose $(200, \Pi_5 - 200)$. Agent $b$ accepts a proposal iff the share offered is greater than or equal than $\Pi_6 - 200$. If both proposals are greater or equal than $\Pi_6 - 200$, the buyer accepts the proposal that gives a higher share for himself. Otherwise, $b$ rejects.

- **At round $t$, with $t \in \{2,4,6\}$**
  Agent $b$ proposes $(\Pi_6 - 200, 200)$. Agents $s_1$ and $s_2$ both accept iff the share offered is greater than $200$. Otherwise, they reject.

**Lemma 1:** Suppose that the initial network is given by:

![Diagram](image.png)

Then, there is a unique PEP equilibrium payoff, which gives a payoff 1300 to the seller and 1200 to the buyer.

**Proof:** Suppose that we are in the 6th round. The equilibrium will then be agent $b$ proposing $(\Pi_6 - 200, 200)$ and agent $s$ accepting, following the structure of an ultimatum game. Now, suppose that we are in the 5th round and we know that a 6th round has to be played. Then, from standard bargaining results (see Rubinstein 1982) we know that proposals will always leave the responder indifferent between accepting or rejecting, and therefore $s$ will leave the
responder indifferent between accepting or rejecting, and will therefore propose \((\Pi_5-(\Pi_6-200), \Pi_6-200)\). If we are in the 5th round and this is the last one, the only equilibrium tells \(s\) to propose \((\Pi_5-200,200)\). Since there exists a 6th round with probability 1/2, after finishing round 4, in expected terms the seller expects to get a payoff of 1/2\((\Pi_5-(\Pi_6-200))+ 1/2(\Pi_5-200) = \Pi_5-(\Pi_6/2)\) and the buyer expects to get \(\Pi_6/2\). In this manner, we can calculate all the proposals in equilibrium.

We now specify the set of strategies leading to the PEP equilibrium.

- At round \(t\), with \(t \in \{1,3\}\), and with \(t=5\) if 5 is not the last period:
  Agent \(s\) proposes \((\Pi_t-(50(5-t)+\Pi_6/2), (50(5-t)+\Pi_6/2))\). Agent \(b\) accepts a proposal iff the share offered is greater than or equal than \(50(5-t)+\Pi_6/2\). Otherwise, \(b\) rejects.

- At round \(t\), with \(t \in \{2,4\}\): Agent \(b\) proposes \(((50(6-t)+\Pi_6/2), \Pi_t-(50(6-t)+\Pi_6/2))\). Agent \(s\) accepts a proposal iff the share offered is greater than or equal than \(\Pi_t-(50(6-t)+\Pi_6/2)\). Otherwise, \(s\) rejects.

- At round \(t\), with \(t=5\), if 5 is the last period: Agent \(s\) proposes \((\Pi_5-200, 200)\). Agent \(b\) accepts a proposal iff the share is equal or greater than 200, otherwise \(b\) rejects.

- At round \(t\), with \(t=6\):
  Agent \(b\) proposes \((\Pi_6-200, 200)\). Agent \(s\) accepts a proposal iff the share is equal or greater than 200, otherwise \(s\) rejects.

**Proposition 2**: Suppose that the initial network is given by one of the two networks below:

![Network Diagram]

Then, there exists a unique PEP equilibrium payoff, in which sellers \(s_1\) and \(s_2\) get a payoff of 1300 and \(b_1\) and \(b_2\) get a payoff of 1200.

**Proof**: Suppose that we are in the 6th round. If there is only a pair left, we know the equilibrium by Lemma 1 and we are done. Alternatively, suppose that all four agents are still in the market.

Now, suppose that we are in the 6th round. Note that responders will never reject in equilibrium (not even reject with some probability). The finite nature of the game implies that after a rejection, agents will always get 200. Then, the fact that a responder rejects with some probability, implies that accepting was no better than rejecting. This implies in turn that both proposers proposed to keep all the pie for themselves and leave 200 for the responder. Then, at most one proposer was able to receive all the pie minus 200. Then, the other proposer would be willing to deviate and propose to give 200+\(e\) to the responder, and this is an offer that would clearly be accepted by at least one responder (as the alternative is accepting 200 or rejecting).

Now, suppose that both agents accept a price \(p_i\) from the same proposer, with the other proposers having proposed \(p_2\). Clearly the other proposer wouldn't be matched and would receive 200. Thus, this proposer would be willing to deviate and propose to get 200+\(e\) for himself, an offer that would clearly be accepted by at least one responder. Therefore, it must be the case that responders accept from different proposers or that both proposers proposed the same price. Thus, we conclude that in equilibrium responders accept from different proposers with probability one or accept the same price proposal \(p\), \(p\) being the proposal of both proposers. Given that after rejection, proposers will get only 200, one can conclude that the proposals in equilibrium will give everything to the proposers and leave only 200 to respondents.

Once we deduce the payoffs for round 6, we can proceed as before by using induction, since proposals will always leave the responder indifferent between accepting or rejecting.

We now specify the set of strategies leading to the PEP equilibrium.
If all players are in the game:

- At round $t$, with $t \in \{1, 3\}$, and for $t=5$ if 5 is not the last period:
  Both sellers propose $(\Pi_t-(50(5-t)+\Pi/2), (50(5-t)+\Pi/2))$. Buyers accept the highest proposal for themselves iff the share offered is greater than or equal to $(50(5-t)+\Pi/2)$, and otherwise they reject.

- At round $t$, with $t \in \{2, 4\}$:
  Both buyers propose $((50(6-t)+\Pi/2), \Pi-(50(6-t)+\Pi/2))$. Sellers accept the highest proposal for themselves iff the share offered is greater than or equal to $\Pi-(50(6-t)+\Pi/2)$, and otherwise they reject.

- At round $t$, with $t=5$, and 5 being the last period:
  Both sellers propose $(\Pi_5-200, 200)$. Buyers accept the highest proposal for themselves iff the share offered is greater than or equal to 200, and otherwise they reject.

- At round $t$, with $t=6$:
  Both buyers propose $(\Pi_6-200, 200)$. Sellers accept the highest proposal for themselves iff the share offered is greater or equal than 200, and otherwise they reject.

If not all players are in the game, then it must be the case that we are in the network described in Lemma 1. Proceed according to the set of strategies defined in Lemma 1. ■

The PEP described above can be generalized in a PEP that exists for any given network, as shown by Theorem 2.

**Theorem 2:** Take any graph $G$ and decompose it as a union of $G^S$, $G^E$ and $G^B$ according to Theorem 1. Then, there exists a PEP in which:

- Sellers in $G^S$ receive 200, buyers in $G^S$ receive 2300.
- Sellers in $G^B$ receive 2300, buyers in $G^B$ receive 200.
- Sellers in $G^E$ receive 1300, buyers in $G^E$ receive 1200.

**Proof:** We will show the result using induction.

Step 0: $n \leq 2, m \leq 2$) We first show the result for a number of sellers $\leq 2$, and a number of buyers $\leq 2$. Only four different graphs are possible with at most two sellers and two buyers. These are the graphs analyzed in the previous resultss (Lemma 1, Proposition 1 and Proposition 2), in which the above statement is true.

Step 1: $n \leq k, m \leq k$) Suppose that the result is true for graphs of sizes $n \leq k-1, m \leq k-1$. We now have to show that the result is true for graphs of sizes $n \leq k, m \leq k$.

Given that in our game a pair of connected agents may reach a agreement at any point in time, while unmatched agents keep playing, to describe the subgame-perfect equilibria of the game we need to know the equilibrium in any possible network that results as a consequence of the deletion of some of the links in the initial network.

We now write the strategies that would support the PEP for a graph of size $n \leq k, m \leq k$.

- Strategies whenever the current graph is $G_i$, a strict subgraph of $G$ (somebody has traded). By the induction step we know of the existence of an PEP in this subgame. this is the PEP strategies will prescribe agents to follow.
- Strategies whenever the graph is $G$ (nobody has traded). Call the current round $t$, with $1 \leq t \leq 6$. Call this set of proposals, price proposal $P$.

**Proposal P:**

- At round $t$, with $t (1, 3, 5)$ with 5 not being the last period.
  Sellers in $G^S$ propose $(200, \Pi_2-200)$. Sellers in $G^B$ propose $(\Pi_2-200, 200)$. Sellers in $G^E$ propose: $(\Pi_t-(50(5-t)+\Pi/2), (50(5-t)+\Pi/2))$.

- At round $t$, with $t (2, 4)$
  Buyers in $G^S$ propose $(\Pi_2-200, 200)$. Buyers in $G^B$ propose $(200, \Pi_2-200)$. Buyers in $G^E$ propose: $((50(6-t)+\Pi/2), \Pi_6-(50(6-t)+\Pi/2))$.

- At round $t$, with $t=5$, with 5 being the last period:
  Sellers in $G^S$ propose $(200, \Pi_5-200)$. Sellers in $G^B$ propose $(\Pi_5-200, 200)$. Sellers in $G^E$ propose $(\Pi_5-200, 200)$.
At round $t$, with $t=6$. Buyers in $G^S$ propose $(\Pi_t-200, 200)$. Buyers in $G^B$ propose $(200, \Pi_t-200)$. Buyers in $G^E$ propose $(\Pi_t-200, 200)$.

Acceptances: If the price proposal has been equal to $P$, then all responders in $G^S$ accept the proposal made by proposers in $G^S$, responders in $G^B$ accept the proposal made by proposers in $G^B$, and responders in $G^E$ accept the proposal made by proposers in $G^E$.

We now write what agents do facing some of the possible unilateral deviations:

Members of $G^S$ (odd round, sellers proposing): If the distribution differs from $P$ in one price only, then call $s_i$ the seller that deviated in its proposal. Buyers in $G^S$ will continue to accept the share of $\Pi_t-200$ for themselves. By the definition of a $G^S$ subgraph, there exists a way to match all buyers in $G^S$ with a set of sellers in $G^S$ which excludes $s_i$. Therefore, all buyers can get $\Pi_t-200$, and the seller that deviated is isolated and gets 200. The rest of buyers all accept the proposal made by sellers in their respective subgraph. Similarly for Members of $G^B$ in an even round, with buyers proposing.

Members of $G^E$ (odd round, sellers proposing): Suppose that seller $s_i$ tries to propose a better share for himself. Relabel agents so that $s_i$ and $b_i$ are linked in a matching involving all agents in $G^E$ (the existence of this matching is guaranteed by the definition of $G^E$). Then, all buyers excepting $b_i$ accept the proposal stated by all sellers excepting $s_i$, and $b_i$ rejects. The rest of buyers all accept the proposal made by sellers in their respective subgraph. This means that in the following period agents $s_i$ and $b_i$ will remain isolated and play as in Lemma 1. Similarly for members of $G^E$ in an even round, with buyers proposing.

Now it is easy to check that what we have above is indeed a Nash Equilibrium, as no agent can unilaterally deviate asking for a larger share and be accepted, and no responder can do better by rejecting.

To define a subgame-perfect equilibrium, we must also state what strategies specify off the equilibrium path. We now explain how strategies can be constructed so that the strategies above conform to a subgame perfect equilibrium. For any distribution of prices, agents have a finite set of actions that consist of either accepting one of the proposals or rejecting all. If less than the maximum possible number of pairs form, then by the induction step we know that there exists a PEP in the resulting subgraph (since this will be a subgraph that has a number of agents strictly smaller than $k$). We define strategies so that if less than the maximum possible number of pairs form, then strategies follow the PEP of the resulting subgraph (which we know exists by the induction step). If all agents reject, then the strategies will prescribe for proposers to propose price distribution $P$ and for responders to accept. Therefore we can conclude that given an action for all responders, the payoffs are immediately determined. This must have at least one NE. We will define the strategies as follows: for any distribution of prices, strategies will tell responders to play according to this NE. However, note though that there may be multiple NE. If this is the case, strategies must specify which of the several NE will be played. Any specification would suffice.

**Proposition 4:** Suppose that the initial network is given by one of the two networks below:

Then, the unique PEP equilibrium payoff will give:

- a payoff of 200 to agents $s_1$ and $s_2$
- a payoff of 2300 to agent $b_1$
- a payoff of 1300 to sellers $s_3$ and $s_4$
- a payoff of 1200 to buyers $b_2$ and $b_3$
**Proof:** Existence is immediate by Theorem 1, as the above networks decompose as a subgraph of type $G^S$ (formed by agents $s_1$, $s_2$ and $b_1$) and a subgraph of type $G^E$ (formed by agents $s_3$, $s_4$, $b_2$, $b_3$). (see Appendix A).

To show uniqueness, as a first step, we will show that in either case sellers $s_1$ and $s_2$ receive a payoff of 200 in equilibrium, while buyer $b_1$ receives 2300. Suppose to the contrary (w.l.o.g.) that $s_1$ receives a payoff higher than 200. This cannot happen in an odd round, through $s_1$ proposing a partition that $b_1$ accepts, since $s_2$ would have undercut by proposing $(200+\varepsilon, \Pi - 200 - \varepsilon)$, a proposal that $b_1$ cannot reject, as it is strictly better than anything $b_1$ can get in the next period. Then, it should be the case that $s_1$ accepted from $b_1$ in an even round. But if this is the case, $s_2$ would also accept, therefore both sellers $s_1$ and $s_2$ would be accepting from $b_1$ in an even round, each reaching agreement with the same probability, and otherwise receiving 200. But this cannot be an equilibrium either, as $b_1$ could propose a higher share for himself (and would still be accepted, since sellers in the following round would get 200). We can conclude then that $s_1$ and $s_2$ get 200 in equilibrium and $b_1$ gets the rest of the pie, starting in any subgame.

Now it is intuitive to see that agents $s_3$, $s_4$, $b_2$, and $b_3$ will play as in Proposition 2, as if the link connecting $b_1$ and $s_3$ would not exist. Indeed, if all four agents are still in the market in the last round, we know that in equilibrium buyer $b_1$ will propose $(\Pi - 200, 200)$ and that this would be accepted by sellers $s_1$ and $s_2$. Clearly, if $s_3$ or $s_4$ rejects, he receives a payoff of 200, so should accept any proposal that is greater than or equal to it. Thus, we can apply the arguments of Proposition 2 here as well. ■

**Proposition 5:** Suppose that the initial network is given by one of the two networks below:

![Network Diagram]

Then, the unique PEP will give:

- a payoff of 200 to all sellers.
- a payoff of 2300 to all buyers.

**Proof:** Existence is immediate by Theorem 1, as the above networks decompose as a unique subgraph of type $G^S$. (see Appendix A).

To show uniqueness, note first that as four pairs will never form, at least one of the sellers must receive a payoff of 200 in equilibrium. Let us call this seller $s_i$. This seller must be linked to at least one buyer, call him $b_i$. Now, since $s_i$ received 200 in equilibrium, this implies that he could not deviate in the first round and propose $200+\varepsilon$ for himself. This implies in turn that $b_i$ could accept a share of 2300 from somebody else (since note that in the network we are analyzing any buyer $b_i$ is linked to at least 2 sellers), call him $s_j$. That is, this implies that seller $s_j$ was proposing 200 for himself and was accepted. But again, if he could not deviate, this implies that all the linked buyers accept 2300 from other sellers. That is, the fact that a seller $s_i$ received the reservation value in equilibrium implies that the sellers linked to $b_i$, with $b_i$ being a buyer linked to $s_i$, also receiving the reservation value. Given the structure of the networks, in this case this implies that all sellers proposed the reservation value for themselves and were accepted. ■
Appendix C - Instructions

Thank you for participating in this experiment. You will receive 500 pesetas for attending the session and appearing on time. In addition, you will make decisions for which you will receive an amount of money; the amount depends on the choices made in the experiment. You will receive a subject number that we will use to identify you during the experiment. Please hold on to this number, as we will need it in order to pay you.

In this experiment, you will be in a group of 7 persons who will engage in anonymous bargaining sessions. As explained below, people will make proposals to divide a sum of money between them. In each session, every person will be connected to one or more other persons. You can only bargain with those people with whom you are connected. An individual can only reach an agreement with at most one other person in a bargaining session.

An example of a diagram of a network (the overall set of possible connections between people) is shown on the board. A network has two sides, a top and a bottom.

Your connection(s) will be constant throughout a bargaining session; however, there will be multiple bargaining sessions and your location on the network may change from one session to the next. The network itself will remain constant for some number of bargaining sessions, but the network will change at some point in the experiment. You will be informed when this occurs and a diagram of the new network will be displayed on the board.

Each bargaining session will consist of up to 5 or 6 rounds, each of which consists of two parts. In the 1st part of the 1st round, each member of the top side of the network will make a proposal for dividing a sum of money with anyone on the bottom side of the network with whom he or she is connected. A proposal is a suggestion of how much money you would receive and how much money a person accepting the proposal would receive. In the 2nd part of the 1st round, individuals on the bottom side of the network respond to the proposals made by those individuals with whom they are connected. One may choose to accept one of these proposals or choose to reject all available proposals. If a proposal is accepted, there is a match and both parties to the match are removed from the network for the remainder of that bargaining session. If there are no more possible matches that can be made, the bargaining session has been completed.

If there are still possible matches, we continue to a 2nd round. In the 1st part of the 2nd round, all persons on the bottom side of the network who have not become matched will make proposals to divide a (smaller) sum of money with connected persons on the top side of the network. In the 2nd part of the 2nd round, each unmatched individual on the top side of the network will choose either to accept one of the proposals made by people with whom he or she is connected or to reject all available proposals. Matches are determined and displayed. Once again, if there are no more possible matches to be made, the bargaining session is over. If there are still potential further matches, the bargaining session will continue to a 3rd round, in which the top side of the network will make proposals to divide a (still smaller) sum of money. A 4th round, where the bottom side of the network would make proposals, would follow if necessary.

If we reach a 5th round of a bargaining session, we will flip a coin to see if a 6th round will be permitted (if necessary) or if the bargaining session ends after the 5th round.

The amount of money to be divided will diminish over the course of a bargaining session, in the following manner:
A proposal made in the 1st round suggests a division of 2500 pesetas
A proposal made in the 2nd round suggests a division of 2400 pesetas
A proposal made in the 3rd round suggests a division of 2300 pesetas
A proposal made in the 4th round suggests a division of 2200 pesetas
A proposal made in the 5th round suggests a division of 2100 pesetas
A proposal made in the 6th round suggests a division of 2000 pesetas

Any individual who remains unmatched at the end of a bargaining session would receive a payoff of 200 pesetas for that session.

Mechanics: A diagram of the network in use will be shown on the board at all times. We designate positions on the network with the letters A-G. When proposals are made, we will indicate all of the proposals on this diagram. The choice of each responder to either accept or reject proposals will subsequently be displayed. If a match has been made, the connection between the two matched parties will be circled.

At the beginning of each bargaining session, you will receive a sheet of paper with a drawing of the network; your location on the network will be circled. You have been given a stack of small pieces of paper with your subject number on them. When you are making a proposal or when you are rejecting or accepting proposals, please do so on one of the small pieces of paper and also fill in your assigned letter in the space provided.

As it may be possible that a responder could respond to more than one proposal, if you choose to accept a proposal you must indicate the letter of the person making this proposal.

In the event of more than one person accepting the same proposal, we will randomly determine which responder becomes matched. If you have accepted a proposal but do not become matched, you must proceed to the next round.

We wish to preserve anonymity throughout the experiment. We therefore ask that you turn in one of the small pieces of paper in each part of each round played, even if you are already matched or if it is not your turn to propose or respond. If this procedure were not followed, other participants might be able to deduce the identity of the person at a location on the network. If it is your turn to propose or respond, please do so. If it is not your turn, we ask that you write “Not my turn” on one of the small pieces of paper.

Payment: Although there will be a number of bargaining sessions, at the end of the experiment we will randomly select (using a die) the results from one of these bargaining sessions for actual payment.

If you have any questions, please ask them now or by raising your hand during the course of the experiment. Communication between participants is strictly forbidden. Are there any questions?
Appendix D – Average Payoffs

Table D1 – Average Payoffs in Treatment 1

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These are actual payoffs for all bargaining rounds.
Table D3 – Average Payoffs in Treatment 3

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*One of the four sessions did not have a period 10.