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Authors
White, M
Tinker, JL
McBride, CK

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Mock galaxy catalogs using the quick particle mesh method

Martin White\textsuperscript{1,2}, Jeremy L. Tinker\textsuperscript{3} and Cameron K. McBride\textsuperscript{4}

\textsuperscript{1} Lawrence Berkeley National Laboratory, 1 Cyclotron Road, Berkeley, CA 94720, USA
\textsuperscript{2} Departments of Physics and Astronomy, University of California, Berkeley, CA 94720, USA
\textsuperscript{3} Center for Cosmology and Particle Physics, Department of Physics, New York University, New York, NY 10003, USA
\textsuperscript{4} Harvard-Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge, MA 02138, USA

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ABSTRACT

Sophisticated analysis of modern large-scale structure surveys requires mock catalogs. Mock catalogs are used to optimize survey design, test reduction and analysis pipelines, make theoretical predictions for basic observables and propagate errors through complex analysis chains. We present a new method, which we call “quick particle mesh”, for generating many large-volume, approximate mock catalogs at low computational cost. The method is based on using rapid, low-resolution particle mesh simulations that accurately reproduce the large-scale dark matter density field. Particles are sampled from the density field based on their local density such that they have $N$-point statistics nearly equivalent to the halos resolved in high-resolution simulations, creating a set of mock halos that can be populated using halo occupation methods to create galaxy mocks for a variety of possible target classes.

Key words: cosmology: large-scale structure of Universe, cosmological parameters, galaxies: halos, statistics

1 INTRODUCTION

The study of the large-scale structure in the Universe is a cornerstone of modern cosmology. In addition to allowing us to understand the structure itself, such studies offer an incisive tool for probing cosmology and particle physics and set the context for our modern understanding of galaxy formation and evolution. Large galaxy surveys play a key role in this enterprise, and ever larger surveys have provided increasing insight and ever tighter constraints on cosmological models. For almost as long as there have been sky surveys, people have used mock catalogs in order to interpret them (e.g. Neyman, Scott & Shane\textsuperscript{1953}, Scott, Shane & Swanson\textsuperscript{1954}, Soneira & Peebles\textsuperscript{1977, 1978}, Shanks\textsuperscript{1979}). With time, surveys have become larger and the type of questions we ask from them have changed significantly, becoming steadily more quantitative. Driven by the need for ever more precise theoretical predictions and the use of increasingly complex and sophisticated data-analysis algorithms, simulations and synthetic data sets have become increasingly important.

Within the modern paradigm, wherein galaxies and quasars form and evolve within the halos of the cosmic web, the natural technique for creating mock catalogs is N-body simulation (see e.g. Springel, Frenk & White\textsuperscript{2006} for a recent review). N-body simulations have been the workhorse of modern cosmology for several decades, and codes with high force resolution can accurately predict the abundance, spatial distribution, profiles and substructure of dark matter halos in representative cosmological volumes (e.g. Springel et al.\textsuperscript{2005}, Kahlen et al.\textsuperscript{2012}). Such runs are often quite expensive, however, and while it is certainly possible (e.g. Fosalba et al.\textsuperscript{2008}, McBride et al.\textsuperscript{2009}, Heitmann et al.\textsuperscript{2010}) the practicality of running very large numbers of them for Monte-Carlo studies is unclear\textsuperscript{1}. This is especially true if the majority of the information can be obtained more cheaply. In this vein there are two important considerations: the total run-time of any given method and the total memory requirement. The latter is in many ways the most stringent if we wish to enable the generation of many simulations on commonly available hardware.

Over the years a large number of approximate methods have been developed which produce halo catalogs of sufficient reliability for many tasks, such as predicting the very large-scale distribution of galaxies in simulations. Many of these methods started out as a means to highlight key characteristics of non-linear structure formation and were implemented numerically (this includes such methods as the adhesion approximation [Gurbatov, Saichev & Shandarin\textsuperscript{1989}], [Weinberg & Gunn\textsuperscript{1990}], [Kofman et al.\textsuperscript{1992}], [Melott, Shandarin & Weinberg\textsuperscript{1994}, Valageas & Bernardeau\textsuperscript{2011}], the log-normal model [Coles & Jones\textsuperscript{1991}], the truncated Zel’dovich approximation [Coles, Melott & Shandarin\textsuperscript{1993}, Melott, Pellman & Shandarin\textsuperscript{1994}], the frozen flow approximation [Matarrese et al.\textsuperscript{1992}], the free-particle approximation [Short & Coles\textsuperscript{2006}], PATCHY [Scoccimarro & Sheth\textsuperscript{2002}, Manera et al.\textsuperscript{2013}], Pinocchio [Monaco, Theuns & Taffoni\textsuperscript{2002}, Monaco et al.\textsuperscript{2013}], remapping Lagrangian perturbation theory [Leclercq et al.\textsuperscript{2013}], PATCHY [Kitaura, Yepes & Prada\textsuperscript{2013}], and machine-learning techniques [Xu et al.\textsuperscript{2013}]). A recent summary of some of

\textsuperscript{1} Or at the very least, it limits the applicability to a small number of groups.
these methods can be found in [Neyrinck (2012)]. One of the key observations is that much of the filamentary cosmic web of structures which arises within N-body simulations is also present in these more approximate methods. Indeed, much of the “work” involved in running an N-body simulation in a large volume is evolving linear theory, or modes which are not far from linear. This gives some hope that approximate and cheap algorithms can capture many of the important properties of full N-body simulations.

The most widely used approximation is the log-normal model, which has been used to create mock catalogs for many large-scale structure surveys in the past decade (e.g., Cole et al. 2005; Percival et al. 2010; Reid et al. 2010; Blake et al. 2011; Beutler et al. 2011, 2012, among others). By contrast, the BOSS team used mocks based on second-order Lagrange perturbation theory in their recent cosmology analyses (PThalos; Manera et al. 2013).

In this paper we present a new, approximate, method for generating mock galaxy catalogs for the first step in a Monte-Carlo simulation of redshift surveys. There are two particular arguments which pushed us to consider the rapid generation of mock catalogs as a means to Monte-Carlo our errors. The first is that many steps in the analysis can be quite non-linear, for example the reconstruction procedure for BAO which involves constrained realizations of interpolating fields and non-linear motions of both galaxy and random points prior to computing the 2-point function. The interaction of these non-linearities with the complex observing geometry can be particularly difficult to model directly. The second is that we are primarily interested in quite large scales, where the complexities of galaxy formation and fully non-linear clustering are mitigated. On smaller scales internal estimates of the covariance matrix can be constructed (e.g. via bootstrap or jackknife), however on large scales these methods do not perform as well (e.g. Norberg et al. 2009 and references therein). For these reasons we wish to investigate a procedure or set of procedures which allow us to generate point sets which mimic the observational samples, at least in terms of low order statistical properties. As is often the case, increasing fidelity for each simulation comes at a price of increased computational complexity which implies fewer realizations (and more noise in the Monte-Carlo, see e.g. Taylor, Joachimi & Kitching 2013) for a fixed computational effort.

The outline of this paper is as follows. In Section 2 we discuss different approaches to producing mock catalogs and the direct N-body simulation method which shall serve as our benchmark. Section 3 details the “quick particle mesh” (QPM) method, the main substance of this paper. Section 4 compares the mock catalogs produced by a variety of different methods, including QPM, to those derived from an N-body simulation. We briefly describe the public implementation of this method, mockFactory, in Section 5. Finally, we discuss limitations, extensions, and comparisons to other methods recently presented in Section 6. Unless otherwise stated, the assumed cosmology is flat ΛCDM with \( \Omega_m = 0.274, \Omega_b = 0.046, \Omega_{\Lambda} = 0.726, h = 0.7, n = 0.95, \) and \( \sigma_8 = 0.8 \).

## 2 MOCK CATALOG METHODS

Modern galaxy formation models assume that galaxies form and remain in the potential wells of dark matter halos. The four basic steps of creating mock galaxy catalogs are (1) predicting the evolution of the underlying mass field, (2) locating and characterizing the properties of dark matter halos, (3) populating the halos with mock galaxies and (4) applying survey characteristics to the box of galaxies. The halo to mock galaxy mapping is often trained by matching small-scale clustering statistics to the observational galaxy sample (e.g., Reid & Spergel 2009; White et al. 2011). The last step involves adding survey-specific realism to the mock galaxy distribution, such as applying radial and angular selection functions to match the geometry. The final goal is to produce a set of points that statistically matches the spatial distribution of observational galaxy samples, and in our case to be able to do this many times so as to characterize the probability distributions of observables.

Characterizing dark matter halos, while not strictly required, is a useful step. By definition, halos represent overdense regions of the mass field (typically 100-300 times the mean density or 200 times the critical density) that arise from non-linear gravitational collapse. Approximate methods of predicting the mass evolution do not attempt to accurately calculate this collapse. Instead the focus is on larger scales dominated by linear and weakly non-linear gravitational evolution. The strongly non-linear interactions within halos, which are represented in the observational galaxy distributions, can be added with analytic prescriptions on top of the halo distribution. If adequate halo catalogs can be generated, the subsequent steps towards survey-specific mock galaxy catalogs are comparatively straightforward.

For this work, we will compare various fast methods of halo catalog generation against the relevant properties of halo catalogs derived from periodic box N-body simulations. We have implemented three approximate methods of mock catalog creation. Two of these represent recent methods used to model galaxy surveys, namely the log-normal (LGN) model and the method of Manera et al. (2013), based on second order Lagrangian Perturbation theory, which we shall refer to as LPT. We discuss the details of our implementations in Appendix A. We compare both of these models to our new method which we refer to as “quick particle mesh” or QPM which we describe in detail in Section 3.

The focus of our investigation resolves halo masses and volumes appropriate to current modern redshift surveys, specifically galaxies in SDSS-III BOSS (Dawson et al. 2012). This is a practical choice; the application of these methods is certainly not restricted to modeling these types of galaxies. Indeed, our method is flexible enough to create mocks for a variety of survey specifics and target types, even within the same mock galaxy distribution.

### 2.1 N-body simulations

We make use of several N-body simulations in this paper, each of the ΛCDM family with the same cosmology (\( \Omega_m = 0.274, \Omega_b = 0.046, \Omega_{\Lambda} = 0.726, h = 0.7, n = 0.95, \) and \( \sigma_8 = 0.8 \)). These (high resolution) simulations will form our benchmark and be the fiducial model of “truth”. The have also been used in [White et al. 2011; Reid & White 2011; White et al. 2012] and more details can be found in those papers.

Our high resolution simulations resolve all of the halos described throughout the paper. Briefly, we used an updated version of the TreePM code described in White (2002) to evolve 3000\(^3\) particles (5.9×10\(^{10}\) h\(^{-3}\) M\(_{\odot}\)) in a box of side 2750 h\(^{-1}\)Mpc. We ran 2 realizations of this simulation, differing only in the random number seed chosen for the initial conditions. We also ran 20 realizations of the same cosmology using 1500\(^3\) equal mass (7.6×10\(^{10}\) h\(^{-3}\) M\(_{\odot}\))
particles in a periodic cube of side length $1500 h^{-1}$Mpc. This second set allow for better estimation of the sample variance at large scales. For each simulation, the initial conditions were generated by displacing particles from a regular grid using second order Lagrangian perturbation theory (Buchert 1989; Moutarde et al. 1991; Hivon et al. 1995) at $z = 75$, where the rms displacement is 10 per cent of the mean inter-particle spacing.

For each output we found dark matter halos using a friends-of-friends algorithm (FoF; Davis et al. 1985) with a linking length of 0.168 times the mean inter-particle spacing. This partitions the particles into equivalence classes roughly bounded by isodensity contours of $100 \times$ the mean density. The position of the most-bound particle and the center of mass velocity are stored for each halo and used in the comparisons described below.

3 QPM: THE METHOD

The QPM method uses a low resolution particle-mesh (PM) N-body solver to evolve particles within a periodic simulation volume as is common in the field. The time steps are set to be quite large and the mesh scale and mean inter-particle spacing exceed the size of all but the largest dark matter halos. In this manner we keep both the run time and the memory requirements modest.

The particle-mesh evolution of $N$ particles within a periodic cube employs fast Fourier transforms on a fixed Cartesian mesh to compute the force. In our configuration, the force mesh has as many nodes as the simulation has particles, and we use $2 h^{-1}$Mpc as the mean inter-particle spacing as our default. We start each simulation at $z = 25$ using second order Lagrangian perturbation theory. The rms particle displacement is 15 per cent of the mean inter-particle spacing at this redshift. The code evolves the particles using a second order leap-frog method, with time steps of $\Delta t = 0.0005$.

At each step the potential on the grid is computed using a $1/k^2$ kernel and the force is derived from the potential using 4th order differencing.

We experimented with different choices for the time step, number of particles and mesh spacing. The choice of time step is a major driver of run time and the choice of mesh spacing is a major factor in memory requirements. Since we are primarily interested in quite large scales we found that even very large time steps and coarse force meshes were sufficient to get convergence to the required accuracy in our mock halo catalogs. Setting the mesh and particle numbers equal was convenient. Our choice of 15 per cent in $\ln a$ provided a good trade-off between adequately resolving the change in the growth of structure during the matter-dominated to A-dominated transition and speed. Steps in $a$ and in the linear growth rate produced similar behavior, we decided on constant steps in $\ln a$ partly out of simplicity.

In QPM we only resolve the density field on large-scales and so must decide how to partition the mass, which in hierarchical theories forms in a bottom-up manner, into a spectrum of dark matter halos using only large-scale information. We have chosen to do this using the local density for each particle at the time of relevance$^\dagger$

\begin{equation}
\mu = \langle \ln(1 + \delta) \rangle
\end{equation}

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{figure1.png}
\caption{The large-scale bias, $b$, of particles selected based on their local density. The probability of selection is a Gaussian in $\ln(1 + \delta)$ with mean $\mu$ and width $\sigma$. We show the bias as a function of $\mu$ for two values of $\sigma$: 0.1 (blue squares and dotted lines) and 0.2 (red triangles and dashed lines).}
\end{figure}

Based on its density we annotate a subset of the simulation particles as mock dark matter halos, recording the position and velocity of these “halos”. Selecting particles based on their local density allows us to work with low particle numbers and hence low computational cost as well as offering a different avenue to explore compared to full simulation. This is similar in spirit to the method used in Cole et al. (1996) or Wechsler et al. (2004), although the technical details differ somewhat.

There is no reason, in principle, why we cannot skip the halo creation step and generate a sample of mock “galaxies” directly by sampling particles. However we have found it very useful to have halo information for our samples, and it allows tighter connection with halo-occupation modeling, as well as the ability to model multiple target samples within the same mock—e.g., red and blue galaxies, or bright and faint galaxies—essentially, and target population that can be modeled within the halo occupation context. Thus we shall always go through halos in this paper.

To keep the run-time as small as possible the density is estimated using Fourier methods on the same mesh as is used for the force calculation. The density is interpolated onto and off of the mesh using cloud-in-cell interpolation, and the density is smoothed with a Gaussian kernel of 1 mesh cell in width (i.e. $\sigma = 2 h^{-1}$Mpc). If the smoothing kernel is too large then a scale-dependent bias is introduced near the acoustic peak in the correlation function at $100 h^{-1}$Mpc. For a $2 h^{-1}$Mpc Gaussian smoothing the scale-dependence is small enough that it is not a concern. Once the density field is computed using all of the particles, the position, velocity, and density of a random subset of the particles are saved.

As we intend these particles to stand in for halos, it might be better to store the average velocity smoothed on some scale rather than the particular velocity of the particle. However, at the scales of interest and the resolutions we choose, we are relatively insensitive to “virial” motions within halos, so we use the particle velocity for simplicity. In fact, comparing the pairwise velocity dispersion

\footnote{Sampling particles with probability $p(\delta_{\text{smth}})$ is the same as computing the correlation function of $(1 + \delta_{\text{smth}}) / \sqrt{1 + \delta_{\text{smth}}}$, where $\delta_{\text{smth}}$ is the density field defined by the particles (i.e. unsmoothed). In this correlation function mode-coupling introduces scale-dependent bias.}

\dagger We are not restricted to using only the local density smoothed on a single scale in deciding where to place halos in our QPM field. We can use the density smoothed on multiple scales, the density field in the initial conditions, or other quantities (e.g. the tidal shear or the initial peak curvature) as well. We have not explored this issue further, but it is an interesting question which set of large-scale observables of the matter field best predicts the existence of a halo in a higher resolution simulation.
of particles in the PM simulations with those in a higher resolution simulation we see that the small-scale velocity field is not well resolved by the PM simulation. Thus we add an additional, Gaussian random velocity of 125 km s\(^{-1}\) to each component of the halo velocity. As this value is dependent on the details of the simulation we make it a free parameter in the model and the software described below. It affects the measured quadrupole moment of the correlation function on small scales and can in principle be adjusted to improve agreement with measurements on small scales, if desired.

The mock halo catalog at each output time is constructed in post-processing. Our goal is to choose particles from the saved subset, with a density-dependent probability, and have them stand in for halos of a given mass. We select the particles and assign halo masses so as to match the mass function and large-scale bias of halos, as determined from high-resolution simulations (e.g. Tinker et al. 2008, 2010). The choice of sampling function is arbitrary, and we tried several. A convenient form is a Gaussian in \(\ln(1 + \delta)\), with a mean \(\mu\) and a width \(\sigma\) which we can adjust to get the desired clustering. Higher \(\mu\) in general leads to a higher large-scale bias, as shown in Fig. 1.

In our fiducial models we held \(\sigma = 0.1\) fixed and adjusted \(\mu(b)\) so as to reproduce the large-scale bias, \(b\), as a function of halo mass (taken from Tinker et al. 2010). The same large-scale clustering can be achieved by simultaneously increasing \(\mu\) and \(\sigma\) (see Fig. 3), but we are more interested in the models with lower \(\sigma\) and holding \(\sigma\) fixed is a convenient choice.

We divided the range of halo masses we wish to produce into \(N_b\) bins spaced equally in log \(M_h\). The results are insensitive to the value of \(N_b\) as long as \(b(M_h)\) doesn’t vary significantly over the bin. For each mass bin we know how many particles we need to sample to match the mass function (Tinker et al. 2008) and we know the Gaussian we need to sample from to match the desired large-scale bias (from the calibration of \(\mu(b)\) above). Thus we simply loop over the particles and select halos in a mass bin from them with the necessary probability. The resulting set of mock halos has the proper number density and large-scale bias as a function of mass. Note that this procedure is equivalent to using Bayes’ theorem in the form \(P(M_h|x)\propto P(x|M_h)P(M_h)\) with \(x = \ln(1 + \delta)\).

It is of course possible to simultaneously adjust both \(\mu\) and \(\sigma\) as a function of halo mass to better match the scale-dependence of halo bias at smaller scales, but we did not find that to be useful for our purposes.

Recently Lassey, Zaldarriaga & Eisenstein (2013) introduced a rapid simulation method which they referred to as “COLA: COMoving Lagrangian Acceleration”. This method accelerates a PM code by solving for the evolution of large-scale structure in a frame that is comoving with trajectories calculated in Lagrangian Perturbation Theory (LPT). While our procedure is quite similar in spirit, we have chosen not to implement this speed-up for three reasons. The first is that the run-time of our simulations is already short (taking only a few more steps than COLA). The second is that the COLA method carries with it memory overhead (holding the first and second order Lagrangian displacements for each particle) and memory is the primary driver for simulation size or volume in our situation. Finally the COLA method, like any PM scheme, resolves halos and halo positions reliably only when the mesh scale is significantly finer than the mean inter-particle separation and the halo virial radius. If run in this regime over very large volumes the code requires a large amount of memory to hold the force mesh, and this severely limits the machines upon which it can be run.

### 4 COMPARISON OF METHODS

In this section we compare several different methods of making mock catalogs with those based upon the halos found in high resolution N-body simulations. We shall treat the high resolution simulations as “truth” for the purposes of the comparisons.

#### 4.1 Mass comparison

While we will primarily be concerned with halo and galaxy catalogs in this paper, we begin by comparing the matter fields produced by LPT and QPM with that of the TPM simulation. For simplicity we use one of the larger box simulations and in making this comparison we ensure that all of the methods use the same phases in the linear theory realization, so we expect the structure to match in position across simulations.

A visual comparison of the density field produced by these three methods is given in Fig. 4, which shows as a greyscale image the mass density in a thin (4\(h^{-1}\) Mpc) slice through the box. The slice is centered on the \(3^\text{rd}\) most massive halo in the 2.75\(h^{-1}\) Gpc box and zooms in from a region 256\(h^{-1}\) Mpc on a side to 16\(h^{-1}\) Mpc on a side. The 256 \(\times\) 256 \(\times 4\)\(h^{-1}\) Mpc slice (top row) shows that all three methods produce the same filamentary structure on large scales (with the random phases matched). The 8\times\) coarser mass resolution in the QPM (middle column) simulation compared to the TPM simulation (left column) and the lack of small-scale power in the LPT simulation (right column) is already evident in the middle row. These trends are more apparent in the bottom row, where we see the QPM code has merged a number of halos together while the LPT code does not produce a bound structure at all. In the original PThalos algorithm (Scoccimarro & Sheth 2002) the mass field was “corrected” by superposing the profiles of bound dark matter halos as measured in high-resolution simulations. In the LPT method of Manera et al. (2013) this is not done (for the mass field) because the focus is on populating halos with galaxies. Were a more accu-
rate mass field desired, it would be straightforward to modify the algorithm to include this step.

In general the matter density field produced by the QPM simulation is highly correlated with that of the TPM simulation on scales above the mesh scale. The power spectra of the matter fields in the TPM and QPM simulations agree to better than 5 per cent to $k \approx 0.35 \, h^{-1}\text{Mpc}^{-1}$ beyond which the QPM simulation has less power than the TPM simulation. The cross-correlation is above 95 per cent in Fourier space for $k < 1 \, h^{-1}\text{Mpc}^{-1}$ and above 95 per cent in configuration space for cubic cells larger than $2.7 \, h^{-1}\text{Mpc}$ (it falls to 87 per cent on the mesh scale of the QPM simulation). The cross-correlation between the initial density field and the field at $z \approx 0.55$ in the TPM simulations is also well reproduced by the QPM and LPT runs, as shown in Fig. 4.

The distribution of the counts-in-cells is very similar for the QPM and TPM runs on scales above $5 \, h^{-1}\text{Mpc}$. For the LPT runs there is a pronounced lack of high density cells, as might be expected from Fig. 3. The discrepancy is an order of magnitude at $\rho = 10 \bar{\rho}$ even for $10 \, h^{-1}\text{Mpc}$ cells.

One way of characterizing the filamentary nature of large-scale structure is through the configuration dependence of the 3-point function (in real space). As an example, the 3-point function for triangles with $r_2 = 2 \, r_1$ is shown in Fig. 5. We see that the 3-point function of the QPM simulation agrees very well with that of the TPM simulation on all scales plotted. The LPT density field is also in good agreement with TPM on larger scales, but appears to be slightly more circular (i.e. less filamentary) for the smaller triangles.

These comparisons suggest that the QPM simulation is well...
resolving the large-scale structure in which the mock halos are to be placed, validating its use as the input to the mock halo creation step.

4.2 Halo comparison

Each of our approximations provides us with a catalog of halo masses, positions and velocities and galaxy (luminosities,) positions and velocities. While our primary interest is in the galaxy catalogs, we first discuss the halos upon which they are built since such galaxy catalogs are limited by the accuracy of the input halo catalogs.

The halo catalogs are necessarily sparser than the particles which defined the mass density field, so in order to have better statistics across a wide range of scales we make use of the 20 lower resolution simulations, with a corresponding 20 mock simulations for each. Throughout we shall plot the average of each statistic over the 20 realizations to reduce sample variance. Fig. 6 shows the real-space halo correlation function (i.e. configuration space 2-point function) for the 4 mock catalogs for a sample of halos with $M > 10^{13}h^{-1}M_{\odot}$, chosen to roughly match the number density of BOSS CMASS galaxies (White et al. 2011) Anderson et al. 2012 Dawson et al. 2013) though other mass ranges look qualitatively similar.

Fig. 6 shows that the large-scale bias of the halos produced by all of the methods is approximately correct. (We are not concerned here with the level of agreement below $10h^{-1}$Mpc scales, though we shall come back to that in the next section.) On large scales we see that the LGN mocks agree almost perfectly with the TPM mocks, as expected. Those mocks are designed to produce a given 2-point function, in this case produced using convolution Lagrangian perturbation theory (CLPT; Carlson, Reid & White 2013). Since CLPT provides a good fit to the halo correlation function we fully expect our LGN mocks to fit as well, and this expectation is born out. The QPM mocks provide an adequate fit to the real-space correlation function, with a slight excess clustering near $20h^{-1}$Mpc. This excess appears visually quite significant due to the choice of $r^2\xi$ as the $y$-axis, but is only 3 per cent in $\xi$ at $20h^{-1}$Mpc and is completely acceptable for our purposes. Finally we see that the LPT mocks tend to undershoot the correlations over a wide range of scales and have a slightly different shape near the acoustic peak ($r \sim 100h^{-1}$Mpc). This is due to our choice of a broad range of halos masses and a single linking length. For a narrower range of halos we can adjust the linking length to obtain a better match, but we found it difficult to simultaneously match the bias and mass function across a wide range of halo masses with a single value of the linking length.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4.pdf}
\caption{The Fourier-space cross-correlation coefficient, $r \equiv P_{ij}/\sqrt{P_iP_j}$, between the initial conditions and density field at $z = 0.55$. The lines are for TPM (red solid), QPM (blue dashed) and LPT (green dotted). Note that the cross-correlation, sometimes called the “propagator”, seen in the TPM simulation is well reproduced by the QPM and LPT approximations on all scales that are of interest here.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5.pdf}
\caption{The real-space 3-point function of the matter density at $z = 0.55$ from the three simulations. We show 3-point configurations with side lengths in the ratio $r_2 = 2r_1$ as a function of the cosine of the included angle ($\mu$). The upper panel shows $r_2 = 10h^{-1}$Mpc and the lower panel $r_2 = 20h^{-1}$Mpc. The lines are for TPM (red solid), QPM (blue dashed) and LPT (green dotted). Note that the matter distribution is more circular, or less filamentary, on small scales for LPT than for QPM and TPM, but on large scales all methods agree very well.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure6.pdf}
\caption{The real-space 2-point correlation function, $\xi(r)$, for halos with $M > 10^{13}h^{-1}M_{\odot}$. The shaded red band shows the 1σ error on the mean of the correlation function from the 20 TPM mocks. The uncertainty on the other lines (QPM: dashed blue, LPT: dotted green, LGN: dot-dashed magenta) is of the same size and is discussed further in the text. The LGN model almost perfectly overlaps the TPM $\xi(r)$, by construction.}
\end{figure}
Figure 7. The redshift-space monopole (upper) and quadrupole (lower) 2-point correlation functions for halos with $M > 10^{13} \, h^{-1} M_{\odot}$. We have divided each spectrum by a smooth model which is the linear theory, real-space spectrum using the smooth fit to the linear theory transfer function of Eisenstein & Hu (1998) and a scale-independent bias of $b = 2$. The amplitude thus reflects the effects of redshift-space distortions as well as non-linearity, and the acoustic oscillations are highlighted. The shaded red band shows the $1\sigma$ error on the mean of the correlation function from the 20 TPM mocks. The uncertainty on the other lines (QPM: dashed blue, LPT: dotted green, LGN: dot-dashed magenta) is of the same size and is discussed in the text.

Figure 8. The redshift-space monopole (upper) and quadrupole (lower) dimensionless power spectra for halos with $M > 10^{13} \, h^{-1} M_{\odot}$. We have divided each spectrum by a smooth model which is the linear theory, real-space spectrum using the smooth fit to the linear theory transfer function of Eisenstein & Hu (1998) and a scale-independent bias of $b = 2$. The amplitude thus reflects the effects of redshift-space distortions as well as non-linearity, and the acoustic oscillations are highlighted. The shaded red band shows the $1\sigma$ error on the mean of the spectra from the 20 TPM mocks. The uncertainty on the other lines (QPM: dashed blue, LPT: dotted green, LGN: dot-dashed magenta) is of the same size.

The situation in redshift-space is shown in Fig. 7 which presents the monopole and quadrupole moments of the correlation function. For the monopole all of the mocks agree relatively well. Once again the LPT mocks show a slight deficit of clustering across a broad range of scales, the QPM mocks overshoot near $20 \, h^{-1} \text{Mpc}$ and the LGN mocks match very well. For the quadrupole the agreement is less good, but still quite acceptable given that the observations tend to have larger errors on the quadrupole than the monopole.

Figure 9. The real-space 3-point function of the halo density at $z = 0.55$ from the four mock catalogs. We show 3-point configurations with side lengths $r_2 = 2r_1 = 20 \, h^{-1} \text{Mpc}$ as a function of the cosine of the included angle ($\mu$). The agreement between the TPM (solid red) and LPT (dotted green) mocks is very good on these scales, and slightly better than the agreement between TPM and QPM (dashed blue). The LGN mocks (dot-dashed magenta) are clear outliers.

An alternate view of the redshift-space statistics is given in Fig. 8 which presents the monopole and quadrupole power spectra divided by a smooth model. The trends are very similar to those in Fig. 7. Figure 9 shows the configuration-space 3-point correlation function for triangles with $r_2 = 2r_1 = 20 \, h^{-1} \text{Mpc}$ as a function of the cosine of the included angle. The TPM and LPT mocks agree quite well in the amplitude and shape of the 3-point function. The QPM mocks agree slightly less well, but still have approximately the right shape, indicating the prevalence of filaments in the cosmic web. (Note that $10 - 20 \, h^{-1} \text{Mpc}$ are the scales where the QPM mocks agree least well with the TPM simulation for the two point function.) The clear outlier is the LGN mocks, where the remapping of the original Gaussian density field does not produce the same filamentary, beaded web as is produced by gravitational evolution and the shape dependence of the 3-point function is very different from that of the other mocks. From this we infer that laying down halos based on a Gaussian field does not produce the right structure in the halo density field. Laying down halos based on the non-linearly evolved field does a slightly better job, but since the mock halos are selected at random based only on local density the filamentary nature of the halo distribution is slightly washed out. Finding overdense regions in the evolved field does best of all, but is the most expensive in terms of memory or computational cost.

Figure 10 shows the fractional uncertainty on the correlation function, computed from the 20 independent simulations, for each of the methods. We expect linearly biased tracers of the matter field to behave in a similar fashion on large scales, and that is born out in this comparison. The trends for the real- and redshift-space statistics are very similar, and largely agree between the 4 mock methods. Recall that with only 20 simulations, if we approximate the individual measurements as Gaussian we would expect an error on our fractional error of 30 per cent, so we are unable to statistically distinguish between these approximate methods. Unfortunately it is too expensive to increase the number of TPM simulations to reduce the error and provide a stronger test. Near the acoustic peak
the fractional error on the real-space correlation function for this volume and number density, assuming linearly biased linear theory and Gaussian statistics, is in good agreement with the numbers shown in Fig. [10] and the error is almost totally dominated by sample variance. Given that all of the methods are doing a good job of modeling the largest scales which dominate the sample variance, it is not unreasonable to expect they produce the same scatter in the correlation function.

4.3 Galaxy comparison

From the halo catalogs we create galaxy catalogs using a halo-occupation-distribution approach (Peacock & Smith 2000; Seljak 2000; Benson et al. 2000; Scoccimarro et al. 2001; White, Hernquist & Springel 2001; Berlind & Weinberg 2002; Cooray & Sheth 2002). We use the HOD parameterization of Tinker et al. (2010).

Once a set of HOD parameter values has been chosen, we populate each mock halo in a given output with mock “galaxies”. The HOD provides the probabilities that a halo will contain a central galaxy and the number of satellites. The central galaxy is placed at the center of the halo (which is the position of the most bound particle in the halo for the FoF halos and the position of the particle for the mock halos), and we place satellites assuming a spherical profile with a concentration determined using the method of Munoz-Cuartas et al. (2010).

We follow standard practice and adjust the HOD parameters for each mock to fit the small-scale projected two-point clustering. As an illustrative example we have chosen to use the projected correlation function of BOSS CMASS galaxies reported in White et al. (2011). In each case we are able to match the clustering between the mocks and the observational measurements, as shown in Fig. [11].

Since the log-normal mocks don’t produce a halo catalog with assigned masses, we don’t include them in this section. The way in which log-normal mocks are typically used is to directly produce the galaxies from the log-normal density field, rather than going through an intermediate step. The comparison is then qualitatively similar to that for the halos which we discussed above and we gain little insight by repeating it here.

The real-space correlation function for each of the mock galaxy catalogs is shown in Fig. [12] which is the analog of Fig. [6]. As in that case we see that the mocks agree with the high-resolution N-body simulation very well over a broad range of scales. The QPM mocks have slightly more power near 20 $h^{-1}$ Mpc but only at the few per cent level. This indicates that we are placing halos correctly, in a statistical sense, within the large-scale mass field and any higher order correlations between halos that we are missing by such a random assignment don’t impact the low-order galaxy statistics.

The situation in redshift-space is shown in Fig. [13] which is the analog of Fig. [7] and presents the monopole and quadrupole moments of the correlation function. The level of agreement is very much as we saw in Fig. [7] indicating that we are modeling fingers-of-god and satellite fractions comparably in all three cases. The complementary view is given in Fig. [14] which shows the monopole and quadrupole of the redshift-space power spectrum.
Finally we show the fractional uncertainty in the real-space, monopole and quadrupole correlation functions vs. scale in Fig. 15. From 20 realizations we expect the error on the error to be about 30 per cent, so we are unable to statistically distinguish these different approximations from the simulations we have.

4.4 Computational cost

For each of the mock catalog methods described above the most time consuming steps involve fast Fourier Transforms. We use the parallel, real Fourier transforms in the FFTW package. The FFT scales with problem size as $N \log N$, so that doubling the linear dimension of the grid increases the run time by approximately an order of magnitude (in addition to requiring about an order of magnitude more memory). The additional cost of moving the particles is small, though with more particles and more clustered particles the time taken to find halos with the FoF algorithm and halo properties (for those cases where this is a step in the process) increases and can become a significant fraction of the total runtime for some methods.

As each of the methods has the same underlying operation as its most computationally demanding step, we can roughly compare “cost” by simply comparing the number of FFTs used for each. The LGN mocks require 4 FFTs (one to convert from $\delta(k)$ to $\delta(x)$ and then 3 to compute the displacements). In our implementation the LPT mocks require 13 FFTs. With our fixed time steps the QPM code takes about 20–30 time steps and requires 2 FFTs per step (plus 13 FFTs for the generation of the initial conditions). Since we use a lower resolution with fewer particles for the QPM run however each step is about 8× faster and requires less memory than the LGN or LPT mocks. For the same particle number then the LGN simulations run a few times faster than the LPT simulations, which are comparable to the QPM simulations which is more than two orders of magnitude faster than the TPM simulation.
applying redshift-space distortions and trimming to the survey geometry. Our implementation of these steps has three separate parts, any of which can be used independently: (1) creation of mock halo files from local densities, (2) application of an HOD to map mock galaxies to halos, and (3) transformation of the periodic box of objects into a realistic catalog on the sky. We make this code, referred to as mockFactory, publicly available.

The first step was the topic of the previous sections. The code selects, from a random sample of dark matter particles with densities, a set of mock halos with masses. The halos match a specified mass function and large-scale bias vs. mass relation, for which we use the fits of Tinker et al. (2008, 2010) for overdensity $\Delta = 200$ halos. The result of this stage is a periodic box of halos, stored as a HaloFile. The operation of the code is controlled through a parameter file, which is ascii text. The performance and memory considerations of the code depend on the size of the PM simulation and the fraction of particles that are retained in the subsample file from which the mock halos are culled. For test runs on a PM simulation with 1280$^3$ particles, of which 10% were retained in the subsample file, the run-time on a typical workstation core is $\sim 50$ minutes to create the mock halos, requiring 5.5 GB in memory (effectively no overhead relative to the size of the input subsample file). The run time scales linearly with the size of the subsample file.

The second part of the code populates a halo catalog (typically the HaloFile described above, but it is possible to use any halo file provided it is in the proper format) with artificial galaxies. This applies a HOD as described in Section 4.3. The central galaxy is placed at the center of the halo and satellites are placed assuming a spherical profile with a concentration determined using the method of Munoz-Cuartas et al. (2010), which can be scaled by a factor in the input parameter file. In the examples used in this paper, we use the central-satellite HOD parameterization used in Tinker et al. (2012), although the code has a number of options for these parameterizations. The code can also adjust the input HOD parameters to match a desired galaxy number density by rescaling all the halo mass values by the same factor. To fill the HaloFile with mock galaxies requires substantially less time and memory: approximately 3 minutes for the example introduced above and only the HaloFile is held in memory.

The third part of the code, called make_survey, takes a periodic box of objects (mock galaxies, quasars, halos or particles from a simulation box) and projects them on the sky, optionally applying various layers of realism to represent a mock survey. Our implementation can perform a number of steps which include:

(a) volume remapping of the periodic box using BoxRemap (Carlson & White 2010),
(b) box translation and rotation,
(c) sky projection using cosmological distances,
(d) modeling redshift distortions using peculiar velocities,
(e) trimming to a survey footprint using a MANGLE mask (Swanson et al. 2008),
(f) downsampling based on angular sky completeness, and
(g) downsampling based on radial selection.

A configuration file controls which of these steps are applied, as well as giving any required parameters. In addition, several tools exist within the code base to assist the user in determining and testing the parameters. The make_survey code is written to be relatively efficient. It reads the input catalog one object at a time, processes each object through all required steps, and outputs any object that makes all cuts before moving on to the next. This results in minimal memory usage as the input catalog is never fully read into memory, nor is the output stored. The runtime is sufficiently fast such that it should not require significant computing resources even for a large number of mock catalogs.

6 SUMMARY

In the rising age of large-scale galaxy redshift surveys, the need for fast and accurate methods to create mock galaxy catalogs is at a premium. In this paper we have presented a new method for creating such mocks, which we call the “quick particle mesh” method (QPM). Our method is based on using rapid, low-resolution particle-mesh simulations that accurately reproduce the large-scale dark matter density field. Particles are sampled from the density field based on their local density such that the sampled particles have N-point statistics nearly equivalent to the halos resolved in high-resolution simulations. Thus our method creates a set of mock halos that can be populated using halo occupation methods to create galaxy mocks for a variety of possible target classes.

We compare the real- and redshift-space clustering statistics of our mock halos and mock galaxies to those of second-order Lagrange perturbation theory (LPT) and the log-normal method (LGN). These two methods are currently the most widely used for creating large sets of mocks. We use high-resolution N-body simulations (TPM) as “truth” in these comparisons, which focus on halo mass ranges applicable to those probed by BOSS-type samples. The halo clustering produced by the QPM method agrees quite well with TPM results. For two-point statistics, our implementation of LPT slightly underpredicts the halo clustering. The LGN method offers an excellent match to the TPM halo clustering, but cannot reproduce the behavior of the density field for higher-order statistics.

There are several benefits of our method compared to other current methods. In comparison to some perturbation-theory methods, QPM offers improvement in terms of both run-time and memory requirements. Although a PM simulation performs more FFTs than LPT, with our particle sampling scheme the grid size required to achieve the same halo mass resolution is less, thus the transforms run significantly faster and require less memory. Although we restrict our comparisons to halo mass ranges $\gtrsim 10^{13} h^{-1} M_\odot$, the lower mass limit of the QPM HaloFile can be set to any required value. Increasing the mass resolution of LPT requires increasing the size of the density grid on which the FFTs are performed. We note, however, that our current implementation of QPM is designed for BOSS-type galaxy samples, thus achieving accurate halo catalogs at $M_{halo} \lesssim 10^{13} h^{-1} M_\odot$ requires additional calibration. When comparing TPM to the LGN catalogs we find the LGN method reproduces the two-point clustering better than either QPM or LPT. The filamentary nature of the field is not well reproduced however. In addition, a single QPM HaloFile can be used to create galaxy mocks for a variety of targets with varying bias values; the LGN simulation is tuned to a specific value of bias, thus must be re-run for any variation of target class. In some situations this is a disadvantage.

The QPM method, as discussed here, uses a simple scheme of determining mock halos from local particle densities. While successful, there were a few discrepancies in the halo clustering that were not present in the mass fields. It is possible that an improved...

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9 http://github.com/mockFactory
10 http://mwhite.berkeley.edu/BoxRemap/
mapping could be found, perhaps using more information from the density field (or initial conditions) or conditionally selecting particles based on previous "draws", and this would mitigate some of these discrepancies. We have not explored this avenue here as we do not believe the discrepancies to be a serious limitation at present – the differences are small and often reduced further once mapped to mock galaxies.

We have released our QPM implementation as part of the mockFactory software package. Users supply their own PM simulations, and the code can create the mock halo file, create the mock galaxy catalog, and convert this galaxy catalog from a periodic cube to a cut-sky angular mock in redshift space. We do not supply the PM code, but there are many public codes that have been thoroughly tested and used throughout the community, e.g. the code of Klypin & Holtzman [1997], PMFAST (Merz et al., 2005), as well as tree-particle-mesh codes that can be run as PM-only, such as GADGET2 (Springel 2005) and the TPM code of Bode & Ostriker (2003). Additionally, if the reader is sufficiently motivated, instructions for creating one’s own PM code are also available.

QPM offers a method that is fast, requires relative little memory, and is highly flexible. It’s low computational requirements and high flexibility make QPM optimal for the next generation of large-scale structure surveys that will push the limits of current methods both in terms of the volume they probe and the bias of the galaxies they target.

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APPENDIX A: MOCK CATALOGS

We have compared the QPM method described in this paper to a number of commonly used approximate methods, in addition to high force and mass resolution simulations. In this appendix we give the technical details for the various mock catalog schemes, other than QPM, presented in this paper.

A1 Log-normal model

A popular method for creating mock catalogs is the log-normal model of Coles & Jones [1991]. This model has been used to generate mock catalogs for analyses of the 2dF (e.g., Cole et al., 2005), SDSS (most recently, Percival et al., 2010; Reid et al., 2010), WiggleZ (Blake et al., 2011), and 6dF (Beutler et al., 2011, 2012) surveys, among others.

In this model a Gaussian density field is generated with a correlation function $\xi_G$ and then the field is exponentiated to obtain a log-normal field which is then sampled (with probability proportional to density) to produce a set of particles. In order to obtain any desired $\xi_LN$ the correlation function for the Gaussian field should obey $\xi_G = \ln(1 + \xi_{LN})$. The density distribution of the resulting particle set is log-normal, while its two point function matches the desired function by construction. These particles thus behave in their low order statistics very similarly to the halos in N-body simulations or galaxies in observational surveys.

Often the log-normal model is used to produce fields in real space, ignoring redshift-space distortions. There are two ways of introducing redshift-space distortions into the model. First, one can follow the reasoning in Coles & Jones [1991] who point out that the log-normal model can be thought of as a kinematical model in which the velocity field is assumed to remain always linear. In this case the continuity equation can be solved for the density, which is an exponential in the velocity divergence, $\theta$. In linear theory $\theta$ is simply proportional to $\delta$, which is assumed to obey Gaussian statistics. Thus we can draw our velocities from the Gaussian field, $\delta_G$, which is exponentiated to give $\delta_{LN}$ by treating it as (proportional to) the velocity divergence and generating $v(k) \propto (k/k)^{\delta_G}(k)$. Such an approach has been used in Kitaura, Gallenani & Ferrara (2012) for example.

A second method assumes a model for the effect of redshift-space distortions on the correlation function (or power spectrum) of the Gaussian field and simply generates a random realization of a Gaussian field from the anisotropic 2-point function. This approach is followed in Beutler et al. [2012] using the Kaiser approximation (Kaiser 1987) for the power spectrum. We follow the first approach rather than the second, because there are some situations where it is useful to have the velocity information explicitly.

Our implementation of a log-normal code takes as input a target, real-space correlation function and a value of $\beta \equiv f(\Omega)/b$. We generate this correlation function using CLPT, as described in Carlson, Reid & White [2013]. The power spectrum of the Gaussian field is computed semi-analytically by integrating $\ln(1 + \xi_{LN})$.
against \( \sin(kr)/(kr) \) and then a Gaussian random field is generated from this power spectrum in the usual way. A large number of points are thrown randomly throughout the volume, and kept with probability proportional to the density. We interpolate the density field to the trial position using “cloud-in-cell” interpolation (e.g. Hockney & Eastwood 1980). We find that the Gaussian power spectrum damps strongly at \( k \approx 1.6 \, h \, \text{Mpc}^{-1} \) but that if we wish to sample the density field with tracers so as to obtain a point set then using grids with a finer spacing provides better agreement between the 2-point function of the generated point set and the target than a coarser grid (for further discussion see Fig. A1). If the analysis is done directly on the gridded field, or if the interpolation scheme doesn’t “clip the peaks”, then the coarser grid is adequate. As the velocity field is generated from the Gaussian velocity divergence the field in redshift-space approaches the prediction of Kaiser (1987) on large scales.

A2 LPT model

The use of second order Lagrangian perturbation theory (LPT; Buchert 1989; Moutarde et al. 1991; Hivon et al. 1995; Scoccimarro 1998) to generate density distributions in which halos are placed is the basis of the PThalos algorithm (Scoccimarro & Sheth 2002). A variant of this approach, in which the halos in the LPT density field are found using the friends-of-friends algorithm (FoF; Davis et al. 1985), was introduced in Manera et al. (2013).

We follow the method outlined in Manera et al. (2013). From an initial power spectrum we generate displacements using LPT. We then displace particles from an initially Cartesian grid along these displacement vectors to form a particle-based realization of the density field. We identify halos using the FoF algorithm and assign masses to the halos by abundance matching to the mass function of an N-body simulation (or a fit to it).

There are several choices which must be made in this algorithm (see also Neyrinck 2012; Leclercq et al. 2013 for related discussion). First we need to decide whether to damp the input power spectrum at high \( k \). While there are good arguments for such a truncation in some contexts, we found that using an undamped spectrum is better in our implementation of this algorithm. This is because we displace particles from a Cartesian grid, and a smoothed spectrum distorts the grid on large scales but leaves vestiges of it clearly visible in the particle distribution on small scales. When coupled with a FoF halo finder working with a spherical linking criterion this introduces undesirable artifacts. A natural choice is to set the grid resolution to match the mean inter-particle spacing. If the particle loading and grid are too coarse we lose both mass resolution and linear resolution. Increasing the resolution of the grid too much can also cause problems however, as at high resolution LPT fails\(^\text{16}\) and leads to artifacts like halos in voids (Buchert et al. 1994; Bouchet et al. 1995; Sahni & Shandarin 1996; Neyrinck 2012; Leclercq et al. 2013). A good compromise is a grid spacing and mean inter-particle spacing of \( O(1 \, h^{-1} \text{Mpc}) \) (see also Neyrinck 2012). We use a 3000\(^3\) grid for the 2.75 \( h^{-1} \text{Gpc} \) boxes and a 1500\(^3\) grid for the 1.5 \( h^{-1} \text{Gpc} \) boxes, with an equal number of particles as grid points.

Having generated an initial density/particle field we need to partition the mass into halos. This step must necessarily be ad hoc because the LPT method, like any perturbative scheme, generates the wrong small-scale power and lacks information on halo formation. These complexities are worse for hierarchical models such as cold dark matter (as compared with e.g. hot dark matter; Melott, Shandarin & Weinberg 1994; Monaco, Theuns & Taffoni 2002). We follow Manera et al. (2013) and use the FoF algorithm. This algorithm has a single parameter, the linking length, and produces a unique partition of the particles into groups (of multiplicity 1 particle or higher). For a model such as CDM it is natural to express the linking length in terms of the mean inter-particle separation, with typical numbers being \( 0.1 \sim 0.2 \) (Davis et al. 1985; Lacey & Cole 1994). In our situation the appropriate linking length to use (and whether it should scale only with the mean inter-particle separation) is less clear. We follow Manera et al. (2013) and empirically adjust the linking length to match the clustering strength of a constant number density sample of halos seen in our high-resolution N-body simulation. Specifically, for the halo catalog produced at any linking length we rank order the halos by particle number and choose those whose number densities lie within a narrow range determined from the abundance of halos in the high resolution simulation. For halos spanning an octave in mass centered on \( \log M = 12.5, 13 \) and \( 13.5 \left( h^{-1} M_{\odot} \right) \) we find number densities of \( \sim 3.24 < n < 2.95, -3.82 < \lg n < -3.45, -4.09 < \lg n < -4.54, (h^{-1}\text{Mpc})^{-1} \) in the 2.75 \( h^{-1} \text{Gpc} \) simulation. As the linking length is increased the large-scale bias of each sample, as determined by the amplitude of the halo-mass cross-spectrum at low \( k \), decreases. We were unable to find a single linking length which matched the large-scale bias for all 3 mass/abundance bins, so we chose the linking length which best matched the \( 10^{13} h^{-1} M_{\odot} \) sample. This linking length was the same as found by Manera et al. (2013), \( b = 0.36 \).

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