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Depth Dependence of Permeability in the Oregon Cascades Inferred from Hydrogeologic, Thermal, Seismic, and Magmatic Modeling Constraints

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Abstract. We investigate the decrease in permeability, $k$, with depth, $z$, in the Oregon Cascades employing four different methods. Each method provides insight into the average permeability applicable to a different depth scale. Spring discharge models are used to infer shallow ($z < 0.1 \text{ km}$) horizontal permeabilities. Coupled heat and groundwater flow simulations provide horizontal and vertical $k$ for $z < 1 \text{ km}$. Statistical investigations of the occurrences of earthquakes that are probably triggered by seasonal groundwater recharge yield vertical $k$ for $z < 5 \text{ km}$. Finally, considerations of magma intrusion rates and water devolatilization provide estimates of vertical $k$ for $z < 15 \text{ km}$. For depths $> 0.8 \text{ km}$, our results agree with the power-law relationship, $k = 10^{-14} \text{ m}^2 (z/1 \text{ km})^{-3.2}$, suggested by Manning and Ingebritsen [Rev. Geophys., 1999] for continental crust in general. However, for shallower depths (typically $z \leq 0.8 \text{ km}$ and up to $z \leq 2$) we propose an exponential relationship, $k = 5 \times 10^{-13} \text{ m}^2 \exp(-z/0.25 \text{ km})$, that both fits data better (at least for the Cascades and seemingly for continental crust in general) and allows for a finite near-surface permeability and no singularity at zero depth. In addition, the suggested functions yield a smooth transition at $z = 0.8 \text{ km}$, where their permeabilities and their gradients are similar. Permeabilities inferred from the hydroseismicity model at Mt. Hood are about one order of magnitude larger than expected from the above power-law. However, higher permeabilities in this region may be consistent with advective heat transfer along active faults causing observed hot springs. Our simulations suggest groundwater recharge rates of $0.5 \leq u_R \leq 1$. 
m/year and a mean background heat-flow of $H_b \approx 0.080$ to $H_b \approx 0.134$ W/m² for the investigated region.
1. Introduction

Permeability, $k$, describes a material's ability to transmit fluids. It is thus a critical material property, relevant to numerous geological processes that depend on mass and/or energy transfer as described for example by Ingebritsen and Sanford [1998]. Examples include transfer of water, steam, hydrocarbons, pore-fluid pressure, and heat. Permeability is a second rank tensor that can be highly anisotropic and heterogeneous, varying over more than 15 orders of magnitude in geological materials [e.g., Freeze and Cherry, 1979]. Because fluid driving forces and viscosities vary much less, permeability is the most important parameter in geological porous media flow, largely determining fluid fluxes. Measuring the permeability of rock cores in laboratory settings [e.g., Brace, 1980, 1984; Saar and Manga, 1999] is done routinely. However, such rock core permeabilities may not reflect field-scale values [e.g., Clauser, 1992; Sánchez-Vila et al., 1996; Hyun et al., 2002] where larger representative elementary volumes [Bear, 1988] may have to be considered that include large-scale fractures or layers. At intermediate field scales, from tens to hundreds of meters, it is frequently possible to conduct pumping or slug tests to determine $k$. However, to investigate permeability distributions at tens to thousands of meters, it is necessary to employ analytical or numerical models that are constrained by (sometimes indirect) observations [Manning and Ingebritsen, 1999].

Because $k$ affects mass and energy transfer, it is in principle possible to use various observations as constraints or boundary conditions in models to estimate $k$, provided that the observations depend on, or determine, mass and/or energy transport. Possible constraints and boundary conditions include classical hydrogeologic parameters such as
surface water infiltration rates, hydraulic head distribution, spring discharge, and no-flow boundaries. Other observations may include subsurface temperature distribution, heat-flow, rock metamorphism, mineral precipitation, chemical or biological fluid-rock interactions, pore-fluid pressure variations and related seismicity, magma intrusions providing heat and volatiles, and concentrations of fluids and isotopes.

Hydrogeologic models are typically underconstrained, particularly at large scales. Therefore, utilizing multiple constraints is desirable. Despite the inherent difficulties of constraining fluid flow models, it is often the regional, large-scale permeability distribution that has important implications for geological processes. These processes and applications include regional groundwater flow patterns, long-term water management, coupled heat and fluid transfer, geothermal energy exploration, carbon sequestration, subsurface waste fluid injection, and mineral, oil, and gas exploration.

The Cascades arc is a volcanic range located along an active, convergent plate boundary. In this paper we determine permeability, \( k \), as a function of depth, \( z \), in the upper crust of the Oregon Cascades (Figure 1). We also find evidence supporting a similar \( k(z) \)-profile in continental crust in general.

We use four different modeling approaches to estimate 1-dimensional permeability (vertical or horizontal) at four different depth scales: 1) A spring discharge model provides insight into (mostly) horizontal permeability values at depths of \( z < 0.1 \) km. 2) Analytic and numerical models of coupled heat and groundwater transfer provide horizontal and vertical \( k \) at depth scales of about \( z < 1 \) km. 3) Studies of seismicity induced by groundwater recharge allow for estimates of vertical \( k \) at approximately \( z < 5 \) km. 4) Finally, magma intrusion and degassing models provide insight into permeabilities at the
largest depth scale considered here of about \( z < 15 \) km. Methods from scale 1) as well as methods and results from scale 3) are based on previous publications \([\text{Manga, 1997, 1999; Saar and Manga, 2003}]\) and are summarized, applied, and interpreted here to fit the objective of this paper. Methods and results from scales 2) and 4) are original to this publication and are thus described in more detail.

We compare our results for permeability as a function of depth for the upper crust of the Oregon Cascades with the permeability-depth curve compiled by \( \text{Manning and Ingebritsen} \) [1999] for continental crust in general and show that there is good agreement for depths larger than about 1 km. For depths smaller than approximately 1 km, we propose an exponential permeability curve that both fits data better and allows for a finite near-surface permeability. In addition, from our study of coupled heat and groundwater transfer (scale 2) we infer regional geothermal background heat-flow and groundwater recharge rates.

2. Four Depth Scales of \( k \)

We investigate permeability, \( k \), at four different depth scales which are considered representative elementary volumes of four different sizes that are in themselves homogeneous but possibly anisotropic. In this section we present methods and results starting at the smallest (shallowest) and proceeding towards the largest (deepest) scale. The four methods employed either provide hydraulic conductivity, \( K \) (in vertical, \( K_z \), or horizontal, \( K_x \), direction) or vertical hydraulic diffusivity, \( D_z \). Both hydraulic conductivity and hydraulic diffusivity are converted to permeability as described in Section 3, so that the change in permeability as a function of depth can be discussed in Section 4. Table 1 lists symbols used in this paper, their units, and their definitions.
2.1. Spring Discharge Model: $k(z < 0.1 \text{ km})$

*Manga* [1997, 1999, 2001] shows that discharge at springs can be used to estimate large-scale near-surface horizontal hydraulic conductivity, $K_x$, and horizontal permeability, $k_x$. We briefly describe our approach and apply the method to discharge data from springs in the study region. We reserve interpretation for the discussion section.

2.1.1. Method

Stacked lava flows form aquifers that typically show high horizontal hydraulic conductivities within the upper blocky parts of each flow [Meinzer, 1927; Brace, 1984]. The dense but jointed interior of a solidified lava flow shows predominantly vertical groundwater flow along cooling joints. *Manga* [2001] suggests that the horizontal blocky layer may be approximated as a confined aquifer of mean constant thickness, $b$, and high horizontal hydraulic conductivity, $K_x$, and the dense interior as an unconfined aquifer providing water from storage according to its specific yield, $S_Y$ (for a sketch see Figure 12 in *Manga* [2001]). As a result, hydraulic head distribution is governed by a (1-dimensional) confined aquifer equation

$$\frac{\partial h}{\partial t} = \frac{K_x b}{S_Y} \frac{\partial^2 h}{\partial x^2} + \frac{u_R(x, t)}{S_Y},$$

(1)

where specific storage, $S_S$, has been substituted by $S_Y/b$. Here, $t$, $x$, $u_R$, $K_x$, and $S_Y$ are time, horizontal dimension, recharge rate, horizontal hydraulic conductivity, and specific yield, respectively. The (variable) hydraulic head in the dense interior layer is denoted $h$. Eq. (1) is equivalent to a linearized Boussinesq equation for unconfined aquifer flow, in which case $b$ and $h$ denote mean constant hydraulic head and small (variable) head, respectively. Eq. (1) is a linear diffusion equation where the hydraulic diffusivity is given...
by

\[ D_x = \frac{K_x b}{S_Y}. \]  

(2)

As usual in a diffusive system, the diffusion time scale, \( \tau \), is characterized by \( \tau = L^2/D_x \), where \( L \) is the aquifer length (Figure 2). Therefore, Eq. (2) becomes

\[ K_x = \frac{S_Y L^2}{b \tau}. \]  

(3)

The aquifer thickness, \( b \), is related to the mean residence time of water in the aquifer, \( \bar{t}_r \), by

\[ \bar{t}_r = \frac{A b S_Y}{Q_s} = \frac{b S_Y}{u_R}, \]  

(4)

where \( A, Q_s \), and \( u_R = Q_s/A \) are the surface area (exposed to recharge) of the unconfined aquifer, the mean spring discharge flux, and the mean groundwater recharge rate, respectively (Figure 2). Substituting Eq. (4) into Eq. (3) yields

\[ K_x = \frac{S_Y^2 L^2}{\tau \bar{t}_r u_R}. \]  

(5)

We determine horizontal hydraulic conductivity, \( K_x \), from Eq. (5). Hydraulic diffusivity, \( D_x \), and other parameters can be estimated from baseflow recession curves of streams [Hall, 1968; Singh and Stall, 1971; Tallaksen, 1995; Amit et al., 2002]. The aquifer length, \( L \), is given by the watershed geometry and \( D_x \) is adjusted until good agreement between measured and calculated stream discharge is achieved [Manga, 1997, 1998]. Mean groundwater recharge, \( u_R \), can be approximated by discharge in runoff-dominated streams [Manga, 1996]. Specific yield, \( S_Y \), may be approximated by connected pore fraction, \( n \). The mean residence time, \( \bar{t}_r \), for these springs is determined from radiogenic tracers [James et al., 2000].
2.1.2. Results

We apply Eq. (5) to two spring-fed streams, Cultus River and Quinn River, in the Oregon Cascades (study region A in Figure 1). The input data from both streams and results for horizontal hydraulic conductivity, $K_x$, are given in Table 2. For the assumed range of specific yield, $0.05 \leq S_Y \leq 0.15$, and the uncertainties implied by the number of significant digits in each input parameter (Table 2), the range of horizontal hydraulic conductivity is about

$$10^{-3} \leq K_x \leq 10^{-2} \text{ m/s.}$$

(6)

2.2. Coupled Heat and Groundwater Transfer Model: $k(z < 1 \text{ km})$

Many studies have discussed heat and groundwater transfer in the subsurface [e.g., Bredehoeft and Papadopulos, 1965; Sorey, 1971; Domenico and Palciauskas, 1973; Woodbury et al., 1987; Woodbury and Smith, 1988; Forster and Smith, 1989; Ge, 1998; Hurwitz et al., 2002]. Similar to the work presented in this section, Deming [1993] investigates coupled heat and groundwater flow at the north slope of Alaska to estimate the local permeability on a large, regional scale. Here, we employ analytical and numerical models to simulate coupled groundwater and heat-flow in a 1-dimensional Cartesian and a 3-dimensional cylindrical coordinate system, respectively. Due to low temperatures present in the region of interest, we assume gravity-driven convection only and no density-driven flow. This allows us to first solve the groundwater flow equation followed by the heat advection-diffusion equation to determine the temperature field. In Section 2.2.3, the temperature distribution is compared with measurements in boreholes to evaluate model input parameters.
2.2.1. Method: 1D flow

The central axis of approximately radially symmetric mountains as well as the vertical axis at the center of saddles between two such mountains may be represented by vertical 1-dimensional (1D) groundwater recharge and flow regimes (vertical lines in Figure 3). Away from the axes flow becomes 2-dimensional (2D) or possibly 3-dimensional (3D). In this section, we employ analytical models of coupled groundwater and heat transfer in a 1D recharge region.

Phillips [1991] derives a solution for the advection-diffusion equation, providing temperature as a function of depth, $T(z)$, in a 1D groundwater recharge area. The solution assumes a constant mean horizontal permeability resulting in a linear decrease in vertical groundwater flow (Darcy) velocity, $u_z$, from a recharge (Darcy) velocity at the water table to $u_z = 0$ at the bottom of the aquifer. Here, we provide an equivalent solution for $T(z)$, Eq. (18), however with inverted $z$-axis so that $z$ is positive-downwards, consistent with the reference frame adopted for this paper (Figure 3). In addition, we introduce a solution to the 1D heat advection-diffusion equation where permeability decreases exponentially with depth (Eq. 17).

For the remainder of this section we will refer to Darcy velocity, $u$, as velocity, with the understanding that the actual interstitial (seepage) velocity is given by $v = u/n$, where $n$ is the connected pore fraction. The vertical mean recharge velocity at the surface, $z = 0$, is given by $v_R = v_z(z = 0) = \Phi \Psi$, where $\Phi$ is the fraction of the mean yearly precipitation, $\Psi$, that infiltrates the ground. At the crest of the (Oregon) Cascades, typically $0.1 \leq \Phi \leq 0.5$ and $\Psi \approx 2$ m/year so that $0.2 \leq v_R \leq 1$ m/year [Ingebritsen et al., 1994].
Following Phillips [1991], for an unconfined aquifer with approximately horizontal bedding, as may be expected near the symmetry axis of a saddle, the horizontal hydraulic head gradient at any depth, \( z \), is equal to its value at the water table. Because the vertical pore-fluid pressure is roughly hydrostatic, the hydraulic head is constant for any \( z \) and Darcy’s law is given by

\[
\mathbf{u}(x, z) = -K_x(x, z) \nabla h(x),
\]

(7)

where \( x = (x, y) \) denotes both horizontal directions and \( K_x(x, z) \) is the horizontal hydraulic conductivity as a function of horizontal and vertical directions. Near the symmetry axis of the saddle, the recharge velocity, \( u_R \), and the curvature of the water table may be assumed constant. Consequently, in a 2D cross section in the \( xz \)-plane (Figure 3), considered for the remainder of this section, Eq. (7) may be reduced to yield the horizontal groundwater flow velocity near the vertical symmetry axis,

\[
u_x(x, z) = K_x(x, z) \frac{x}{R},
\]

(8)

where \( R = - (\partial^2 h / \partial x^2)^{-1} \) is the (constant) radius of curvature of the water table and thus \( x/R = \nabla h(x) \). We use topography (Figure 4) to obtain \( R \approx 100 \pm 20 \) km.

For an incompressible fluid, \( \nabla \cdot \mathbf{u} = 0 \). Thus, for the case of constant mean horizontal hydraulic conductivity, \( K_x \), the horizontal and vertical velocities are given by

\[
u_x(x) = K_x \frac{x}{R}
\]

(9)

and

\[
u_z(z) = - \int \frac{\partial u_x}{\partial x} dz = K_x \left( b - \frac{z}{R} \right),
\]

(10)
respectively. For Eq. (10) the boundary condition \( u_z(z = b) = 0 \) was applied to determine the integration constant. Here, \( b \) is the total vertical thickness of the saturated zone (Figure 3b). Eq. (10) shows that for the case of (vertically) constant horizontal hydraulic conductivity the vertical groundwater flow velocity decreases linearly from the recharge velocity, \( u_R = u_z(z = 0) \), to zero at the bottom of the aquifer at \( z = b \).

In contrast, for the case of (vertically) exponentially decreasing horizontal hydraulic conductivity,

\[
K_x(z) = K_{sx} e^{-z/\delta},
\]

as a function of depth, \( z \), and with skin depth, \( \delta \), from a near-surface value, \( K_{sx} \), the horizontal and vertical velocities are given by

\[
u_x(x, z) = \frac{K_{sx} x}{R} e^{-z/\delta}
\]

and

\[
u_z(z) = -\int \frac{\partial u_x}{\partial x} dz = \frac{K_{sx} \delta}{R} \left( e^{-z/\delta} - 1 \right) + u_R,
\]

respectively. The boundary condition \( u_z(z = 0) = u_R \) was applied in Eq. (13) to provide the integration constant, where the recharge velocity, \( u_R \), is determined in Appendix A. Substituting \( u_R \) from Eq. (A3) in Appendix A into Eq. (13) results in

\[
u_z(z) = \frac{K_{sx} \delta}{R} \left( e^{-z/\delta} - e^{-b/\delta} \right).
\]

Therefore, as expected, for a medium with exponentially decreasing hydraulic conductivity, the vertical groundwater flow velocity also decreases exponentially, from a recharge velocity of \( u_R \) at \( z = 0 \) with a skin depth (characteristic length scale) of \( \delta \).
Figure 5 shows groundwater flow streamlines based on Eqs. (9, 10, 12, and 14) for both constant and (in vertical direction) exponentially decreasing horizontal hydraulic conductivity. Input parameters for Eqs. (9, 10, 12, and 14) are discussed in Section 2.2.3 and Table 3. Figure 5 shows that the exponential-$K$ model causes streamlines to divert more rapidly into horizontal directions, allowing for a more rapid decrease in vertical groundwater flow velocity with depth than the constant-$K$ model.

Substituting Eq. (14) into the steady-state 1D vertical heat advection-diffusion equation,

\[
\gamma u_z \frac{\partial T}{\partial z} = D_m \frac{\partial^2 T}{\partial z^2} ,
\]  

and integrating with respect to $z$ and with boundary condition

\[
\frac{\partial T}{\partial z} (z = b) = \left( \frac{\partial T}{\partial z} \right)_b
\]

yields

\[
\frac{\partial T}{\partial z} = \left( \frac{\partial T}{\partial z} \right)_b \exp \left[ \frac{\gamma K_x \delta}{D_m R} \left( (b + \delta - z)e^{-b/\delta} - \delta e^{-z/\delta} \right) \right]
\]

for exponentially decreasing hydraulic conductivity. Here, \((\partial T/\partial z)_b\) is the (conductive) background temperature gradient at the bottom of the aquifer and \(\gamma = (\rho_w c_w)/(\rho_m c_m)\).

To obtain \(T(z)\), we integrate Eq. (17) numerically with respect to \(z\) and with boundary condition \(T(z = 0) = T_R\), where \(T_R\) is the (near-surface) recharge temperature.

In contrast to Eq. (17), for the case of constant hydraulic conductivity, i.e., linearly decreasing \(u_z\), Eq. (10) is substituted into Eq. (15) and integrated twice with respect to \(z\) and with the same boundary conditions as for Eq. (17), resulting in

\[
T = T_R - \left( \frac{\partial T}{\partial z} \right)_b \left[ \sqrt{ \frac{\pi D_m R}{2 \gamma K_x} } \operatorname{erfc} \left( b - z \right) \right] .
\]
Here, erfc is the complementary error function. $D_m$ in Eqs. (15, 17, and 18), $\rho_m$, and $c_m$ are mixed thermal diffusivity, mixed density, and mixed specific heat of the water-rock complex as defined in Appendix B.

In Section (2.2.3) we compare results for $T(z)$ from the exponential-$K$ (integral of Eq. 17) and constant-$K$ (Eq. 18) solutions and estimate groundwater recharge rates, $u_R$, characteristic depth scale, $\delta$, aquifer thickness, $b$, and background heat flux,

$$H_b = -K_{Tr} \left( \frac{\partial T}{\partial z} \right)_b.$$  

(19)

Here, $K_{Tr}$ is the thermal conductivity of the rock alone, assumed to be on average about $K_{Tr} \approx 2 \text{ Wm}^{-1}\text{°C}^{-1}$, based on measurements on cores and cuttings from the Cascades [Blackwell et al., 1982; Ingebritsen et al., 1994].

2.2.2. Method: Radial flow

As described previously, owing to relatively low temperatures, we assume only gravity-driven advection and no density-driven flow. This allows us to first solve the steady-state groundwater flow (hydraulic head diffusion) equation,

$$\frac{\partial}{\partial r} \left( K_r \frac{\partial h}{\partial r} \right) + \frac{K_r}{r} \frac{\partial h}{\partial r} + \frac{\partial}{\partial z} \left( K_z \frac{\partial h}{\partial z} \right) = 0,$$  

(20)

for anisotropic and heterogeneous hydraulic conductivities, $K_{rz}(rz)$, in a 3D-cylindrical coordinate system using only hydrogeologic and no thermal boundary conditions. Here, $h$ is the variable hydraulic head and $K_r$ and $K_z$ are the hydraulic conductivities in the radial, $r$, and vertical, $z$, directions, respectively. This coordinate system is chosen because of the radial symmetry of Mt. Hood (Figure 6), where the center of symmetry is approximated by the summit of the volcano. The outer vertical no-flow boundary is chosen based on the occurrence of apparent groundwater-recharge induced earthquakes (see Section 2.3 as
well as Saar and Manga [2003]) and on the existence of a borehole geotherm that shows a slight concave-down temperature profile (triangles in Figure 8) suggesting upward flow of warmer water [Bredehoeft and Papadopulos, 1965]. The simulation domain has a radius of 7 km and a depth of 1.3 km.

We solve Eq. (20) numerically by employing a finite difference algorithm with successively under-relaxed iterations [Saar, 2003]. The obtained hydraulic head field, \( h(r, z) \), in the \( rz \)-plane is then converted into a Darcy velocity field, \( u(r, z) \), using a finite difference approach [Saar, 2003].

We determine the temperature distribution by solving the steady-state heat-advection-diffusion equation (without heat sources or sinks) in cylindrical coordinate form,

\[
\frac{D_m}{\gamma} \left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} \right) = -u_r \frac{\partial T}{\partial r} - u_z \frac{\partial T}{\partial z},
\]

where \( T \), \( u_r \), and \( u_z \) are temperature and Darcy velocities in the radial and vertical directions, respectively.

Eq. (21) is also solved using a finite difference approximation and successively under-relaxed iterations [Saar, 2003]. Boundary conditions for Eqs. (20) and (21) are no groundwater- and heat-flow (Dirichlet) boundary conditions, \( \partial h / \partial r = 0 \) and \( \partial T / \partial r = 0 \), at the central axis and at the outer vertical boundary. The lower boundary has mixed boundary conditions with no groundwater flow, \( \partial h / \partial z = 0 \), but constant heat-flow (Neumann) boundary condition, \( \partial T / \partial z = (\partial T / \partial z)_b = \) constant. Hydraulic head and temperature distribution along the upper boundary are a function of elevation, where the water table is assumed to be similar to the topography and temperature is approximated by the mean adiabatic lapse rate (MALR).
Hurwitz et al. [2003] suggest that water tables in volcanoes with ice-caps may be much lower than the ground surface. However, for our approximate radial model the uncertainty associated with the unknown position of the water table is small compared with the overall model dimensions.

2.2.3. Results for 1D and radial flow

Figure 7a shows measured [Blackwell, 1992] and modeled temperature-depth profiles for the groundwater recharge region at Santiam Pass, Oregon. Solutions to both the constant (Eq. 18) and the exponentially decreasing (integral of Eq. 17) horizontal hydraulic conductivity model are provided. Both models suggest a very similar aquifer thickness, \( b \approx 1000 \text{ m} \), and background heat flux, \( H_b \approx 0.080 \text{ W/m}^2 \) (Table 3). However, the constant-\( K \) model requires a much lower recharge rate of \( u_R \approx 0.1 \text{ m/year} \) compared with the exponentially decreasing \( K \) model which allows for \( u_R \approx 0.5 \text{ m/year} \). The larger recharge velocities in the latter case (thin solid curve in Figure 7a) are due to near-surface horizontal hydraulic conductivities of \( K_{sx} = 5.8 \times 10^{-6} \text{ m/s} \) that are about one order of magnitude larger than in the case of constant horizontal hydraulic conductivity where \( K_x = 3.5 \times 10^7 \text{ m/s} \) (Figure 7b). The mean absolute error between temperature data, \( T_D \), and calculated temperatures, \( T_C \), is given by \( \epsilon = |T_D - T_C|/N \), where \( N \) is the number of measurements (including the surface temperature). The constant-\( K \) model allows for a lower minimized error of \( \epsilon \approx 0.68 \degree \text{C} \) compared with \( \epsilon \approx 0.92 \degree \text{C} \) for the exponential-\( K \) model. However, recharge rates of \( u_R \approx 0.5 \text{ m/year} \), as suggested by the exponential model, are a more significant improvement because other studies [e.g., Ingebritsen et al., 1994] suggest values of about \( 0.2 \leq u_R \leq 1 \text{ m/year} \). In addition, recharge rates of \( u_R \approx 1 \).
m/year are also supported by the results from our radial model discussed in the next paragraph.

Figure 8 shows measured and calculated temperature-depth profiles for the three geothermal boreholes that fall within the radially symmetric model region at Mt. Hood. Results agree well with the two deeper profiles but the shallow profile shows some deviations from measurements that are probably dominated by limitations in reducing the three dimensions into a 2D cross section (i.e., owing to deviations from radial symmetry). Minimizing the misfit, $\epsilon$, between model and data yields a horizontal near-surface hydraulic conductivity, $K_{xs} = 10^{-6}$ m/s, that is about 30 times larger than the vertical near-surface hydraulic conductivity, $K_{zs} = 3 \times 10^{-8}$ m/s. Both conductivities are assumed to decrease exponentially with depth according to a skin depth of $\delta = 250$ m. The simulation results also suggest a background heat-flow of $H_b \approx 0.134$ W/m$^2$ and a recharge velocity of $u_R \approx 1$ m/year (Table 3).

2.3. Groundwater-recharge-induced Seismicity Model: $k(z < 5 \text{ km})$

Saar and Manga [2003] suggest that an increase in seismicity in late summer at Mt. Hood, Oregon, is caused by seasonal groundwater recharge during spring due to snow melt on the flanks of the volcano. They show that a statistically significant correlation exists between groundwater recharge and seismicity. Groundwater recharge is approximated by discharge in runoff-dominated streams. The recharge in this region provides a natural pore-fluid pressure signal of narrow temporal width and potentially relatively large amplitude. The pore-fluid pressure pulse diffuses and can cause an effective stress decrease on preexisting faults at depth, effectively “unclamping” the fault and triggering earthquakes. Saar and Manga [2003] use the time lag between groundwater recharge and
increased seismicity at a mean earthquake depth of 4.5 km to estimate (vertical) hydraulic diffusivity, \( D_z \), for the depth range of \( 0 < z < 5 \times 10^3 \) m, assumed to be homogeneous.

2.3.1. Method

The solution to the 1-dimensional (pressure) diffusion equation for periodic sinusoidal boundary conditions,

\[
P'(t, z = 0) = P_0 \cos \left( \frac{2\pi t}{\psi} \right),
\]

in a half-space is given by \citep{Turcotte:2002}

\[
\frac{P'}{P_0} = \exp \left( -z \sqrt{\frac{\pi}{\psi D_z}} \right) \cos \left( \frac{2\pi t}{\psi} - z \sqrt{\frac{\pi}{\psi D_z}} \right),
\]

where \( P_0, P', \psi, t, \) and \( z \) are near-surface pore-fluid pressure amplitude, pore-fluid pressure amplitude at depth (in excess to hydrostatic pressure), period, time, and depth, respectively. The cosine term in Eq. (23) describes the phase lag of the pressure peak as a function of depth. \cite{Saar:2003} show that the argument in the cosine-term is likely to be zero (or a multiple of \( 2\pi \)) and can thus be used to determine vertical hydraulic diffusivity,

\[
D_z = \frac{\psi z^2}{4\pi t^2}.
\]

Eq. (24) assumes pore-fluid pressure diffusion in the saturated zone only and is thus sensitive to the location of the water table and the mean earthquake depth. As mentioned in Section 2.2.2 \cite{Hurwitz:2003} suggest that water tables in stratovolcanoes may be relatively deep. However, as described by \cite{Saar:2003} a low water table reduces both the diffusion depth, \( z \), to earthquakes and the available diffusion time, \( t \), in Eq. (24) cancelling the effects of each reduction. In addition, uncertainties in earthquake depths
are large (±2 km) for the unrelocated earthquakes and are considered in the following section.

2.3.2. Results

We use Eq. (24) to determine the large-scale vertical hydraulic diffusivity, $D_z$, for the upper 5 km of crust at Mt. Hood, Oregon. The periodicity of groundwater recharge due to snow melt in spring is $\psi = 1$ year. The mean earthquake depth, $z = 4.5 \pm 2$ km, and time lag, $t = 151 \pm 7$ days (determined by cross correlations), provide the remaining parameters for Eq. (24), resulting in a hydraulic diffusivity of [Saar and Manga, 2003]

$$D_z = 0.30 \pm 0.22 \text{ m}^2/\text{s}.$$  \hspace{1cm} (25)

Hydraulic diffusivities of about 0.3 m$^2$/s for continental crust agree with values compiled by Talwani and Acree [1984/85] and are in the upper range for fractured igneous rocks [Roeloffs, 1996]. In fact, Rothert et al. [2003] report hydraulic diffusivities between 0.3 and 2 m$^2$/s for deeper (9.1 km), fractured crystalline crust at the German Continental Deep Drilling site (KTB) and diffusivities between 0.004 and 0.01 m$^2$/s for shallower (5.4 km), less-fractured crust.

2.4. Magma Intrusion and Degassing Model: $k(z < 15 \text{ km})$

The Cascades range is a volcanic arc located within the North American plate at its convergent boundary with the Juan de Fuca plate. Subduction zone related magmas intrude into the crust where they may (partially) solidify, assimilate crustal material, and/or erupt. Devolatilization of (mostly) water from solidifying magma provides a fluid source in the crust that, due to conservation of mass, is transmitted to the Earth’s surface, unless consumed in metamorphic reactions. Due to the large uncertainty range in magmatic
volatile content considered in Section 2.4.2, we neglect possible effects of metamorphic reactions on reducing the fluid flux. The volumetric fluid flux, $Q$, can be used to estimate vertical hydraulic conductivity, $K_z$, on a depth scale of about 15 km, i.e., from the estimated main magma intrusion depth to the surface (Figure 9). As before, within this region, $K_z$ is assumed homogeneous.

### 2.4.1. Method

The volumetric magma flux is given by

$$Q_M = q_M W,$$

where $q_M$ is the volumetric magma intrusion rate per kilometer arc length and $W$ is the estimated arc width (Figure 9). We assume that only water devolatilizes. Therefore, the volume flux of water, $Q_w$, released by the solidifying magma is given by

$$Q_w = \frac{q_M W \rho_M \phi_w}{\rho_w},$$

where $\phi_w$, $\rho_M$, and $\rho_w$ are weight fraction of water content in the magma, and the densities of magma and water, respectively. In one (vertical) dimension, Darcy's law reduces to

$$|q_z| = \frac{Q_w}{A} = \left| -K_z \frac{\partial h}{\partial z} \right|,$$

where $A$ and $z$ are the total cross-sectional area perpendicular to the fluid flow direction and depth, respectively. Here, $A = WL$, where $L = 1$ km is the arc length considered for the volumetric magma intrusion rate, $q_M$, described previously. The hydraulic head, $h = h_e + h_p$, is the sum of elevation head, $h_e$, and pressure head, $h_p = P \rho_w^{-1} g^{-1}$, where $P$ is the pore-fluid pressure. At depth, $z$, the magma and its exsolved volatiles, such as water, are under lithostatic pressure so that $P = P_t \approx \rho_r g z$, where $\rho_r$ is the mean rock density. At the surface ($z = 0$), $P$ is given by the hydrostatic pressure, $P = P_h = \rho_w g z = 0$. 


Therefore, if we set the arbitrary reference elevation to the magma intrusion depth, \( d \),

then \( h(z = d) = h_p = \rho_r d/\rho_w \) and \( h(z = 0) = h_e = d \). As a result, for any intrusion depth

the hydraulic gradient is given by

\[
\frac{\partial h}{\partial z} = \frac{h_p - h_e}{d} = \frac{\rho_r}{\rho_w} - 1. \tag{29}
\]

Substituting Eqs. (27) and (29) into Eq. (28) and recognizing that \( A = WL \), yields the vertical hydraulic conductivity

\[
K_z = \frac{Q_w}{A \left( \frac{\rho_r}{\rho_w} - 1 \right)} = \frac{q_M \rho_M \phi_w}{L \rho_w \left( \frac{\rho_r}{\rho_w} - 1 \right)}. \tag{30}
\]

Applying \( L = 1 \) km, used for the volumetric magma intrusion rate per kilometer arc length, \( q_M \), and rearranging yields

\[
K_z = \frac{q_M \phi_w}{\left( \frac{\rho_r}{\rho_M} - \frac{\rho_w}{\rho_M} \right)} \text{ km}^{-1}. \tag{31}
\]

Therefore, \( K_z \) is independent of the assumed arc width, \( W \). Assuming that, independent

of subsurface temperature, \( \rho_r \approx \rho_M \) reduces Eq. (31) to

\[
K_z = \frac{q_M \phi_w}{\left( 1 - \frac{\rho_w}{\rho_M} \right)} \text{ km}^{-1}. \tag{32}
\]

Near the ground surface \( \rho_w/\rho_M \approx 1/3 \) and with increasing temperature at depth \( \rho_w/\rho_M \to 0 \) because \( \rho_w \) decreases and \( \rho_M \) remains relatively constant compared to \( \rho_w \). Consequently, the term in the parentheses in Eq. (32) ranges between 2/3 and 1 for low and high temperatures, respectively, thus varying by a maximum factor of 1.5. In contrast, the much wider range of possible water fractions in the magma, \( 0.002 \leq \phi_w \leq 0.07 \), allows for variations in \( K_z \) by a maximum factor of 35. Therefore, we assume a constant water-magma density ratio of \( \rho_w/\rho_M = 1/4 \) reducing Eq. (32) to

\[
K_z = \frac{4}{3} q_M \phi_w \text{ km}^{-1}. \tag{33}
\]
Eq. (33) is independent of intrusion depth, \( d \), because as depth increases, so does the lithostatic pressure, \( P_L \). However, \( d \) determines the depth scale for which \( K_z \) is determined. As before, within this depth scale, \( K_z \) is assumed homogeneous.

**2.4.2. Results**

We use Eq. (33) to determine overall (assumed homogeneous) vertical hydraulic conductivity for the upper 15 km of the crust in the Cascades. As input data we assume a range of magma intrusion rates of \( 9 \leq q_M \leq 50 \text{ km}^3 \text{ Ma}^{-1} \text{ km}^{-1} \) [Ingebritsen et al., 1989; Blackwell et al., 1990]. Sisson and Layne [1993] and other authors [e.g., Grove et al., 2002] suggest a magma water content of about 0.2 to over 3 wt.-% and up to 7 wt.-% for the Cascades arc including the Mt. Shasta region in northern California. In addition, \( \text{CO}_2 \) may also be a significant volatile in subduction-related magmas, where carbonates may have been derived from subduction of marine sediments. To account for the uncertainty in volatile content, we apply a wide concentration range from 0.2 to 7 wt.-% (i.e., \( 0.002 \leq \phi_w \leq 0.07 \)) and assume it is all water. Substituting these values into Eq. (33) results in a range of vertical hydraulic conductivities of approximately

\[
9 \times 10^{-13} \leq K_z \leq 2 \times 10^{-10} \text{ m/s}
\]

for the deepest depth scale (15 km) considered.

**3. Conversions between \( D \), \( K \), and \( k \)**

In order to compare results from the four methods employed in this study we convert hydraulic diffusivity, \( D \), and hydraulic conductivity, \( K \), to permeability, \( k \), assuming scalar quantities for the remainder of this paper. Transferring \( D \) to hydraulic conductivity,

\[
K = DS_S
\]
requires assuming a value for specific storage,

\[ S_S = \rho_w g (\alpha + n\beta), \]  

(36)

where \( n \) is pore fraction, \( \beta = 4.8 \times 10^{-10} \text{ m}^2/\text{N} \) is the compressibility of water, and \( \alpha \) is the bulk aquifer compressibility at constant vertical stress and zero lateral strain [Wang, 2000].

Further assumptions about temperature (and pressure) dependent (dynamic) viscosity, \( \mu \), and water density, \( \rho_w \), have to be made when converting hydraulic conductivity, \( K \), to permeability, \( k \), using

\[ k = \frac{\nu K}{g}, \]  

(37)

where the kinematic viscosity is given by \( \nu = \mu/\rho_w \). Estimating \( \alpha \), \( \rho_w \), and \( \mu \) is not trivial and can lead to large uncertainties in inferred permeability values, particularly when converting from hydraulic diffusivity to permeability, where, in addition to \( \nu \), \( S_S \) has to be estimated. We assume \( \alpha \approx 10^{-10} \text{ m}^2/\text{N} \) for fractured igneous rock [Roeloffs, 1988; Domenico and Schwartz, 1998] and \( n \approx 0.01 \) [Ingebritsen et al., 1992; Hurwitz et al., 2003] yielding \( S_s \approx 10^{-6} \text{ m}^{-1} \).

Similar to Germanovich et al. [2000], we approximate kinematic viscosity as a function of depth-dependent mean temperature, \( T_m \), for Eq. (37) by

\[ \nu(T_m) \approx \frac{0.032 \text{ Pa} \cdot \text{s} \circ \text{C}}{\rho_w (1 - \alpha_w T_m)(15.4 \circ \text{C} + T_m)}, \]  

(38)

where \( \alpha_w = 10^{-3} \circ \text{C}^{-1} \) is the coefficient of thermal expansion of water and \( \rho_w = 10^3 \text{ kg/m}^3 \) and \( \rho_w = 0.5 \times 10^3 \text{ kg/m}^3 \) are the water densities for mean depths (and associated pressures and temperatures) \( z_m \leq 3 \text{ km} \) and \( z_m \approx 10 \text{ km} \), respectively. For calculation of \( z_m \) and depth ranges (\( z \)-ranges) over which our permeability results from the previous section are applicable, we refer to Section 4 in conjunction with Table 4. The mean temperature is
then given by \( T_m \approx T_R + z_m(\partial T/\partial z)_b \) with a typical recharge temperature of \( T_R \approx 5 \, ^\circ C \) (Figures 7a and 8) and a background temperature gradient of \( (\partial T/\partial z)_b \approx 50 \, ^\circ C/\text{km} \) (Figures 7a and 8 as well as Section 4.2). As we will show later, the depth ranges over which calculated permeabilities are applicable are independent of the actual permeability values. However, the \( z \)-ranges do depend on the characteristics of the assumed decrease in permeability with depth (exponential versus power-law relationship) and on the method for calculating the mean (arithmetic versus harmonic). Table 4 summarizes the hydraulic conductivities discussed in the previous section and the resulting permeabilities for the different depth scales.

4. Discussion

In Section 4.1 we discuss results from the previous two sections and suggest a curve describing permeability as a function of depth. In Section 4.2 we address mean background heat-flow values determined in Section 2.2.

4.1. Heterogeneity and Anisotropy of Permeability

Permeability, \( k \), typically decreases with depth due to compaction, metamorphism, and/or filling of pore spaces and fractures by precipitating minerals, particularly in hydrothermal regions [Blackwell and Baker, 1988; Nicholson, 1993; Ingebritsen and Sanford, 1998; Fontaine et al., 2001]. In some cases, however, fractures can remain open due to earthquakes providing long-term (10s to 100s of years) high-\( k \) pathways [Rojstaczer and Wolf, 1992]. The characteristics of the reduction of \( k \) with depth, \( z \), are difficult to determine and are controversial [e.g., Gangi, 1978; Brace, 1980, 1984; Deming, 1993; Morrow et al., 1994; Huenges et al., 1997; Germanovich et al., 2001]. Several publications [e.g.,
Belitz and Bredehoeft, 1988; Williams and Narasimhan, 1989; Manning and Ingebritsen, 1999; Shmonov, 2002; Hurwitz et al., 2003; Shmonov, 2003] as well as this study suggest a non-linear decrease of \( k \) with \( z \) in the Cascades region and for continental crust in general. Non-linear \( k(z) \)-profiles allow for high near-surface permeabilities and related high groundwater recharge rates so that the observed near-surface isothermal and inverted temperature-depth profiles are possible (Figures 7, 8, and 10). At the same time, a non-linear \( k(z) \)-profile can reduce permeabilities to \( k \leq 10^{-16} \text{ m}^2 \) at \( z \geq 4 \text{ km} \), thus allowing for conduction-dominated heat transfer at these depths, as also required by observations [e.g., Ingebritsen, 1992]. A transition from advection- to conduction-dominated heat transfer at \( k \approx 10^{-16} \text{ m}^2 \) has been suggested by numerous authors [Norton and Knight, 1977; Smith and Chapman, 1983; Manning et al., 1993; Hayba and Ingebritsen, 1997; Huenges et al., 1997; Manning and Ingebritsen, 1999; Clauser et al., 2002; Hurwitz et al., 2003].

In Section 2.2, we invoked non-linearity by decreasing permeability exponentially with depth. Figure 5 shows streamlines for constant (thin curves) and exponentially decreasing (bold curves) \( k \), where recharge rates for the latter are about five times larger than for constant \( k \) (thin lines near the surface in Figure 7a). The exponential \( k(z) \)-curve results in larger horizontal groundwater flow components at smaller depths, rapidly reducing vertical groundwater flow velocities and allowing for conduction-dominated heat transfer at relatively shallow depths.

However, while exponential functions of (vertical) permeability, such as

\[
k_z(z) = k_{zs} e^{-z/\delta}, \tag{39}
\]

tend to provide reasonable near-surface (denoted with the subscript “s”) values, including \( k_z(z = 0) = k_{zs} \) at zero-depth, they can also reduce permeability to unrealistically low
magnitudes at depths greater than about 2 km (Figure 11b). In contrast, power laws, such as

\[ k_z(z) = k_{zd}(\frac{z}{d})^{-\lambda}, \quad (40) \]

provide larger, and thus more realistic, permeabilities at greater depths, but approach infinity for \( z \to 0 \) and result in a singularity for \( z = 0 \). They are thus defined for a finite depth, \( d \), where \( k_z(z = d) = k_{zd} \).

Due to these advantages and limitations of either function, we suggest Eq. (39) for \( 0 \leq z \leq 0.8 \) km and Eq. (40) for \( z > 0.8 \) km. Eq. (40) is equivalent to the curve suggested by Manning and Ingebritsen [1999] if \( \lambda = 3.2 \) and \( k_{zd} = 10^{-14} \) m\(^2\) at \( d = 1 \) km are chosen. In general, units of \( z \) and \( d \) have to be identical (e.g., both m or both km). Our results from Section 2.2 suggest \( \delta \approx 250 \) m and \( k_{zs} \approx 5 \times 10^{-13} \) m\(^2\) for Eq. (39). For these values of \( \lambda, k_{zd}, \delta, \) and \( k_{zs} \), as well as for a depth of \( z \approx 0.8 \) km, the vertical permeabilities and their gradients,

\[ -\frac{k_{zs}}{\delta} e^{-z/\delta} \approx -\frac{k_{zd} \lambda D^\lambda}{z^{\lambda+1}}, \quad (41) \]

are similar, thus enabling the suggested smooth transition from Eq. (39) to Eq. (40) at \( z = 0.8 \) km (Figure 11b).

The methods employed and conversions discussed in Sections 2 and 3 provide estimates of mean permeability at four depth scales, mostly in vertical, but in two cases also in approximately horizontal, directions. The spring discharge model (Section 2.1) of study region A (Figure 1) yields an estimate of mean horizontal permeability, \( \kappa_x \), because the method considers individual near-surface aquifers that consist of slope-parallel, and thus roughly horizontally-layered, lava flows. Within such solidified lava flows (or aquifers),
water can flow primarily through the (about parallel and horizontal) pathways of highest permeability. Thus, the mean horizontal permeability is given by the arithmetic mean \([Maasland, 1957]\),

\[
\overline{k}_x = \frac{1}{b} \sum_{i=1}^{N} k_x b_i,
\]

where the subscript \(i\) denotes the \(i^{th}\) layer or pathway, \(b_i\) is the thickness of layer \(i\), and \(b\) is the total thickness of all layers.

In contrast, vertical groundwater flow across horizontal lava flows (or aquifers) is passing through sections of varying permeability in series, so that mean vertical permeabilities are typically determined by the harmonic mean \([Maasland, 1957]\),

\[
\overline{k}_z = b \left( \sum_{i=1}^{N} \frac{b_i}{k_{z,i}} \right)^{-1},
\]

where \(\overline{k}_z\) is dominated by the lowest permeabilities along that path. Hence, in volcanic or sedimentary settings, where slope-parallel, and thus approximately horizontal, layers, pathways, and aquifers are common, typically \(10 \leq \overline{k}_x/\overline{k}_z \leq 1000\), at least near the surface \([e.g., Deming, 1993]\) and in the absence of near-vertical fractures.

A further consequence of \(\overline{k}_z\) being determined by the harmonic mean (Eq. 43) and \(k_z(z)\) generally decreasing with depth is that our results for \(\overline{k}_z\) from Section 2 apply predominantly towards the lower portions of the considered depth ranges where the lowest permeabilities are encountered. Replacing the discrete layer formulation in Eq. (43) with the definite integral from zero to \(b\) of the continuous functions, Eqs. (39) and (40), results in the harmonic mean vertical permeabilities

\[
\overline{k}_z = \frac{bk_{zs}}{\delta (e^{b/\delta} - 1)}
\]
and

\[ k_z = k_{zd} \left( \frac{D}{b} \right)^{\lambda} (\lambda + 1) \]  

(45)

suggested for \( 0 \leq z \leq 0.8 \) km and \( z > 0.8 \) km, respectively. Setting Eq. (39) equal to Eq. (44) and Eq. (40) equal to Eq. (45) provides the depths

\[ z_z = -\delta \ln \left[ \frac{b}{\delta (e^{b/\delta} - 1)} \right] \]  

(46)

and

\[ z_z = b(\lambda + 1)^{-1/\lambda}, \]  

(47)

where \( k_z \) and \( k_{zd} \) are reached, respectively. Over- and underbars in Eqs. (44-47) and in later equations denote functions applicable to shallow (based on Eq. 39) and deeper (based on Eq. 40) depths, respectively. As before, \( b \) is the considered depth scale. Eqs. (46) and (47) are independent of the actual permeability value but depend on the characteristics of the decrease in permeability with depth (exponential versus power-law relationship). We use Eq. (46) for depth scales, \( b \), shallower than about 1 km and Eq. (47) for \( b \) greater than approximately 1 km. Parameter values of \( 200 \leq \delta \leq 300 \) m, \( 1 \leq \lambda \leq 10 \), and \( b \pm 0.25 b \) are based on results from Section 2. These ranges in parameters are used to calculate the depth ranges (\( z \)-ranges in Table 4 and Figure 11b) applicable to the shallower, \( k_z \), and deeper, \( k_{zd} \), permeability results obtained in Section 2.

For the shallow-most model (Section 2.1 and study region A in Figure 1) the arithmetic mean horizontal permeability is determined. Here, each horizontal layer’s permeability is assumed to decrease exponentially with depth according to Eq. (39). Similar to before, we replace the discrete formulation of the arithmetic mean (Eq. 42) with the definite vertical integral from zero to \( b \) of the continuous function (Eq. 39) resulting in the arithmetic
mean horizontal permeability given by

\[ \bar{k}_x = \frac{\delta k_{zs}}{b} \left(1 - e^{-b/\delta}\right). \] (48)

Setting Eq. (39) and Eq. (48) equal yields the depth,

\[ z_x = -\delta \ln \left[ \frac{\delta}{b} \left(1 - e^{-b/\delta}\right) \right], \] (49)

where \( \bar{k}_x \) is reached. This depth, \( z_x \), is thus only based on the suggested ranges of maximum near-surface aquifer depths (thicknesses), \( b = 50 \text{ m} \pm 25 \% \), and skin depths \( 200 \leq \delta \leq 300 \text{ m} \) (Table 4), where values for \( \delta \) are inferred from our heat-flow model (Section 2.2). As before, the overbars in Eqs. (48) and (49) denote functions applicable to shallower depths.

Figure 11b shows our results for \( \bar{k}_x, \bar{k}_z, \) and \( \bar{k}_z \) from Section 2 at depths \( z_x, z_z, \) and \( z_z \) determined using Eqs. (49), (46), and (47), respectively. This approach allows us to estimate depths, for which our permeability results are applicable, based on the shape of the \( k(z) \)-curve (exponential versus power-law) and the method of calculating the mean (arithmetic versus harmonic) as also described in Table 4. Therefore, while this technique does not use the actual (near-surface) permeability values, it requires assuming a model describing the change of permeability with depth (Eqs. 39 and 40). While the equations appear to describe \( k_z(z) \)-profiles in principle, the appropriate parameters are not well-constrained and depend largely on the local geology. We thus use a wide range of values for \( \delta, \lambda, \) and particularly \( b \), as suggested previously, to calculate depth ranges (i.e., uncertainties) over which the mean permeabilities are applicable (Table 4).

We recognize that our depth ranges are partially based on assuming that Eq. (40) applies for mean depths \( z_m > 1 \text{ km} \) and are thus dependent on the curve suggested by
Manning and Ingebritsen [1999], with $\lambda = 3.2$. However, we can use a wide range of $1 \leq \lambda \leq 10$ values to calculate a range of $z_z$ using Eq. (47). For shallower mean depths of $0 \leq z_m \leq 1$ km and for vertical and horizontal permeabilities, we employ our suggested Eqs. (46) and (49), respectively, that are instead based on Eq. (39). Here, we use $200 \leq \delta \leq 300$ m to determine ranges of $z_z$ and $z_x$. For either method we estimate independently a large uncertainty range in $b \pm 25\%$. Thus, while $z_z$ is not completely decoupled from $\lambda = 3.2$, the depth range is nonetheless based on an independent estimate of the maximum depth scale, $b$, a wide range of $\lambda$, and the reasonable assumption that the mean horizontal and vertical permeabilities are given by the arithmetic and the harmonic means, respectively, if approximately horizontal layers and no vertical fractures are present.

Given the previous considerations, we suggest that our results for $k_z(z)$ and $k_x(z)$ in the Oregon Cascades are largely independent from, but comparable to, the permeability-depth profile suggested by Manning and Ingebritsen [1999] for the continental crust. However, we propose that for depths smaller than about 1 km, Eq. (39) with $k_z \approx 5 \times 10^{-13}$ m$^2$ and $\delta \approx 250$ m is more appropriate at least for the Cascades and possibly for continental crust in general.

For example, Shmonov et al. [2003] suggest a near-surface permeability of approximately $k_s \approx 2.75 \times 10^{-13\pm1.9}$ m$^2$ that decreases to $1.67 \times 10^{-20\pm1.5}$ at 40 km depth based on experimental data on the permeability of samples of amphibolite and gneiss from the Kola Ultradeep Borehole. Furthermore, Patriarche et al. (manuscript in preparation, 2004) invoke an exponential relationship between depth and hydraulic conductivity analogous to Eq. (39) with an average $\delta \approx 0.24$ km to satisfy calibrations for both measured hydraulic heads and $^4$He concentrations in the sedimentary Carrizo aquifer in Texas. Because of
the similarity of their \( \delta \) value to ours \( (\delta = 0.25 \text{ km}) \), their \( K(z) \)-curve (converted to 
\( k(z) = K \nu/g \approx 10^{-7} K \)) has a similar shape as our exponential \( k(z) \)-curve. Their zero-
depth permeability of \( 5 \times 10^{-11} \text{ m}^2 \) is about two orders of magnitude larger than ours, is based on a near-surface hydraulic conductivity of the aquifer of about \( 5 \times 10^{-4} \text{ m/s} \), and plots within the dotted region in Figure 11b.

However, measurements from the German Continental Deep Drilling program (KTB) \cite{Huenges1997} show near-surface permeabilities of \( 5 \times 10^{-18} \) to \( 3 \times 10^{-16} \text{ m}^2 \) that are much lower than those suggested by the \( k(z) \)-curve from Manning and Ingebritsen \cite{Manning1999} and significantly lower than even our suggested exponential \( k(z) \)-curve. Such low permeabilities may be expected in near-surface crystalline rocks at the KTB site. These rocks appear to show lower hydraulic diffusivities at shallower depths, probably due to higher fracture densities at greater depths, as reported by Rothert et al. \cite{Rothert2003} and discussed briefly in Section 2.3.2. Huenges et al. \cite{Huenges1997} state that no clear depth-dependence was observed. This, or even inverted \( k(z) \)-profiles, may be expected if near-surface permeabilities are so low that a reduction in permeability at depth, due to compaction of highly-fractured sections, can be negligible.

The range of measured (near-surface) permeability values, as well as the variance in depth dependence of permeability may serve as a reminder that a wide range of lithology-dependent permeabilities \cite[e.g.,][]{Freeze1979} is expected, particularly near the ground surface (Figure 11a). Our suggested exponential \( k(z) \)-curve falls within this wide range of permeabilities but of course cannot reflect all possible lithologies or degrees of fracturing. Instead, general \( k(z) \)-curves serve as permeability estimates at various depths over large spatial (and temporal) scales where more lithology-specific determinations are
not feasible, as discussed further in Sections 5 and 6. However, a maximum permeability value may be reached at about $k \approx 10^{-7} \text{ m}^2$ for gravel (Figure 11a). A more typical upper limit for continental crust in general may be $k \approx 10^{-12} \text{ m}^2$ \cite{Manning and Ingebritsen, 1999, Table 1 and Figure 8} at least in the vertical direction across horizontal layers. Such a low maximum average permeability further supports our exponential $k_z(z)$-relationship with the suggested finite near-surface value of about $k_{zs} \approx 5 \times 10^{-13} \text{ m}^2$ (Figure 11b) for vertical groundwater flow across sub-horizontal lava flows or sedimentary layers and in the absence of near-vertical fractures.

A further advantage of using Eq. (39) for depths shallower than about 1 km (or even shallower than 2 km) is that the function provides a finite near-surface permeability of $k_{zs}$ as depth approaches zero. In contrast, Eq. (40) diverges towards infinity for $z \to 0$ and results in a singularity at $z = 0$. This limitation of Eq. (40) may not be critical for global studies of permeability, where overall estimates over larger depth scales ($z > 0.5$ km) may be desired. In fact, for $z > 2$ km, Eq. (40) is preferred because at greater depths its permeability gradient is smaller than the one for Eq. (39), providing more realistic deeper permeabilities. Therefore, we propose a transition from Eq. (39) to Eq. (40) at $z = 0.8$ km. This transition is relatively smooth as permeabilities and their gradients (Eq. 41) are comparable at $z = 0.8$ km (Figure 11b) for the suggested values of $k_{zs} \approx 5 \times 10^{-13} \text{ m}^2$, $\delta \approx 250 \text{ m}$, $k_{zd} \approx 10^{-14} \text{ m}^2$, and $\lambda = 3.2$ (the latter two values are adopted from Manning and Ingebritsen [1999] and corroborated by this study). However, because of the similarity in permeabilities obtained from Eqs. (39) and (40) for the suggested parameter values of $k_{zs}$, $\delta$, $k_{zd}$, and $\lambda$ and for $0.5 < z < 2$ km (Figure 11b) only Eq. (39) or only Eq. (40) may be used for $z < 2$ km and for $z > 0.5$ km, respectively. In contrast, if
$k(z \geq 0 \text{ km})$ is of interest, both equations should be employed with a transition at $z = 0.8 \text{ km}$.

However, if sub-vertical fractures and faults are present, high-$k_z$ pathways can alter the relationship between depth and permeability. Hence, we suggest that our elevated values of $k_z \approx 5 \times 10^{-15} \text{ m}^2$ at $z_m \approx 3 \text{ km}$ (Table 4 and Figure 11b), determined by the hydroseismicity method (Section 2.3) for study region C3 (Figure 1), are due to faults located on the southern flanks of Mt. Hood, such as the White River fault. These faults, that are located away from the volcano’s central axis, show predominantly normal-fault focal mechanisms [Jones and Malone, 2002] and thus suggest tectonically-driven (possibly hydrologically-triggered [Saar and Manga, 2003]) earthquakes, rather than magma-flow-induced seismicity that is typically characterized by non-double-couple focal mechanisms [Julian, 1983; Dreger et al., 2000]. Locally high values of $k$ allow for advective heat transfer and hot springs, where water can reach the surface faster than heat diffusion time scales required for thermal equilibration with the surrounding rock. Indeed, the only two off-centered hot spring areas (Meadows Spring and Swim Warm Springs) observed at Mt. Hood to date [Nathenson, 2003] are located on the southern flanks of the volcano close to the earthquakes’ epicenters (Figure 6). Therefore, we suggest that the slightly elevated permeabilities inferred for mean applicable depths of about $z_m = 3 \text{ km}$ in study region C3 (Figure 1 and Table 4) reflect the permeability of a large representative elementary volume that is dominated by normal faults that are being kept permeable by hydroseismicity [Rojstaczer and Wolf, 1992; Townend and Zoback, 2000]. In addition, vertical faults are approximately parallel to each other, thus providing parallel pathways whose combined mean permeability is dominated by the highest-$k$ pathways which is determined by the
larger arithmetic mean (Eq. 42), rather than by the smaller harmonic mean (Eq. 43) [Rovey, 1998].

Scale-dependence of permeability is controversial [Brace, 1980, 1984; Clauser, 1992; Sánchez-Vila et al., 1996; Renshaw, 1998; Rovey, 1998; Townend and Zoback, 2000; Zlotnik et al., 2000; Hyun et al., 2002; Becker and Davis, 2003; Zimmermann et al., 2003]. Permeability tends to increase as the representative elementary volume increases from laboratory rock-core measurements, over in-situ field measurements, to large-scale regional models, as those considered in this study. This relationship may be the case because large-scale heterogeneities, that may include high permeability pathways, are more likely to be sampled if larger volumes are considered. For example this appears to be the case in the hydroseismicity model described in the previous paragraph. Therefore, Hsieh [1998] suggests to call this sampling effect on inferred permeability values a “sampling bias” rather than a “scale effect.” However, an upper bound appears to be reached at the upper field and regional modeling scale [Clauser, 1992; Sánchez-Vila et al., 1996], possibly because most heterogeneities have been included [Renshaw, 1998].

4.2. Basal Heat-flow, $H_b$

Figure 10 shows a map of vertical near-surface heat-flow for study region E in Figure 1. The near-surface results are based on a multi-quadratic interpolation technique [Nielson, 1993] applied to 209 geotherm boreholes and $K_T = 2 \text{ W}^{-1}\text{C}^{-1}\text{m}^{-1}$ [Ingebritsen et al., 1988; Blackwell, 1992; Ingebritsen et al., 1993]. High precipitation ($\sim 2 \text{ m/ year}$) and infiltration rates ($\sim 1 \text{ m/ year}$) in the Oregon Cascades cause isothermal (and sometimes inverted) near-surface geotherm profiles (Figures 7a, 8, and 10) effectively masking the background heat-flow [Ingebritsen et al., 1989, 1992]. We minimize the mean absolute
error, $\epsilon$, between data and calculated temperature values and infer a thermal gradient of about $(\partial T/\partial z)_b \approx 40 \, ^\circ C/km$ (Figure 7a) at Santiam Pass, located between two volcanoes, Three-Fingered Jack and Mt. Washington (Figure 4a). Connecting the deepest two data points yields $(\partial T/\partial z)_b \approx 90 \, ^\circ C/km$. However, the last data point was taken as drilling progressed [Blackwell, 1992]. Deeper portions of the linear temperature-depth profiles at Mt. Hood volcano (Figure 8) yield $(\partial T/\partial z)_b \approx 65 \, ^\circ C/km$. Therefore, we infer a mean background heat-flow, $H_b$, for the study region of approximately

$$0.080 \leq H_b \approx k_{Tr} \left( \frac{\partial T}{\partial z} \right)_b \leq 0.130 \, W/m^2,$$

(50)

where the lower and upper heat-flow values are inferred from the study of Santiam Pass and Mt. Hood, respectively (Table 3). These values are consistent with the more extensive studies by Blackwell et al. [1982] and Ingebritsen et al. [1994]. As expected for a region showing active volcanism, heat-flow is elevated with respect to global average values for continents of about $0.065 \, W/m^2$ [Pollack et al., 1993].

5. Implications

As stated in the introduction, permeability largely determines subsurface fluid fluxes as well as energy transfer. Examples of subsurface fluids include water, steam, CO$_2$, noble gases, and hydrocarbons. Energy transfer may include pore-fluid pressure diffusion and advective heat transport. In addition, transfer of heat and mass can cause fluid-rock interactions and metamorphic reactions. Therefore, understanding spatial (and temporal) variations in permeability distributions has implications for studies involving groundwater, geothermal, ore, and hydrocarbon resources as well as for research addressing environmental contamination, carbon sequestration, and hydrologically induced earthquakes.
Of particular interest for the Cascades investigated in this paper is the magnitude of advective heat transfer beneath the volcanoes which depends on the permeability distribution. This, in turn, has implications for background heat-flow estimates, the shape and size of magmatic plumbing systems underneath subduction-related volcanoes, and the feasibility of geothermal energy exploration in the Oregon Cascades and geologically similar regions.

Spatial (and temporal) scales of permeability distribution, and related mass and energy transfer patterns, can be highly variable as discussed in Section 4.1. For small-scale, near-surface studies of fluid flow it is thus important to determine site-specific permeabilities employing for example permeameter or pumping tests. However, if the study region is large (hundreds to thousands of meters) model-based approximations of permeability as a function of depth as shown in Figure 11b may be a more representative description of the subsurface. In the latter case, the $k(z)$-relationship should be valid from shallow to large depths and should vary smoothly so that inferred models can be applied to all depths and abrupt refractions of flow lines are kept to a minimum. Both conditions are fulfilled for the composite $k(z)$-curve suggested in this study and summarized in Eq. (51).

6. Conclusions

Because large-scale hydrogeologic models are generally underconstrained, it is desirable to utilize multiple direct and indirect observations to improve simulations, as described in the introduction. In this study, we use classic hydrogeologic boundary conditions and parameters as well as temperature and seismic data, and estimates of magma intrusion rates. Further improvements in future work may include additional constraints such as water chemistry and multi-phase fluid flow, particularly at greater depths.
We employ analytical and numerical modeling techniques to estimate permeability at different depth scales for the Oregon Cascades. Our results suggest that for depths shallower than about 1 km, permeability decreases exponentially with depth, with a skin depth of $\delta \approx 0.25$ km, from a near-surface value of $k_{zs} \approx 5 \times 10^{-13}$ m$^2$.

For depths larger than about 1 km, permeability appears to decrease according to a power law as suggested by Manning and Ingebritsen [1999] with an exponent of $\lambda \approx 3.2$ and a permeability of $k_{zd} \approx 10^{-14}$ m$^2$ at depth $z = 1$ km. Therefore, we propose

$$k_z(z) = \begin{cases} 
5 \times 10^{-13} \text{ m}^2 \exp \left( \frac{-z}{0.25 \text{ km}} \right) & \text{for } 0 \leq z \leq 0.8 \text{ km} \\
10^{-14} \text{ m}^2 \left( \frac{z}{1 \text{ km}} \right)^{-3.2} & \text{for } z > 0.8 \text{ km}
\end{cases}$$

(51)

where the parameters for $z > 0.8$ km are adopted from Manning and Ingebritsen [1999] and are corroborated by this study. For studies that consider only depths < 2 km or depths > 0.5 km the respective exponential or power-law functions can be employed without invoking a transition from one to the other at 0.8 km depth (see similarity between the functions at $0.5 < z < 2$ km in Figure 11b). However, if estimates of $k_z$ are desired at all depths, the advantage of using both functions in Eq. (51) with a transition at $z = 0.8$ km is that they provide realistic average vertical permeabilities for both small ($z < 0.5$ km) and large ($z > 2$ km) depths which neither function alone could both achieve, as discussed in Section 4.1. In addition, the functions in Eq. (51) yield a relatively smooth transition at $z = 0.8$ km, where their permeabilities and their permeability gradients are similar.

We also determine horizontal permeabilities at depths shallower than about 1 km. At least near the surface, and in the absence of vertical fractures, horizontal permeabilities are typically one to three orders of magnitude larger in volcanic or sedimentary settings where slope-parallel, and thus approximately horizontal, layers, pathways, and aquifers are common.
We note a slight divergence from Eq. (51) for our hydroseismicity model that suggests about one order of magnitude higher permeability values. However, higher permeabilities in this region may be consistent with advective heat transfer along active faults causing observed hot springs.

Finally, our coupled heat and groundwater transfer models suggest groundwater recharge rates of $0.5 \leq u_R \leq 1$ m/year near the crest of the Cascades and mean background heat-flow values of about $H_b \approx 0.080$ to $H_b \approx 0.134$ W/m$^2$ beneath the study region. The elevated background heat-flow values are associated with volcanic centers that may have magma sources at shallower depths.

Lithological variations and the degree of fracturing or dissolution can cause a wide range of permeabilities as shown for example in Figure 11a. Therefore, it is important to emphasize that the general relationship between depth and permeability suggested here (solid bold curve in Figure 11b and Eq. 51) provides only a large-scale average estimate. This estimate was developed for the volcanic (Oregon) Cascades but may also apply to continental crust in general including sedimentary basins where near-horizontal layers are present. However, the suggested $k(z)$-relationship is not a substitute for more lithologically-specific measurements of permeability for a given region of interest. However, frequently such direct permeability measurements are technically or economically not feasible at greater depths or over large regional scales. In such cases, estimates of the depth dependence of permeability, as presented here, can be useful.
Appendix A: Determination of the recharge velocity, $u_R$, for exponentially decreasing hydraulic conductivity in a 1D recharge region

In the $xz$-plane (Figure 3), the recharge velocity, $u_R$, in Eq. (13) is given by the horizontal divergence (denoted $\nabla_x \cdot$) of the vertical integral over the depth of the saturated zone, $b$, of Eq. (7). For exponentially decreasing hydraulic conductivity, $K_x(z)$ in Eq. (7) is given by Eq. (11) resulting in

$$u_R = \nabla_x \left( -K_{sx} \nabla h(x) \int_0^b e^{-z/\delta} dz \right). \quad (A1)$$

Solving the definite integral in Eq. (A1) and expanding the divergence term yields

$$u_R = \nabla_x \left[ -K_{sx} \delta \left( 1 - e^{-b/\delta} \right) \right] \cdot \nabla_x h(x)
- K_{sx} \delta \left( 1 - e^{-b/\delta} \right) \nabla^2_x h(x). \quad (A2)$$

At the maximum of the water table at $x = 0$, $\nabla_x h = 0$ and thus Eq. (A2) reduces to

$$u_R = \frac{K_{sx} \delta}{R} \left( 1 - e^{-b/\delta} \right), \quad (A3)$$

where, as before, $R = -\left( \partial^2 h/\partial x^2 \right)^{-1} = \nabla^2_x h(x)$ is the (constant) radius of curvature of the water table in the $xz$-plane.

Appendix B: Definition of mixed thermal diffusivity, $D_m$

For a given pore fraction, $n$, mixed properties, $\xi_m$, of the water-rock complex are approximated by

$$\xi_m = n\xi_w + (1 - n)\xi_r, \quad (B1)$$

where $\xi_w$ and $\xi_r$ are the properties in question of pure water and pure rock, respectively. Water-rock complex properties to be substituted into Eq. (B1), where subscripts remain as given in Eq. (B1), are mixed density, $\xi = \rho$, mixed specific heat, $\xi = c$, and mixed...
thermal conductivity, $\xi = K_T$. Mixed thermal diffusivity is then given by

$$D_m = \frac{K T_m}{\rho_m c_m}.$$ (B2)

Throughout this analysis we assume local thermal equilibrium between water and rock and that $n$ is sufficiently small so that Eq. (B1) applies.

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**Figure 1.** Shaded relief map of the study region in Oregon, USA. Red boxes indicate particular study regions used for the four methods and scales described in Section 2: A) spring discharge model, B) coupled heat and groundwater flow model in a 1-dimensional recharge area, C) radial coupled heat and groundwater flow model, D) magma intrusion model, E) region for heat-flow interpolation based on 209 geotherm profiles (white circles).

**Figure 2.** Spring discharge model of a simplified aquifer. Symbols are defined in the main text.

**Figure 3.** a) Schematic peaks (e.g., Three Fingered Jack and Mt. Washington, Oregon; see Figure 4a) and a saddle (e.g., Santiam Pass, Oregon; see Figures 1 and 4a) may represent groundwater recharge regimes with 1-dimensional (1D) vertical groundwater flow. The saddle can also constitute a (local) groundwater discharge regime. b) Schematic cross section A-A’ (see panel a) showing a typical recharge geometry. Parameters are defined in the main text.

**Figure 4.** Radius of curvature of the ground surface at Santiam Pass, Oregon. a) Shaded relief map between latitudes 44.25 and 44.55, color coded to elevation. Black lines indicate locations of the 12 cross sections shown in panel b. The geothermal borehole location is indicated by the gray arrow. Weather station 357559 (U.S. Department of Commerce, National Oceanic & Atmospheric Administration, NOAA) is given by the white arrow. b) East-west elevation profiles (thin black lines) used to estimate the minimum (R=80 km) and maximum (R=120 km) radius of curvature (bold red curves). The borehole is shown as the gray vertical line.
Figure 5. Groundwater flow streamlines beneath a 1D recharge region for (vertically) exponentially decreasing horizontal hydraulic conductivity, Eqs. (12) and (14) with $\delta = 270$ m and $K_{sx} = 5.8 \times 10^{-6}$ m/s (solid curves), and constant hydraulic conductivity, Eqs. (9) and (10) with $K_x = 3.5 \times 10^{-7}$ m/s (dashed curves). The exponential-$K$ model, allows for recharge velocities of $u_R \approx 0.5$ m/year (large solid arrow) compared with $u_R \approx 0.1$ m/year (small dotted arrow) for the constant-$K$ model (Figure 7a). As a result, the exponential-$K$ model shows larger horizontal flow vector components at shallower depths reducing vertical flow velocities more rapidly with depth than the constant-$K$ model.

Figure 6. Shaded relief map of Mt. Hood, Oregon (study region C in Figure 1), subsurface temperature distribution (color coded) based on a multiquadratic interpolation of thirteen geotherms (black bars), and earthquake locations (red circles). The three geotherms closest to the summit of Mt. Hood provide the measured $T(z)$ data in Figure 8. The relief map is vertically offset by 3 km (arrow) from the temperature map for better visualization of the temperature data. Elevated temperatures beneath Mt. Hood appear to be offset towards the south (see vertical bar through the summit for comparison), consistent with the occurrence of the only two hot spring areas, Meadows Spring and Swim Warm Springs (yellow bars on elevation contour plot) observed to date at Mt. Hood [Nathenson, 2003]. Most earthquakes also occur beneath the southern flanks of the volcano (see projection of earthquakes onto map) and show mostly normal-fault focal mechanisms (see Section 2.3 and [Jones and Malone, 2002; Saar and Manga, 2003]).
Figure 7.  a) Temperature versus depth models, $T(z)$ (bold curves), and vertical groundwater flow velocity profiles, $u_z(z)$ (thin curves) at Santiam Pass, Oregon. Dashed and solid curves indicate results based on constant and exponential horizontal hydraulic conductivity profiles, respectively (see panel b). The $T(z)$-profiles are calculated from Eq. (18) and from numerical integration of Eq. (17) for constant and exponential hydraulic conductivity profiles, respectively. White-filled circles are temperature measurements from drill hole SP 77-24 [Blackwell, 1992] (for location see Figures 1 and 4). The dark-filled circle is the mean annual surface temperature at nearby NOAA weather station 357559 (white arrow in Figure 4a) at Santiam Pass, Oregon, averaged over the last 10 available complete years (1975 - 1984) at an elevation of 1449 m. Precipitation rate at the same weather station is about 2.2 m/year. The gray bold line connects the deepest two data points at a slope of about 90 °C/km. The slope of the models at $z \approx 1000$ m is about 40 °C/km. b) Horizontal hydraulic conductivity, $K_x$, versus depth, $z$, for the constant $K_x = 3.5 \times 10^{-7}$ m/s model (long-dashed line) and the exponential $K_x = K_{sx} \exp(-z/\delta)$ model (solid line) with $K_{sx} = 5.8 \times 10^{-6}$ m/s and $\delta = 270$ m. The short-dashed curve is the permeability-depth curve suggested by Manning and Ingebritsen [1999] converted to $z$ in m and hydraulic conductivity in m/s, assuming a constant temperature of about 20 °C for estimates of water viscosity. Parameters are provided in Tables 1 and 3.
**Figure 8.** Temperature, \( T \), versus depth, measured from the surface at each of the three geotherms at Mt. Hood, Oregon that are nearest to the summit (see inset in this figure and black bars on elevation contour plot in Figure 6). The solid, dashed, and dotted curves are results from the 3D-radial simulation. Measurements are indicated by triangles, circles, and squares. Two geotherms (circles and squares) show typical recharge dominated profiles with isothermal upper section, similar to the one shown in Figure 7a. The geotherm depicted by the triangles shows a mostly linear (conductive) profile with a slight concave-downward bend at shallower depths, suggesting a groundwater discharge region. Input parameters are discussed in the main text.

**Figure 9.** Schematic illustration of magma intrusion underneath the Cascades range volcanic arc (not drawn to scale) [after Hildreth, 1981]. Symbols are defined in the main text.

**Figure 10.** Map of near-surface temperature gradients in study region E (Figure 1), based on an interpolation of geotherm data from drill holes (circles). Isothermal and inverted temperature gradients exist in high precipitation and groundwater recharge areas such as the high-relief volcanoes. Cold groundwater is heated at depth, the heat is advected and discharged at hot springs [Ingebritsen et al., 1989; Ingebritsen et al., 1992].
Figure 11.  a) Approximate (near-surface) range of permeability values [after Freeze and Cherry, 1979]. The (Oregon) Cascades primarily consist of basalt and basaltic andesite where the youngest (< 2.3 Ma) rock units of the High Cascades show high near-surface permeabilities of \( k \approx 10^{-14} \) m\(^2\) [Ingebritsen et al., 1992; Ingebritsen et al., 1994], consistent with the \( k \)-range indicated for permeable basalts.  

b) Permeability, \( k \), as a function of depth, \( z \). Shown are our results for study regions A,B,C, and D (Figure 1 and Table 4) for horizontal, \( x \) (thin boxes), and vertical, \( z \) (bold boxes), permeabilities. The depth ranges are determined as described in the main text and in the caption to Table 4. Box widths and heights reflect approximate variations in permeability and applicable depth ranges, respectively, calculated from uncertainties in input parameters as described in Section 4.1 and Table 4. Superimposed is the exponential profile of Eq. (39), denoted Exp, with \( \delta = 0.25 \) km and \( k_{zs} = 5 \times 10^{-13} \) m\(^2\) for vertical (bold curve) and \( k_{xs} = 5 \times 10^{-10} \) m\(^2\) for horizontal (thin curve) permeabilities. Also shown is the power law profile of Eq. (40), denoted Pwr, with \( \lambda = 3.2 \) and \( k_{zd} = 10^{-14} \) m\(^2\) at \( d = 1 \) km as suggested by Manning and Ingebritsen [1999] for vertical (bold line) permeabilities. \( k_{zd} = 10^{-11} \) m\(^2\) is used for horizontal (thin line) permeabilities. Solid lines indicate the permeability-depth curves suggested with a transition from exponential to power-law profile at a depth of 0.8 km (dashed horizontal line). The dotted area indicates the expected range between vertical and horizontal permeability. At depth this range may be expected to reduce due to compaction resulting in \( k_x \rightarrow k_z \) which is not shown in panel b as the width of the dotted area remains constant. However, our results at \( z \approx 0.5 \) km suggest similar vertical (study region B) and horizontal (study region C1) permeabilities.
### Table 1. Definition of symbols in equations

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Table 2. Spring model input and results for Quinn River and Cultus River.

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<td>(-\log K_x ) (m/s)</td>
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\( a \) Manga [1997]

\( b \) James et al. [2000]

\( c \) Manga [1997]

\( d \) Ingebritsen et al. [1992]

\( e \) Manga [2001]
Table 3. Parameters for coupled groundwater and heat-flow models

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<tr>
<th>Symbol</th>
<th>Units</th>
<th>1Dc</th>
<th>1De</th>
<th>3D-radial</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_z(z)$</td>
<td>(m/s)</td>
<td>$K_{sz}$</td>
<td>$K_{sz}e^{-z/\delta}$</td>
<td>$\eta K_{sz}e^{-z/\delta}$</td>
</tr>
<tr>
<td>$K_{sz}$</td>
<td>(m/s)</td>
<td>$3.5 \times 10^{-7}$</td>
<td>$5.8 \times 10^{-6}$</td>
<td>$3 \times 10^{-8}$</td>
</tr>
<tr>
<td>$\delta$</td>
<td>(m)</td>
<td>$\infty$</td>
<td>270</td>
<td>250</td>
</tr>
<tr>
<td>$\eta$</td>
<td></td>
<td>1</td>
<td>1</td>
<td>30</td>
</tr>
<tr>
<td>$b$</td>
<td>(m)</td>
<td>1000</td>
<td>1000</td>
<td>1200</td>
</tr>
<tr>
<td>$h$</td>
<td>(m)</td>
<td>$x^2/R$</td>
<td>$x^2/R$</td>
<td>TOPO</td>
</tr>
<tr>
<td>$u_R$</td>
<td>(m/yr)</td>
<td>0.1</td>
<td>0.5</td>
<td>1</td>
</tr>
<tr>
<td>$T_r$</td>
<td>(°C)</td>
<td>5</td>
<td>5</td>
<td>MALR</td>
</tr>
<tr>
<td>$\left(\frac{\partial T}{\partial z}\right)_b$</td>
<td>(°C/m)</td>
<td>0.042</td>
<td>0.040</td>
<td>0.067</td>
</tr>
<tr>
<td>$H$</td>
<td>(W/m²)</td>
<td>0.84</td>
<td>0.080</td>
<td>0.134</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>(°C)</td>
<td>0.68</td>
<td>0.92</td>
<td>$\sim$1</td>
</tr>
</tbody>
</table>

The anisotropy ratio of horizontal to vertical hydraulic conductivity is given by $\eta = K_x/K_z$.

For the radial model the recharge temperature is determined by the mean adiabatic lapse rate (MALR) and the upper hydraulic head, $h$, boundary is approximated by topography (TOPO).

For the one-dimensional models with constant hydraulic conductivity (1Dc) and exponentially-decreasing hydraulic conductivity (1De), $h$ is determined by the radius of curvature of the water table, $R = 10^5 \pm 2 \times 10^4$ m (Figure 4), and the horizontal distance, $x$, from the maximum $h$ in the recharge region.
Table 4. Conversions from hydraulic conductivity, $K$, to permeability, $k$, and parameters for calculation of respective applicable depth ranges ($z$-ranges)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Units</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method</td>
<td>spring dis.</td>
<td>heat-flow</td>
<td>heat-flow</td>
<td>heat-flow</td>
<td>hydrosis.</td>
<td>magma intr.</td>
<td></td>
</tr>
<tr>
<td>Section #</td>
<td>2.1</td>
<td>2.2</td>
<td>2.2</td>
<td>2.2</td>
<td>2.3</td>
<td>2.4</td>
<td></td>
</tr>
<tr>
<td>Study Area</td>
<td>$x$</td>
<td>$x$</td>
<td>$z$</td>
<td>$z$</td>
<td>$z$</td>
<td>$z$</td>
<td></td>
</tr>
<tr>
<td>EA, EH, PH</td>
<td>EA</td>
<td>EA</td>
<td>EH</td>
<td>EH</td>
<td>PH</td>
<td>PH</td>
<td></td>
</tr>
<tr>
<td>$\delta_{\text{min}}, \delta_{\text{max}}$</td>
<td>km</td>
<td>0.2, 0.3</td>
<td>0.2, 0.3</td>
<td>0.2, 0.3</td>
<td>0.2, 0.3</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\lambda_{\text{min}}, \lambda_{\text{max}}$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1, 10</td>
<td>1, 10</td>
<td></td>
</tr>
<tr>
<td>$b \pm 0.25b$</td>
<td>km</td>
<td>0.05 ± 0.0125</td>
<td>1 ± 0.25</td>
<td>1.2 ± 0.3</td>
<td>1.2 ± 0.3</td>
<td>4.5 ± 1.125</td>
<td>15 ± 3.75</td>
</tr>
<tr>
<td>$z$-range</td>
<td>km</td>
<td>0.02 - 0.03</td>
<td>0.3 - 0.5</td>
<td>0.4 - 0.9</td>
<td>0.6 - 1.1</td>
<td>1.7 - 4.4</td>
<td>5.6 - 14.8</td>
</tr>
<tr>
<td>$(\partial T/\partial z)_b$</td>
<td>°C/km</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>$z_m$</td>
<td>km</td>
<td>0.025</td>
<td>0.4</td>
<td>0.65</td>
<td>0.85</td>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>$T_m$</td>
<td>°C</td>
<td>6</td>
<td>25</td>
<td>38</td>
<td>48</td>
<td>155</td>
<td>505</td>
</tr>
<tr>
<td>$P_m$</td>
<td>MPA</td>
<td>0.25</td>
<td>4</td>
<td>6.5</td>
<td>8.5</td>
<td>30</td>
<td>100</td>
</tr>
<tr>
<td>$\rho_m$</td>
<td>kg/m$^3$</td>
<td>$10^3$</td>
<td>$10^3$</td>
<td>$10^3$</td>
<td>$10^3$</td>
<td>$10^3$</td>
<td>$0.5 \times 10^3$</td>
</tr>
<tr>
<td>$\mu_m$</td>
<td>Pa · s</td>
<td>$1.5 \times 10^{-3}$</td>
<td>$8 \times 10^{-4}$</td>
<td>$6 \times 10^{-4}$</td>
<td>$5 \times 10^{-4}$</td>
<td>$2 \times 10^{-4}$</td>
<td>$5 \times 10^{-5}$</td>
</tr>
<tr>
<td>$\nu_m$</td>
<td>m$^2$/s</td>
<td>$1.5 \times 10^{-6}$</td>
<td>$8 \times 10^{-7}$</td>
<td>$6 \times 10^{-7}$</td>
<td>$5 \times 10^{-7}$</td>
<td>$2 \times 10^{-7}$</td>
<td>$1 \times 10^{-7}$</td>
</tr>
<tr>
<td>$K_{\text{min}}$</td>
<td>m/s</td>
<td>$10^{-3}$</td>
<td>$10^{-6}$</td>
<td>$3 \times 10^{-7}$</td>
<td>$3 \times 10^{-7}$</td>
<td>$8 \times 10^{-8}$</td>
<td>$9 \times 10^{-13}$</td>
</tr>
<tr>
<td>$K_{\text{max}}$</td>
<td>m/s</td>
<td>$10^{-2}$</td>
<td>$10^{-6}$</td>
<td>$3 \times 10^{-6}$</td>
<td>$3 \times 10^{-6}$</td>
<td>$5 \times 10^{-7}$</td>
<td>$2 \times 10^{-10}$</td>
</tr>
<tr>
<td>$k_{\text{min}}$</td>
<td>m$^2$</td>
<td>$1.5 \times 10^{-10}$</td>
<td>$8.2 \times 10^{-14}$</td>
<td>$1.8 \times 10^{-14}$</td>
<td>$1.5 \times 10^{-15}$</td>
<td>$1.6 \times 10^{-15}$</td>
<td>$9 \times 10^{-21}$</td>
</tr>
<tr>
<td>$k_{\text{max}}$</td>
<td>m$^2$</td>
<td>$1.5 \times 10^{-9}$</td>
<td>$8.2 \times 10^{-14}$</td>
<td>$1.8 \times 10^{-13}$</td>
<td>$1.5 \times 10^{-15}$</td>
<td>$1.0 \times 10^{-14}$</td>
<td>$2 \times 10^{-18}$</td>
</tr>
</tbody>
</table>

Conversions from hydraulic conductivity, $K$, to permeability, $k$, in horizontal, $x$, and vertical, $z$, directions, based on Eqs. (37) and (38). To calculate the mean kinematic viscosity, $\nu_m = \mu_m/\rho_m$ in Eq. (38) the mean temperature, $T_m = T_R + z_m(\partial T/\partial z)_b$, is determined where $T_R \approx 5$ °C is a typical recharge temperature (Figures 7 and 8) and $(\partial T/\partial z)_b \approx 50$ °C/km (Figure 7 and Section 4.2) in all cases. Here, the applicable mean depth, $z_m$, is the average of the $z$-range. The latter is calculated using combinations of minimum/maximum values for the aquifer depth, $b \pm 25$ %, and minimum/maximum skin depths, $\delta$, or exponents, $\lambda$, for exponential, $E$, or power-law, $P$, functions, respectively. The following refers to approximately horizontal layers without vertical fractures. The exponential solution, $E$, for the $z$-range is used for shallower depths ($b \leq 2$ km), both for approximately horizontal (parallel) pathways (Eq. 49) and for approximately vertical (serial) pathways (Eq. 46) that are calculated using the arithmetic mean, $A$, and the harmonic mean, $H$, permeability expressions, respectively. The power-law solution, $P$, is reserved for mean depths $z_m > 1$ km and is determined only for approximately vertical (serial) pathways (Eq. 47) that are characterized by harmonic mean, $H$, permeability values. Resulting viscosities agree to within the implicit uncertainties with compilations by Grigull et al. [1990]. Hydraulic conductivity for column 5 (Section 2.3) is based on results for vertical hydraulic diffusivity, $D_z$, and assuming $S_g \approx 10^{-6}$ m$^{-1}$ in Eq. (35) as discussed in Section 3. Permeability results are plotted in Figure 11b, where permeability uncertainties for columns 2 and 4 have been assigned ±1/2 order of magnitude based on the uncertainties calculated for $k$ in columns 1, 3, 5, and 6.
$u_R \approx 0.5 \text{ m/year}$

$u_R \approx 0.1 \text{ m/year}$

$K_x(z) = K_{sx} e^{z/\delta}$

$K_x = \text{constant}$