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METHOD FOR DETERMINING THE ORBITAL ANGULAR MOMENTUM IN K⁻-D CAPTURE

Donald H. Miller

April 29, 1960
In a recent communication, Day, Snow, and Sucher predicted that $K^-$ mesons when stopped in liquid hydrogen or deuterium would be captured from high-lying $S$ atomic orbitals of the $K^-$-$p$ or $K^-$-$d$ system. They found that the Stark-effect mixing of angular-momentum states induced by the strong electric fields experienced by the neutral $K^-$-$p$ or $K^-$-$d$ system as it passed within the Bohr orbitals of other atoms of the liquid allowed rapid capture through the $S$-wave-$K$-nucleon interaction. Their prediction has been used to draw important conclusions about the properties of hyperons and the nature of the low-energy, $S$-wave-$K$-nucleon interaction. In addition, estimates indicate that Stark-effect mixing determines the course of the antiproton annihilation reaction at rest.

Because of the wide applicability of the prediction, an experimental test is highly desirable. The obvious experiment of looking for $K$-shell x-rays when $K^-$ mesons are stopped in liquid $H_2$ or $D_2$ is technically difficult and probably will not be done in the near future.

It may be noted that several of the $K^-$-$d$ capture reactions are sensitive to the orbital angular momentum of the $K^-$-$d$ system. This is implied for example in the calculations of Pais and Treiman on the $\Sigma$-$n$-hyperfragment production rate. Their work shows that if the bound $\Sigma$-$n$ system existed and its characteristics were known,
the production rate for stopped \( K^- \) mesons would provide a clear
determination of the orbital from which capture occurs. It is the purpose
of this note to point out that the \( K^- \)-d orbital angular momentum de-
termines the rates for another class of reactions that is more accessible
experimentally, the nonmesic capture reactions in deuterium,

\[
K^- + d \rightarrow \Sigma^- + p \quad (1a) \\
\Sigma^0 + n \quad (1b) \\
A + n, \quad (1c)
\]

where the rate for reaction (1a) is twice that for (1b) by charge independence.

We shall confine our attention to reaction (1a) since it is easily recognized
in a deuterium-filled bubble chamber and is known to occur at a rate of
\( \sim 0.7\% \) when stopped \( K^- \) mesons are captured. \(^6\) For \( K^- \) capture at rest, the
center-of-mass momentum of the \( \Sigma^-p \) system is 511 MeV/c, and production is
inhibited unless the \( K^- \) is absorbed while the two nucleons are within
a distance \( \sim 0.4(10^{-13}) \) cm. The small size of this effective interaction
volume implies that the rate for (1a) will be \( \alpha |\phi_nS(r=0)|^2 \) or \( \alpha |\psi_nP(r=0)|^2 \)
depending upon the atomic orbital from which capture occurs. On the other
hand, mesic absorption can occur over the entire volume of the deuteron
and will proceed through the \( S\)-wave \( K^-\)-nucleon interaction for capture
from either \( S \) or \( P \) atomic orbitals. \(^5\) Therefore, if capture occurs
predominantly from \( S \)-orbitals as predicted by Day, Snow, and Sucher,
the fraction of nonmesic absorptions observed at rest and inflight must be
a continuous and slowly varying function of \( K^- \) momentum.
Making the explicit assumption that the behavior of the nonmesic transition amplitude is determined by the centrifugal barrier in the initial state, and that only S- and P-wave absorption is important at low momentum, we have

$$\Gamma_{nm}(q) = (A + Bq^2) pcN.$$  

(2)

Here $q$ is the center-of-mass momentum of the initial $K^- d$ system in units of $\mu_{Kd} c$ where $\mu_{Kd}$ is the $K^- d$ reduced mass and $c$ the velocity of light; $p$ is the final center-of-mass momentum measured in units of its threshold value, $p_0 = 511$ Mev/c; and $N$ is the atomic density of liquid deuterium. The quantities $A$ and $B$ are to be determined from measurement of the in-flight production cross sections. The capture rate from $nS$ or $nP$ orbitals may then be estimated by using

$$\Gamma_{nm}^{nS} = cA \left| \phi_{nS}(r=0) \right|^2$$  

(3a)

$$\Gamma_{nm}^{nP} = cB \left( \frac{1}{\mu_{Kd}} \right)^2 \left| \nabla \phi_{nP}(r=0) \right|^2$$  

(3b)

Since the absolute transition rate cannot be measured, Eqs. (3a) and (3b) must be compared with the mesic absorption rate. To obtain the latter, we assume that the absorption rate in deuterium is essentially the sum of the absorption rates for the free neutron and proton. The mesic absorption rate from an $n^f$ orbital is then given by

$$\Gamma_{n}^{n^f} = 4\pi c \left( \frac{1}{\mu_{Kp}} \right) \left( \frac{b_0 + 3b_1}{2} \right)^2 \left\langle \phi_{n^f}(r) \right\rangle^2_{av},$$  

(4)

where $\mu_{Kp}$ is the reduced $K^- p$ mass, and the average over the nucleon density distribution is evaluated by using the Hulthen form for the deuteron wave function. The quantities $b_0$ and $b_1$ are the imaginary parts of the
T = 0 and T = 1 zero-energy scattering lengths evaluated by Dalitz and Tuan. Dividing Eqs. (3a) and (3b) by Eq. (4), we obtain for the fraction of nonmesic absorptions from S or P atomic orbitals:

\[ R^S(0) = 0.123 \left(10^{26}\right) \text{A} \]  
\[ R^P(0) = 0.0241 \left(10^{26}\right) \text{B}. \]  

At present, only the fraction of nonmesic absorptions occurring at rest is known. Since either Eq. (5a) or (5b) should essentially account for the observed value, \( R(0) \approx 0.007 \), \( A \) and \( B \) may be determined and corresponding lower limits for \( \Gamma_{nm}(q) \) estimated from Eq. (2). To obtain lower limits for the fraction of nonmesic absorptions expected in flight, \( \Gamma_{nm}(q) \) may be divided by the mesic absorption rate as given by

\[ \Gamma_{m}(q) \approx q_c \left(\frac{\sigma_0 + 3\sigma_1}{2}\right) N, \]

where \( \sigma_0 \) and \( \sigma_1 \) are the absorption cross sections for the \( T = 0 \) and \( T = 1 \) states of the \( K^-\)p system. Typical results are listed in Table I for the two assumptions of S- or P-orbital capture at rest.

### Table I

<table>
<thead>
<tr>
<th>( K^- ) laboratory momentum (Mev/c)</th>
<th>S-orbital capture</th>
<th>P-orbital capture</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0.007</td>
<td>0</td>
</tr>
<tr>
<td>200</td>
<td>0.019</td>
<td>0.016</td>
</tr>
<tr>
<td>300</td>
<td>0.017</td>
<td>0.032</td>
</tr>
</tbody>
</table>
The ease with which the experiment can yield a definitive result depends upon the extent to which A is not equal to B in Eq. (2). In the analogous reaction \( \pi^+d\rightarrow p+p \) B is much greater than A because of the anomalously small S-wave, \( \pi \)-nucleon interaction. In the present case, the S-wave, \( K^\pi \)-nucleon interaction is very large, and \( \text{if} \ A \ is much greater than B, a measurement of low statistical accuracy would indicate that ordinary collisional and radiative de-excitation mechanism were inadequate to account for the observed nonmesic rate at rest.
FOOTNOTES

7. The general properties for such a transition amplitude have been discussed by K. A. Brueckner and K. M. Watson Phys. Rev. 86, 923 (1952) and K. M. Watson Phys. Rev. 88, 1163 (1952) in connection with the reaction \( p+p \rightarrow \pi^+ + d \).
8. It may be expected that final-state interactions will redistribute the reaction products among the available final states but will not seriously affect the absolute transition rate. T. B. Day, G. A. Snow, and J. Sucher in Univ. of Maryland Technical Report 167, March 1960 have estimated the effects of multiple scattering on the absorption rate at 200 Mev/c \( K^- \) laboratory momentum and found them to be small.
9. At zero and 200 Mev/c, the necessary parameters for the \( T = 0 \) and \( T = 1 \) absorption channels were taken from reference 2 for the experimentally preferred (a+) and (b+) solutions. At 300 Mev/c, we used the \( \sigma_0 \) and \( \sigma_1 \) absorption cross sections of P. Norden, R. Tripp,
A. Rosenfeld and F. Solmitz reported by L. Alvarez at the Ninth International Conference on High Energy Physics, Kiev, Russia 1958.