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Moving Slot Concept
for
Automated Highway Control

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ABSTRACT

The objective of this report is to optimize performance of Automated Highway Systems through management of space accounting for interaction between entrance and exit processes. To accomplish this objective, we develop a comprehensive framework, including a new integrated highway model called the moving slot model, and operational strategies, called slot/lane assignment rules. The model manages highway space to maximize capacity accounting for safety and vehicle maneuvers. Operational strategies minimize space requirements by forming vehicles into specific patterns of destinations through entry and lane-change control such that vehicles can exit successfully. This research aims to expedite the application of Automated Highway Systems without significantly altering system configurations while optimizing performance in terms of capacity and travel time.

In the moving slot model, an operational unit, called a slot in a one-lane highway and a stack in a multi-lane highway, contains the minimal space for accommodating vehicles and supporting necessary maneuvers without affecting other units. This design provides independence among operational units and can vary with system parameters such as number of vehicles in a platoon. This not only reduces the complexity of system control but also makes the framework adaptable to various system requirements. We provide both theoretical and simulation results for system performance of a simplified highway under the framework as well as simulation results for varied system configurations.
EXECUTIVE SUMMARY

This research provides an efficient and effective design for the operation of highway systems, especially under heavy traffic conditions, to minimize travel time by operating vehicles at a maximum speed and to double the capacity of conventional highways (i.e. 4000 vehicles per hour per lane or more). A “slot” is developed as the fundamental unit of analysis, which is analogous to a packet in data communication. Each lane transports a continuous series of slots, each of which is composed of multiple vehicles traveling in close proximity. The fundamental decisions in system operation include release of vehicles from entrances to highways, assignment of vehicles to slots, and assignment of vehicles to highways.

Slot assignment rules minimize ramp space by allowing only one maneuvering process per slot passing a ramp. Lane assignment rules minimize the number of lane changes such that ramps can be closely spaced and throughput is increased. Various system configurations are studied by simulation, providing criteria for designing vehicles waiting space at entrances and locating ramps. Overall, this research provides design methodologies that balance entrance and exit requirements on highway through the management of space within highway lanes.
Chapter 1

Introduction

Traffic is getting worse, particularly in urban areas. The 2003 Annual Urban Mobility Report from Texas Transportation Institute shows that the congestion results in 5.7 billion gallons of wasted fuel and 3.5 billion hours of lost productivity in 2001, which cost the nation $69.5 billion and $4.5 billion, respectively. Capacity increase, due to constructing new highways, has not kept pace with demand for travel. Between 1980 and 1999, route miles of highways increased 1.5 percent while vehicle miles of travel increased 76 percent. The extra time needed for rush hour travel has tripled over two decades. The national average Travel Time Index for 2001 was 1.39 (meaning a rush hour trip took 39 percent longer than a non-rush hour trip). The national average in 1982 was only 1.13. The volume of freight movement alone is forecast to nearly double by 2020. Congestion has been a problem for big cities and increasingly common in small cities and some rural areas as well.

Automated Highway Systems (AHS) were conceived as a promising way to relieve highway congestion without the support of excessive infrastructure. The remarkable advances of automation and communication technologies and vehicular control algorithms have also been made to realize the system while attaining three major goals: capacity, safety, and stability (Shladover, 1991). While the hardware supporting the concept becomes mature, achievable system performance relies on management of traffic flow, vehicle spacing and complex vehicle maneuvers.

1.1 Motivation of the Research

A critical factor affecting the capacity increase in highways is the maneuvering behavior of vehicle, including entry, lane change, and exit. In conventional highways, drivers control and make all decisions when driving. In intelligent vehicle/highway systems, driving decisions are partially or totally made by high-tech communication and control devises. As a result, both capacity and safety are increased.
In an automated highway, capacity increases when vehicles are grouped according to characteristics such as destinations. Space between vehicles is determined based on safety. However, space between groups of vehicles has not been studied in detail. Specifically, the utilization of space when vehicles enter or leave the system or change lanes is not fully analyzed in the literature. The capacity of an AHS is limited by the ability to manage the space of the system under every maneuvering process. This research begins with studying basic vehicle maneuvering processes and finally attempts to create a comprehensive framework such that highway systems can be operated at the maximum performance in terms of safety and throughput while requiring minimal support of infrastructure.

A group of vehicles in an AHS is called a platoon. Within a platoon, the space between adjacent vehicles is called the intraplatoon distance, denoted by $L_{\text{intra}}$. And the space between adjacent platoons is called the interplatoon distance, denoted by $L_{\text{inter}}$. Let $N$ be the number of vehicles in a platoon, $V$ be the speed of a platoon, $L_v$ be the mean length of a vehicle, and $L_s$ be the total length of a platoon. Ideally, the capacity can be formulated as

$$\text{Highway Capacity} = \frac{NV}{L_s} \quad (1.1.1)$$

$$L_s = L_{\text{inter}} + NL_v + (N-1)L_{\text{intra}} \quad (1.1.2)$$

Equation (1.1.1) represents the capacity when there are no vehicle maneuvers and all platoons have $N$ vehicles. Vehicle maneuvers occur during entry, exit, and lane changes. To meet safety requirements, vehicles must become a single platoon before maneuvering. Hence, a platoon may split into several smaller platoons and extra space is needed. However, if this additional space is not well managed, waste arises. For example, the distance between adjacent platoons could be longer than one interplatoon distance, but too short to accommodate an additional vehicle. Or,
consecutive platoons have numbers of vehicles smaller than \( N \) but not sufficient space to admit more vehicles. In these conditions, the average space per vehicle occupied is increased and the capacity is decreased.

Figure 1.1.1 provides detailed information on space utilization of the platoon model. A “Green Zone” represents the space that can permit an entering vehicle, and the “Red Zone” represents the space that is already occupied by vehicles or used for safety purpose and, therefore, no vehicle maneuver is allowed. Both zones are identified with respect to the front end of a changing vehicle. When a vehicle needs to change lane, it must wait until it is located in a green zone. As more vehicles enter a lane, the green zones decline in size, and some platoons may become full. At this point, vehicles may need to wait for several platoons to pass before they encounter a green zone and complete their lane changes. The lane number increases from right to left with the rightmost lane as lane 1. A box stands for a vehicle. The speed on lane \( i \) is denoted by \( V_i \) with \( V_i \leq V_{i+1} \).

A vehicle driving on lane 1 with target lane 3, it must traverse lane 2. For the sake of safety, it is assumed that vehicles requesting lane changes will be a single platoon during the whole process. Hence, the minimum space for the vehicle to move into lane 2 is \( 2L_{\text{inter}} + L_{\text{SA}} + L_v \) (denoted by \( L_{\text{tra}} \)), where \( L_{\text{SA}} \) is the distance for acceleration/deceleration to the speed of target lane. The length of the green zone is the length of the gap, space between adjacent platoons, subtracted by \( L_{\text{tra}} \) and starts at a distance of \( L_{\text{inter}} \) after the preceding platoon and ends at a distance of \( L_{\text{inter}} \) ahead of...
the following platoon. The two distances switch when vehicles change to a slower lane. When a gap of length $S'$ is greater than $L_{\text{inter}}$, but less than $L_{\text{tra}}$, it is too short for a vehicle to change lane. Hence, $S'$ is wasted.

One might argue that merging adjacent small platoons into a big platoon can create more space. However, during the negotiation before the merging, communication delay is experienced. Some adjacent platoons may even fail to merge because of the limitation on platoon size. Hence, to accomplish the task, a platoon separation may occur before a platoon merge. However, the split of a platoon may cause other platoons to slow down or speed up to make room for lane changing vehicles. This causes more delay. The same situation happens for vehicles separating from platoons. Moreover, platoons that can merge may not be close to lane changing vehicles, which have to wait. During the waiting, some intermediate vehicles may take the space and make the waiting even longer. If we don’t allow this, a space kept for a long time unused is also a waste. These conditions become worse when the traffic flow is close to the lane capacity. When a space adjustment process involves more platoons, the performance of the system declines drastically.

The above observations are summarized as follows: (in the platoon model)

1. A vehicle maneuver usually leads to a space adjustment process, especially in heavy traffic.
2. A space adjustment process causes delay.
3. A space adjustment process may involve many platoons.
4. A space adjustment process still cannot guarantee efficient utilization of space.

The concept of platooning provides the possibility of capacity increase provided that space is properly managed. Hence, a new vehicle following model,
based on platooning, is desirable to provide a reasonable capacity while accounting for necessary vehicle maneuvers and utilizing space efficiently.

Besides the space management, the application of AHS on current highways is another concern in developing the model. We focus on the ramp configuration and try to minimize the space on ramps required in entry and exit processes and the space between adjacent ramps such that current ramps can be adopted in AHS without being altered significantly.

In a word, the research attempts to expedite the application of the concept of Automated Highway Systems. Hence, the focus is on the system performance under heavy traffic conditions and the conversion of current highways without significantly altering system configuration, especially related to ramps.

### 1.2 Definitions of the Research

The research consists of the following tasks.

1. A new vehicle following model will be designed to effectively exploit highway space and minimize interference with traffic due to vehicle maneuvers. This model defines an operational unit, called a slot in one-lane systems or a stack in multi-lane systems. A unit consists of several pieces of space. Each space is intended for a specific purpose, either for storing vehicles or for implementing vehicle maneuvers. Under different system configurations and operational scenarios, varied vehicle maneuvers require different amounts of space. Therefore, the model will be designed to vary with these situations.

2. To implement the model on AHS, we propose slot assignment rules. The slot assignment rules state that vehicles will join slots serving vehicles with
similar characteristics, such as destinations. Slot assignment rules are designed to achieve desirable system performance and minimize ramp space by minimizing number of maneuvering processes occurred at ramps. Furthermore, assistant operations are also proposed. For example, the release improvement mechanism is applied at entrances to increase the capacity.

3. To apply the moving slot model in multi-lane AHS, lane assignment rules are proposed. The rules state that vehicles will use specified lanes on different segments of a highway based on varied origin/destination demands. Lane assignment rules are designed to minimize space between adjacent ramps by minimizing number of lane changes. By applying lane assignment rules, the relationships between entrances and exits are, therefore, created by the vehicle routings.

4. A complete framework consisting of the new vehicle following model and slot/lane assignment rules will be applied on a fully automated highway system. A one-lane AHS will be studied first, since most vehicle maneuvers can be studied from the system. The processes of entry, joining, merging, separation, and exit will also be defined. Then, a two-lane AHS is studied to evaluate the lane change behavior. Most results can be applied to a multilane AHS.

5. Simulation is conducted to study the space for storing vehicles at an entrance under various slot assignment rules, verifying the system performance from analytic models. Two interesting topics are also studied: a dynamic lane assignment rule and the location of exit.
1.3 Contributions of the Research

In this research, we provide a complete framework including a model optimizing the highway space and corresponding operational strategies optimizing other system performance such as space on/between ramps under various scenarios. We also simulate the impacts of varied system configurations, such as location of exits, on the system performance. The detailed contributions are listed below.

- We develop a prototype vehicle-following model, called the moving slot model, to be applied on automated highway systems, especially under heavy traffic conditions. This model optimizes the utilization of highway space such that the capacity is maximized while interference among vehicle maneuvers is minimized. Under this model, the highway space is divided into slots. A slot is an independent operational unit. An independent unit reduces the complexity of system control and time delay due to communication. The design of a slot varies with space requirements from vehicle maneuvers. Hence, the model can also be adapted to various system configurations.

- In the moving slot model, the travel time is minimized by operating at the maximum legally allowable speed of 30m/s. The capacity is twice that of conventional highways and can be further increased by increasing $N_{max}$, or increasing normal or emergence acceleration/deceleration rates. Since slots are operated at the maximum speed, no speed difference exists among highway lanes. Hence, in multi-lane AHS, an operational unit is called a stack consisting of well-aligned slots, one on each lane. Lane change processes in one stack won’t affect those processes in other stacks. The Random-Join-And-Leave slots are used to minimize the waiting time of lane changes.

- The space at ramps for vehicles joining or leaving slots is minimized by applying slot assignment rules that are devised to allow only one entry process at an entrance and one exit process at an exit for each passing slot. In this
design, current highway ramps can be converted into dedicated ramps in AHS without lengthening the space.

- Applying slot assignment rules integrated with lane assignment rules minimizes the distance between ramps for lane changes. Given a fixed slot speed, the distance is determined by the number of lane changes in a stack. Thus, this design results that current ramps won’t be shut down because of insufficient spacing when highways are equipped automated systems. Therefore, capacity won’t be limited by number of ramps and the number of dedicated ramps to be constructed is reduced.

- Mathematical representations of distributions of number of vehicles released under slot assignment rules (i.e. SS, SSRIM, EJSS, EJSSRIM, and GSRIM) are derived by assuming that there is no shortage of vehicles at entrances. The capacity of an entrance is represented by the mean release rate. Among five slot assignment rules, the rule of GSRIM is the best in terms of the highest release rate at an entrance and the fastest recovery from loss of traffic flow, but requiring the longest distance between ramps. A family of sorted-slot based rules, SS, SSRIM, EJSS, and EJSSRIM, can be applied alternately in AHS depending on number of lanes at an entrance and the distance to the immediate downstream ramp.

- A greedy method, in which vehicles with long trips occupy upper slots in stacks and leave maximal space to admit vehicles in the lowest slot, is developed to establish lane assignment rules. Lane assignment rules optimize the use of lanes in AHS. This method is verified by simulated mean release rates from a dynamic lane assignment rule comparable to those solved analytically.
This research also provides simulation results about the impacts of vehicle waiting space on the capacity of entrance and the relative location of exits to adjacent entrances on the system performance. As a result, the mean arrival rate at an entrance can be specified based on the applied slot assignment rule and the vehicle waiting space under a service level represented by percentage of blockage of vehicles from entering the entrance. When an exit is located from close to the upstream entrance to the downstream entrance, the numbers of lane changes decreases over several sections but increases significantly in one or two following sections and the system throughput, the sum of total flow rates at ramps, has an insignificant increment.

The moving slot model adapting to various operational scenarios optimizes space in AHS. The application of the model in accordance with slot/lane assignment rules on current highways minimizes the travel time and provides at least twice capacity of conventional highways without altering system configurations, especially of ramps.

1.4 Organization of the Report

In this chapter, we introduced the focus of the report (i.e. the importance of space management on AHS) and then we proposed a framework to implement the concept. In Chapter 2, we review previous work on vehicle following models and basic concepts and operations of automated highway systems. In Chapter 3, a general description about the design of the moving slot model, slot assignment rule, and lane assignment rules are provided. The systems studied in the following chapters are also defined. Chapter 4 presents the application of the moving slot model and slot assignment rules on a one-lane AHS. Chapter 5 extends applications to a two-lane AHS and describes similarities to a multi-lane AHS. Chapter 6 provides simulation results of cases of limited vehicle waiting space at ramps, number of lane changes in a multi-lane system, a dynamic lane assignment rule, and varying locations of ramps. Chapter 7 concludes the research.
Chapter 2

Literature Review

In Section 2.1, many car-following models addressing space-speed relationships on current highways are reviewed to realize that human driving behavior is a major restriction of capacity increase on current highways. In Section 2.2, automated highway systems are proposed to boost highway performance in both capacity and safety. The system performance is based on the trade-off between safety and capacity. Operational strategies in automated highway systems to optimize space utilization and the impact of platoon formation at entrances are described in Section 2.3 and Section 2.4, respectively.

2.1 Car-Following Models

Car following models have been extensively studied to derive the capacity of a highway lane and predict traffic behavior around incidents. Car-following models of a single lane state an explicit relationship between the inter-vehicle spacing and vehicle speed. A driver dynamically responds to stimuli and makes decisions to control his vehicle such that safety is sustained. In the following, we describe basic single-lane car-following models with linear and nonlinear characteristics.

GM models

Among various car-following models, a series of models developed by Chandler et al. (1958), Herman et al. (1959), Herman and Potts (1959) and Gazis et al. (1959, 1961) at the GM Research Laboratories have received much attention over the years. The GM model is the generalized model. As a matter of fact, other models developed by Pipes (1953, 1967), Forbes et al. (1958), and Forbes (1963), are special cases of the GM model.

The GM model is a stimulus-response car-following model. It states simply that, in a row of vehicles on a single lane with no passing, the acceleration or
deceleration response of the \( n^{\text{th}} \) vehicle at time \((t + T)\), where \( T \) represents the time delay, is proportional to the \( p^{\text{th}} \) time derivatives of positions of this vehicle and its preceding vehicle. Mathematically, the model can be represented as

\[
\frac{d^2 x_n(t + T)}{dt^2} = \lambda \left[ \frac{d^p x_{n-1}(t)}{dt^p} - \frac{d^p x_n(t)}{dt^p} \right]
\]  

(2.1.1)

in which \( \lambda \), the gain factor, is considered as a function of speed and space between vehicles (Gazis et al. 1961), namely

\[
\lambda = \frac{\lambda_{l,m} \left( \frac{dx_n(t)}{dt} \right)^m}{\left[ x_{n-1}(t) - x_n(t) \right]^l}
\]  

(2.1.2)

where \( \lambda_{l,m} \) is the sensitivity coefficient that is a constant to be determined experimentally. Herman (1992) suggested that a driver could hardly estimate the physical quantities represented by values of the parameters \( p \) greater than unity. For example, a driver cannot know the acceleration rate of a vehicle ahead. In addition, following a vehicle by keeping a constant space ahead, i.e. \( p=0 \), is shown theoretically to be unstable. Both are verified by experiment.

In general, it can be shown that solutions with \( p \) even are all unstable, and those with \( p \) odd are asymptotically stable. Therefore, the above equations become,

\[
\frac{d^2 x_n(t + T)}{dt^2} = \lambda \left[ \frac{dx_{n-1}(t)}{dt} - \frac{dx_n(t)}{dt} \right]
\]  

(2.1.3)

\[
\lambda = \frac{\lambda_{l,0}}{\left[ x_{n-1}(t) - x_n(t) \right]}
\]  

(2.1.4)
Hence, the simplified model states that the acceleration or deceleration response of a driver at time \((t+T)\) is proportional to the relative speed, with respect to its preceding vehicle, at time \(t\) multiplied by the gain factor. The gain factor is equal to the sensitivity coefficient divided by the space between these two vehicles. In this model, the non-zero relative speed is the only stimulus and the response is to accelerate or decelerate.

Chandler et al. (1958) experimented on the GM model. Experimental data, shown in Table 2.1.1, indicated that \(T\) is about 1.5 seconds and \(\lambda\) is about \(0.37\text{sec}^{-1}\). They were also concerned with the stability of the motion of a long line of vehicles. When the product of \(T\) and \(\lambda\) exceeds \(\frac{1}{2}\), it becomes unstable. The data implies that drivers tend to control vehicles on the verge of instability.

<table>
<thead>
<tr>
<th>Drivers</th>
<th>(\lambda (s^{-1}))</th>
<th>Average speed (m/s)</th>
<th>Average spacing (m)</th>
<th>(\lambda T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.74</td>
<td>19.8</td>
<td>36</td>
<td>1.04</td>
</tr>
<tr>
<td>2</td>
<td>0.44</td>
<td>16</td>
<td>36.7</td>
<td>0.44</td>
</tr>
<tr>
<td>3</td>
<td>0.34</td>
<td>20.5</td>
<td>38.1</td>
<td>1.52</td>
</tr>
<tr>
<td>4</td>
<td>0.32</td>
<td>22.2</td>
<td>34.8</td>
<td>0.48</td>
</tr>
<tr>
<td>5</td>
<td>0.38</td>
<td>16.8</td>
<td>26.7</td>
<td>0.65</td>
</tr>
<tr>
<td>6</td>
<td>0.17</td>
<td>18.1</td>
<td>61.1</td>
<td>0.19</td>
</tr>
<tr>
<td>7</td>
<td>0.32</td>
<td>18.1</td>
<td>55.7</td>
<td>0.72</td>
</tr>
<tr>
<td>8</td>
<td>0.23</td>
<td>18.7</td>
<td>43.1</td>
<td>0.47</td>
</tr>
</tbody>
</table>

Table 2.1.1 Results of car-following experiment by Chandler et al. (1958)

The general properties of the GM model are:

1. It is a deterministic stimulus-response model. It assumes that the relative speed as well as the distance between adjacent vehicles can be perceived precisely and drivers can response precisely.
2. Its result is symmetric with respect to the relative speed; that is, with other things remaining the same, the magnitude of the response is only
dependent on the magnitude of the stimulus. For example, if a driver responds by accelerating at $\alpha$ m/s$^2$ when the relative speed is $V$ m/s, then under similar conditions of vehicle spacing and speed, the response to a relative speed of -$V$ m/s will be -$\alpha$ m/s$^2$.

3. The response of the driver is based on only one stimulus, namely, the relative speed. Once the relative speed is zero, a driver neither accelerates nor decelerates, irrespective of the distance between adjacent vehicles.

The non-linear characteristics of the GM model were studied by May and Keller (1967). They used the same speed-density equations derived by Gazis et al (1961) and applied on both freeway data and tunnel data. However, reaction time could not be identified with the macroscopic data.

Ozaki (1993) used a sequential procedure to estimate the non-linear GM model. First, he used regression analysis to estimate reaction times for 4 different acceleration and deceleration actions. As a result, the reaction time depends on the actions and situations, in particular, between acceleration and deceleration decisions. These reaction times were used to estimate car following parameters with correlation analysis. Ozaki estimated separate sets of parameters for acceleration and deceleration decisions.

Lee (1966) used an integral transform technique to mathematically consider a ‘memory function’ defining the way in which a following driver processes his information. His model is a variation of the GM model. The driver reacts to the relative leader speed over a period of time rather than at an instant. The mathematical model is:

$$x_n^\varepsilon(t) = \int_0^t M(t-\tau)[x_{n-1}'(\tau) - x_n'(\tau)]d\tau$$  \hspace{1cm} (2.1.5)
Where, $M(.)$ is a memory function shown in Figure 2.1.1. Drivers can only recall events encountered over a certain period of time. As a matter of fact, the most recent event affects more on a driving decision and variety exists from driver to driver.

![Figure 2.1.1 Memory function in GM model by Lee (1966).](image)

Lee proposed several functional forms of the memory function and analyzed the stability of the resulting response to periodic changes of the leader speed. Darroch and Rothery (1972) empirically estimated the shape of the memory function using spectral analysis. They found that a dirac-delta function, which corresponds to the linear GM model, is a reasonable approximation.

Herman and Rothery (1965) and Bexelius (1968) hypothesized that drivers follow vehicles in front of their leader as well as the immediate leader. Assuming different sensitivities to the relative speed with respect to each one of these leaders. Herman and Rothery (1965) report inconclusive results regarding the effect of the second-nearest leader on the subject behavior.

Other Car-Following models

Newell (1961) proposed a stimulus-response relationship that the velocity of a following car at time $t$ is a nonlinear function of the inter-vehicle spacing at time $t-T$, i.e.,
for some appropriate function $G$. He began with a simple, but physically reasonable, model by assuming $G$ as a differential equation and showed that results concluded from this model were also valid qualitatively for a large class of functions. This model can explain all the results previously derived for linear car-following models and also the nonlinear phenomena.

Newell (2002) proposed another simplified model that the trajectory of a vehicle would approximately follow the trajectory of the preceding vehicle by appropriate values of time delay and distance deviation. Figure 2.1.2 shows that the trajectory of the $n^{th}$ vehicle varies depending on the trajectory of the $(n-1)^{th}$ vehicle. The following will start to accelerate at a time delay of $\tau_n$ and a space advance of $d_n$ with its preceding vehicle, and the vehicle spacing will lengthen from $S_n$ to $S'_n$. Newell made a detailed comparison of various prior models and concluded his model could be a special case of existing models but with fewer parameters and a different logic. Soyoung et al. (2003) verified the model by an observation of vehicles’ behavior after being released at a signalized intersection with a long queue.
Komentani and Sasaki (1958) developed a model based on the assumption that the subject speed is determined so as to keep a minimum safe spacing, and is therefore a function of the leader space headway and the leader speed. They proposed linear and quadratic (in the subject speed) formulations of the model and studied the stability of the predicted motion of the subject in response to disturbances in the speed of the leader.

Car-following models mentioned above depend on parameters representing reaction time and sensitivity coefficient. These two human dependent factors restrict the increase of capacity on current highways. In current highways, both capacity and safety are substandard when confronting growing demand. Without excessive support of infrastructure, capacity increase on current highways depends on automation of vehicles and highway systems, i.e. lowering human-control portion of driving tasks. Hence, the concept of automated highway systems was proposed and made feasible by the modern advanced technology.
2.2 Automated Highway Systems

The concept of Automated Highway Systems (AHS) has been proposed to increase capacity and safety in highway transportation systems (Varaiya, 1993). Organizing vehicles into closely spaced units, called platoons, increases the capacity of AHS. The tight spacing between vehicles within a platoon prevents intraplatoon collisions at high relative velocities, while the large gaps between platoons prevent interplatoon collisions.

The AHS architecture presented by Varaiya and Shladover (1991) is shown in Figure 2.2.1. This architecture consists of five hierarchical layers: network, link, coordination, regulation, and physical layer. The network and link layers manage the distribution of flow between lanes, routes of vehicles, and general system parameters. The other three layers build safety-critical systems. The coordination layer organizes maneuvers by means of which vehicles and platoons make coordinated movements. Maneuvers include lane change, platoon merging and separation, and entry and exit of vehicles. The regulation layer controls movements of vehicles to execute maneuvers, while the physical layer reflects the operation of sensors and vehicle controls.
Hsu et al (1991) presented three basic platooning maneuvers, which are necessary in normal conditions.

1. Merge: a following platoon accelerates to form a single platoon with its predecessor.
2. Split: a platoon separates into two individual platoons separated by a safe interplatoon space (or more).
3. Change-lane: a single vehicle moves from one lane to another, where it may (i) join into a long space between two platoons and remain a free agent, or (ii) join ahead of a platoon and then be merged by the platoon, or (iii) join behind a platoon and subsequently accelerate to merge with it, or
(iv) cause the platoon to split (to full platoon separation), and enter into the gap. Then, all merge into one platoon by two successive merge operations.

Vehicle Following Concepts

In AHS, vehicle following concepts were proposed in terms of spacing between vehicles, degree of automation, and support of infrastructure. A fully automated highway system requires support from automation functionalities of vehicles and infrastructure. In the following, we describe two basic on fully automated highway systems.

1. Moving-Cell Model

Under the concept, an infrastructure based control system creates and maintains vehicle “cells” in space and time. Thus, a vehicle is not following its preceding vehicle but a virtual moving point specified by the control system. Cells can be thought of as moving roadway segments, each of which holds at most one vehicle at any time. Basically, the length of a cell is fixed. Vehicles that need more space may be assigned multiple cells. For instance, a truck may be assigned two cells. Under this design, the control becomes easier in the moving-cell model.

The moving-cell concept was introduced by Godfrey (1968) to simplify the merging problem in automated systems. Wilkie (1970) assumed that vehicles occupy positions on a guideway within hypothetical ‘cells’ that move at a constant velocity when studying on the longitudinal control problem. The length of a moving cell consists of the length of a vehicle, the distance of deceleration when incidence happens multiplied by a safety factor, and the distance for response (Rumsey 1974). The safety issue is assured in the concept of the moving cell by setting the safety factor and the capacity of the system is inversely proportional to the length of a cell.
The major advantages of the model are:

1. Each vehicle is controlled by using only information about its velocity and position.
2. Merging strategies are readily implemented.
3. There is no platoon instability problem because of individual vehicle perturbations.

The capacity and merging problems of the synchronous moving-cell system were studied by Rumsey and Powner (1974). In the paper, they pointed out the difficulties of the control of a string of vehicles. They were mainly instabilities of the system caused by complexity of communication among vehicles and the control system. Some related issues on dual-mode systems were also studied by Stefanek and Wilkie (1973) and Stefanek (1972).

The increment of capacity in a moving-cell model is still not good enough. Therefore, the current interest in automated highways has focused more on the platoon model.

2. Platoon Model

Shladover (1979) proposed and studied the platoon model. In this model, longitudinally adjacent vehicles are spaced either very close to or very far from each other. Both distances are decided by a relatively minor damage with a small relative speed if accidents occur. As a result, vehicles are clustered as groups of vehicles called platoons driving on an automated highway lane. This rule provides a significant capacity increase and safety under the support of advanced technologies. Therefore, great capacity increase can only be realized on fully AHS. In the rest of this chapter, researches about the system performance under the platoon model and the approaches to maximize the performance are reviewed.
Capacity

Safety and capacity are correlated in highway systems. In AHS, spacing between vehicles is determined by the consideration of avoiding collision or reducing damage of collision. More space is reserved to meet a strict safety requirement. To evaluate various space requirement under different scenarios becomes a popular way to estimate capacity in AHS. In the following, we briefly review previous works on capacity in terms of longitudinal spacing, spacing of lane change, and traffic flow theory.

- Longitudinal Spacing

Tsao and Hall (1994) developed a probabilistic model for analyzing longitudinal collision/safety between an abruptly decelerating vehicle and its immediate follower on an Automated Highway System. The input parameters are the distance between the two vehicles, their common speed prior to the failure, the reaction delay of the following vehicle and a bivariate distribution for these two deceleration rates. The output includes the probability of a collision and the probability distribution of the relative speed at time of collision. These safety consequences were used to balance the desire to increase AHS capacity with the safety requirements.

A vehicle failure under different longitudinal-separation rules will result in collisions of different severity with different probability. In the paper, the collision speed, i.e. relative speed between two colliding vehicles at the time of collision, was used as a surrogate for collision severity and only the initial collision after a failure was considered. They used this model to compare the safety consequences associated with the platooning and "free agent" longitudinal-separation rules and demonstrated that the free-agent rule implemented with a potential technology of fast and accurate emergency deceleration, under some reasonable conditions, can avoid collisions while offering a high freeway capacity previously thought possible only under the
platoon model. They conclude that platooning leads to more frequent small collisions, but less frequent severe collisions.

The safety spacing was also studied by Sun and Ioannou (1995) and Kanaris et al. (1996, 2001) under different AHS concepts and vehicle maneuvering conditions. They considered the worst-case braking scenario, because different braking scenarios imply different spacing requirements and different capacity levels. In this study six AHS operational concepts were considered. For each concept, the minimum inter-vehicle spacing was derived and used for collision-free vehicle following, under different road conditions. For architectures involving platoons, the alternative constraint of bounded energy collisions was also used to calculate the spacing that can be applied, if collisions at a limited relative velocity are allowed. In every case, the minimum spacing in turn, was used to calculate the maximum possible capacity that could be achieved for each operational concept.

Vehicle braking performance data derived from road tests performed by MHTSA and by the leading consumer magazines. The choice of timing parameters was based on sensor-actuator communication technology limitations and was supported by vehicle tests performed by the authors and by other researchers in the PATH program.

The capacity estimates for each vehicle-following concept considered are summarized in Table 2.2.1 and Table 2.2.2.
<table>
<thead>
<tr>
<th>Capacity without platooning</th>
<th>5% mixing of buses</th>
<th>5% mixing of trucks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Dry</td>
<td>Wet</td>
</tr>
<tr>
<td>Autonomous vehicles with class identification</td>
<td>3736</td>
<td>2516</td>
</tr>
<tr>
<td>Autonomous vehicles without class identification</td>
<td>3631</td>
<td>2432</td>
</tr>
<tr>
<td>Infrastructure supported with class identification</td>
<td>4730</td>
<td>2923</td>
</tr>
<tr>
<td>Infrastructure Managed with class identification</td>
<td>5472</td>
<td>3197</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Moving-Cell model</th>
<th>2.5% buses+2.5% trucks</th>
<th>5% buses+5% trucks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Dry</td>
<td>Wet</td>
</tr>
<tr>
<td>Autonomous vehicles with class identification</td>
<td>3596</td>
<td>2391</td>
</tr>
<tr>
<td>Autonomous vehicles without class identification</td>
<td>3488</td>
<td>2314</td>
</tr>
<tr>
<td>Infrastructure supported with class identification</td>
<td>4492</td>
<td>2755</td>
</tr>
<tr>
<td>Infrastructure Managed with class identification</td>
<td>5155</td>
<td>2997</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Capacity with platooning</th>
<th>10-car platoons</th>
<th>20-car platoons</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Dry</td>
<td>Wet</td>
</tr>
<tr>
<td>Autonomous platoons without coordinated braking</td>
<td>6090</td>
<td>5652</td>
</tr>
<tr>
<td>Infrastructure supported platoons without coordinated braking</td>
<td>6312</td>
<td>5843</td>
</tr>
<tr>
<td>Infrastructure Managed platoons without coordinated braking</td>
<td>5434</td>
<td>5947</td>
</tr>
<tr>
<td>Autonomous platoons with coordinated braking</td>
<td>7217</td>
<td>4531</td>
</tr>
<tr>
<td>Infrastructure supported platoons with coordinated braking</td>
<td>7531</td>
<td>4652</td>
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<tr>
<td>Infrastructure Managed platoons with coordinated braking</td>
<td>7704</td>
<td>4718</td>
</tr>
<tr>
<td>Autonomous platoons with delayed braking</td>
<td>7060</td>
<td>4468</td>
</tr>
<tr>
<td>Infrastructure supported platoons with delayed braking</td>
<td>7359</td>
<td>4586</td>
</tr>
<tr>
<td>Infrastructure Managed platoons with delayed braking</td>
<td>7525</td>
<td>4649</td>
</tr>
</tbody>
</table>

Table 2.2.1 Capacity comparisons with mixing vehicles from Kanaris et al. (1996)
<table>
<thead>
<tr>
<th>Capacity without platooning</th>
<th>0% mixing of vehicles</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Dry</td>
</tr>
<tr>
<td>Autonomous vehicles</td>
<td></td>
</tr>
<tr>
<td>with class identification</td>
<td>4116</td>
</tr>
<tr>
<td>Autonomous vehicles</td>
<td></td>
</tr>
<tr>
<td>without class identification</td>
<td>4116</td>
</tr>
<tr>
<td>Infrastructure supported</td>
<td></td>
</tr>
<tr>
<td>with class identification</td>
<td>5400</td>
</tr>
<tr>
<td>Infrastructure Managed</td>
<td></td>
</tr>
<tr>
<td>with class identification</td>
<td>6437</td>
</tr>
<tr>
<td>Moving-Cell model</td>
<td>4047</td>
</tr>
</tbody>
</table>

Table 2.2.2 Capacity comparisons without mixing of vehicles from Kanaris et al. (1996)

These results indicate that the capacity is reduced by 30% to 40% by going from dry road to wet road conditions under each concept. The capacity is also reduced by about 10% if all vehicles are required to use lower but similar braking force during emergency stopping.

Mixing of different classes of vehicles reduces capacity by about 11% in the case of mixing 2.5% buses and 2.5% trucks with passenger vehicles and by about 23% for 5% buses and 5% trucks. Platooning with coordinated braking gives the highest capacity. The moving-cell model gives the lowest. The use of vehicle-to-vehicle communication for notifying vehicles about the onset of braking used in the Free Agent and Platooning based concepts helps increase capacity considerably.

- Spacing of Lane Change

A vehicle occupies a space during a period of time when driving on a highway. When it is cruising, the space occupied consists of the length of the vehicle and the safety distance required. For example, let \( n \) vehicles be in a platoon, \( L_{\text{inter}} \) be the minimum safety distance between adjacent platoons, and \( L_{\text{intra}} \) be the safety distance between adjacent vehicles in a platoon, \( L_v \) be the average length of a vehicle, then the average space a vehicle take up, \( s_i \), is,
When a vehicle changes lane, the space required depends on operational scenarios. If a lane-changing vehicle has to become a single platoon, a platoon of one vehicle, then the space required is $2L_{\text{inter}} + L_{\text{v}}$ on both lanes. However, in an optimistic case, a vehicle can change lane without any extra space reserved for safety, the space required is 0 on current lane and $s_l$ on the target lane.

The requirement of space and time spent in these two vehicle maneuvers for all vehicles are considered as a load in a multilane AHS, since the space and time of a system is fixed, we can maximize the load under the system configuration to achieve maximum capacity. This idea was proposed by Hall (1995).

He created a workload model in which longitudinal flow and lateral flow put different loads on a lane. Then, the model was used to derive an optimization formula whose initial objective was to maximize throughput through control of lane change behavior. The final objective function was to balance the workload of sections of lanes. In this paper, he indicated that under 1000 meter-seconds lane change requirement, the nominal capacity of 7200 vehicles per hours with mean trip length of 20 km and vehicle speed of 30m/s can be attained. However, when the lane change requirement reaches 3000 meter-seconds, the capacity may be comparable to that of a conventional highway. The model proposed was an idealized model, which did not account for temporary variation in flow and flux and the discrete nature of highway entrances and exits.

Tsao et al. (1997) developed stochastic/analytic models in which he compared three different types of vehicle-following rules: free agent, platooning and slotting by formulating the gap length distributions for each case and platoon size distribution for the concept of platooning. The slotting rule here can be thought of as
the moving cell model. The derived lane-change completion time was used to identify the lateral capacity. It provides another way to evaluate the impact of lane change on the highway capacity. The slotting model and the platooning model were outperformed at four different flow levels. And they indicated that the speed differential of 3m/s provided approximately the best lane-change completion time. From the paper, it is obvious that the minimization of the times of lane changes can lead to the maximization of the capacity.

Hossein et al. (2000) analyzed the kinematics of vehicles involved in a lane change/merging maneuver and studied conditions under which collision could be avoided. Given a lane change/merging scenario, they calculated a minimum longitudinal spacing which vehicles involved in this maneuver should have so that no collision, of any type, would occur during the process. Three longitudinal acceleration scenarios: constant longitudinal speed, switching longitudinal acceleration, and modified switching longitudinal acceleration, were applied. They found that switching and modified switching scenarios could expanded safety region during a merging maneuver. They suggested that the minimum longitudinal derived mathematically could be used to assess the safety of lane change maneuvers.

Simulation is another way to estimate capacity in AHS when considering complex longitudinal and lateral vehicle maneuvers. The capacity of an automated highway with platooning and lane changing has been investigated by Rao et al. (1994) using the SmartPath simulator.

- Traffic theory of AHS

Broucke and Varaiya (1996) proposed a traffic flow theory for automated highways. The theory formulates a traffic management plan including vehicle activities, speed of vehicle, entry flow and exit flow for each vehicle on each section of a highway. A vehicle activity occupies a certain length of highway space. Let \( n(i,t,\theta) \) be number of vehicle type of \( \theta \) in section \( i \) at time \( t \), \( \pi(\alpha,i,t,\theta) \) be the
fraction of $n(i,t,\theta)$ engaged in the activity $\alpha$, $\lambda(\alpha)$ be the space required in activity $\alpha$. Thus, $n(i,t)$ vehicles engaged in $\pi(i,t)$ will occupy space, in section $i$ during period $t$, of

$$\sum_{\alpha} \sum_{\theta} \lambda(\alpha) \pi(\alpha,i,t,\theta) n(i,t,\theta)$$

(0.0.2)

The state of the system at time $t$ is defined as $n(t)=\{ n(i,t,\theta) \}$. Let $f(i,t,\theta)$ and $g(i,t,\theta)$ be the entry and exit flow of vehicle type of $\theta$ in section $i$ at time $t$, respectively. Therefore, for all $t$ and section $i$,

$$n(i,t+1,\theta) = \rho(i,t) n(i,t,\theta) + [1 - \rho(i-1,t)] n(i-1,t,\theta) + f(i,t,\theta) - g(i,t,\theta)$$

(0.0.3)

If $1 \leq i \leq I$, for all $t$ and $\theta$, the boundary conditions are,

$$n(0,t,\theta) = 0$$

(0.0.4)

$$n(I+1,t,\theta) = 0$$

(0.0.5)

They call $u(t)=[\pi(t), \nu(t), f(t), g(t)]$ a traffic management plan, where $\nu(t)$ is the speed of vehicles in section $i$. The system studied is a one-lane AHS with two main assumptions: one activity at a section and safety needs space, which is a minimum safe space that a vehicle will occupy under an activity. It is formulated as an optimization problem with constraints of flow conservation, non-negativity, speed limit, and boundary conditions. The set of achievable flows is shown as a convex polygon and a greedy rule is also proposed to achieve capacity, but does not minimize the travel time.
The solution is in steady state. Under platoon design, the maximum flow is 4186 vehicles per hour with interplatoon distance of 60 meters, intraplatoon distance of 1 meter, vehicle length of 5 meters, platoon size of 15 vehicles. The speed of vehicles is assumed to be 25 m/s and acceleration/deceleration rates are $\pm 2 \text{ m/s}^2$. This rate was shown in Caywood et al. (1977) that maximum values of acceleration and jerk should not exceed 0.2 g and 0.2 g/s, respectively, if the ride is to be acceptable.

Queueing Characteristics at Ramps of AHS

Queueing at entrance has long been an interesting topic in highway performance. Hall et al. (2001) evaluated the entrance capacity and queueing delay through the use of simulation and analytical modeling to compare different types of AHS operating concepts. The most promising concept is infrastructure assisted/supported with platooned entry. However, the comparable capacity at exit must be achieved and the release of platooned vehicles involves more issues to be determined. In this paper, neither lane assignment rule nor platoon formation rule is taken into account.

2.3 Lane Assignment Rules

This section covers that operating strategies, called lane assignment rules, are proposed to utilize lanes of a highway in order to maximize highway throughput or minimize travel time. The estimate of capacity has been made on AHS. However, an achievable capacity depends on practical operational rules applied. In the last section, Hall (1995) proposed a workload model to optimize load on a highway. He concluded that balancing loads among lanes and sections of a highway seems to be an optimal solution and the maximum capacity is achieved by deliberately assigning flows to lanes and later diverting them among sections of a highway. This idea was used to develop lane assignment rules in his following papers.

Hall and Lotspeich (1996) formulated a highway as a flow network with discrete segments as nodes. Three highway segments were on-ramp, off-ramp,
neither on-ramp nor off-ramp. The workload model was again applied in the LP formulation of an integrated automated highway system for maximization of the total flow with a fixed O/D demand. They concluded that under some conditions, lane change requirement over 1000 m-s or irregular O/D demands, increasing the number of AHS lanes beyond 2 or 3 only provide incremental capacity gains. In all cases, increasing the number of lanes provided decreasing marginal returns, due to the added overhead for lane changes.

This observation arises in practical cases. Entry flow is restricted by the capacity of an entrance. When entrances are all operated at full capacity, more highway lanes do not contribute at all. Vehicles need more lane changes on a multi-lane highway. This lane changes may affect the performance at immediately downstream ramp. When number of highway lanes increases, the range of influence extends farther downstream. Hence, the system throughput won’t increase linearly with the number of highway lanes.

Hall and Caliskan (1997) presented the dynamic version of the model and the results were similar. In this paper, an application of the model on the Hollywood Freeway is made. The freeway has frequent on and off ramps coupled with short trips. It is the case in urban area. They found that if a lane change could be achieved in 100 meter-seconds (e.g., 20 m additional space over 5 seconds), then the capacity could reasonably be more than double compared to that of a conventional highway.

Both papers mentioned lane assignment rules, which directly relates to the number of lane changes required. Hence, the assignment of vehicles into lanes in varied segments of a highway is not only balancing load of lanes but also reducing number of lane changes. These two factors all result in the increase of capacity. Therefore, the capacity of AHS can be maximized through an operational rule minimizing total number of lane changes including entry and exit.
The minimal number of lane changes is achieved by vehicles staying on the rightmost lane after entering. However, we cannot allow all vehicles to stay on the rightmost lane on a multi-lane highway if there is excessive demand. Therefore, some vehicles have to use left lanes. Vehicles on left lanes will eventually go back to the right lane when exiting. Its path is illustrated in Figure 2.3.1 as path (A).

Figure 2.3.1 Path of vehicles

Figure 2.3.1 also shows the intercepts of vehicle paths. When paths intercept, one vehicle is obstructed by another. This may incur loss of capacity due to entry obstruction and exiting failure due to exit obstruction. The system throughput is therefore reduced. Based on the idea, Ramaswamy et al. (1995) proposed non-partitioned and partitioned lane assignment strategies. The latter set of strategies is based on reducing path intercepts with respect to origins, destinations, and both. Figure 2.3.2 illustrates the destination monotone strategy, which states vehicles with farther destinations should be assigned to left lanes. The origin monotone strategy states that vehicles with nearer destinations should use right lanes. Figure 2.3.3 shows the monotone strategy, which is the combination of origin and destination monotone strategies.

Figure 2.3.2 Destination Monotone strategy

Figure 2.3.3 Monotone strategy
The lane assignment problem was also viewed as an optimization problem under capacity, nonnegativity, and conservation constraints. Two objectives were used here: minimize the total travel time and balance the workload. The model was applied on a 3-lane AHS. They concluded that the partitioned strategies could provide the optimal solution to one-entrance system and near-optimal solutions to more realistic system.

2.4 Platoon Formation Rules

A platoon needs more space during a separation process. More space with fixed number of vehicles results in the reduction of capacity. Therefore, if we group vehicles with similar destinations, platoons of these vehicles can remain intact longer, which increases capacity. However, due to the randomness of vehicle destinations, the smaller the range of destination a platoon requires, the smaller the number of vehicles in a platoon will be. Platoons of small size also reduce capacity.

With this idea, Hall and Chin (2002) studied the platoon formation rules applied on vehicles waiting at entrances. Four rules were proposed and their analytic models and simulation results were presented. Queueing at entrance and trip length distributions were included when modeling. The results were based on a one-entrance system. They proposed four rules:

1. Destination Group (DG): all downstream exits are divided into entrance lanes and vehicles enter the lane serving a range of exits including their destinations.
2. Dynamic Grouping (DYG): the range an entrance lane serves is based on the current vehicles and limited to a number specified by the system. A vehicle enters a lane such that the range of destinations is not larger than the number.

3. Dynamic Grouping Range (DGR): similar to DYG, but the range number of an entrance lane is determined by maximizing throughput.

4. Dynamic Grouping and Platoon Splitting (DGPS): vehicles in a platoon are sorted in a non-increasing order of their destinations.

Among these four strategies, in the rule of DG, vehicles stay longer in a platoon and the throughput is the largest in most situations. However, the rule of DGPS draws attentions when considering a system with multiple ramps. The other three rules may cause conflict or difficulty from entrance to entrance. For example, two entrances use DG, the range of a platoon released from one entrance may not match a range of a lane at another entrance. However, the DGPS can work among entrances. A platoon released from one entrance can admit vehicles if their destinations don’t break the order in the platoon.

In this paper, Hall and Chin also proposed a mechanism to sort vehicles into entrance lanes in terms of their destinations. Figure 2.4.1 shows the sorting that has multiple stages depending on ranges of destinations required in an operating strategy. Initially, vehicles enter an entrance lane serving their destinations. Then, vehicles are sorted into smaller ranges of destinations by driving forward to join proper lanes. In the following figure, vehicles with destinations in range 1 stay on the leftmost lane at stage 1. Then, those with destinations in a sub-range 1b are sorted into the middle lane at stage 2. The last stage is the stage for releasing vehicles. Vehicles with destinations in range 2 have to wait until all vehicles at stage 2 are released or sorted into stage 3 to be sorted. Hence, the more stages results in longer wait.
Hall and Chin provide a good start to design rules of vehicles forming platoons at entrances and these rules must be applicable among entrances without causing conflict. Therefore, we can study a highway of multiple entrances and multiple lanes.

![Figure 2.4.1 Multi-stage sorting](image_url)

**2.5 Summary**

By studying conventional car-following models, we realize that human factors restrict capacity increases on current highways. Automated highway systems provide a promising way to boost capacity while maintaining safety. These systems are becoming feasible under support of modern technology. The increase of capacity in AHS relies on platooning. In the literature, an automated highway generally achieves two to three times capacity of conventional highways.

Highway capacity depends on operational strategies such as lane assignment rules and platoon formation rules. Lane assignment rules maximize capacity by balancing load of lanes and minimizing weaving traffic. Platoon formation rules increase capacity by grouping vehicles with similar destinations so as to keep platoons stay intact to reduce space required for vehicle maneuvers.
In the next chapter, we design a new vehicle following model, in which space for safety and vehicle maneuvers is managed, complementing operational strategies minimizing space required by vehicle maneuvers.
Chapter 3

Moving Slot Model and Operating Strategies

In this chapter, a new vehicle following model, called the moving slot model, is presented. It is designed to facilitate the management of space in AHS operated in urban areas during rush hours. This model integrates the moving cell model and the platoon model, while taking space required by vehicle maneuvers into account. Corresponding operating strategies are also devised to achieve and maximize the system performance. These strategies minimize space requirements on/between ramps by regulating vehicles at entrances and highway lanes through platoon formation such that traffic at exits is also controlled. The design of the model and operational strategies are described in this chapter and applied in subsequent chapters.

Under this model, highway space is divided into slots. Vehicles join slots on the rightmost lane when entering and move among slots on different lanes when changing lanes. In Section 3.1, we discuss the components of a slot and focus on the one whose length varies with operational conditions. There are two sets of operational strategies. One is called slot assignment rules, which define which vehicle should join which slot at which position when entering an AHS or changing lanes. The other is called lane assignment rules, which regulate the use of lanes to minimize the weaving traffic that reduces the system performance. Maneuvering areas caused by lane changes can be used to determine the distance between ramps. These are discussed in Section 3.2 and 3.3, respectively. Section 3.4 provides numeric values of basic parameters and detailed definitions of systems. Section 3.5 concludes this chapter.

3.1 Description of the Moving Slot Model

In the moving slot model, slots are continuously located on an automated lane. Slots can be categorized by functions and vehicle types served. Different lanes may have different types of slots. A slot is an operating unit. The system
performance is, therefore, based on the design of slot. For the sake of capacity, a slot contains a platoon. A platoon is a group of vehicles closely spaced. A slot also contains space to meet requirements of vehicle maneuvers.

Space management is the core of the moving slot model, in which every piece of space of an AHS is reserved for a purpose. A slot contains the minimal space for accommodating a set of vehicles and supporting necessary maneuvers. By this design, a slot can be regarded as an independent operating unit in an AHS. The independence of a slot is defined that, in a one-lane AHS, vehicle maneuvers in one slot won’t affect those in other slots and, in a multi-lane AHS, a lane change process involving two slots won’t affect processes of other slots. This not only ensures safety but also reduces the complexity of system control.

Slot Creation and Maintenance

At the beginning of a highway, a dedicated entrance or a section of the highway is used to form slots. Slots have constant length and speed. Hence, a group of vehicles will be released into the system at a constant rate. A slot creation area must have sufficient space to store and sort vehicles to provide the maximum flow for AHS.

Slots are maintained on an automated highway lane by leading vehicles communicating to each other and keeping constant space between them and the other vehicles will follow their preceding vehicles. The central control system tracks positions of slots and assigns leading vehicles when necessary.

The Length of a Slot ($L_s$)

A slot contains a platoon and the space for vehicle maneuvers. Let $L_{\text{inter}}$ be the interplatoon distance, $L_{\text{intra}}$ be the intraplatoon distance, $N_{\text{max}}$ be the maximum allowable number of vehicles in a slot, $L_v$ be the average length of a vehicle, $V_i$ be the speed on lane $i$, and $a$ be the acceleration/deceleration rate assumed to be a constant. The first three terms in Equation (3.1.1) represent the length of a platoon of $N_{\text{max}}$
vehicles. The fourth term is called the speed adjustment distance, denoted by $L_{SA}$. It represents the space that vehicles need to adjust speed when changing lanes. The calculation is shown in Equation (3.1.2).

$$\begin{align*}
L_\gamma &= N_{\text{max}} L_\gamma + (N_{\text{max}} - 1)L_{\text{intra}} + L_{\text{inter}} + L_{SA} + L_f \\
L_{SA} &= \left(\frac{V_2 - V_1}{a}\right)\left(V_2 - \frac{V_2 + V_1}{2}\right), \quad V_2 > V_1
\end{align*}$$

(3.1.1) (3.1.2)

$$\text{Capacity on lane } i = \frac{N_{\text{max}} V_i}{L_\gamma}$$

(3.1.3)

The last term in Equation (3.1.1), denoted by $L_f$, is designed to provide the flexibility of lane changes, which is defined as vehicles’ ability choosing positions in both slots involved in a lane change. Vehicles move among slots when changing lanes. When more space is reserved in a slot, lane changes can be more flexible. The most flexible lane change occurs when vehicles can move from positions of one slot to positions of another slot without restriction. The amount of space also depends on operational scenarios. In the following, we consider three operational scenarios and evaluate the minimum space providing sufficient flexibility for each scenario. $L_f$ is determined under a worst case with full slots in each scenario. The first two scenarios are applied in one-lane AHS when designing slot assignment rules and the last one is used in two-lane AHS.

We first assume that, based on safety, a group of vehicles requesting a lane change will form a single platoon in advance and the space in the target slot is also adjusted such that this platoon keeps one interplatoon distance ahead and one behind upon joining the slot. We also assume that one maneuver occurs at a time per slot.
(1) End-Join-And-Leave:

In this scenario, vehicles can only join or leave a slot at the rear or front end. In Figure 3.1.1, we show that a vehicle will leave a slot at the rear end. The black box represents a lane change vehicle and small white boxes are staying vehicles and a long rectangle represents a slot. The vehicle becoming a single platoon needs one interplatoon distance ahead and one behind. The slot needs one more interplatoon distance, since there is already one inside. The \(L_{SA}\) is also behind the vehicle, because it will move to a slower lane. In this situation, the \(L_f\) is defined below.

\[
L_f = L_{inter} - L_{intra}
\]  

\[(3.1.4)\]

![Figure 3.1.1 The length of a slot for maneuvering positions at the rear of a slot](image)

(2) Middle-Join-And-Leave:

In this scenario, vehicles can join or leave a slot at any position. Figure 3.1.2 shows that the second to last vehicle will leave the slot. Two more interplatoon distances are needed to allow this lane change, since there are three platoons at the time of maneuver. In this situation, the \(L_f\) is defined below.

\[
L_f = 2(L_{inter} - L_{intra})
\]  

\[(3.1.5)\]
(3) Random-Join-And-Leave:

In this scenario, vehicles can separate at any position of one slot and join at any position of the other slot. Figure 3.1.3 shows the upper slot is full and the lower slot has space for one vehicle. The worst case would be the last vehicle in the upper slot requests a lane change to be the first vehicle in the lower slot. All vehicles in the lower slot move to one interplatoon distance after the position of the lane change vehicle, which is located one interplatoon distance behind the preceding platoon. In this situation, the $L_f$ is defined below.

$$L_f = 2(L_{\text{inter}} - L_{\text{intra}}) + (N_{\text{max}} - 1)(L_r + L_{\text{intra}})$$  \hspace{1cm} (3.1.6)
Figure 3.1.4 shows the maximum achievable capacity of the above three scenarios based on assumptions of an interplatoon distance of 60m, an intraplatoon distance of 1m, mean vehicle length of 5m, slot speed of 30m/s, speed difference of 10m/s, and acceleration/deceleration rate of 2 m/s$^2$.

As a result, the first scenario has the highest capacity because of the shortest length of a slot. However, in reality, the operation of slots in this scenario is limited. For example, if vehicles can only separate at the rear of a slot, then vehicles with farther destinations cannot join a slot whose last vehicle is destined to a closer exit. However, this scenario might perform well when vehicles have short-trip origin/destination patterns.

The third scenario is the most flexible scenario and has capacity from 4000 to 6000 vehicles per hour per lane, for $N_{\text{max}}$ from 10 to 30. This number is two to three times that of a conventional highway. Hence, this scenario, though reserving much space for the flexibility of lane change, can still provide high capacity.
The moving cell model can be regarded as a special case of the moving slot model with one vehicle per slot. Moreover, the moving slot model can be viewed as a variation of the platoon model under frequent access, egress, and lane changes, because the moving slot model is designed for platoons implementing frequent vehicle maneuvers. However, in the platoon model, the system performance may be worse when vehicle maneuvers cannot be executed due to lack of space, or vehicles miss maneuver timing because of communication delay. Therefore, managing space in advance is essential to the success of an AHS applied under a heavy traffic condition.

**Selection of Speed in One-Lane AHS**

The optimal speed on a highway lane is determined with respect to capacity and the total travel time, and is limited by the maximum legally allowable speed. In general, the maximum speed on a highway in urban areas is set at 65 to 70 mph; hence, 30 m/s is used as an average. Capacity of a highway depends on speed and density of vehicles. Given a fixed density, capacity increases with speed, and vice versa. However, an increase of speed usually results in a decrease of density, because a higher speed requires a longer inter-vehicle distance to drive safely. The total travel time is defined as the average travel time per vehicle multiplied by the flow rate and the average travel time is the mean trip length divided by the average vehicle speed.

In the moving slot model, the length of a slot contains two speed-related parameters: $L_{SA}$ and $L_{inter}$. The interplatoon distance is determined under the situation that when a vehicle comes to a full stop, the following vehicle won’t collide. Let $V$ be the slot speed and $\alpha$ be the emergency deceleration rate assumed to be a constant, then

$$L_{inter} = \frac{V^2}{2\alpha} \quad \text{(3.1.7)}$$
In addition to assumptions and parameters used in Figure 3.1.4, we assume that the distance at an entrance ramp can support vehicles to accelerate to 20m/s upon joining a slot, the emergency constant deceleration rate is 7.5m/s², and the mean trip length is 20km. Then, by using Middle-Join-And-Leave slots as an example,

\[ \text{Capacity} = \frac{10V}{0.45V^2 - 15V + 282} \]  \hspace{1cm} (3.1.8)

\[ \text{Total travel time} = \frac{200000}{0.45V^2 - 15V + 206} \]  \hspace{1cm} (3.1.9)

The results of Equations (3.1.8) and (3.1.9) are shown in Figure 3.1.5.

The speed producing the maximum capacity of 5331 vehicles per hour is at 17m/s (i.e. 38.25 mph). To operate a highway at this speed seems not agreeable,
since people expect to save their time by driving on highways. At the speed of 30m/s, the capacity of 4122 vehicles per hour is around twice that on a conventional highway, although less than that at 17m/s. Furthermore, the total travel time can be reduced 56.2% meaning that every driver can save half of the time driving on the highway.

In addition to speed, capacity can also be increased by other factors such as \( N_{\text{max}} \) and normal and emergency acceleration/deceleration rates while the total travel time can only be reduced by increasing speed. For instance, Figure 3.1.4 shows the relationship between capacity and \( N_{\text{max}} \). Therefore, in this research, the criterion to choose an operational speed of a slot is to minimize the total travel time while keeping an acceptable capacity. Slots in one-lane AHS are operated at the speed of 30m/s.

**Speed Difference in Multi-Lane AHS**

In a one-lane highway, we consider speed difference when vehicles laterally move into the highway lane from an entrance ramp. Given a limited ramp space and a constant slot speed, we reserve a space in each slot to allow released vehicles to accelerate to the slot speed. In a two-lane system, we need to consider the speed difference between lanes. A space for adjusting speed would also be inserted into a slot when the speed difference arises.

From results shown in Figure 3.1.5, the maximum capacity on one lane occurs at the speed of 17m/s. In this situation, \( L_{\text{SA}} \) equals zero because the distance on an entrance ramp can support vehicles to accelerate to this speed. If we consider only capacity, then 17m/s would also be selected on other lanes and speed difference among lanes would be zero. However, from the conclusions of one-lane AHS, speed is selected to minimize the total travel time if the corresponding capacity is acceptable. Hence, we assume that slots on highway lanes are all operated at the speed of 30m/s and there is also no speed difference among lanes.
Under this assumption, for the Middle-Join-And-Leave slots, the capacity is 8679 vehicles per hour on two-lane AHS. However, if this type of slot is applied in multi-lane AHS, the opportunity of lane changes is limited. Hence, we adopt Random-Join-And-Leave slots in multi-lane AHS to maximize the flexibility of lane changes and the corresponding capacity is 7578 vehicles per hour in two-lane AHS. This capacity is around 1.9 times of that in conventional two-lane highways and acceptable. Note: these numbers are derived based on $N_{max}=10$ and increase with $N_{max}$.

**Operational Units in Multi-Lane AHS (Stacks)**

With the same speed, in some situations, vehicles have no opportunities to change lane, even though the target slot is not full. In Figure 3.1.6, a dashed rectangle stands for a slot, an empty box represents a vehicle, and a box with slashes represents a vehicle requesting a lane change. The figure shows two vehicles, one on each lane, cannot change lane because they are adjacent to vehicles or areas reserved for safety. These vehicles have to wait until other vehicles in the same slots depart, then they can move to their positions to complete lane changes while target slots are still not full. A lane change may take a long time to wait and complete when vehicles in the slots have distant destinations. These unsuccessful lane changes prevent vehicles from exiting and block vehicles from entry.

![Figure 3.1.6 Not-aligned slots in two-lane](image)

Figure 3.1.6 Not-aligned slots in two-lane
To minimize the waiting time for a lane change, slots on both lanes are perfectly aligned to form a one-to-one relationship. In general, a lane change needs two conditions to initiate and complete: the target slot has sufficient space to admit vehicles and space in both slots involving in this lane change should be adjusted in time for vehicles to move to the specified position. The first condition is common and the second condition is satisfied by this design of well-aligned slots, because, in two-lane AHS, Random-Join-And-Leave slots can support a lane change between any positions in two slots, which are aligned perfectly. Moreover, since slots are independent, both slots can adjust space for a lane change at the same time. Hence, the waiting time for a lane change is minimized. Figure 3.1.7 shows well-aligned slots.

![Figure 3.1.7 Well-aligned slots in two-lane](image)

In one-lane AHS, a slot is an operational unit. A vehicle maneuver in one slot won’t affect maneuvers in other slots. In two-lane AHS, we define an operational unit called a stack. A stack consists of two well-aligned slots. The slot on the left lane is called the upper slot and right lane the lower slot. The slot type used in a stack allows any vehicle in one slot to move to any position in the other slot. Likewise, vehicle maneuvers in one stack won’t affect maneuvers in other stacks.

The creation of stacks is similar to the creation of slots. At the beginning section of a two-lane AHS, vehicles are stored and sorted by their destinations into highway lanes. This stack-creation area can be a section of a highway or a dedicated
ramp. We assume stacks are always full upon entering the system under a heavy traffic condition.

Substitution of $L_{SA}$ and $L_f$

These two distances are reserved in a slot for lane changes. Broadly, lane changes include vehicles moving from entrances to the rightmost lane of a highway, among highway lanes, and from the rightmost lane to exits. When there is a speed difference, we need $L_{SA}$. However, the speed difference is not necessarily the same between lanes. For example, a larger speed difference may exist between entrances and the rightmost lane. Hence, slots on different lanes may have different $L_{SA}$ based on the speeds on adjacent lanes. The $L_{SA}$ in slots on one lane is determined based on the larger speed difference with adjacent lanes.

A slot can only execute one maneuver at a time. This rule gives us the chance to partly substitute $L_{SA}$ for $L_f$. For example, a slot on the rightmost lane may have an entry process followed by a lane change process to the second lane. The former process requires a $L_{SA}$ and the latter requires both $L_{SA}$ and $L_f$. However, $L_{SA}$ in different processes may not be the same. Suppose the former process needs a longer $L_{SA}$. Therefore, the $L_{SA}$ in the slot is defined by the former process. Then, in the latter process, part of $L_{SA}$ can be used for $L_f$. This management of space reduces the length of a slot and, in turn, increases the capacity.

3.2 Slot Assignment Rules

In the moving slot model, vehicles join slots when entering an AHS and maneuver among slots on different lanes until exit. Slots can be designed to serve vehicles with certain characteristics to attain certain objectives. In the following, we discuss issues in the design of slot assignment rules.
**Vehicle Type**

Different types of vehicles have different performance characteristics. For example, trucks, in general, have slower acceleration/deceleration rates than passenger cars. A slot containing both trucks and passenger cars is not a good design because they require different amounts of space in emergencies and changing lanes. This would make space management difficult and results in bad utilization of space and lower throughput. Therefore, we assume that a slot will contain one type of vehicle.

At an entrance, vehicles are grouped and released into the system to a slot that accepts their type. For example, slots can be classified to serve trucks, buses, or passenger cars. We can use different entrance lanes for different types of vehicles or we can assign entrances specific to vehicle types. Because the acceleration/deceleration rates vary, the ramp for released vehicles to accelerate to meet the speed requirement upon joining can vary in length. For entrances serving all kinds of vehicles, we need a ramp long enough to take the slowest vehicles into account. For vehicle type-specific entrances, we can design the ramp space just for the specific type.

If slots of different vehicle types are on the same automated lane, the slot length may vary with the vehicle types, or it is roughly fixed with some slots serving fewer vehicles than others. The interplatoon distance, intraplatoon distance, and speed adjustment distance also depend on vehicle types.

If there is only one lane in an AHS, vehicles are designed to run at the same speed. We may operate the system at a lower speed to ensure for all vehicles. For a multi-lane AHS, one type of vehicle can be assigned to one lane and the lane speed can increase without violating the law to increase the throughput. However, the $L_{SA}$ should be designed based on the speed difference and acceleration/deceleration rates of vehicle types.
Range of Vehicle Destinations

A slot could serve all vehicles without restriction on their destinations. However, it may be beneficial to use slots serving limited ranges of destinations. A brief discussion is in the following. Note: the destination of a vehicle is represented by an exit number. Hence, a vehicle of destination 3 will leave at the exit 3.

1. Distinct range of destinations:
   A slot serves a distinct range. For example, 3 slots serve destination ranges of (1~3), (4~6), and (7~9), respectively. During a section between the exit 1 and the exit 3, slots serving (1~3) admit and release vehicles, while other slots admit vehicles only. If vehicles join slots only at the rear but leave at any position, slots serving (1~3) need more space than others during the section. In the following section between the exit 4 and the exit 6, slots serving this section need more space than others. This implies that the space in slots serving one section can be shifted to slots serving the next section and so on. This management reduces the requirement of $L_{SA}$ and increases capacity. After passing the exit 3, slots serving (1~3) will be assigned to serve (10~12).

2. Overlapped range of destinations:
   A slot serves a range, which is partially or totally overlapped with another. For example, a slot serves destinations of (1,2), another (1~4) and the other (1~ last destination). When slots pass exit 2, slots serving (1,2) will be assigned to serve (3~6) and slots serving (1~4) serve (3~4). In this arrangement, we actually classify slots by serving immediate downstream 2 exits, 4 exits, and all exits. It is designed especially for short trip-length highways. We can also design for long trip length highways by overlapping more downstream exits.

3. Range of destinations by lanes:
   The above design is applied to slots on one lane. In a multilane AHS, slots on different lanes may serve different ranges. If the range on one lane is (3~7), then
slots on the lane are assigned to serve any subset of (3,...,7). When the lane is assigned a new range to serve, say (5~8), slots release vehicles of destinations 3 and 4 and admit vehicles of destination 8.

**Sequence of Vehicle Destinations**

A slot could serve vehicles based on the order of destinations. For instance, a slot has vehicles sorted as (7,3,2,2,1) and a vehicle with destination of 5 will join the position between 7 and 5 without breaking the order of destinations in the slot. In a descending sequence, exiting vehicles will always appear at the tail of a slot. Therefore, in the exiting process, we need only 2 interplatoon distances to ensure safety. As stated in Section 3.1, capacity increases in this case. Vehicles can also be in ascending sequence of destinations and exit at the front of a slot.

Design based on range and sequence of destinations can be combined into practical use. For example, slots serve range of (4~7) and vehicles have to be in an ascending sequence. However, this combination results in more restrictions on selection of slots to join. For example, a slot serves a descending order of destinations of (3~6) can neither admit a vehicle with destination of 7 nor a vehicle with destination of 4 if the vehicle misses the timing to join the position without breaking the order. Hence, vehicles may spend more time in entering a system or changing lanes.

**3.3 Lane Assignment Rules**

The lane assignment rule specifies the lane a vehicle should use in each section of an AHS, from entrance to exit, based on the destination or characteristics of the vehicle. Generally speaking, vehicles having a longer trip should use different lanes from those having a shorter trip. When the traffic flow is low, all vehicles can drive on the rightmost lane to avoid unnecessary lane changes. With an increase in traffic flow, the space on the rightmost lane must be reserved for vehicles entering from downstream entrances. As a result, some vehicles should move to the left lanes in advance.
It is advantageous to use the left lanes for vehicles traveling to distant destinations, because this creates more space for vehicles entering the system. Vehicles must pass through all of the right lanes of the assigned left lane. Thus, to minimize the time of lane changes into the target lane minimizes the time staying on middle lanes and, in turn, maximizes the time staying on the target lane. Furthermore, space in middle lanes can be released earlier to accommodate more vehicles assigned to these and left lanes.

In the following, we present strategies derived from the concept of workload (Hall, 1995) and the concept of path intercepts (Ramaswamy, 1995). Strategies are developed in terms of vehicle destination and vehicle location.

**General Rule**

From these two concepts, during a section of highway, vehicles are arranged to balance “loads” among lanes while keeping long-trip vehicles on left lanes and short-trip on right lanes to reduce path intercepts and increase throughput. Hence, a general lane assignment rule based on these concepts is illustrated in Figure 3.3.1.

Lanes are numbered from right to left, beginning with Lane 1, and the exit shown is the exit 1. Each lane serves a distinct range of destinations. To balance loads among lanes, ranges may be altered after an exit or entrance. In the figure, ranges step up by one after the entrance. Lane 1 initially serves destinations 1 to 3, but changes to destinations 2 to 4 after passing the exit 1. Vehicles with destination 4 on Lane 2 and vehicles with destination 8 on Lane 3 change to right lanes when passing the segment marked by two vertical dashed lines. Vehicles with destination 1 will exit. Thus, under this rule, vehicles move to their target lanes after entering and move to the right in steps as they approach their destinations.
Figure 3.3.1 A general lane assignment rule

Figure 3.3.2 shows the same rule as that in Figure 3.3.1 except the locations of ramps. Vehicles with destination 1 will leave first and then, vehicles move to lanes assigned by the rule.

---

Light Traffic Rule

In a light traffic condition, vehicles with long trips can still stay on right lanes to minimize number of lane changes. When the traffic becomes heavier, these vehicles start to move to left lanes to increase the system throughput. The general rule can be modified to provide this flexibility in assigning vehicles to lanes, shown in Figure 3.3.3.
The main idea is to reduce the number of lane changes. Each lane has a lower bound on its serving range, but no upper bound. This design allows slots on Lane 1 to serve all vehicles and vehicles move to left lanes only when necessary. The lower bounds increase with lane numbers and have the tendency of moving vehicles with more distant destinations to the left lanes.

When traffic flow on a lane approaches a specified limit, vehicles with long trips are instructed to move left. When vehicles approach their destinations, they are permitted to move right. Otherwise, they stay on their current lanes. In Figure 3.3.3, if the right lane becomes congested, vehicles destined to the exit 5 and above may move left. Likewise, if Lane 2 becomes congested, vehicles destined to the exit 9 and above may move left.

**Trip Length Specific Rule**

Lane assignment rules can be designed to favor some specific patterns of trip length distributions. In Figure 3.3.4, we show a lane assignment rule designed for short-trip origin/destination patterns.
This rule reduces the total number of lane changes of vehicles ready for exiting. The range of destinations on each lane is fixed over several sections of the highway. The range changes until the slots on Lane 1 are empty, and when the range changes, entire platoons on left lanes tend to move right, rather than individual vehicles. Lane 3 initially carries all vehicles destined to the exit 11 or above. Over several entrances, the range remains the same, meaning that traffic gradually increases as vehicles destined to this range enter the system. Meanwhile, Lane 1 initially carries vehicles destined to exits 1 to 5, but as exits are passed, traffic declines. Eventually, no continuing traffic remains on Lane 1, and all vehicles on lane 2 are now instructed to move right, and all vehicles destined to exits 11 to 15 on Lane 3 are instructed to move right too. This frees some space on Lane 3, which can carry more vehicles for long trips.

This rule works well when vehicles have short trips. In this condition, most vehicles drive on the rightmost lane and vehicles assigned to left lanes accumulate gradually without exceeding capacity of lanes. However, when demand for long trip increases, flow rates of left lanes reach capacity quickly and vehicles cannot move to their assigned lanes. This rule must be adjusted to account for the capacity constraint. Then, frequent changes of destination ranges on lanes make this rule actually a special case of the general rule shown in Figure 3.3.1.

**Maneuvering Areas**

Maneuvering areas arise when vehicles enter, change lanes, and exit. A maneuvering area appears around a ramp. The size of a maneuvering area depends
on the lane assignment rule applied on the ramp. To minimize maneuvering areas is to minimize space between ramps. If ramps have limited capacity, then the throughput per unit length on a highway can be increases by densely located ramps. Based on the assumption of one maneuver at a time per slot and constant slot speed, a maneuvering area is determined by the number of lane changes occurred. In the following, we discuss maneuvering areas in detail.

We specifically call a maneuvering area around an entrance an entering zone and around an exit an exiting zone. Figure 3.3.5 shows an entering zone with the applied lane assignment rule. For simplicity, the ranges are unaltered.

An entering zone represents a segment where vehicles released move to their target lanes specified by lane assignment rules. In general, the zone widens on the left lanes, as random delays may occur when traversing each intermediate lane. Lane 1 serves the range of (1~3), but also temporarily serves ranges of left lanes. Likewise, Lane 2 serves its own range and temporarily the range of Lane 3. Hence, the entering zone on one lane accounts for the staying time of vehicles assigned to left lanes and the staying time of vehicles on one lane should include that on right lanes. For example, a vehicle with destination 9 spends 4 seconds on Lane 1 and 5 seconds on Lane 2. The maneuvering area on Lane 3 for this vehicle starts at time of the 9th seconds until it completes the lane change.

There are two types of entry modes (or called release modes). One is “slot-searching release” and the other is “slot-assigned release.” The former is implemented by regular release and the released vehicles begin looking for an
appropriate slot on the right lane to merge. In the latter case, vehicles are released only when there are appropriate slots approaching. In slot-searching release, the length of the ramp needs to be longer. Therefore, we adapt slot-assigned release mode in our systems.

An exiting zone occurs when slots pass an exit and vehicles move to exit. Figure 3.3.6 shows an exiting zone with the applied lane assignment rule.

![Figure 3.3.6 An exiting zone](image)

In this lane assignment rule, vehicles moving to left lanes will start together at a certain position marked by a vertical dashed line in the figure. This position is where vehicles with destination 1 begin to exit. This design allows vehicles on left lanes to have more space to change lanes, since space on Lane 1 will be freed first. Then vehicles, with destination 4, moving to Lane 1 free space for vehicles with destination 8 to move left. This exiting mode is called segment-wise exiting, since vehicle will move to the rightmost lane from segment to segment specified by lane assignment rules.

Figure 3.3.7 shows another exiting mode called “continuous exiting.” In this mode, vehicles stay on their specified lanes until their destinations appear immediately downstream. Then, vehicles move to exit.
Figure 3.3.7 An entering zone for a continuous exiting mode

An exiting zone under the segment-wise exiting mode shifts to the right compared to that in continuous exiting mode. This design avoids overlapping of maneuvering areas when the space between an entrance and an exit is limited. Figure 3.3.8 shows maneuvering areas on a highway.

Figure 3.3.8 A system view of maneuvering areas

3.4 Conclusions

In the moving slot model, a slot is a basic operating unit, which is continuously located on an automated lane. The components of a slot include a platoon of vehicles, interplatoon distance, intraplatoon distance, speed adjustment distance, and the space representing the flexibility of lane change. A slot has the shortest length when vehicles are only allowed to join or leave at end positions and the longest length while vehicles can join and leave at any position of slots. Given a fixed platoon size, shorter slots theoretically lead to a larger capacity. However, the operational restriction reduces the accessibility of vehicles to a system and, in turn, reduces the throughput.
The criterion to select slot speed is based on the minimization of total travel time while keeping an acceptable capacity. Hence, the maximum legally allowable speed of 30m/s is applied. This results in no speed difference among lanes in multi-lane AHS. Therefore, Random-Join-And-Leave slots are used to minimize the waiting time of lane changes. The capacity, under this operation, is around twice that of conventional highways and can be further increased by increasing $N_{\text{max}}$, or increasing normal or emergency acceleration/deceleration rates.

Slot assignment rules regulate vehicles accessing slots by their types, destinations and specific sequences in slots. Lane Assignment rules specify the use of lanes based on vehicle destinations in multi-lane AHS. Vehicles with long trips will be assigned to left lanes to create more space on right lanes for vehicles having short trips. This not only increases the throughput but also minimizes the number of lane changes.

The moving slot model is integrated with slot assignment rules and lane assignment rules in a multi-lane AHS. Vehicles from entrances are assigned target lanes. Lane change processes to target lanes may involve multiple selections of slots. Hence, in this sense, vehicles following lane assignment rules execute a series of slot assignment rules. In a word, vehicles are controlled by slot assignment rules at entrances, on highway lanes until exit. To meet these rules, some system performance measures, such as release rates at entrances and lane changes on highway lanes, may decline. However, other performance measures, such as easy and successful exit, may increase. On the other hand, if no slot assignment rule is applied, the performance at entrances may increase, since no restriction is applied. However, because some maneuvering processes may not be implemented well, the system performance might worsen. Therefore, a trade-off between entrance performance and exit performance or other system performance measures depends on designing slot/lane assignment rules.
Maneuvering areas on a highway, especially near ramps, depend on lane changes. They are specifically called entering zones and exiting zones in this chapter. These areas can be used to determine the ramp spacing. Smaller maneuvering areas allow closer ramps and increase the number of vehicles released per unit distance.

In the following chapters developing operational strategies, we assume that vehicles have the same type of passenger car, and their characteristics, such as acceleration/deceleration rate and vehicle length, are all the same. Slots are able to serve vehicles destined to all downstream exits. Table 3.4.1 lists numeric values for basic parameters used in this research. The first element, if any, in the parenthesis of a parameter represents its symbol and the second element, if any, represents unit: m is meter and s is second. The interplatoon distance in the table is determined by preventing a vehicle from collision with its preceding vehicle, which comes to a full stop suddenly. Vehicles are assumed to maneuver at the constant speed or constant acceleration/deceleration rates listed in the table. Therefore, with a limited ramp space of 100 meters, the speed upon joining for vehicles at entrances would be 20m/s, the speed adjustment distance in a slot, \( L_{SA} \), would be 25m, and the time for a vehicle to move laterally to an adjacent lane is 4 seconds based on the width of a lane, 8 meters. The \( t_I \) is assumed to be a constant for an exiting group of any size. The length of a slot able to allow vehicles to join at ends is 203m and that allowing vehicles to join or leave at any position is 262m. These two types of slots are used in one-lane AHS to implement different slot assignment rules.
<table>
<thead>
<tr>
<th><strong>Vehicle</strong></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>vehicle length($L_v$, m)</td>
<td>5</td>
</tr>
<tr>
<td>vehicle speed($V$, m/s)</td>
<td>30</td>
</tr>
<tr>
<td>acceleration/deceleration rate</td>
<td>2</td>
</tr>
<tr>
<td>at normal conditions($a$, m/s$^2$)</td>
<td>2</td>
</tr>
<tr>
<td>deceleration rate at</td>
<td>7.5</td>
</tr>
<tr>
<td>emergence conditions($\alpha$, m/s$^2$)</td>
<td></td>
</tr>
<tr>
<td>time of lateral movement($t_l$,s)</td>
<td>4</td>
</tr>
<tr>
<td><strong>Slot</strong></td>
<td></td>
</tr>
<tr>
<td>intraplatooon distance($L_{intra}$, m)</td>
<td>1</td>
</tr>
<tr>
<td>interplatoon distance($L_{inter}$, m)</td>
<td>60</td>
</tr>
<tr>
<td>Maximum number of vehicles</td>
<td>10</td>
</tr>
<tr>
<td><strong>Highway</strong></td>
<td></td>
</tr>
<tr>
<td>number of entrances</td>
<td>10</td>
</tr>
<tr>
<td>number of exits</td>
<td>20</td>
</tr>
<tr>
<td>ramp space for acceleration/</td>
<td>100</td>
</tr>
<tr>
<td>deceleration(m)</td>
<td></td>
</tr>
<tr>
<td>The width of a lane(m)</td>
<td>8</td>
</tr>
</tbody>
</table>

Table 3.4.1 Numeric values of basic parameters used in this research
Chapter 4
One-Lane Automated Highway Systems

Studying one-lane AHS can build a foundation for dealing with multi-lane AHS, since most traffic conditions occur in one-lane highways. The moving slot model and slot assignment rules are applied on one-lane AHS. Slot assignment rules are proposed to specify positions in slots for vehicles at entrances to join without breaking the specific patterns of destinations that minimize space required at exits and allow vehicles to exit successfully. This application reduces flow rates at entrances but guarantees successful exits without lengthening ramp space. Hence, space for vehicles to exit at current highway ramps is also sufficient in AHS. In this regard, slot assignment rules minimize the infrastructure construction when current highways are converted into automated systems.

In Section 4.1, we discuss the drawback of releasing vehicles to fill up slots simply by their arrival sequences, First-Come-First-Serve (FCFS). This rule, called Random Slot, maximizes flow rates at entrances. However, vehicles randomly positioned in a platoon regardless of destinations may take a long time and more space to leave a highway successfully. Hence, five slot assignment rules are proposed in Section 4.2. The theoretical maximum capacity at an entrance is found for each slot assignment rule by assuming that slots are always full when passing entrances. In Section 4.3, maneuvering processes are clarified and ramp spacing under slot assignment rules are calculated. Then, the system throughput, depending on ramp spacing and mean release rate under each slot assignment rules, is also derived. Section 4.4 concludes the chapter.

4.1 Random Slot Rule

This rule states that a slot serves vehicles without restriction of destination and vehicles waiting at entrances are served FCFS. Therefore, the number of
vehicles released depends on the number of vehicles in queue and the available space in a slot. By assuming that vehicles are always available for release, an entrance has the maximum release rate under this rule. However, an exit may need a long ramp space for all exiting vehicles to leave successfully.

A slot under this rule is called a random slot, which can support vehicles to join or leave at any position of a slot. An exiting group is a group of adjacent exiting vehicles. Vehicles requesting to exit in a random slot may not be contiguous. There may exist more than one exiting group and, hence, multiple exiting processes are required. In this section, we first define an exiting process and then we derive the distribution of number of exiting groups in a random slot given an exiting probability. Finally, the required length of ramp at an exit is calculated.

**Exiting Process**

An exiting process consists of a space adjustment process for an exiting group becoming a single platoon and a lateral movement from the highway lane to an exit ramp. In the following illustration, a rectangle box stands for vehicles staying on the lane and a black square box shows a vehicle requesting to exit. A dashed rectangle represents the boundaries of a slot.

Figure 4.1.1 (a)-(d) shows the exiting process of one vehicle. In (a), a slot is approaching an exit and a vehicle inside requests to exit. The vehicle becomes a single platoon in (b) and moves laterally to the exit ramp in (c). In (d), remaining vehicles in the slot merge into a single platoon and the vehicle keeps the same speed as the slot on the ramp.
Figure 4.1.1 An exiting process of one vehicle

Figure 4.1.2 (a)-(g) shows the process that two non-contiguous vehicles exit. In (a) a slot approaches an exit and two vehicles at different positions request to exit. The first vehicle becomes a single platoon in (b) and moves to the exit ramp in (c). The second vehicle becomes a single platoon by separating from trailing vehicles in (d) and then separating from the preceding vehicle in (e). The second vehicle moves to the exit ramp in (f). Remaining vehicles merge in the slot and exiting vehicles merge into a platoon in (g)

Figure 4.1.2 An exiting process of two vehicles
Figure 4.1.3 provides exiting processes of slots on an exit. An exiting group on an exit ramp will keep at the same speed to avoid collision with other exiting groups.

Distribution of number of exiting groups

Let $N$ be the number of vehicles in a slot, $K$ be the number of exiting groups ranging from 0 to $\lceil N/2 \rceil$, which is the smallest integer greater than or equal to $N/2$, $p(K,N)$ be the probability of $K$ exiting groups in $N$ vehicles, $p(k)$ be the probability that there are $k$ exiting groups by considering all possible $N$, and $p_e$ be the probability that a vehicle requests to exit. The detailed derivation is shown in a recursive way as follows:

(a) $K=0$, no vehicle exits,

$$p(0,N) = (1 - p_e)^N \tag{4.1.1}$$

(c) $K=1$, one exiting group may have number of vehicles from 1 to $N$,

$$p(1,N) = \sum_{i=1}^{N} (N-i+1)p_e^i(1-p_e)^{N-i} \tag{4.1.2}$$

(d) $K=2$, two exiting groups are separated by a separator, which is a group of vehicles that do not exit. In calculating the probability, $p(2,N)$, we distinguish cases by the beginning location of the separator. In this case, the location of the separator begins from 2 to $N-1$. When the beginning
location moves from position $s$ to $s+1$, the vehicle at position $s$ must be in the left-hand-side exiting group. Since only one exiting group is at each side of the separator, the left-hand-side exiting group can have number of vehicles from 1 to $s$, all including the vehicle at location $s$ and the probability of the right-hand-side exiting group is $p(1,N-s)$. Hence,

$$p(2,N) = \sum_{s=2}^{N-1} \left( \sum_{i=1}^{s-1} p_c^i (1-p_c)^{s-i} \right) p(1,N-s)$$  \hspace{1cm} (4.1.3)

(e) $K=3$, three exiting groups and two separators. This case can be reduced into a one-separator problem if we fix one of two separators. When the left separator is fixed, there will be one exiting group at the left-hand side of the separator and two at the right-hand side. The left separator can be fixed at position beginning from 2 to $N-3$. Hence,

$$p(3,N) = \sum_{s=2}^{N-3} \left( \sum_{i=1}^{s-1} p_c^i (1-p_c)^{s-i} \right) p(2,N-s)$$ \hspace{1cm} (4.1.4)

(f) Recursively, $k$ exiting groups need $k-1$ separators and the leftmost separator begins at position from 2 to $N-2k+3$. Hence,

$$p(k,N) = \sum_{s=2}^{N-2k+3} \left( \sum_{i=1}^{s-1} p_c^i (1-p_c)^{s-i} \right) p(k-1,N-s), k > 1$$ \hspace{1cm} (4.1.5)

And, finally

$$p(k) = \sum_{n=2k-1}^{N_{\text{max}}} \Pr[N = n] p(k,n)$$ \hspace{1cm} (4.1.6)

Figure 4.1.4 shows the distributions of number of exiting groups with different exiting probabilities.
The number of vehicles in a slot is assumed to be uniformly distributed over integers 0 to \(N_{\text{max}}\). When \(p_e\) equals 0.1 or 0.9, most slots have a smaller number of requests and similar distributions occur when \(p_e\) ranges from 0.5 to 0.7: 30% and 15% of slots having 2 and 3 exiting groups. Hence, in a random slot, a short ramp arises when the exiting probability is either very high or very low. In an AHS, a high traffic flow with a high exiting probability for an exit may cause a spillback and damage the whole system. Thus, a small exiting probability is preferred to reduce the requirement for ramp space at an exit.

Table 4.1.1 shows the probabilities of number of exiting groups with \(p_e=0.1, 0.2,\) and 0.3. If an exit is built to serve 3 exiting groups and we assume that an exiting group of larger size is served first, then 4 and 5 exiting groups in a random slot result in one and two exiting groups of one vehicle that fail to exit, respectively. The number of vehicles unable to exit is 0.2, 1.5, and 4.6 per hour for \(p_e=0.1, 0.2,\) and 0.3, respectively. The mean flow in the above case is 2061 vehicles per hour. If
the system is operated at full capacity, they are 1, 9.6, and 29.4. The probabilities are shown in Table 4.1.2. The numbers are even greater if an exit can only serve 2 exiting groups. Vehicles unable to exit will stay in slots and reduce release rates of entrances.

<table>
<thead>
<tr>
<th>Number of exiting groups</th>
<th>$p_e$ (0.1)</th>
<th>$p_e$ (0.2)</th>
<th>$p_e$ (0.3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.6238</td>
<td>0.4155</td>
<td>0.297</td>
</tr>
<tr>
<td>1</td>
<td>0.3014</td>
<td>0.3773</td>
<td>0.3764</td>
</tr>
<tr>
<td>2</td>
<td>0.0671</td>
<td>0.1662</td>
<td>0.2358</td>
</tr>
<tr>
<td>3</td>
<td>0.0074</td>
<td>0.0375</td>
<td>0.0796</td>
</tr>
<tr>
<td>4</td>
<td>0.00034</td>
<td>0.0035</td>
<td>0.0108</td>
</tr>
<tr>
<td>5</td>
<td>0.000004</td>
<td>0.0008</td>
<td>0.0004</td>
</tr>
</tbody>
</table>

Table 4.1.1 Probabilities of number of exiting groups in a random slot

<table>
<thead>
<tr>
<th>Number of exiting groups</th>
<th>$p_e$ (0.1)</th>
<th>$p_e$ (0.2)</th>
<th>$p_e$ (0.3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.3486</td>
<td>0.1074</td>
<td>0.0282</td>
</tr>
<tr>
<td>1</td>
<td>0.4304</td>
<td>0.346</td>
<td>0.196</td>
</tr>
<tr>
<td>2</td>
<td>0.1855</td>
<td>0.3698</td>
<td>0.4081</td>
</tr>
<tr>
<td>3</td>
<td>0.0331</td>
<td>0.1536</td>
<td>0.2963</td>
</tr>
<tr>
<td>4</td>
<td>0.0022</td>
<td>0.0224</td>
<td>0.0681</td>
</tr>
<tr>
<td>5</td>
<td>0.0003</td>
<td>0.0008</td>
<td>0.0033</td>
</tr>
</tbody>
</table>

Table 4.1.2 Probabilities of number of exiting groups in a full random slot

Length of Ramp

Based on the exiting processes shown above, the length of ramp for exiting is derived. Let $L_{ex}$ be the length of exit ramp, $n_{lc}$ be the number of exiting groups, and $t_s$ be the time to adjust space. We assume the first exiting group will finish adjusting space upon arriving an exit. The length of ramp for $n_{lc}$ exiting groups is

$$L_{ex} = V(t_i + (n_{lc} - 1)(t_i + 2t_s(V, a, L_{inter})))$$

(4.1.6)

$$t_s = \frac{-V + \sqrt{V^2 + aL_{inter}}}{a/2}$$

(4.1.7)
For simplicity, the numeric value of \( t \) is rounded up to an integer, 2 seconds. The length of ramp for one exiting group is 120 meters and increments 240 meters per exiting groups.

Hence, an exit capable of serving 3 exiting groups must have a ramp of 600 meters and some vehicles still fail to exit and cause a reduction of throughput. Furthermore, in addition to the space for exiting processes, an exit needs other space for deceleration or storing vehicles. The space is minimized by allowing only one exiting group per slot passing an exit. Obviously, the rule of random slot cannot meet the requirement, which is achieved in the next section by developing slot assignment rules.

### 4.2 Slot Assignment Rules

In the random slot rule, release rates are maximized because vehicles are released without any restriction but slot capacity. However, this causes the requirement of a long ramp distance for vehicles to exit. The basic method to reduce this requirement is to allow only one exiting group in a slot passing an exit. This is achieved by arranging vehicles into special patterns. These patterns are formed and maintained at time of releasing vehicles, because vehicles on a one-lane highway cannot switch their positions.

In this section, we propose five slot assignment rules. They all reduce the number of exiting groups per slot to one. However, the system throughput can decrease, because more constraints are implemented on the release of vehicles from entrances. Hence, capacities of an entrance under varied slot assignment rules are compared. The capacity of an entrance is represented by the mean release rate derived by assuming that there is no shortage of vehicles to be released. The mean release rate is derived from the distribution of number of vehicles released for each slot assignment rule. The inputs are the distribution of destinations of vehicles at
entrances, the distribution of destinations of vehicles in a slot, and the distribution of number of vehicles in a slot. Vehicles are assumed to be First-Come-First-Serve.

Slot Assignment Rules

We proposed five slot assignment rules attaining the objective minimizing space for vehicles to exit. They are described in detail as follow:

1. Sorted Slot (SS)

In a slot, vehicles destined to farther downstream exits will stay ahead of those destined to nearer downstream exits. A slot of this fashion is called a sorted slot. In this rule, exiting vehicles will always stay at the rear of a slot while a group of released vehicles will join a slot at a position without breaking the original sequence in the slot.

The joining position is determined by the destination of the first vehicle at an entrance. For example, an upcoming slot has 3 vehicles destined to exit numbers 3, 7, and 9 and the first vehicle in queue is destined to exit number 6. Then the released group will join the slot between exit numbers 3 and 7. If the first vehicle is destined to exit number 7, then positions between exit numbers 3 and 7 and between exit numbers 7 and 9 are feasible. Under this circumstance, the released group will be formed such that more waiting vehicles can be released and join the corresponding position. To implement this rule, a slot has to be designed to allow vehicles that can join at any position.

2. End-Join Sorted Slot (EJSS)

This rule is a variation of Sorted Slot. A group of released vehicles can only join at the rear or front of a slot. The destination of the first vehicle in queue has to beyond the range of destinations in a slot to get released. The length of a slot is shorter than that in SS because of the limitation of joining positions. Specifically, when a group of released vehicles can only join at both ends of a slot, the platoon
already in a slot does not have to separate. At the time that a new group of vehicles joins a slot, there are two platoons, which need two interplatoon distances. The length of a slot can be reduced by one $L_{\text{inter}}$ compared to Sorted Slot

**Release Improvement Mechanism (RIM)**

Before presenting the following slot assignment rules, we propose a mechanism applied at entrances to increase the number of vehicles released without breaking the First-Come-First-Serve rule. In slot assignment rules, released vehicles are required in certain patterns of destinations. The mechanism can arrange vehicles into those patterns. However, this requires at lease two lanes at an entrance.

Hall and Chin (2002) proposed a similar mechanism, in which vehicles wait on two entrance lanes and are sorted into lanes where a certain pattern of destinations is formed. When there are more lanes, vehicles have more choices to join a lane. The detail is stated in Section 2.4. Figures 4.2.1-4.2.4 show the process that vehicles are sorted at an entrance through RIM.

In Figure 4.2.1, we show the use of lanes at an entrance. One lane is used for storing vehicles and the other is used to sort vehicles. Vehicles on the waiting lane (WL) are chosen into the release Lane (RL) while forming a platoon with sorted destinations. In Figure 4.2.2, a box marked with a number represents a vehicle and its exit number. The order of exit numbers on WL is 3, 7, 5, and 9. We assume that exit numbers 3 to 7 is the range of this release. Hence, the vehicle with exit number 7 will be directed into RL first and followed by vehicles with exit numbers 5 and 3. The vehicle with exit number 9 will be used to identify the range of the next release based on the order of exit numbers in the next coming slot. In Figure 4.2.3, vehicles already formed a sorted platoon are waiting to be released. In Figure 4.2.4, the sorted platoon is released and vehicles are sorted into RL while vehicles enter into WL. RIM can be applied at an entrance with multiple lanes. For example, three lanes at an
entrance, the middle lane can be used as RL and others as WLs. With the support of RIM, the following slot assignment rules are proposed.

3. Sorted Slot with Release Improvement Mechanism (SSRIM)
   This rule is the SS complemented the Release Improvement Mechanism to increase release rates of entrances.

4. End-Join Sorted Slot with Release Improvement Mechanism (EJSSRIM)
   This rule is the EJSS complemented the Release Improvement Mechanism to increase release rates of entrances.
5. Grouped Slot with Release Improvement Mechanism (GSRIM)

In this rule, vehicles with the same destinations are contiguous in a slot. Hence, a slot has one exiting group. A slot with this fashion of destination is called a grouped slot. The lengths of a sorted slot and a grouped slot are the same. However, other than a sorted slot, an exiting group in a grouped slot may appear at any position. Hence, the RIM works differently. Vehicles in queue with exit numbers within the range of this release are directed into RL sequentially, one group of the same exit number at a time. The range of a release is determined by the destination of the first vehicle and destinations in a slot. For example, a slot has 3 vehicles with exit numbers 3, 7, and 5. The first vehicle in queue is destined to exit number 4. Then, the range of this release is all exit numbers except 5 and vehicles join the position between exit numbers 3 and 7.

Derivation of distributions of number of vehicles released

For each slot assignment rule, the capacity of an entrance is derived from the distribution of destinations of vehicles in a slot, the distribution of destinations of vehicles in queue, and the distribution of number of vehicles in a slot, and we assume there is no shortage of vehicles in queue to be released. To be fair, vehicles in queue are served First-Come-First-Served. Note that the destination of a vehicle is represented by the exit number the vehicle is destined to.

The basic rational to derive the probability that \( r \) vehicles are released is that, given the destination of the first vehicle in queue, we first consider the conditions of a slot to allow this release. This includes destinations and the available space in the slot. Then, the following \((r-1)\) vehicles can be released if they are in a certain pattern of destinations required by the applied slot assignment rule and the slot has enough space to admit. When a slot has more than \( r \) empty space, a condition that the \((r+1)\)th vehicle cannot be released is accounted. Due to complexity, special cases are generally considered first and a recursive approach is often taken.
The probability of a vehicle with destination of \( d_i \) in a slot is denoted by \( D_i \). The probability of \( j \) vehicles in a slot is denoted by \( N_j \). The probability of a waiting vehicle with destination of \( d_i \) is denoted by \( p_i \). \( U \) denotes the last exit of a highway.

- **SS**

  In this rule, vehicles in a slot are sorted in a non-increasing order of their destinations. To keep vehicles sorted, vehicles released from an entrance also need to be sorted. Hence, the range of destinations of a release is determined by the destination of the first vehicle in queue as the upper bound, and the lower bound is the greatest destination in a slot but less than the upper bound. Since released vehicles can join at any position of a slot, the first vehicle at an entrance can always be released as long as the passing slot is not full. The following vehicle has to be within the range to be released. Once a following vehicle is eligible to be released, the upper bound is replaced by the destination of the vehicle, while the lower bound keeps the same. For example, a slot has 4 vehicles with destinations of 2, 5, 10, and 12. The destination of the first vehicle in queue is 9. Hence, the bounds are (9,5) for this release. If the destination of the second vehicle in queue is 7, then the bounds become (7,5) and so on. The process will continue until one following vehicle in queue has a destination out of bounds or a slot has no more space to admit vehicles.

  To derive the probability of \( r \) vehicles being released, we use the following steps.

  1. Given the probability of a range of destinations for a release.
  2. Calculate the probability of a non-increasing sequence of \( r \) destinations within the range defined by step 1.
  3. Sum up all results from steps 1 and 2.

  Let \( S \) be number of vehicles a slot can admit. If there are \( r \) vehicles released, two situations are to be considered.

  1. \( S > r \): the destination of the \( (r+1)^{th} \) vehicle is out of bounds.
  2. \( S = r \): the destination of the \( (r+1)^{th} \) vehicle does not matter.
Let $d_u$ be the upper-bound destination of a release and $d_l$ be the lower-bound destination. In the situation of $S > r$, the probability of $(d_u, d_l)$ is

$$p(u, l, r) = p_u \begin{cases} \sum_{n=0}^{N_{max} - r - 1} N_n \left( \sum_{i=2u}^{l} D_i \right)^n, & u = 1 or u \leq l \\ \sum_{n=1}^{N_{max} - r - 1} (1 - (1 - D_i)^n) N_n, & u = l + 1 \end{cases} (4.2.1)$$

In the situation of $S = r$, the probability of $(d_u, d_l)$ is

$$\bar{p}(u, l, r) = p_u \begin{cases} N_{max}^{-r} \left( \sum_{i=2u}^{l} D_i \right)^n, & u = 1 or u \leq l \\ N_{max}^{-r} \left( (1 - \sum_{i=r+1}^{u} D_i)^{N_{max} - r} \right), & u = l + 1 \end{cases} (4.2.2)$$

The situation that all vehicles in a slot have destination numbers greater than that of the first vehicle in queue is included in $u \leq l$. Both formulas are valid for $r < N_{max}$. When the release range of $(d_u, d_l)$ is determined, the following question is how many possible ways for $r-1$ vehicles to form as a valid group being released.

Let $q(u, l, r)$ be the probability of the upper bound of $d_u$, the lower bound of $d_l$, and the release number of $r$. We enumerate $r$ numbers between $u$ and $l$ in a non-increasing order and sum up probabilities.

In the situation of $S = r$, the following formula can be used to get the probabilities of enumerations of $r-1$ vehicles released.

$$q(u, l, r-1) = q(u-1, l, r-1) + p_u q(u, l, r-2), \forall r \geq 2 (4.2.3)$$
\[ q(m, m-1, n) = \sum_{i=0}^{n} p_m^{-i} p_{m-1}^i \] \hspace{1cm} (4.2.4)

\[ q(u, l, 1) = \sum_{i=0}^{u} p_i \] \hspace{1cm} (4.2.5)

\[ q(u, l, 0) = 1 \] \hspace{1cm} (4.2.6)

The formula (4.2.6) does not mean that the probability of no release is one. It is simply the basis for the recursive computation.

In the situation of \( S > r \), the destination of the \((r+1)^{\text{th}}\) vehicle is considered as a stopping criteria of release. The destination of the \(r^{\text{th}}\) vehicle can be any number within the range of \((d_u, d_l)\). If it is \(d_k\), then the destination of the \((r+1)^{\text{th}}\) vehicle has to be one outside the range of \((d_k, d_l)\) to stop adding more vehicles and make the previous \(r\) vehicles released. The probability of enumerations of \(r-1\) vehicles, with the destination of the \(r^{\text{th}}\) vehicle as \(d_k\) and the release range of \((d_u, d_l)\), is \(q(u, k, r-2) p_k\). Therefore, the probability of enumerations of \(r-1\) vehicles being released is

\[ \sum_{k=l}^{u} q(u, k, r-2) p_k (1-\sum_{i=l}^{k} p_i) \forall r \geq 2 \] \hspace{1cm} (4.2.7)

Hence, the probability of \(r\) vehicles released given the release range of \((d_u, d_l)\) is,

\[ p(u, l, r) \sum_{k=l}^{u} q(u, k, r-2) p_k (1-\sum_{i=l}^{k} p_i) + \bar{p}(u, l, r) q(u, l, r-1) \forall r \geq 2 \] \hspace{1cm} (4.2.8)

By summing up all possible release ranges, the probability of \(r\) vehicles released can be obtained. Since the above formulas are only valid for \( r \geq 2 \), we need to consider other cases.
1. The case of \( r = 0 \):
   The probability is equal to the probability that a slot is full.

2. The case of \( r = N_{\max} \):
   For the maximum number of vehicles in a release, a slot has to be empty and
   \( r \) vehicles in queue are in non-increasing order of destinations.

   \[ N_{0q}(U,1,N_{\max}) \]  \hspace{1cm} (4.2.9)

3. The case of \( r = 1 \):
   The situations to be considered also are \( S=r \) and \( S>r \).

   \[ N_{N_{\max}-1} + \sum_{u=1}^{U} \sum_{l=1}^{U} p(u,l,1)(1-\sum_{i=1}^{l} p_i) \]  \hspace{1cm} (4.2.10)

   EJSS
   This rule is designed to reduce the space reserved in a slot for lane change
   while keeping features of a sorted slot. By doing this, the throughput increases.
   However, the operation that vehicles can join only at ends of a slot becomes more
   restricted for releasing vehicles.

   A release exists in the following two cases.
   1. Rear-End Join: Destinations of all vehicles in a slot are longer than or equal
      to the destination of the first vehicle in queue. Vehicles join at the rear of a
      slot.
   2. Front-End Join: Destinations of all vehicles in a slot are closer or equal to the
      destination of the first vehicle in queue. Vehicles join at the front of a slot.

   The number of vehicles released also depends on the available space in a slot
   and the sequence of destinations of vehicles in queue. Two special cases are when
slots are full or empty. Both cases are the same as those in SS. General cases are described as follows:

The probability of no vehicle being released is when the destination of the first vehicle in queue is within the range of destinations of vehicles in a slot. The destination of the first vehicle cannot be the first or the last exits and a slot needs at least two vehicles to form a range of destinations.

\[ N_{\text{max}} + \sum_{u=2}^{U-1} p_u \sum_{n=2}^{N_{\text{max}}-1} N_n \sum_{i=1}^{U-1} \sum_{i=1}^{U-1} \left( \left( \sum_{i=1}^{U} D_i \right)^n - \left( \sum_{i=1}^{U-1} D_i \right)^n - \left( \sum_{i=1}^{U-1} D_i \right)^n + \left( \sum_{i=1}^{U-1} D_i \right)^n \right) \] (4.2.11)

The probability of one vehicle being released consists of two cases:

- Rear-end join:

\[ \sum_{u=1}^{U-1} p_u \left( \sum_{n=1}^{N_{\text{max}}-2} N_n \left( \sum_{i=1}^{U-1} D_i \right)^n + N_{\text{max}} \sum_{i=1}^{U-1} \sum_{i=1}^{U-1} \left( \sum_{i=1}^{U-1} D_i \right)^n \right) \] (4.2.12)

- Front-end join:

\[ \sum_{u=2}^{U} p_u \left( \left( \sum_{n=1}^{N_{\text{max}}-2} N_n \sum_{i=1}^{U} D_i \right)^n \right) \left( 1 - \sum_{i=1}^{U-1} p_i \right) \left( \sum_{i=1}^{U-1} \sum_{i=1}^{U-1} \left( \sum_{i=1}^{U-1} D_i \right)^n \right) \left( \sum_{i=1}^{U-1} \sum_{i=1}^{U-1} \left( \sum_{i=1}^{U-1} D_i \right)^n \right) \] (4.2.13)

There are repeated calculation in the above two formula when vehicles in a slot and vehicles from an entrance have the same destinations. There are also invalid elements in Front-End Join when all vehicles destined to the same destinations and a slot still has extra space. The stopping criteria is not valid. They are listed below.

\[ \sum_{u=2}^{U} p_u \left( \sum_{n=1}^{N_{\text{max}}-2} N_n D_u^a (1 - p_u) \right) - N_{\text{max}} \sum_{u=2}^{U-1} p_u D_u^a N_{\text{max}} \] (4.2.14)
The probability of $r$ vehicles released is, $2 \leq r \leq N_{\text{max}} - 1$,

- $S=r$
  
  - Rear-end join:

\[
\sum_{u=1}^{U} p_u q(u, 1, r-1) N_{r_{\text{max}}-r}^{N_{r_{\text{max}}-r}} (\sum_{i=1}^{U} D_i)^{N_{r_{\text{max}}-r}}
\]

(4.2.15)

- Front-end join:

\[
\sum_{u=2}^{U} p_u N_{r_{\text{max}}-r}^{N_{r_{\text{max}}-r}} (\sum_{i=1}^{u} q(u, l, r-1)
\]

\[
((\sum_{i=1}^{l} D_i)^{N_{r_{\text{max}}-r}} - (\sum_{i=1}^{l-1} D_i)^{N_{r_{\text{max}}-r}} - q(u, u, r-1) D_u^{N_{r_{\text{max}}-r}})
\]

(4.2.16)

The last term in the parenthesis is the repeated term.

- $S>r$
  
  - Rear-end join:

\[
\sum_{u=1}^{U} p_u \sum_{l=1}^{u} q(u, l, r-2) p_l (1 - \sum_{i=1}^{l} p_i) \sum_{n=1}^{N_{r_{\text{max}}-1}} N_n^{N_{r_{\text{max}}-1}} (\sum_{i=1}^{U} D_i)^n
\]

(4.2.17)

- Front-end join (The last term is the invalid term):

\[
\sum_{u=2}^{U} p_u \left( \sum_{l=1}^{u} \sum_{n=1}^{N_{r_{\text{max}}-r-1}} N_n^{N_{r_{\text{max}}-r-1}} (\sum_{i=1}^{l} D_i)^n - (\sum_{i=1}^{l-1} D_i)^n \right) \sum_{k=1}^{u} q(u, k, r-2) p_k (1 - \sum_{i=1}^{k} p_i)
\]

\[-q(u, u, r-2) p_u (1 - p_u) \sum_{n=1}^{N_{r_{\text{max}}-r-1}} D_u^n
\]

(4.2.18)
SSRIM

In this rule, vehicles in queue can be released if their destinations are within the release range determined by the first vehicle. However, the identification of release range differs from Sorted Slot. The upper bound destination of a release is the destination, in a slot, greater then or equal to the destination of the first vehicle and the lower bound is smaller than or equal to the destination of the first vehicle. For example, the destinations in a slot are 3, 7, and 9 and the destination of the first vehicle is 5. Then, the release range is 3 to 7. Note: vehicles to be released are not sorted initially but will be sorted by RIM before being released. Hence, RIM provides a relief of the constraint of sorted vehicles in queue.

The probability of \( r \) vehicles being released is,

\[
\sum_{u=1}^{U} \sum_{l_1=1}^{u} \sum_{l_2=u}^{U} \left( \sum_{n=2}^{N_{\text{max}}-r-1} N_n D_{l_1,l_2,n} + N_{N_{\text{max}}-r} D_{l_1,l_2,N_{\text{max}}-r} \right) \forall r > 1 \quad (4.2.19)
\]

Where \( D_{l_1,l_2,n} \) denotes the probability that a slot has \( n \) vehicles, \( n > 1 \), and the release range of \((l_1, l_2)\) and is defined as follows:

\[
D_{l_1,l_2,n} = \left( (1 - \sum_{i=l+1}^{l_1-1} D_i)^n - (1 - \sum_{i=l}^{l_1-1} D_i)^n - (1 - \sum_{i=l+1}^{l_2} D_i)^n + (1 - \sum_{i=l}^{l_2} D_i)^n \right) \quad (4.2.20)
\]

\[
D_{k,k,n} = 0 \quad (4.2.21)
\]

\[
D_{k-1,k-1,n} = 1 - (1 - D_k)^n - (1 - D_{k-1})^n + (1 - D_k - D_{k-1})^n \quad (4.2.22)
\]

The invalid terms appear when the stopping criteria are not appropriate. For example, if \( l_1 = u \), then the stopping criteria of \( (1 - \sum_{i=l_1}^{l_2} p_i) \) is not correct. We should...
consider all possible $l_j<l_1$ and use the stopping criterion of $(1-\sum_{i=1}^{l_j} p_i)$. The corrective step is also taken on $l_i=\neq 1$ and $l_i=\neq U$. Repeated items are also identified. When $S=r$, $p_u^r$ repeats in cases of $(l_1,l_2)=(A,u)$ and $(l_1,l_2)=(u,B)$. The above correction also produces repeated items in cases of $(l_1,l_2,l_3)=(A,u,B)$ and $(l_2,l_1,l_2)=(A,u,B)$. One of them should be removed.

Special cases in the rule are:

1. $r=0$: the probability of no release is equal to the probability of a full slot.
2. $r=N_{\text{max}}$: the probability of maximal release is equal to that of an empty slot.
3. $r=N_{\text{max}}-1$: this arises when a slot has 1 vehicle whose destination is the only division for two release ranges. There is no constraint of release if the vehicle in a slot is destined to 1 or U. The last term is subtracted because of repetition.

\[
N_i((D_1 + D_u) + \sum_{i=2}^{U-1} D_i((\sum_{j=1}^{i} p_j)^{N_{\text{max}}-1} + (\sum_{j=1}^{i} p_j)^{N_{\text{max}}-1} - p_i^{N_{\text{max}}-1}))(4.2.23)
\]

4. One vehicle in a slot for $r=1$ to $N_{\text{max}}-2$: the case that slots have one vehicle is accounted here because it makes the whole derivation clear and we can easily understand the presence of invalid terms.

\[
\sum_{i=2}^{U-1} D_i((\sum_{j=1}^{i} p_j)^r(1-\sum_{j=1}^{i} p_j) + (\sum_{j=1}^{U-1} p_j)^r(1-\sum_{j=1}^{U-1} p_j) - p_i^r(1-p_i))(4.2.24)
\]

- **EJSSRIM**

In this rule, RIM helps to increase the number of vehicles released. The probability of no vehicle being released is the same as the case without RIM. The probability of maximal number of release is equal to the probability of an empty slot.
The probability of \( r \) vehicles to be released is, \( 1 \leq r < N_{\text{max}} \),

- **Rear-end join:**

\[
\sum_{u=1}^{U} \left( \sum_{j=u}^{U} p_j \right)^r \left( \sum_{n=1}^{N_{\text{max}}-r+1} N_n \left( \sum_{i=u}^{U} D_i \right)^n - \left( \sum_{i=u+1}^{U} D_i \right)^n \right) \left( 1 - \sum_{j=u}^{U} p_j \right) + N_{N_{\text{max}}-r} \left( \left( \sum_{i=1}^{U} D_i \right)^{N_{\text{max}}-r} - \left( \sum_{i=u+1}^{U} D_i \right)^{N_{\text{max}}-r} \right) \tag{4.2.25}
\]

- **Front-end join:**

\[
\sum_{u=1}^{U} \left( \sum_{j=u}^{U} p_j \right)^r \left( \sum_{n=1}^{N_{\text{max}}-r+1} N_n \left( \sum_{i=1}^{u-1} D_i \right)^n - \left( \sum_{i=1}^{u-1} D_i \right)^n \right) \left( 1 - \sum_{j=u}^{U} p_j \right) + N_{N_{\text{max}}-r} \left( \left( \sum_{i=1}^{U} D_i \right)^{N_{\text{max}}-r} - \left( \sum_{i=1}^{u-1} D_i \right)^{N_{\text{max}}-r} \right) \tag{4.2.26}
\]

\[
\sum_{u=1}^{U} p_u^r \left( N_{N_{\text{max}}-r} D_u^{N_{\text{max}}-r} + \sum_{n=1}^{N_{\text{max}}-r-1} N_n D_u^n \left( 1 - p_u \right) \right) \tag{4.2.27}
\]

- **GSRIM**

In this rule, vehicles of same destinations will stay contiguous in a slot. Because there may be more than one vehicle with a destination, a destination in a grouped slot is called a destination group like an exiting group. Destination groups in a slot can be in any order. Hence, a slot of \( n \) destination groups has \( n! \) possible sequences. We assume they have the same probabilities.

Let \( SD_e \) be the set of all downstream exits of entrance \( e \) and \( SDS \) be the set of destinations in a slot. The set of valid destinations of a release, \( SVD_e \), is \( SD_e \) minus \( SDS \) plus two destinations, in \( SDS \), beside a joining position. Other destinations belong to the set of invalid destinations denoted by \( SID_e \). \( SD_e = SVD_e + SID_e \). The first vehicle in queue having destination in \( SDS \) will decide the join position and \( SID_e \). Then, the process continues to either further reduce \( SID_e \) when a following
vehicle in queue has another destination but adjacent to the first one in SDS or stop once the first vehicle of destination in SID is found. It also stops when a slot has no more space to admit. All vehicles chosen will be released through RIM to meet the rule. For example: there are 5 downstream exits. Hence, SDc={1,2,3,4,5} and SDS={2,4,5}. If the sequence of vehicle destinations in queue is (2,1,3,2,5,4,…), then SVDc={1,2,3,4} and SIDc={5} and the first four vehicles can be released in the order of (2,2,1,3).

The derivation in this rule is quite complicated because the stopping criteria vary from case to case. We first list the probability formula of r vehicles released by cases distinguished by stopping criteria provided number of vehicles and destination groups in a slot. Then, we derive the probability that a slot has certain number of vehicles and certain destination groups. Finally, the probability of r vehicles released can be derived by combining previous two probabilities.

Suppose we know the first vehicle in queue having destination in SDS is destined to d_i and d_j and d_k are destinations of vehicles in the front and back of the vehicle, respectively. The SDS is represented by (d_1, d_2, ..., d_n). If r vehicles are released, the following conditions have to be taken into account.

Case 1: The available space in a slot is equal to r, i.e. S=r. In this case, we don’t need SIDc as the stopping criteria. The probability is

\[
(P_{SDS}^r + \sum_{i=1}^{n} \sum_{j=1}^{i} P_{SDS'}^{j-1} p_i (P_{SDS'} + p_j)^{r-j})
\]

\[
+ \sum_{i=1}^{n} \sum_{j=1}^{i} \sum_{l=1}^{j} P_{SDS'}^{l-1} p_i \sum_{j=l+1}^{n} (P_{SDS'} + p_i)^{l-1-j} p_j (P_{SDS'} + p_i + p_j)^{r-l})
\]

\[
P_{SDS} = \sum_{d_i \in SDS} p_i \quad (4.2.29)
\]

\[
P_{SDS'} = 1 - P_{SDS} \quad (4.2.30)
\]
Case 2: One released vehicle has a destination in $SDS$ and space in a slot is greater than $r$. Two possible situations may happen and cause different $SID_c$.

(A) If $d_i$ is at the end positions of a sequence in slot, the probability is

$$\sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{l=1}^{r} P_{SDS}^{l-1} p_i (P_{SDS} + p_i)^{r-l} (P_{SDS} - p_i - p_j)$$

(B) Otherwise, the probability is

$$\sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} \sum_{l=1}^{r} P_{SDS}^{l-1} p_i (P_{SDS} + p_i)^{r-l} (P_{SDS} - p_i - p_j - p_k)$$

Case 3: Two released vehicles have destinations in $SDS$. The available space in a slot is greater than $r$. The probability is

$$\sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{l=1}^{r} \sum_{k=1}^{r-l} P_{SDS}^{l-1} p_i (P_{SDS} + p_i)^{r-l} p_j (P_{SDS} + p_i + p_j)^{r-l} (P_{SDS} - p_i - p_j)$$

Since every sequence of destinations in $SDS$ is equally likely, the probability for each case mentioned above is shown below provided that released destinations are given.

$$\begin{cases} \frac{2}{n}, & \text{the last item in case 1 and case 3} \\ \frac{2}{n(n-1)}, & \text{in case 2(A)} \\ \frac{1}{n(n-1)}, & \text{in case 2(B)} \end{cases}$$
To derive the probability that a slot has a $SDS$ of $(d_1, d_2, \ldots, d_n)$ and $N$ vehicles, the following three steps are used.

1. List all possible combinations of number of vehicles destined to each $d_i$ in $SDS$. For example, a slot has 4 destination groups and 6 vehicles, then the possible sets of numbers of vehicles are $(1,1,2,2)$ and $(1,1,1,3)$. The following algorithm is used to derive all combinations of vehicle numbers to hold the “Exactness of $SDS$. “ The Exactness of $SDS$ means that at least one vehicle is destined to each $d_i$ in $SDS$. Let $n$ be number of destination groups in a slot. The following formula is used to get each possible list of number of vehicles.

$$g_k(N,n) = \sum_{i=1}^{\min(N-n,n)} g_{k+1}(N-n,i)$$ \hspace{1cm} (4.2.35)

The $g_k(N,n)$ can be translated into a list of number of vehicles. If $n=1$, all vehicles would be in a group of the same destination. If two arguments are of the same value, i.e. $N=n$, then each vehicle is assigned one distinct destination. The $k$ is the index number starting from 0 and used to identify the number of iterations. The initial set of vehicle numbers for $g_0(N,n)$ is a list of $n$ entries with all 1’s. When $k$ increases by 1, the set will add its first $n$ entries by 1. The formula recursively implements until $n=1$ or $N=n$.

For example, the number of vehicles is 8 and the number of destination groups in a slot is 4. The initial set of vehicle numbers is $(1,1,1,1)$. By the formula above, there are four cases to be considered.

A.) $g_1(4,1)$: the remaining vehicles would be in a destination group. Hence, the set of vehicle number would be $(5,1,1,1)$.

B.) $g_1(4,2)$: two sub-cases are considered. The initial set is changed to $(2,2,1,1)$.
b1.) $g_2(2,1)$: the final set becomes $(4,2,1,1)$.

b2.) $g_2(2,2)$: the final set becomes $(3,3,1,1)$.

C.) $g_1(4,3)$: by formula, it is equal to $g_3(1,1)$ with the set of $(2,2,2,1)$.

Hence, the final set is $(3,2,2,1)$.

D.) $g_1(4,4)$: the set is $(2,2,2,2)$.

2. For each list from the step 1, we calculate possible combinations of destinations. For example, there is a list of (a, b, c), this means the slot has $SDS=\{d_1, d_2, d_3\}$ and one group has a vehicles, one has b vehicles, and the other has c vehicles. However, each group can have a, b, or c vehicles. Hence, in the above example, (1,1,2,2) can have $4!/(2!*2!)=6$ and (1,1,1,3) has $4!/3!=4$ combinations of destinations.

3. For each result from step 2, we calculate all possible combinations of vehicles matching it. In this step, we simply remove the repetition of vehicles contributing to same lists of destinations. In the example of step 1, (1,1,2,2): $6!/(2!*2!)$ and (1,1,1,3): $6!/3!$.

To get the probability of $r$ vehicles being released, we need to sum up all possible $SDS$ with no more than $N_{\text{max}}-r$ vehicles. Some special situations listed below reduce the time of computation.

1. There is no release when the passing slot is full.
2. The first vehicle can always be released if the passing slot is not full.
3. When the number of destination groups is less than 3, vehicles can be released until the passing slot is full. In the case, there would be no SID as the stopping criterion. And through RIM, vehicles can be released as many as possible.
4. The number of vehicles in a slot is at least equal to the number of destination groups in the slot. Then the calculation can begin from 3 destination groups.
to \( N_{\text{max}} - 2 \) destination groups, if the number of downstream exits is greater than or equal to \( N_{\text{max}} - 2 \). Since \( N_{\text{max}} - 1 \) destination groups means only one vehicle space in slot is available and will be occupied by the first vehicle in queue. The number of vehicles in a slot requiring consideration is also from 3 to \( N_{\text{max}} - 2 \).

Experiments

The system is an entrance with 10 downstream exits. All input probability distributions (i.e. destination of waiting vehicles, destination of vehicles in slot, and number of vehicles in a slot) are uniform. Parameters used are stated in Section 3.4. Figure 4.2.5 shows the probability distributions of number of vehicles released.

The distribution of number of vehicles released under Random Slot is uniform because the number of vehicles in a slot is uniformly distributed. Random Slot produces the maximum mean release rate because passing slots are always filled. GSRIM is close to Random Slot because a slot having less than 2 vehicles can
always be filled up and restriction on release is minimal among other rules. SS can release one vehicle with probability of 65%, because a vehicle can join at any point of a slot; the first vehicle in queue will only be rejected when a slot is full with probability of 10%. SS and EJSS have similar performance when slots have less than 5 vehicles. EJSS has a very high probability of no release, same as EJSSRIM. However, when a release is feasible, the number of vehicles released depends largely on the destination pattern of vehicles in queue, especially when slots have few vehicles. The RIM can largely increase the number of vehicles released, especially when a slot is nearly empty. SSRIM and EJSSRIM perform similarly when more than 6 vehicles are released (i.e. less than 4 vehicles in a slot).

Table 4.2.1 shows mean release rates of slot assignment rules proposed in this section and compared to the rule of Random Slot. Obviously, a random slot has highest release rate since there is no restriction of release. Among others, the rules with RIM increase release rates by double or even more if a rule puts severe constraints on the release process, like EJSS. GSRIM performs close to Random Slot since this rule has the least constraint on release.

<table>
<thead>
<tr>
<th>Mean Release Rate (vehi/hr)</th>
<th>SS</th>
<th>SSRIM</th>
<th>EJSS</th>
<th>EJSSRIM</th>
<th>GSRIM</th>
<th>RANDOM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>525.3</td>
<td>1106.4</td>
<td>397.6</td>
<td>1026.2</td>
<td>1923.9</td>
<td>2045.5</td>
</tr>
</tbody>
</table>

Table 4.2.1 Release rates of slot assignment rules

- Heavy traffic condition

A heavy traffic condition is created by assuming that slots released from the first entrance are carrying the maximum number of vehicles. The system has one automated highway lane and 10 pairs of entrance and exit followed by 10 more exits for the last entrance to implement lane assignment rules. Parameters used are stated in Section 3.4. The origin/destination patterns used are the uniform distribution, exponential distribution with mean trip lengths of 10km and 20km, and a bell-shape distribution shown in Figure 4.2.6.
In this bell-shape distribution, exiting probabilities is low at the first three exits and increase from exit 4 to exit 6, and then drop to exit 8. This distribution is similar to that around an urban area during morning commute time.

The performance measures to compare are mean release rates of entrances, highway flow between each pair of entrance and exit, and throughput. Their calculations are stated in the following, and Figure 4.2.7 shows the system configuration.

At an entrance, vehicles are assumed to arrive in a Poisson process with mean rate \( \lambda \) and the probability of a vehicle from entrance \( i \) to exit \( j \) is denoted by \( D_{ij} \). The mean release rate of an entrance \( i \), \( \bar{r}_i \), is derived from distribution of number of vehicles released. The probability of \( k \) vehicles released is represented by \( R(k) \), then
\[ \bar{r}_i = \sum_{k=i}^{N_{\text{up}}} kR(k) \quad (4.2.36) \]

- **Exit**

At an exit \( e \), a vehicle will exit with the probability denoted by \( p_e \) and every exiting vehicle can leave successfully by application of slot assignment rules.

\[ p_e = \frac{\sum_{\text{upstream entrance}} \bar{r}_i D_{ie}}{\sum_{\text{upstream entrances}} \bar{r}_i \sum_{\text{downstream exits}} D_{ij}} \quad (4.2.37) \]

- **Highway Flow**

The highway flow is represented by the distribution of number of vehicles in a slot. In a section immediately after an entrance, the probability of \( r \) vehicles in a slot is calculated by \( i \) vehicles released from the entrance provided \((r-i)\) vehicles in a slot before passing the entrance. Note: the number of vehicles in a slot before and after a ramp is represented by \( N' \) and \( N \), respectively. Hence, the distribution after an entrance is

\[ N(r) = \sum_{i=0}^{r} R(i) N'(r-i) \quad (4.2.38) \]

and the distribution after an exit is

\[ N(i) = \sum_{k=i}^{N_{\text{up}}} N'(k) \binom{k}{i} p_e^{k-i} (1-p_e)^i \quad (4.2.39) \]
Slot assignment rules proposed are designed for ease of the exiting process, but constraining the release process. Except for a random slot, the release rate of an entrance is less than the exit rate of its immediate upstream exit, because slots are not always filled after passing an entrance. Therefore, slots have more space when passing more exits. More space in a slot increases release rates at downstream entrances. Table 4.2.2-4.2.4 shows the release rates of entrances, highway flow rates between pairs of entrance and exit, and system throughput under the uniform distribution of destination.

The system throughput is the sum of flow rates at ramps. EJSS and EJSSRIM have higher release rates at the first entrance because of shorter slots. By observing the release rate at the second entrance, GSRIM has the highest numeric value. This means that the rule can recover the capacity loss most quickly that others and EJSS is the worst in this respect. This fact also arises in other distributions of destinations. Hence, GSRIM is supposed to hold the highest highway flow than others. However, due to the high flows at the first entrance, EJSSRIM outperforms in this regard and system throughput.

<table>
<thead>
<tr>
<th>Entrance/Exit Number</th>
<th>SS</th>
<th>SSRIM</th>
<th>EJSS</th>
<th>EJSSRIM</th>
<th>GSRIM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean Release Rate</td>
<td>Mean Exit Rate</td>
<td>Mean Release Rate</td>
<td>Mean Exit Rate</td>
<td>Mean Release Rate</td>
</tr>
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<td>5320.2</td>
</tr>
<tr>
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<td>176.5</td>
<td>215.4</td>
<td>57.8</td>
</tr>
<tr>
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<td>198.9</td>
<td>226.2</td>
<td>203.0</td>
<td>226.7</td>
<td>92.0</td>
</tr>
<tr>
<td>4</td>
<td>214.8</td>
<td>238.8</td>
<td>218.4</td>
<td>239.5</td>
<td>115.9</td>
</tr>
<tr>
<td>5</td>
<td>228.4</td>
<td>253.1</td>
<td>232.1</td>
<td>254.0</td>
<td>134.8</td>
</tr>
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<td>242.1</td>
<td>269.2</td>
<td>246.1</td>
<td>270.4</td>
<td>151.4</td>
</tr>
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<td>287.6</td>
<td>261.4</td>
<td>289.1</td>
<td>167.0</td>
</tr>
<tr>
<td>8</td>
<td>272.6</td>
<td>308.6</td>
<td>278.5</td>
<td>310.5</td>
<td>182.5</td>
</tr>
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<td>332.8</td>
<td>297.8</td>
<td>335.4</td>
<td>198.7</td>
</tr>
<tr>
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<td>310.4</td>
<td>361.0</td>
<td>319.9</td>
<td>364.4</td>
<td>216.1</td>
</tr>
</tbody>
</table>

Table 4.2.2 Mean flow rates (vehicles/hour) at ramps in uniform distribution of destination
Throughput (veh/lhr) SS SSRIM EJSS EJSSRIM GSRIM

| Uniform | 9006.7 | 9067.6 | 9656.1 | 9780.2 | 9229.7 |
| EXPO(10) | 16199.7 | 18450.2 | 16457.9 | 18851.3 | 20145.5 |
| EXPO(20) | 13825.2 | 14603.0 | 13541.1 | 14356.5 | 15843.1 |
| Bell-Shape | 9450.1 | 9635.4 | 10145.7 | 10363.1 | 10064.8 |

Table 4.2.3 Throughput in uniform distribution of destination

<table>
<thead>
<tr>
<th>Between Rampss</th>
<th>SS</th>
<th>SSRIM</th>
<th>EJSS</th>
<th>EJSSRIM</th>
<th>GSRIM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4122.1</td>
<td>4122.1</td>
<td>5320.2</td>
<td>5320.2</td>
<td>4122.1</td>
</tr>
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<td>4092.5</td>
<td>5111.9</td>
<td>5113.1</td>
<td>4118.8</td>
</tr>
<tr>
<td>3</td>
<td>4071.5</td>
<td>4080.1</td>
<td>4934.9</td>
<td>4939.3</td>
<td>4118.0</td>
</tr>
<tr>
<td>4</td>
<td>4060.1</td>
<td>4071.9</td>
<td>4776.7</td>
<td>4786.4</td>
<td>4117.1</td>
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<td>4492.5</td>
<td>4519.0</td>
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<td>4398.0</td>
<td>4112.6</td>
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<td>4037.0</td>
<td>4231.1</td>
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<td>4024.2</td>
<td>4104.3</td>
<td>4171.2</td>
<td>4106.5</td>
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<td>4008.8</td>
<td>3978.4</td>
<td>4062.9</td>
<td>4101.7</td>
</tr>
</tbody>
</table>

Table 4.2.4 Highway flow rates between ramps in uniform distribution of destination

On a highway, frequent access and egress occur when vehicles have short trips. In this situation, a full slot is more likely to become an empty one when passing an exit. Hence, GSRIM is expected to perform well in terms of the release rate and the highway flow by its quick recovery feature. The release rates and highway flows in the case of the exponential distribution of destinations with mean trip length of 10km (EXPO10) are shown in Table 4.2.5 and Table 4.2.6, respectively. Interestingly, EJSS performs closely to SS compared to the case of uniform distribution, so does EJSSRIM to SSRIM. With short trip length, a huge initial flow created in EJSS is diminished after the first few exits. The restriction of release in EJSS is relieved when slots have few vehicles with long trips and vehicles, at entrances, with short trips. The above two reasons plus a short slot length in EJSS makes SS and EJSS, or SSRIM and EJSSRIM, perform closely, especially in downstream sections of the highway.
The release rate in the case of short trip length is not increasing monotonously. They increase initially then decline to the end of the system, except in EJSS. The probability to leave at the last exits is high because all vehicles remain on the highway must leave the system eventually. Hence, these vehicles dominate in downstream sections of the highway. In general, slots become less stuffed downstream. However, the accumulation of these vehicles consumes space. Therefore, the fast the accumulation, the earlier the trend of release rates drop and the peak value appears. The accumulation depends on the recovery of flow. Hence, GSRIM has the peak release rate at the entrance 3, SSRIM entrance 5, and SS and EJSSRIM entrance 7.

The highway flow in the case of short trip length is not decreasing monotonously either. The highway flow has the opposite trend as the release rate. A lower release rate implies that slots have less space to admit vehicles. Less space in slots implies a lower exit rate in the upstream exit. A large exit rate usually results in more space in a slot after passing its immediate downstream entrance. Hence, the highway flow increases when the release rate decreases. Among slot assignment rules, the earlier the release rate reaches its peak, so does the highway flow to its lowest value. The lowest highway flow in GSRIM appears in the section after the entrance 3, SSRIM entrance 4, SS and EJSSRIM entrance 5, and EJSS entrance 6.

<table>
<thead>
<tr>
<th>Entrance</th>
<th>SS</th>
<th>SSRIM</th>
<th>EJSS</th>
<th>EJSSRIM</th>
<th>GSRIM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4122.1</td>
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<td>4122.1</td>
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<td>528.1</td>
<td>641.9</td>
<td>320.7</td>
<td>337.7</td>
<td>931.8</td>
</tr>
<tr>
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<td>576.0</td>
<td>747.7</td>
<td>379.3</td>
<td>444.3</td>
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<td>427.3</td>
<td>572.6</td>
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<td>806.2</td>
<td>468.9</td>
<td>681.6</td>
<td>913.1</td>
</tr>
<tr>
<td>6</td>
<td>613.9</td>
<td>794.3</td>
<td>500.2</td>
<td>737.6</td>
<td>864.6</td>
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<td>614.7</td>
<td>771.7</td>
<td>520.9</td>
<td>751.3</td>
<td>817.4</td>
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<td>744.6</td>
<td>533.6</td>
<td>745.2</td>
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<td>686.2</td>
<td>546.8</td>
<td>721.3</td>
<td>687.0</td>
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</tbody>
</table>

Table 4.2.5 Mean Release Rates in EXPO10 distribution of destination
Table 4.2.6 Highway flow rates between ramps in EXPO10 distribution of destination

<table>
<thead>
<tr>
<th>Between Ramps</th>
<th>SS</th>
<th>SSRIM</th>
<th>EJSS</th>
<th>EJSSRIM</th>
<th>GSRIM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>4122.1</td>
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<tr>
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<td>3535.1</td>
<td>3612.4</td>
<td>3937.2</td>
</tr>
<tr>
<td>4</td>
<td>3071.6</td>
<td>3457.8</td>
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<td>3358.8</td>
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<tr>
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<tr>
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<td>2904.5</td>
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<td>4010.6</td>
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<tr>
<td>7</td>
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<td>2911.1</td>
<td>3521.2</td>
<td>4030.8</td>
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<tr>
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<td>2959.4</td>
<td>3634.1</td>
<td>4047.8</td>
</tr>
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<td>3193.4</td>
<td>3758.9</td>
<td>3031.4</td>
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<td>4061.8</td>
</tr>
<tr>
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<td>3811.9</td>
<td>3117.4</td>
<td>3846.9</td>
<td>4073.2</td>
</tr>
</tbody>
</table>

The system performance of EXPO10 compared to the case of the uniform distribution is shown in Table 4.2.7. GSRIM performs best with the system throughput increased by 118.3% and the mean highway flow rate declined by only 2.3% whereas, the system throughput increases only 70.4% and the mean highway flow rate loses 26.2% in EJSS, which is the worst slot assignment rule in this regard. A high flow rate in a highway produces a high throughput. Because it increases exit rates, a high exit rate leads to more space in slots, which, in turn, increases release rates. The increase of exit rates and release rates results in the increase of system throughput.

Table 4.2.7 Percentage of change of Highway flow rates and Throughput compared to the uniform distribution of destination.

<table>
<thead>
<tr>
<th>Percentage of change</th>
<th>SS</th>
<th>SSRIM</th>
<th>EJSS</th>
<th>EJSSRIM</th>
<th>GSRIM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Highway flow rate(-%)</td>
<td>19.3</td>
<td>9.7</td>
<td>26.2</td>
<td>17.8</td>
<td>2.3</td>
</tr>
<tr>
<td>Throughput (%)</td>
<td>79.9</td>
<td>103.5</td>
<td>70.4</td>
<td>92.7</td>
<td>118.3</td>
</tr>
</tbody>
</table>

The results of the exponential distribution of destination with mean trip length of 20km (EXPO20) are shown in Table 4.2.8 and Table 4.2.9, respectively. The system performance is between the previous two distributions. The throughput
increases 71.7% and the highway flow loses 1.1% in GSRIM whereas 40.2% and 21.3% in EJSS.

<table>
<thead>
<tr>
<th>Entrance</th>
<th>SS</th>
<th>SSRIM</th>
<th>EJSS</th>
<th>EJSSRIM</th>
<th>GSRIM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4122.1</td>
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<td>180.4</td>
<td>188.2</td>
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<td>501.5</td>
<td>230.0</td>
<td>249.7</td>
<td>648.4</td>
</tr>
<tr>
<td>4</td>
<td>468.4</td>
<td>529.8</td>
<td>260.8</td>
<td>295.8</td>
<td>642.5</td>
</tr>
<tr>
<td>5</td>
<td>480.5</td>
<td>544.4</td>
<td>287.0</td>
<td>340.1</td>
<td>632.3</td>
</tr>
<tr>
<td>6</td>
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<td>551.7</td>
<td>311.1</td>
<td>381.9</td>
<td>621.6</td>
</tr>
<tr>
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<td>493.2</td>
<td>554.7</td>
<td>333.6</td>
<td>418.8</td>
<td>611.0</td>
</tr>
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<td>555.1</td>
<td>354.8</td>
<td>449.9</td>
<td>600.5</td>
</tr>
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<td>554.1</td>
<td>375.6</td>
<td>476.1</td>
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<td>502.4</td>
<td>552.5</td>
<td>397.3</td>
<td>499.6</td>
<td>580.3</td>
</tr>
</tbody>
</table>

Table 4.2.8 Mean Release Rates in EXPO20 distribution of destination

<table>
<thead>
<tr>
<th>Between Ramps</th>
<th>SS</th>
<th>SSRIM</th>
<th>EJSS</th>
<th>EJSSRIM</th>
<th>GSRIM</th>
</tr>
</thead>
<tbody>
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</table>

Table 4.2.9 Highway flow rates between ramps in EXPO20 distribution of destination

In the bell-shape distribution (Bell-Shape), the exiting probability begins to rise at the exit 4 and tops at exit 6. Hence, the release rate begins to increase at the entrance 5 for all slot assignment rules, but only tops at the entrance 7 in GSRIM because of quick recovery of flow, 8 in SS and SSRIM, and still increases to the last entrance in EJSS and EJSSRIM. The highway flow rate drops from the exit 4 to exit
7. During this section, EJSS loses 22.8% of the highway flow while GSRIM loses only 1.3% and SS loses 10.5%. The results are shown in Table 4.2.10 and 4.2.11.

<table>
<thead>
<tr>
<th>Entrance</th>
<th>SS</th>
<th>SSRIM</th>
<th>EJSS</th>
<th>EJSSRIM</th>
<th>GSRIM</th>
</tr>
</thead>
<tbody>
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<td>4122.1</td>
<td>5320.2</td>
<td>5320.2</td>
<td>4122.1</td>
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<td>42.0</td>
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<td>403.8</td>
<td>295.6</td>
<td>338.7</td>
<td>374.5</td>
</tr>
</tbody>
</table>

Table 4.2.10 Mean Release Rates in Bell-Shape distribution of destination

<table>
<thead>
<tr>
<th>Between Ramps</th>
<th>SS</th>
<th>SSRIM</th>
<th>EJSS</th>
<th>EJSSRIM</th>
<th>GSRIM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4122.1</td>
<td>4122.1</td>
<td>5320.2</td>
<td>5320.2</td>
<td>4122.1</td>
</tr>
<tr>
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<td>4120.9</td>
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<td>5280.1</td>
<td>4121.9</td>
</tr>
<tr>
<td>3</td>
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<td>4120.8</td>
<td>5248.0</td>
<td>5248.2</td>
<td>4121.9</td>
</tr>
<tr>
<td>4</td>
<td>4120.5</td>
<td>4120.8</td>
<td>5221.1</td>
<td>5221.5</td>
<td>4121.9</td>
</tr>
<tr>
<td>5</td>
<td>4087.1</td>
<td>4093.1</td>
<td>5025.7</td>
<td>5027.8</td>
<td>4116.4</td>
</tr>
<tr>
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<td>3992.7</td>
<td>4635.6</td>
<td>4645.1</td>
<td>4097.3</td>
</tr>
<tr>
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<td>3758.1</td>
<td>4032.0</td>
<td>4076.1</td>
<td>4069.3</td>
</tr>
<tr>
<td>8</td>
<td>3620.7</td>
<td>3727.8</td>
<td>3733.1</td>
<td>3821.0</td>
<td>4082.2</td>
</tr>
<tr>
<td>9</td>
<td>3717.2</td>
<td>3833.4</td>
<td>3695.8</td>
<td>3820.8</td>
<td>4101.2</td>
</tr>
<tr>
<td>10</td>
<td>3767.1</td>
<td>3883.1</td>
<td>3644.7</td>
<td>3802.4</td>
<td>4098.1</td>
</tr>
</tbody>
</table>

Table 4.2.11 Highway flow rates between ramps in a Bell-Shape distribution of destination

This system is in a heavy traffic condition, in which the RIM increases the release rate only slightly. However, in a short trip-length distribution of destination, the increase becomes large because a slot is more likely to become empty after passing an exit. GSRIM is the best slot assignment rule in terms of the release rate and the highway flow rate. However, without the RIM, SS is the best rule. In terms of the system throughput, EJSS and EJSSRIM perform very well because they can create initial large flow into a highway owing to a short slot length. However, the
selection of rules on an entrance also depends on a very important factor: the ramp spacing, which is discussed in the next section.

4.3 Ramp Spacing

A ramp and its adjacent ramps should be separated at a distance such that vehicle maneuvers can be executed successfully. If a space between ramps is limited and vehicles from the upstream ramp fail to complete necessary maneuvers, then vehicle maneuvers at the downstream ramp may also fail. For example, at an entrance, the distance to the next ramp must be long enough for released vehicles to join slots. Or, the ramp may be blocked from releasing vehicles or vehicles in a slot may fail to exit. On the other hand, if the spacing is too long, the throughput declines because vehicles cannot access or egress a highway. Thus, an appropriate spacing between ramps optimizes system performance.

With respect to a highway configuration, capacity is limited by number of lanes, capacities of ramps, and density of ramps. If a section of a highway can support 4000 vehicles per hour and an entrance can release 1000 vehicles per hour, we want to create 4 entrances to achieve the maximum capacity of this section without considering the upstream flow. However, this ideal situation is based on no interference among these entrances. If two entrances are spaced too close such that vehicles released from the upstream entrance become obstacles for the downstream entrance to release vehicles. Then, the downstream entrance is operated under its capacity. So is the section.

Slot assignment rules differ at positions that vehicles join or leave slots. The variation results in various requirements for space. In this section, we compare the ramp spacing in each slot assignment rule proposed in section 4.2 and convert to throughput for a uniform and steady state highway system.
Maneuvering Areas

A system with one automated lane and dedicated ramps is shown in Figure 4.3.1. Five areas, labeled by bold letters and separated by vertical dashed lines, stand for 5 different maneuvering processes occurring in a one-lane AHS. The joining process is the process that vehicles laterally enter into a slot. The merging process occurs when two platoons in a slot become a single platoon. The separation process is the opposite of the merging process and the leaving process is the opposite of the joining process. The space adjustment process is the process that vehicles move to create a space at a certain position required by the vehicles released from an entrance. Generally, a space adjustment process consists of couples of separation and merging processes and an exiting process consists of couples of space adjustment and leaving processes. The calculation of maneuvering time of each process is based on the definition described in Section 3.4.

Area A: vehicles in a slot adjust a space for vehicles released from an entrance. The area contains processes of space adjustment and joining.
Area B: all vehicles in a slot merge into one platoon after the joining process.
Area C: vehicles in a slot adjust a space for the first group of exiting vehicles.
Area D: A series of leaving and space adjustment processes until all requests are satisfied or no more space is allowed for vehicles to exit.
Area E: all vehicles in a slot merge into a single platoon.

These five areas are stated in detail in the following slot assignment rules. Since maneuvering areas also depends on the joining position, it is calculated based on the worst case that the joining position is in the middle of a platoon.
• Random Slot:
  Vehicles released from an entrance will join at the tail of the platoon in a slot. Area A is minimized because no space adjustment but one joining process is required. Area B is also minimized because the space to merge platoons is one interplatoon distance. The first exiting group may be in the middle of the platoon. Thus, Area C needs to account for the formation of a single platoon of exiting vehicles, which requires two separation processes of an interplatoon distance. Area D has been discussed in section 4.2 and is the longest distance since exiting vehicles can be at any position within a platoon. Area E accounts for a merging process of two interplatoon distances.

• SS, SSRIM:
  A group of sorted vehicles can join a sorted slot at any position. Area A accounts for a separation process of two interplatoon distances if released vehicles join at middle positions, or a space adjustment process of moving back for vehicles joining at the front and one joining process. Area B accounts for two merging processes of an interplatoon distance. Area C is minimized because of only one exiting group at the tail of a platoon, which needs one separation process of an interplatoon distance. Area D is also minimized because only one leaving process is undergoing. Area E is negligible since no merging is needed.

• EJSS, EJSSRIM:
  A group of sorted vehicles joins only at both ends of the platoon in a slot. Area A needs to account for a space adjustment process and a joining process. Area B is minimized to account for only one merging process of an interplatoon distance. Area C is minimized to account for one separation process of an interplatoon distance. Area D is also minimized with one leaving process. Area E is eliminated. This rule combines most advantages of a random slot and a sorted slot. Thus the minimal ramp spacing is expected.
• **GSRIM:**

A group of vehicles may join at any position of the platoon in a slot. Area A accounts for a separation process of two interplatoon distances if released vehicles join in the middle and a joining process. Area B accounts for two merging processes of an interplatoon distance. Area C needs two separation processes of an interplatoon distance. Area D is minimized due to one exiting group in a slot. Area E accounts for a merging process of two interplatoon distances.

**Throughput Analysis:**

A section of highway has higher throughput if more ramps arise during this section. Therefore, space between ramps is reserved only for necessary maneuvers can increase the number of ramps. Specifically, as long as these five maneuvering areas are continuously located, this section can achieve the maximum throughput provided all ramps are operated at full capacity.

A one-lane AHS is operated in a steady state. Release rates of entrances are all equal and all vehicles can exit successfully. If a section of the highway has a length of $L_H$, then more entrances able to be located within the system means the system has higher throughput. We assume the capacity of this section is not a constraint for simplicity. The ramp spacing, denoted by $L_r$, is defined as the total distance that a pair of entrance and exit needs to complete five processes described above. Let $R_e$ be the mean release rate of entrances, then the throughput is equal to $\left( \frac{L_H}{L_r} \right) R_e$.

Table 4.3.1 shows the throughput comparison of slot assignment rules. The throughput is defined as the total number of vehicles served by the system hourly. In the table, the numeric value of each area is listed for each rule and release rates comes from the results of an example in Section 4.2. The length of a one-lane AHS of 30km is assumed. The rule of GSRIM performs best. However, the EJSSRIM
outperforms the others. Although it releases the smallest number of vehicles into the system, the distance between ramps is the shortest such that the largest number of releasing ports can be added to make up the deficit. However, the construction of densely spaced entrances may not be feasible, especially in urban areas. The ramp spacing in rules of Random, about one mile, and GSRIM, about a half mile, are close to the current configuration of a highway. GSRIM provides almost twice the throughput as Random and, again, becomes a promising slot assignment rule in one-lane AHS.

<table>
<thead>
<tr>
<th></th>
<th>Random</th>
<th>SSRIM</th>
<th>EJSSRIM</th>
<th>GSRIM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area A</td>
<td>150</td>
<td>270</td>
<td>210</td>
<td>270</td>
</tr>
<tr>
<td>Area B</td>
<td>60</td>
<td>120</td>
<td>60</td>
<td>120</td>
</tr>
<tr>
<td>Area C</td>
<td>120</td>
<td>60</td>
<td>60</td>
<td>120</td>
</tr>
<tr>
<td>Area D</td>
<td>1170</td>
<td>150</td>
<td>150</td>
<td>150</td>
</tr>
<tr>
<td>Area E</td>
<td>120</td>
<td>0</td>
<td>0</td>
<td>120</td>
</tr>
<tr>
<td>L_r (meter)</td>
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<td>600</td>
<td>480</td>
<td>780</td>
</tr>
<tr>
<td>Release rate (vehicles/hr)</td>
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<td>1106.4</td>
<td>1026.2</td>
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<td>Throughput (vehicles/hr)</td>
<td>37879.6</td>
<td>55320</td>
<td>64137.5</td>
<td>73996.2</td>
</tr>
</tbody>
</table>

Table 4.3.1 Comparison of throughput of L_L=30km

4.4 Conclusions

This chapter describes the applications of the moving slot model on a one-lane AHS. In the Random Slot rule, vehicles can be released to fill up a slot without regard to their destinations. Hence, under the rule, an entrance provides the maximum capacity. However, the downside of the rule is that a slot may require multiple exiting processes due to the randomness of vehicle destinations. Hence, a long ramp at exits is required. The other drawback is that the system throughput is likely restricted by capacity of entrances due to a longer ramp spacing. Slot assignment rules are, therefore, designed to minimize the ramp space at exits and reduce space between ramps.
That a slot executes only one exiting process when passing an exit minimizes the space requirement at exits. Therefore, exiting vehicles in a slot must be contiguous to form a single exiting platoon. In a one-lane highway system, vehicles cannot change their relative positions on the highway lane. Hence, this formation is implemented and maintained by releasing vehicles from entrances to join slots at certain positions.

Five slot assignment rules are proposed: SS, SSRIM, EJSS, EJSSRIM, and GSRIM. RIM is a mechanism applied on an entrance to improve the release rate by arranging sequences of vehicle destinations required by slot assignment rules without breaking the First-Come-First-Serve rule. RIM needs at least two lanes at an entrance. The selection of slot assignment rules at an entrance depends on the configuration of the entrance and the space between adjacent ramps. If the entrance has one lane, only SS and EJSS can be implemented. When there are more lanes, RIM can be executed and all slot assignment rules are available for selection.

Generally, among slot assignment rules, GSRIM provides the highest release rate because of minimal constraint on releasing vehicles, but requires the longest lane space to complete maneuvering processes because destination groups are not sorted. Vehicles in the other four rules are sorted in slots. They can be applied alternately on a highway system. Slots in SS and EJSS differ in their positions allowing vehicles to join or leave and, hence, the space requirement for vehicle maneuvers. The application of RIM depends on the number of lanes at an entrance. For example, if an entrance has two lanes and the distance to the next ramp is longer than 600 meters, we use SSRIM, or SS for a one-lane entrance. If the distance is less than 600 meters but longer than 480 meters, EJSS for single-lane entrance and EJSSRIM for multiple-lane entrances can be selected. Though, in the example, the calculation of ramp spacing is based on a homogeneous highway. The analysis of the relationship between the system throughput and ramp spacing under varied slot assignment rules can be used in more complex system definitions including detailed time and space required for various vehicle maneuvers.
We study lane change behavior of vehicles in the moving slot model applied in an AHS by categorizing lane changes with respect to their positions of initiation and completion, prioritizing them with respect to their impacts on the system throughput, and minimizing them by means of applying slot/lane assignment rules. The determination of lane assignment rules is formulated as an optimization problem and solved by a greedy method.

Vehicles changing lanes cause maneuvering areas. These areas are critical in measuring the system throughput. The minimization of maneuvering areas depends on the management of lane changes through the application of slot/lane assignment rules. Given a constant stack speed, the maneuvering areas between ramps are determined by the numbers of lane changes. We first describe four activities of lane changes in two-lane AHS and their influences on the system throughput. The maneuvering areas under the rule of Random Stack are studied. Then, we use slot assignment rules derived from the results of one-lane AHS and integrate them with lane assignment rules developed to minimize space on/between ramps. Finally, we describe similar results applicable in multi-lane AHS.

5.1 Analysis of Lane Change

A detailed lane change activity is studied in this section. They are classified by the direction of movement and prioritized by the impact on the system throughput.

5.1.1 Activities of Lane Change

In two-lane AHS, there are four lane change activities, which are identified by the positions of initiation and completion.
1. Lane Change from entrances to the right lane, also called lane 1:
   A group of vehicles conforming to the applied slot assignment rule is released into the system and joins a slot on the right lane. This is also called the entry process and has been studied in Chapter 4.

2. Lane Change from the lane 1 to the left lane, also called lane 2 (LC12):
   A LC12 is initiated when some vehicles in a released platoon are assigned to the left lane. The whole process is illustrated in Figures 5.1.1-5.1.4. The numbers represent the lanes assigned; 1 is the right lane and 2 the left lane. Boxes without numbers are vehicles already in the passing slot. The boundary of a stack is represented by a dotted rectangle. In Figure 5.1.1, three vehicles are released and the middle one is assigned to the left lane. Figure 5.1.2 shows that the middle vehicle forms a single platoon before changing lane. This consists of two consecutive processes. The leading vehicle in the released platoon separate from the middle vehicle and merges with vehicles ahead, and then, the trailing vehicle separate to keep an interplatoon distance away from the middle vehicle. Figure 5.1.3 shows the middle vehicle changes lane. The vehicle keeps the longitudinal speed and is assumed to execute an acceleration process followed by a deceleration process, both at same rate, to complete the lateral movement. Figure 5.1.4 shows all vehicles in slots merge into platoons after the lane change process.

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Figure 5.1.1 A group of vehicles assigned to different lanes

Figure 5.1.2 The lane changing vehicle forms a single platoon

Figure 5.1.3 The vehicle changes lane
3. Lane Change from lane 2 to lane 1 (LC21):

The LC21 process is similar to the LC12 except the direction is reversed.

4. Lane Change from lane 1 to exits (LC1E):

The LC1E has been studied as the exiting process in Section 4.2. By application of slot assignment rules, only one LC1E takes place when a stack passes an exit.

5.1.2 Lane Change Area

A lane change area is an area where vehicles change lanes. It is also called a maneuvering area in Section 3.3. As in Chapter 4, the lane change area between ramps determines the ramp spacing. In a two-lane AHS, the ramp spacing is determined by the total distance needed to complete four types of lane change activities listed above. Stacks execute one entry process at an entrance and one exiting process at an exit in common. Therefore, LC12 and LC21 become the major variables in the determination of lane change area.

We cannot specify which area is for which lane change activity. In reality, they may occur alternately. However, this may not be an efficient way to utilize space. In terms of throughput, these lane change processes are not equally emphasized. Therefore, we want to prioritize them to achieve better performance. When space between ramps is limited, the LC1E should be implemented first, because vehicles unable to exit directly reduce the exit rate. LC12 is executed second, because when a LC12 fails, more vehicles stay on lane 1 and the entry rates at downstream entrances are declined. LC21 is implemented last, because vehicles that fail in LC21 may succeed in downstream lane change areas, hence, no direct impact is on the throughput.
The priority is, however, not robust. A LC12 may fail because the upper slot has no space to admit vehicles. Hence, a LC21 may instead take place. By doing this, space in upper slot can be freed for the failed LC12 to use. Therefore, LC12 and LC21 may occur in alternation. However, we will always regard LC1E as the top priority. Note: we don’t need a priority rule if space is not limited.

Although we reserve sufficient space between ramps, some lane changes may still fail in current sections because of the limitation of slot capacity. If there are \( m \) vehicles requesting for LC12 and \( n \) for LC21, the upper slot has \( I \) vehicles and lower slot \( J \) vehicles, the number of vehicles that fail to change lanes is either

\[
\text{max}\{m-n-(N_{\text{max}}-I), 0\} \quad \forall m > n \quad \text{or} \quad \text{max}\{n-m-(N_{\text{max}}-J), 0\} \quad \forall n > m.
\]

If vehicles destined to the immediate downstream exit can leave early, the number of successful vehicles in LC21 can be increased, because more space is available for lane changes.

**Allocation of Lane Change Area**

Figure 5.1.5 illustrates an allocation of lane change areas. The area used for the entry process is not shown. The area of LC1E is reserved around an exit whereas the areas of LC12 and LC21 can be overlapped in the remaining space between ramps. For LC12 and LC21, both lanes have the same lane change areas because a lane change involves slots on both lanes. However, at an exit, the lane change area is on the right lane only. In the figure, the right lane has a greater lane change area than that on the left lane.

As a matter of fact, in a multi-lane AHS, the lane change area on one lane is greater than that on its left lane, because more lane changes are executed on right lanes. For example, vehicles traverse right lanes before their target lanes. Since lane change areas determine ramp spacing, and in turn the system throughout. Hence, the
lane change area on the rightmost lane determines the system throughput. The system performance of a multi-lane highway is restricted by the rightmost lane and cannot be increased by simply adding more lanes.

5.1.3 Time of Lane Change

A lane change process consists of a space adjustment process and a lateral movement. Hence, the time of a lane change is the sum of the times of two constituent processes. The following Figures show the lane change process and, for simplicity, a lane change platoon contains only one vehicle. In Figure 5.1.6, a vehicle just finished a lane change and stays in the upper slot while another vehicle is waiting for a space adjustment process to begin. This vehicle first separates from the preceding vehicles and forms a platoon with its following vehicles in Figure 5.1.7. Then, the following vehicles separate again to allow the vehicle to become a single platoon ready for a lateral movement in Figure 5.1.8. In Figure 5.1.9, the vehicle moves to the upper slot. Finally, vehicles in slots return into platoons. In Figure 5.1.7, independent vehicle maneuvers occur concurrently in both slots. Note: the longitudinal movement of vehicles is not shown among figures.

Figure 5.1.6 A lane change just completes

Figure 5.1.7 Space adjusting for the next lane change

Figure 5.1.8 Adjustment of space is completed for the lane change

Figure 5.1.9 The second lane change is completed
The same process repeats until there are no more vehicles requesting lane changes. Then vehicles in both slots form single platoons.

Before the lateral movement, a group of lane change vehicles has to become an independent platoon. Hence, one interplatoon distance of 60 meters has to be created at both sides of the platoon. Hence, the time to form a lane change platoon will be around 4.2 seconds. The lateral movement assumed to be an acceleration followed by a deceleration takes 4 seconds provided the width of lane is 8 meters. By considering the times of communication and negotiation among control systems, for simplicity, we assume 10 seconds as the total time of a lane change. We also assume that a group of vehicles has the same mechanical characteristics and a platoon is functioning as a single vehicle.

The 10-second lane change time won’t affect the analysis implemented in the research, since it is used as an input parameter to calculate ramp spacing under various operational rules. The ramp spacing is, therefore, determined by another variable: the number of lane changes. In two-lane AHS, the number of lane changes depends on slot capacity and the applied lane-slot assignment rules, which are discussed in detailed in the following section.

**5.2 Operational Rules**

In one-lane AHS, slot assignment rules are designed to minimize space requirement on ramps by sorting vehicles into certain patterns of destinations. In two-lane AHS, slots on different lanes serve different range of destinations assigned by lane assignment rules. Slot assignment rules developed for one-lane AHS are also applied on multi-lane AHS to minimize not only the space on ramp but also the ramp spacing. In the following, we first consider the rule of Random Stack, which is used to compare proposed slot/lane assignment rules. Then, a greedy methodology is developed to set up lane assignment rules so as to maximize the throughput. A case of a heavy traffic condition is experimented.
5.2.1 Random Stack Rule

A random stack means that vehicles in slots are randomly allocated regardless of their destinations. Hence, the lower slot in a stack can always be filled by vehicles at entrances. To meet the requirement of one exiting process per stack per exit, a single platoon of exiting vehicles in a stack is formed by lane change processes.

Figure 5.2.1 shows exiting vehicles in a stack. Boxes with numbers represent exiting vehicles and numbers are only used to identify exiting vehicles. An exiting vehicle may appear at any position in a random stack.

![Figure 5.2.1 A general formation of exiting vehicles in a stack](image)

To form a single platoon of exiting vehicles, these five vehicles in the above figure can only be clustered together via lane changes. Exiting vehicles in the upper slot, like number 2 and 4, can do LC21 to join one of exiting vehicles in the lower slot. However, there are two ways for exiting vehicles in the lower slot, number 1, 3, and 5, to get together. One is that vehicles execute LC12 followed by LC21. The other, shown in Figure 5.2.2, is that vehicles between two exiting vehicles execute LC12. The former can move vehicles to any positions in the lower slot but needs more lane changes than the latter. Note: the detailed separation and merging of vehicle maneuvers before a lane change are not shown. Arrows represent the moving directions of vehicles only.
Suppose that $U$ vehicles are in the upper slot and $L$ vehicles in the lower slot. Then the first operation needs $U+2(L-1)$ lane changes while the second operation needs only $U+(L-1)$. However, the first operation can be improved by moving some exiting vehicles in the lower slot to join an exiting vehicle in the upper slot, and then, move down to the lower slot together. In this operation shown in Figure 5.2.3, one exiting vehicle in the lower slot will be chosen as the seed to form an exiting platoon. The number of lane changes, therefore, is reduced to $U+(L-1)$, which is equal to that in the second operation. Vehicles can change lanes as platoons. Therefore, $U$ and $L$ represent numbers of groups of exiting vehicles in the upper slot and the lower slot, respectively.

Figure 5.2.2 One process of formation of a platoon of exiting vehicles

Suppose that $U$ vehicles are in the upper slot and $L$ vehicles in the lower slot. Then the first operation needs $U+2(L-1)$ lane changes while the second operation needs only $U+(L-1)$. However, the first operation can be improved by moving some exiting vehicles in the lower slot to join an exiting vehicle in the upper slot, and then, move down to the lower slot together. In this operation shown in Figure 5.2.3, one exiting vehicle in the lower slot will be chosen as the seed to form an exiting platoon. The number of lane changes, therefore, is reduced to $U+(L-1)$, which is equal to that in the second operation. Vehicles can change lanes as platoons. Therefore, $U$ and $L$ represent numbers of groups of exiting vehicles in the upper slot and the lower slot, respectively.
One special case is when \( U = 0 \). The number of lane changes is \((L-1)+1=L\). In this case, one exiting vehicle will stay in the lower slot as the seed and other exiting vehicles will move to the upper slot to form a platoon, and then, move back together to join the seed. The other is the case of \( L = 0 \) and the number of lane changes needed is \( U \).

There is no capacity constraint in the above operations. Because, in two-lane AHS, a slot can contain 2 platoons, each of \( N_{\text{max}} \) vehicles, or 3 platoons of \( 2N_{\text{max}}-1 \) vehicles. Two cases causing maximum vehicles in slots during operations of lane changes are discussed in the following.

Both slots in a stack are full. When the first and last vehicles in the lower slot request to exit and \( N_{\text{max}} - 2 \) vehicles in between move to the upper slot. The upper slot has 2 or 3 platoons of \( 2N_{\text{max}}-2 \) vehicles, which is acceptable. When all vehicles in the upper slot request to exit, they can move to the lower slot as the second platoon. The lower slot has 2 platoons, each of \( N_{\text{max}} \) vehicles. However, if there are still other exiting vehicles in the lower slot, then a second exiting platoon must be formed and more than one exiting processes are required.

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**Figure 5.2.3 An improved process of formation of a platoon of exiting vehicles**

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In summary, a random stack in a two-lane AHS needs U+L-1 land changes in addition to one release process and one exiting process. When more than $N_{max}$ vehicles request to exit, 2 exiting processes are required.

Figure 5.2.4 shows the cumulative probability distribution of number of lane changes of a random stack in a homogeneous highway with the exiting probability of a vehicle, denoted by $p_e$, equal to 0.1. In a homogeneous highway, entrances have the same release rates and exits have the same exiting rates and the exiting probability of a vehicle is also the same along the highway. Under this circumstance, the number of lane changes of a random stack is calculated given the probability of number of vehicles in a stack and the exiting probability of a vehicle. The number of exiting groups is calculated by the same approach used in Chapter 4. Since $N_{max}$=10, ranges of U and L are (0,…,5). As a result, around 57% of stacks execute more than one lane changes and, on average, a stack executes 1.7 lane changes.

![Figure 5.2.4 homogeous highway, $p_e=0.1$](image-url)
Figures 5.2.5 and 5.2.6 shows results of different exiting probabilities. About 50% of stacks execute 3 and 5 lane changes to form an exiting platoon and, on average, a stack executes 3.2 and 5.2 lane changes in $p_e$ equal to 0.2 and 0.5, respectively.

Since the same methodology used in one-lane AHS is applied, the number of lane changes increases with $p_e$ when $p_e<0.5$ and decreases when $p_e>0.5$.

Table 5.2.1 shows that a highway exhibits a bell-shape pattern of exiting probabilities. This pattern stands for a section of highway passing a business district during rush hours. Stacks enter the system with uniform distribution of number of vehicles from 0 to 20. The exit rate is the product of $p_e$ and the upstream highway flow. The entry rate of an entrance is close to the exit rate of immediate upstream exit. To satisfy 99.8% of stacks completing all lane changes, shown in the last row of the table, the distance of 9 lane changes is necessary when $p_e \geq 0.25$.

<table>
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<th>4</th>
<th>5</th>
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<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>entry rate</td>
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<td>407.5</td>
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<td>1629.9</td>
<td>2035.9</td>
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<td>815.1</td>
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<tr>
<td>exit rate</td>
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<td>815.1</td>
<td>1630</td>
<td>2037.7</td>
<td>2444.7</td>
<td>2035.3</td>
<td>1222.3</td>
<td>815.1</td>
<td>407.5</td>
</tr>
<tr>
<td>No of lane changes (99.8%)</td>
<td>3</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>9</td>
<td>9</td>
<td>8</td>
<td>7</td>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.2.1 System performance in Random Stack under the Bell-Shape distribution
A limited space can only support a limited number of lane changes between ramps. Sometimes, all exiting vehicles may not be able to form a single platoon. Hence, the exit rate decreases. Vehicles unable to exit will take up space in stacks and prevent vehicles from being released. Hence, the entry rate decreases as well. These vehicles incur a higher $p_e$ in the following sections of a highway and lead to more lane changes requiring more space if $p_e$ is smaller than 0.5. However, a higher $p_e$, greater than 0.5, reduces the number of lane changes. Accumulated vehicles that fail to exit in upstream sections may leave a highway at some exit and cause a high exiting rate, which the exit cannot deal with. In this situation, a spillback may occur to damage the system or some vehicles fail to exit again. Furthermore, the travel time increases when vehicles fail to leave a highway at assigned exits. Hence, minimizing the number of lane changes can minimize the ramp spacing and maximize the system throughput while keeping system operation at a normal condition, that is, no spillback and minimal accumulated vehicles unable to exit.

5.2.2 Slot Assignment Rules: SSRIM and GERIM

In Chapter 4, we proposed five slot assignment rules: SS, SSRIM, EJSS, EJSSRIM, and GSRIM. In a two-lane AHS, a stack can allow lane changes to occur at any position. Therefore, EJSS and EJSSRIM are not applied because vehicles join only at end positions. A multi-lane AHS needs higher flow rates at entrances to maintain capacity. Hence, the RIM is adapted and SSRIM and GSRIM are slot assignment rules applied in multi-lane highway systems.

Slot assignment rules should be implemented on both lanes. If the lower slot is a sorted slot and the upper slot is a random slot, then when vehicles in the upper slot move to the lower slot, excessive lane changes will occur, because vehicles in the upper slot are sorted into the lower slot through lane changes. For example, vehicles destined to the exits 5 and 6 are assigned to the lower slot, then vehicles to the exit 5 will change lanes before vehicles to exit 6. Contiguous vehicles to exits 5
and 6 may not be able to move together because their sequence of destinations might not correspond to those in the lower slot.

In this condition, sequences of destinations reduce the possibility that vehicles change lanes in platoons. However, if both slots are sorted slots, then in the same example, a group of vehicles to exit 5 can move as a platoon to the lower slot followed by a group of vehicles to exit 6. Regardless of capacity constraint, only two lane changes are enough. The same conditions occur in grouped slots. Therefore, we will implement both sorted slots and grouped slots in a stack in multi-lane AHS.

5.2.3 Lane Assignment Rules

In a multi-lane AHS, lane assignment rules and slot assignment rules are incorporated to maximize system performance. If we apply only slot assignment rules, more lane changes may occur because exiting vehicles are likely to be in upper slots. If we apply only lane assignment rules, slots having random sequences of destinations require excessive space at exits. In random stacks, one vehicle may move right and left several times without any purpose. In contrast, under lane assignment rules, vehicles only move left to their assigned lanes upon entering a highway and move right toward their exits. Vehicle routes are traceable.

In a two-lane AHS, vehicles at an entrance will be assigned to one lane. Hence, the lane assignment rule is that, at each entrance, a destination number is used to compare vehicle destinations. If a vehicle’s destination is greater than or equal to this number, then it will be assigned to the left lane; otherwise, to the right lane. Vehicles in stacks change lanes by comparing their destinations to the destination number when passing the entrance. The lane assignment rule at an entrance is established by determining a destination number called the lane division. Lane assignment rules on a highway are represented by a set of lane divisions of entrances.
The determination of lane divisions can be formulated as an optimization problem with the objective of maximizing the system throughput. A two-lane AHS can be represented as a network flow model consisting of pairs of entrance and exit, which is shown in Figure 5.2.7. Nodes A and C represent an entrance and an exit, respectively. Node B represents lane changes occurred between Nodes A and C. Let $f_{i,l}$ be the mean flow rate immediate upstream the entrance $i$ on lane $l$, $f_{in,i}$ and $f_{out,i}$ be the mean flow rates from entrance $i$ and exit $i$, and $LC_{12,i}$ and $LC_{21,i}$ be the mean flow rates from lane 1 to lane 2 and lane 2 to lane 1 between entrance $i$ and exit $i$, respectively. All quantities are non-negative.

Let $EN$, $EX$ be numbers of entrances and exits in the system, respectively, $f_{max}$ be the maximum flow on a lane, $p_{e,i}$ be the exiting probability of exit $i$, $OD_{i,j}$ be the probability that a vehicle at entrance $i$ is destined to exit $j$, and $d_{i}$ be the lane division at entrance $i$. Equation (5.2.1) is the objective function of maximizing the total sum of mean flow rates at ramps. Equation (5.2.2) is the flow conservation constraint on node B and Equation (5.2.3) is the flow conservation constraint considering nodes A and C. The mean flow rate at entrance $i$, $f_{in,i}$, is based on the upstream flow conditions and the slot assignment rule applied at the entrance. Equations are listed in Section 4.2. Equations (5.2.4)–(5.2.5) are used to calculate the mean flow rate at exit $i$, $f_{out,i}$, by assuming all exiting vehicles can leave successfully. Equations (5.2.6)–(5.2.8) are capacity constraints for nodes C, B, and A, respectively.

Figure 5.2.7 A network flow model of a pair of entrance and exit
\[
\text{Max} \sum_{i=1}^{EN} f_{in,i} + f_{out,i} \quad (5.2.1)
\]

\[f_{2,i+1} = f_{2,i} + LC12_i - LC21_i \quad \forall i \geq 1 \quad (5.2.2)\]

\[f_{1,i+1} = f_{1,i} + f_{in,i} - LC12_i + LC21_i - f_{out,i} \quad \forall i \geq 1 \quad (5.2.3)\]

\[
p_{e,i} = \frac{\sum_{k}^{i} f_{in,k}[OD_{k,i}]}{\sum_{k=1}^{i} \sum_{j \in k} f_{in,k}[OD_{k,j}] \quad \forall i \geq 1} \quad (5.2.4)\]

\[f_{out,i} = p_{e,i} (f_{1,i} + f_{in,i} - LC12_i + LC21_i) \quad \forall i \geq 1 \quad (5.2.5)\]

\[f_{1,i} + f_{in,i} - LC12_i + LC21_i \leq f_{\text{max}} \quad \forall i \geq 2 \quad (5.2.6)\]

\[f_{2,i} + LC12_i \leq f_{\text{max}} \quad \forall i \geq 2 \quad (5.2.7)\]

\[f_{1,i} + f_{in,i} \leq f_{\text{max}} \quad \forall i \geq 2 \quad (5.2.8)\]

The decision variables are lane divisions and all flow rates are functions of lane divisions at upstream entrances. The objective of maximizing throughput is achieved by establishing appropriate lane divisions. The system throughput is defined as the sum of flow rates in and out of a highway. The flow rates at exits are defined as the product of the upstream highway flow and the exiting probability, because vehicles can exit successfully under slot assignment rules. The exiting probability shown in Equation (5.2.4) is derived from the OD patterns and flow rates at entrances. The upstream highway flow is equal to total flow rates at upstream entrances subtracting those at upstream exits. Hence, given OD patterns, maximizing flow rates at entrances maximizes system throughput.

The flow rate at an entrance is determined by the distribution of destinations of vehicles from the entrance and slot assignment rules (i.e. SSRIM and GSRIM). Given the distribution of vehicle destinations at an entrance, the flow rate increases with fewer vehicles or smaller number of destinations in the lower slot. Capacity
constraint and flow conservation are considered in developing slot assignment rules, detailed explanation is stated in Chapter 4.

Therefore, the maximization of system throughput based on a transportation network model can be translated into the maximization of entrance capacities by determining lane divisions so that the number of vehicles and the number of destinations in the lower slot is minimized.

Based on above inferences, a greedy method for establishing lane assignment rules in a steady state two-lane AHS is stated as follow:

**The lane division at an entrance is determined such that, after completion of lane changes, vehicles with larger destination numbers fill upper slots.**

From the greedy method, a lane division is determined by the distribution of the smallest destination in the upper slot containing vehicles with destinations greater than or equal to those in the lower slot of a stack. Specifically, in cases of \( n < N_{\text{max}} \), \( n \) denotes the number of vehicles in a stack, the lane division is the immediate downstream destination number. In cases of \( n \geq N_{\text{max}} \), the lane division is the \( N_{\text{max}} \)th destination provided that vehicle destinations in a stack are sorted in a descending order. We derive the distribution of the lane division in the following.

The probability of the \( N_{\text{max}} \)th vehicle destined to \( i \) is derived from cases classified by the number of vehicles destined to destination \( i \), denoted by \( k \), in a stack of \( n \) vehicles. We recursively discuss this classification. Suppose there is only one vehicle destined to destination \( i \), then there must exist \( (N_{\text{max}}-1) \) vehicles destined to destinations greater than \( i \) and the remaining \( (n-N_{\text{max}}) \) vehicles destined to destinations smaller than \( i \). Hence, the probability in this case is,
\[
\binom{n}{N_{\text{max}} - 1} \left( \sum_{j=i+1}^{\text{EX}} p_j \right)^{n_{\text{max}} - 1} \left( n - \frac{N_{\text{max}}}{2} + 1 \right) p_i \left( \sum_{j=1}^{i-1} p_j \right)^{n-N_{\text{max}}} \quad (5.2.9)
\]

When \( k=2 \), after sorting, these two vehicles destined to destination \( i \) can be in the \((N_{\text{max}}-1)^{\text{th}}\) and \( N_{\text{max}}^{\text{th}} \) or \( N_{\text{max}}^{\text{th}} \) and \((N_{\text{max}}+1)^{\text{th}}\) positions. In the first case, \((N_{\text{max}}-2)\) vehicles are destined to destinations greater than \( i \) and \((n-N_{\text{max}})\) vehicles are destined to destinations smaller than \( i \), while in the second case, \((N_{\text{max}}-1)\) vehicles are destined to destinations greater than \( i \) and \((n-N_{\text{max}}-1)\) vehicles are destined to destinations smaller than \( i \). The probability in this case is,

\[
\left( \binom{n}{N_{\text{max}} - 2} \left( \sum_{j=i+1}^{\text{EX}} p_j \right)^{n_{\text{max}} - 2} \left( n - \frac{N_{\text{max}}}{2} + 2 \right) p_i^2 \left( \sum_{j=1}^{i-1} p_j \right)^{n-N_{\text{max}}} \right) + \left( \binom{n}{N_{\text{max}} - 1} \left( \sum_{j=i+1}^{\text{EX}} p_j \right)^{n_{\text{max}} - 1} \left( n - \frac{N_{\text{max}}}{2} + 1 \right) p_i^2 \left( \sum_{j=1}^{i-1} p_j \right)^{n-N_{\text{max}} - 1} \right) \quad (5.2.10)
\]

Therefore, \( k \leq n - N_{\text{max}} + 1 \), \( k \leq N_{\text{max}} \), the probability that the destination of the \( N_{\text{max}}^{\text{th}} \) vehicle is \( i \), provided that \( k \) vehicles destined to destination \( i \) in a stack of \( n \) vehicles, denoted by \( L(n,k,i) \), is

\[
L(n,k,i) = \sum_{h=1}^{k} \binom{n}{N_{\text{max}} - h} \left( \sum_{j=i+1}^{\text{EX}} p_j \right)^{n_{\text{max}} - h} \left( n - \frac{N_{\text{max}}}{k} + h \right) p_i^k \left( \sum_{j=1}^{i-1} p_j \right)^{n-N_{\text{max}} + h - k} \quad (5.2.11)
\]

The index, \( h \), represents the number of vehicles destined to destination \( i \) in the first \( N_{\text{max}} \) vehicles sorted by destinations. The above equation keeps the same except the range of \( h \) varying with the range of \( k \). \( h \) ranges from \( N_{\text{max}} - n + k \) to \( k \) if \( N_{\text{max}} \geq k > n - N_{\text{max}} + 1 \) and from \( N_{\text{max}} - n + k \) to \( N_{\text{max}} \) if \( n \geq k > N_{\text{max}} \).

By considering all \( n \) and \( k \), the probability of the \( N_{\text{max}}^{\text{th}} \) vehicle destined to destination \( i \) provided that \( n \geq N_{\text{max}} \), denoted by \( L(i) \), is

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Hence, the lane division is

\[
L(i) = \sum_{n=N_{\text{max}}}^{2N_{\text{max}}} N^n \sum_{k=1}^{n} L(n, k, i) \quad (5.2.12)
\]

\[
LD = \sum_{n=0}^{N_{\text{max}}-1} N^i + \sum_{i=1}^{E_X} iL(i) \quad (5.2.13)
\]

Given the steady state flow conditions on highway lanes and entrances, we can calculate lane divisions from upstream to downstream entrances. The steady state flow conditions include distribution of number of vehicles and distribution of destinations of vehicles in a stack and distribution of vehicle destinations at entrances. However, to apply equations derive in Chapter 4, we need flow conditions of lower slots.

That a vehicle with destination \( i \) in the lower slot of a stack needs three conditions:

1. Number of vehicles in a stack is greater than \( N_{\text{max}} \).
2. Destinations of vehicles in the upper slot are greater than or equal to \( i \).
3. At least one vehicle in the lower slot is destined to \( i \).

We narrow down the problem by giving a stack with \( n \) vehicles, \( n > N_{\text{max}} \) and we want to derive the probability that at least one of the last \( n-N_{\text{max}} \) vehicles after being sorted by destinations is destined to destination \( i \). In other words, among all combinations of destinations of these \( n \) vehicles, we want to find out the number of combinations that at least one vehicle is destined to destination \( i \) and at least \( N_{\text{max}} \) vehicles are destined to destinations greater than or equal to \( i \).

To avoid repetition, we again divide all combinations by the number of vehicles destined to destination \( i \), denoted by \( k \). For \( k > N_{\text{max}} \), we are sure that at least one vehicle with destination \( i \) appears in the lower slot. Hence, other \( (n-k) \) vehicles
can be destined to any destinations but \( i \). For \( k \leq N_{\text{max}} \), we need at least \((N_{\text{max}}-k+1)\) vehicles destined to destinations greater than \( i \). To avoid repetition, we further divide combinations by the number of vehicles with destinations greater than \( i \). If \((N_{\text{max}}-k+h)\) vehicles are destined to destinations greater than \( i \), then the remaining, \((n-N_{\text{max}}-h)\), vehicles will be destined to destinations smaller than \( i \).

Let \( p(n,k,i) \) be the probability that \( k \) of \( n \) vehicles are destined to destination \( i \) in the lower slot. Hence, \( 1 \leq k \leq N_{\text{max}} \),

\[
p(n,k,i) = \binom{n}{k} p_i^k \left[ \sum_{h=1}^{n-N_{\text{max}}} \binom{n-k}{N_{\text{max}}-k+h} \left( \sum_{j=i+1}^{E} p_j \right)^{N_{\text{max}}-k+h} \left( \sum_{j=1}^{i-1} p_j \right)^{n-N_{\text{max}}-h} \right] \]

(5.2.14)

and, \( n \geq k > N_{\text{max}} \),

\[
p(n,k,i) = \binom{n}{k} p_i^k (1 - p_i)^{n-k} \]

(5.2.15)

By summing over \( k \) and \( n \), we derive the probability that a vehicle in the lower slot is destined to destination \( i \). The probability of \( i \) vehicles in the lower slot equals the probability of \((i+N_{\text{max}})\) vehicles in a stack. Therefore, equations in Section 4.2 can be used to derive flow rates at entrances applying slot assignment rules.

The procedure of determining lane assignment rules under the greedy method maximizes throughput while keeping all constraints. Hence, this method solves the optimization problem. The lane division of an entrance is the mean of the distribution of the \( N_{\text{max}} \)th vehicle destination in a stack. Theoretically, the mean is a real number. However, vehicles are assigned to destinations represented by integers. Hence, a lane division is selected to be the largest integer less than or equal to the
mean. For example, the mean destination of the $N_{\text{max}}$th vehicle is 7.4 and then the lane division is 7. This adjustment ensures upper slots are full.

From the definition of the greedy method, one might argue that a smaller lane division can also fill upper slots and, hence, there are multiple optimal solutions. The following proof shows that the optimal lane division at an entrance is unique.

**Proof: Uniqueness of Optimal Lane Division**

The proof is also done by contradiction. Let $\overline{d}_i$ be another optimal lane division for entrance $i$ and $\overline{d}_i > d_i^\ast$. Since other lane divisions remain the same, $\overline{d}_i > d_i^\ast$ means more vehicles are assigned to the right lane. This not only wastes space in upper slots but also reduce space to vehicles in lower slots. Hence, $\overline{d}_i > d_i^\ast$ is not an optimal solution.

However, any $\overline{d}_i < d_i^\ast$ can also fill upper slots and be the optimal solution. As a matter of fact, a smaller $\overline{d}_i$ causes vehicles with farther destinations to stay on the right lane. These vehicles will reduce release rates due to the slot assignment rules. In SSRIM, long-trip vehicles in lower slots reduce ranges of destinations for releasing vehicles at entrances. In GSRIM, long-trip vehicles in lower slots increase the number of destination groups and prevent vehicles, at entrances, with these destinations from being released. The detailed description of these slot assignment rules is in Chapter 4. Therefore, $\overline{d}_i < d_i^\ast$ is also not an optimal solution. Thus, the uniqueness is proved.

The above derivation shows that unique lane divisions determined by the greedy method maximize system throughput. However, they also minimize intercepts of vehicle routes on a highway, which is the necessary condition of lane assignment rules described in Section 3.3. Under this condition, vehicles won’t move
alternately to left and right lanes on a highway. This condition mathematically that 
\[ d_{i+1}^* \geq d_i^* \], given \( d_i^* \) is the optimal lane division at entrance \( i \). The proof is done by contradiction.

**Proof: Minimizing Vehicle Intercept by the Greedy Method**

If \( d_{i+1}^* < d_i^* \), then vehicles destined to destinations between \( d_{i+1}^* \) and \( d_i^* \) will move to the left lane at entrance \( i+1 \). This implies that there is space to admit vehicles in upper slots before stacks pass entrance \( i+1 \). Since there is no entrance in between, vehicles in stacks won’t increase. In other words, space to admit vehicles in stacks won’t decrease. Thus, \( d_i^* \) is not an optimal lane division, because more vehicles at entrance \( i \) can be assigned to the left lane and fill up the space. The original statement is violated and the proof is done.

The proposed greedy method not only maximizes system throughput but also achieves the necessary condition of lane assignment rules. Therefore, the greedy method can be used to establish lane assignment rules in AHS.

**Experiments**

In the following, we study a two-lane AHS under a heavy traffic condition. The system has 10 pairs of entrance and exit followed by 10 more exits for the last entrance to implement the lane assignment rule. The heavy traffic condition is created by assuming stacks are initially full. Four OD patterns are applied and defined as follow:

1. UNIFORM: uniform OD patterns.
2. EXPO10: a 2-km shifted exponential OD patterns with the mean trip length of 10km. Ramp spacing is assumed to be 4km.
3. EXPO20: a 2-km shifted exponential OD patterns with the mean trip length of 20km. Ramp spacing is assumed to be 4km.
4. Bell-Shape: a bell-shape OD patterns shown in Figure 4.2.6.
Table 5.2.2 shows results in UNIFORM and SSRIM. In the table, a pair of entrance and exit shares the same number. The $p_e$ and mean exit rate belong to an exit and mean release rate and lane division belong to an entrance. The highway flow is the flow between an entrance and an exit. The initial highway flow is 7578 vehicles per hour for two lanes by assuming full stacks entering the system. The first lane division is 10 so as to balance loads between lanes.

<table>
<thead>
<tr>
<th>Entrance/Exit Number</th>
<th>$p_e$</th>
<th>Mean Release Rate (vehicles/hour)</th>
<th>Lane Division</th>
<th>Mean Exit Rate (vehicles/hour)</th>
<th>Highway Flow (vehicles/hour)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.050</td>
<td>7578.0</td>
<td>10</td>
<td>378.9</td>
<td>7578.0</td>
</tr>
<tr>
<td>2</td>
<td>0.053</td>
<td>297.5</td>
<td>11</td>
<td>423.8</td>
<td>7497.5</td>
</tr>
<tr>
<td>3</td>
<td>0.056</td>
<td>350.4</td>
<td>11</td>
<td>444.7</td>
<td>7453.3</td>
</tr>
<tr>
<td>4</td>
<td>0.059</td>
<td>383.6</td>
<td>12</td>
<td>469.0</td>
<td>7422.9</td>
</tr>
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<td>5</td>
<td>0.062</td>
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<td>7395.8</td>
</tr>
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<td>6</td>
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<td>569.0</td>
<td>15</td>
<td>706.9</td>
<td>7240.8</td>
</tr>
</tbody>
</table>

Table 5.2.2 UNIFORM and SSRIM in 2-lane AHS

In the uniform OD pattern, vehicles at an entrance have equal probabilities for any downstream exit. Hence, $p_e$ increases due to accumulation of exiting vehicles. The flow rate at an exit equals the product of $p_e$ and the highway flow in the immediate upstream section. Unlike that in the rule of Random Stack, the highway flow is less than the highway capacity because of the slot assignment rule, SSRIM, applied at entrances. Mean exit rates increase with $p_e$. Mean release rates also increase because of more space to admit vehicles in stacks when passing more ramps. Flows at ramps range from 298 to 707 vehicles per hour.

In Table 5.2.2, the lane divisions increase steadily from 10 to 15. The lane division of an entrance is half the number of downstream exits. This is reasonable because two lanes share the highway flow equally under uniform OD patterns.
<table>
<thead>
<tr>
<th>Entrance/Exit Number</th>
<th>$P_e$</th>
<th>Mean Release Rate (vehicles/hour)</th>
<th>Lane Division</th>
<th>Mean Exit Rate (vehicles/hour)</th>
<th>Highway Flow (vehicles/hour)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.050</td>
<td>7578.0</td>
<td>10</td>
<td>378.9</td>
<td>7578.0</td>
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</tr>
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<td>575.7</td>
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<tr>
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<td>0.091</td>
<td>626.6</td>
<td>15</td>
<td>737.1</td>
<td>7549.5</td>
</tr>
</tbody>
</table>

Table 5.2.3 UNIFORM and GSRIM in 2-lane AHS

Table 5.2.3 shows results in UNIFORM and GSRIM. Some results are similar to SSRIM, such as the exiting probability and lane divisions. However, because of GSRIM, mean release rates are higher from 8% to 24.8% and also mean exit rate from 1% to 4.3% compared to those in UNIFORM and SSRIM. The increase of highway flows is close to the increase of exit rates, because GSRIM can recover flows quickly. The system throughput, the sum of release rates and exit rates of all ramps, increases 3.3%. Flows at ramps range from 370 to 740 vehicles per hour. In UNIFORM, flow rates can be handled at current highway ramps.

Table 5.2.4 shows results in EXPO20 and SSRIM. The first lane division is 5 based on balancing loads between lanes. In practice, vehicles destined to the lane division may stay in lower slots because upper slots are full. Hence, some adjacent entrances may have the same lane divisions because the flow rate to this destination is high in this section of highway. Lane divisions increase more steadily because the variation among exiting probabilities is smaller, compared to those in UNIFORM and SSRIM.
The exiting probabilities, higher than those in UNIFORM and SSRIM, result in higher exit rates. Higher exit rates make more space to admit vehicles in stacks and, in turn, result in higher release rates. The system throughput increases around 65% compared to the uniform case. Stacks become more unoccupied when passing more ramps, because slot assignment rules add constraints at entrances while allowing vehicles to leave easily and successfully. Hence, the mean highway flow declines quickly. It drops 9.8%, while 4.4% in the uniform case, from the section within the first pair of entrance and exit to that within the 10th pair.

Table 5.2.5 shows results under EXPO20 and GSRIM. Compared to those in EXPO20 and SSRIM, the system throughput increases 8.3% and the highway flows increase from 5.9% to 9.6% along the highway and the lane divisions are similar.
Table 5.2.6 shows results in EXPO10 and SSRIM. The lane division begins at 3 and increases quickly. The increase of lane divisions depends on the accumulation of vehicles with long trips in upper slots. In EXPO10, 18% of vehicles, at the first entrance, destined to the last exit. These vehicles stay in upper slots along the highway. A higher release rate accelerates this accumulation. When vehicles have shorter trips, stacks are likely to have more space to admit vehicles and, hence, the mean release rate increases and the system throughput also increases, 26% compared to that in EXPO20 and SSRIM.

<table>
<thead>
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<th>Lane Division</th>
<th>Mean Exit Rate (vehicles/hour)</th>
<th>Highway Flow (vehicles/hour)</th>
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Table 5.2.6 EXPO10 and SSRIM in 2-lane AHS
Table 5.2.7 shows results in EXPO10 and GSRIM. The lane division is 3 at the first entrance and 18 at the last entrance. High release rates in GSRIM accelerate the accumulation of long-trip vehicles in upper slots. Compared to those in EXPO10 and SSRIM, the highway flow increases within a range from 7.1% to 12.9%. The system throughput increases 10%.

<table>
<thead>
<tr>
<th>Entrance/Exit Number</th>
<th>$P_e$</th>
<th>Mean Release Rate (vehicles/hour)</th>
<th>Lane Division</th>
<th>Mean Exit Rate (vehicles/hour)</th>
<th>Highway Flow (vehicles/hour)</th>
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Table 5.2.8 Bell-Shape and SSRIM in 2-lane AHS

Table 5.2.8 shows results in Bell-Shape and SSRIM. The maximum exiting probability is 0.18 at the exit 6. This pattern is studied because it is similar to the origin/destination pattern of a highway in the urban areas during rush hours. Initially, the exiting probability is low, most of vehicles stay on the highway and lane divisions keep the same. Mean exit and release rates are low at these ramps. The lane division begins at 9. The change of lane division starts at entrance 5 when the mean release rate increases, because the $p_e$ increases at the exit 4. The flow rate at an exit brings up the release rate of its downstream entrance. However, the highway flows drop 2.8% and 3.4% between entrances 5 to 6 and 6 to 7, respectively, because the loss at an exit cannot be made up by the immediate downstream release rate.

Table 5.2.9 shows results in Bell-Shape and GSRIM. GSRIM and SSRIM have many similar performance measures in this system, especially before the exit 4.
GSRIM can recover the highway flow quickly. Hence, the increase of the release rate at entrance 5 is 26% compared to that in SSRIM and the highway flows drop only 0.25% and 0.2% between entrances 5 to 6 and 6 to 7, respectively.

<table>
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<tr>
<th>Entrance/Exit Number</th>
<th>$P_e$</th>
<th>Mean Release Rate (vehicles/hour)</th>
<th>Lane Division</th>
<th>Mean Exit Rate (vehicles/hour)</th>
<th>Highway Flow (vehicles/hour)</th>
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Table 5.2.9 Bell-shape and GSRIM in 2-lane AHS

In summary, the lane division is designed to balance loads of lanes at the first entrance and to maximize the system throughput at the following entrances. The accumulation of long-trip vehicles in upper slots causes the change of lane divisions from entrance to entrance. Higher flow rates at ramps accelerate the effect of accumulation. Therefore, in EXPO10 and GSRIM, the lane divisions range from 3 to 18.

The highway flow gradually declines toward downstream. The higher the exiting probability the more the flow will lose, because the slot assignment rules, SSRIM and GSRIM, are not designed for releasing vehicles conveniently but for the success of exit. From this point of view, the selection of slot assignment rules depends on their abilities to retain the highway flow through increasing release rates and recovering flows.

In terms of the system throughput, in GSRIM, it increases 3%, 8.3%, and 10% compared to SSRIM, in UNIFORM, EXPO20, and EXPO10, respectively. In
SSRIM, it increases 26% and 65% by comparing EXPO10 to EXPO20 and EXPO20 to UNIFORM, respectively. Hence, both rules can perform better in shorter trip length origin/destination patterns while GSRIM outperforms SSRIM.

5.3 Multi-Lane Automated Highway Systems

The operations in multi-lane AHS become much more straightforward after studying one-lane and two-lane AHS. The operational unit in multi-lane AHS is still a stack, since no speed difference can still keep capacity high while minimizing the total travel time. Slots in a stack are also aligned so that the waiting time for a lane change is minimized. The length of a slot also supports the maximum flexibility of lane changes.

The slot assignment rules we can apply are Random Slot, SSRIM, and GSRIM. In Random Slot, the number of lane changes to form an exiting platoon is in the order of the sum of groups of exiting vehicles in each slot. SSRIM and GSRIM outperform Random Slot in the number of lane changes needed, since more lanes cause fewer destinations on each lane. This makes sorting or grouping vehicles by their destinations much easier and also increases release rate. An extreme case is that one destination on one lane and the lowest slot is always empty when passing an entrance. In this condition, SSRIM and GSRIM have the same release rates, and then, the comparison between them is based on the number of lane changes. In this example, the number of lane changes is at most number of lanes. Hence, SSRIM will be as good as GSRIM.

Lane assignment rules in multi-lane AHS are represented by a set of lane divisions for each entrance. Let $d_{i,j}^*$ be the optimal lane division between lane $j$ and lane $j+1$ at entrance $i$. Then $d_{i,j}^* \leq d_{i,j+1}^*$ and $d_{i,j}^* \leq d_{i+1,j}^*$. These two statements can be proved by similar procedures in two-lane AHS. The former states that vehicles with farther destinations are assigned to left lanes. The latter states that space in
upper slots is occupied first. The uniqueness of the optimal solutions can also be proved in the same way shown in Section 5.2.3. Therefore, the lane divisions are established to fill stacks from left lanes to right lanes to maximize the system throughput and minimize the number of lane changes.

The lane changes can be executed independently among lanes. Hence, the number of lane changes does not increase linearly with number of lanes. For example, in a four-lane AHS, lane changes between the two right lanes can occur simultaneously with those between the two left lanes. Another example, in a three-lane AHS, an entry process and vehicle maneuvers among two left lanes are also independent to each other.

The performance of a multi-lane AHS is limited by the design of the moving slot model. The flow rate at an entrance is limited by slot speed and slot capacity. Hence, to fully take advantage of a multi-lane system, the system configuration and operational model and strategies must be modified.

5.4 Conclusions

We study lane change behavior of vehicles in the moving slot model applied in AHS. Four activities of lane changes are presented: lane changes from entrances to the right lane, lane changes from the right lane to the left lane (LC12), lane changes from the left lane to the right lane (LC21), and lane changes from the right lane to exits (LC1E). In terms of their impacts on the system throughput, space between ramps will be reserved for LC1E first followed by LC12 and LC21.

In a two-lane AHS, the rule of Random Stack maximizes the releases rate. However, a single exiting platoon can only be formed through a series of lane changes. Since exiting vehicles may appear at any position in a stack, a long distance between ramps is required to complete these lane changes. If vehicles fail to exit, a higher exiting probability leads to a higher exit rate, which may cause a spillback to
damage the highway if the exit cannot handle this increased flow. Therefore, SSRIM and GSRIM are applied.

Lane assignment rules are integrated with slot assignment rules. The lane assignment rules are established by specifying a destination number, called a lane division, for each entrance. At an entrance, vehicles with destinations up to the lane division will be assigned to the right lane, otherwise the left lane. Vehicles in a stack will move to lanes by comparing their destinations and the lane division. A greedy method is proposed to derive optimal lane divisions for entrances. A lane division is chosen as the destination that, in steady state, the upper slot is just full to maximize the system throughput.

In multi-lane AHS, many conclusions made in one-lane and two-lane AHS can be applied. A stack of well aligned slots across lanes form an operational unit running at a speed limit of 30m/s. Since random stack rules require more lane changes to form a single exiting platoon, the current ramp spacing may not be able to support this rule. Hence, only SSRIM (Sorted Slot with Release Improvement Mechanism) and GSRIM (Grouped Slot with Release Improvement Mechanism) are applied on a multi-lane AHS. In SSRIM, vehicles are sorted by their destinations and vehicles with long trips are assigned to stay in upper slots. In GSRIM, vehicles with the same destinations are located together. SSRIM actually performs very close to GSRIM, because most vehicles move to upper slots such that the lowest slot will always be empty to allow the maximum number of vehicles released from an entrance. The lane assignment rule can be derived in the same way as that in two-lane AHS.
Chapter 6

Simulation

Simulations conducted in this chapter focus on three issues: verification of analytic models, derivation of number of lane changes, and comparison of system performance under different configurations.

The simulation model is described in Section 6.1 and detailed experimental designs are stated in Section 6.2. Section 6.3 provides verification of analytic models, including slot assignment rules and lane assignment rules developed in Chapter 4 and Chapter 5, respectively. Section 6.4 studies the number of lane changes, which might not be solved analytically because of capacity constraint and complex sequences of vehicle destinations in both slots of a stack. In Section 6.5 we study the relationships between the limited vehicle waiting space and the entrance capacity under various slot assignment rules. This study provides criteria in designing entrances under the application of AHS. In Section 6.6 we alter the location of an exit relative to adjacent entrances and study the impact on system performance. Section 6.7 concludes this chapter.

6.1 Simulator Design

The simulation model represents a two-lane automated highway that implements the moving slot model, slot assignment rules, and lane assignment rules. The model has 10 pairs of entrances and exits followed by 10 more exits, such that lane assignment rules can be applied at the last entrance. Simulations are conducted in the AweSim environment.

Simulation processes are driven by three major events: creation of stacks, vehicle entry, and vehicle exit. Lane change is inspected after events of vehicle entry.
and exit. Given the ramp locations and specifications of stack, the occurrence of events is predictable and the system can, therefore, be driven by time. System states include number of vehicles in a stack, numbers and sequences of vehicles in each slot in each stack, numbers of vehicles released at entrances and their sequences, numbers of vehicles departing at exits, and lane divisions at entrances. Note: microscopic processes like vehicle movements are not simulated in this model.

Creation of Vehicles and Stacks/Slots

Vehicle arrivals are assumed to be by Poison processes. Arrival time and destination are assigned to each vehicle. Vehicles are served First-Come-First-Served. Vehicles are sorted only when entrances implement the Release Improvement Mechanism (RIM), stated in Chapter 4.

Stacks are created at the first entrance at a constant rate defined by the ratio of the stack length to the stack speed. A stack is assigned the creation time and full numbers of vehicles in both slots to simulate the most congested traffic condition. Vehicles’ sequences in both slots are determined by the applied slot assignment rules. The first entrance is capable of storing and sorting vehicles as stacks into the highway system.

Vehicle Exit

When a stack passes an exit, vehicles in the lower slot are scanned and removed if they are destined to this exit. By applying slot assignment rules, vehicles can leave the system successfully. In some cases, exiting vehicles may stay in the upper slot. In these situations, all vehicles in the lower slot must exit because, by the lane assignment rules, vehicles in the lower slot have shorter trips than those in the upper slots. Since the length of a slot in a two-lane highway is designed to be able to accommodate 2 platoons of full size, one more lane change is implemented so that exiting vehicles in the upper slot can move to the lower slot and exit successfully.
Vehicle Entry

When a stack passes an entrance, vehicles are released. The release process depends on slot assignment rules (i.e. SS, SSRIM, EJSS, EJSSRIM, and GSRIM). These rules are described in detail in Section 4.2.

Lane Change Process

Vehicles in stacks change lanes according to both slot assignment rules and lane assignment rules. Vehicles are scanned after stacks pass entrances and exits to initiate lane changes if there are requests. Requests for lane changes may come from newly released vehicles or vehicles failing to change lanes in upstream sections. One lane change is allowed to occur in a stack at one time.

The occurrence of a lane change process considers available space and sequences of destinations in both slots of a stack. A vehicle can change lane if the target slot can admit it. Then this vehicle joins a position without breaking the existing destination pattern in the target slot. Given the ramp spacing, the stack speed, and the time of a lane change, we derive the maximum number of lane changes allowed in a stack. A vehicle cannot finish a lane change between ramps if there is no available space in the target slot or the number of lane changes exceeds the maximum value. However, it may be completed in downstream sections when feasible conditions exist. Note: the time of a lane change is regarded as a constant in simulation and a simplified calculation is provided in Section 5.1.3.

Generation of Statistics

We collect data for the first 10 pairs of entrance and exit. Data includes numbers and sequences of vehicles in both slots of a stack, lane divisions and numbers of vehicles released when a stack passes entrances, and numbers of lane changes of a stack passing pairs of entrances and exits. The information of stacks is updated with every process and the occurrence time. From these data, we can track
vehicle movements from entry to exit and stack state, including numbers and
sequences of vehicles in both slots, from time to time. Statistics include mean release
rates at entrances, mean lane divisions for entrances, mean number of lane changes
during each pair of entrance and exit, and mean highway flows.

The steady state of a simulation system under heavy traffic is ensured by
examining two conditions: (1) the simulated mean release rate is equal to the analytic
solution and (2) the difference between the mean release rate and the mean exit rate
must be equal to the difference of mean flow rates entering and leaving a section of
highway. By testing various simulation times, 2000-hour simulations are conducted
and data is collected after 1000 hours. The number of simulation runs is determined
by keeping a 99% confidence interval of the mean release size within a width of 0.1.
The mean release size represents the mean number of vehicles released per slot.

The above simulation system is applied in the following experiments. However, in the experiment of “Limited Vehicle Waiting Space,” only an entrance is
simulated under various slot assignment rules and varying ramp spacing. In the
experiment of “Location of Exit,” the relative location of exit changes in the
simulation.

6.2 Experimental Design

Common parameters used for all experiments are listed in Table 3.4.1. Ramp
spacing of 4 km is common for a pair of entrance and exit. We assume that a lane
change takes 10 seconds. Therefore, the maximum number of lane changes between
ramps is 13. Four experiments are described in detail in the following.

Verification

We first design simulation experiments to verify analytic models of slot
assignment rules and lane assignment rules. In the simulation model, the release
processes exactly conform to applied slot assignment rules, which are SSRIM and GSRIM in a two-lane AHS. In deriving distributions of number of vehicles released, we assume that there is no shortage of vehicles at entrances. To ensure this assumption, arrival data are created under a Poisson process with a time interval shorter than that between adjacent stacks visiting an entrance. These data are then used as input files to release vehicles when stacks pass entrances. SSRIM and GSRIM are described in Section 4.2.

To verify the lane assignment rules established by the proposed greedy method described in Section 5.2.3, we propose a dynamic lane assignment rule to maximize the system throughput by maximizing the release rates at entrances. This rule states that a stack can choose a lane division based on the set of current vehicle destinations when passing an entrance such that vehicles with long trips will occupy the upper slot and the number of vehicle destinations in the lower slot is minimized. Specifically, a lane division is chosen such that the first $N_{\text{max}}$ vehicles with the largest destination numbers in a stack will stay in the upper slot and the other vehicles are in the lower slot. For example, after vehicles are released from an entrance, a stack has vehicles with destinations (2, 4, 11, 3, 12, 4, 9, 1, 5, 15, 8, 2). After lane changes, we intend to put vehicles with destinations (15, 12, 11, 9, 8, 5, 4, 4, 3, 2) in the upper slot and the other two vehicles with destinations (2, 1) in the lower slot. Hence, the lane division for this stack at the entrance is the destination number 2.

**Number of Lane Changes**

In multi-lane AHS, the ramp spacing can be determined by the number of lane changes in a stack given a constant time of lane change. This experiment uses fixed lane divisions derived analytically in Section 5.2.3 and slot assignment rules of SSRIM and GSRIM to derive the number of lane changes, which depends on capacities and sequences of vehicle destinations in both slots and applied slot assignment rules. We assume that one lane change occurs at a time within a stack.
In simulation scenarios, vehicles with destination numbers equal to lane divisions can stay in either slot of stacks. However, to free space in lower slots for admitting more vehicles, we intentionally assign these vehicles to upper slots, which are not full.

Vehicle Waiting Space

In reality, the vehicle waiting space at an entrance is limited. Hence, the entrance capacities of analytic models under slot assignment rules are theoretical maximums. In this experiment, we identify the waiting space required at an entrance implementing slot assignment rules, including SS, SSRIM, EJSS, EJSSRIM, and GSRIM. Hence, a more practical capacity at an entrance can be obtained in terms of the applied slot assignment rules and the vehicle waiting space.

Location of Exit

In congested traffic, stacks are likely to be full after passing an entrance. Lane changes are less likely to occur until stacks pass exits and space is freed by exiting vehicles. Therefore, given two adjacent entrances with fixed spacing, the location of an exit between them affects number of lane changes completed. Unsuccessful lane changes lower system performance because they cause more restriction on or occupy space for admitting vehicles. In this experiment, we consider two extreme cases: an exit is located close to either the upstream entrance or the downstream entrance and system performance is compared.

6.3 Verification of Analytic Models

The mean release rates in GSRIM and SSRIM are shown in Table 6.3.1 and Table 6.3.2, respectively.
### GSRIM-Release Rate (vehicles per hour)

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Table 6.3.1 Mean release rates by lane divisions from fixed and dynamic assignments in GSRIM

### SSRIM-Release Rate (vehicles per hour)

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<th>Entrance Number</th>
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<tr>
<td>2</td>
<td>297.5</td>
<td>297.5</td>
<td>1502.3</td>
<td>1498.9</td>
<td>767.8</td>
<td>732.5</td>
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<td>3</td>
<td>350.4</td>
<td>347.9</td>
<td>1817.7</td>
<td>1790.8</td>
<td>966.5</td>
<td>934.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>383.6</td>
<td>381.6</td>
<td>1538.1</td>
<td>1521.2</td>
<td>1055.3</td>
<td>1044.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>409.6</td>
<td>408.6</td>
<td>1482.4</td>
<td>1474.1</td>
<td>1062.1</td>
<td>1052.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>436.5</td>
<td>434.8</td>
<td>1370.2</td>
<td>1368.0</td>
<td>1045.8</td>
<td>1032.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>463.5</td>
<td>462.5</td>
<td>1243.5</td>
<td>1238.9</td>
<td>1027.1</td>
<td>1011.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>494.7</td>
<td>492.8</td>
<td>1167.6</td>
<td>1161.8</td>
<td>1007.5</td>
<td>1001.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>528.3</td>
<td>526.3</td>
<td>1085.4</td>
<td>1065.9</td>
<td>978.7</td>
<td>973.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>569.0</td>
<td>565.4</td>
<td>1015.6</td>
<td>1013.7</td>
<td>958.5</td>
<td>956.6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 6.3.2 Mean release rates by lane divisions from fixed and dynamic assignments in SSRIM

Columns titled “Fixed” represent cases using fixed lane divisions and release rates are derived analytically while “Dynamic” columns represent cases choosing lane divisions dynamically with release rates derived by simulation. OD patterns are defined the same as those in Chapter 5 with symbols of UNIF equal to UNIFORM and BS equal to Bell-Shape. Results from both “Fixed” and “Dynamic” cases under the same scenario (i.e. OD patterns and slot assignment rules) are comparable.
because lane assignment rules are designed to maximize system throughput in both cases.

The mean lane divisions assigned dynamically are shown in Table 6.3.3. They are equal to analytical solutions of the mean destinations of the \( N_{\text{max}} \)th vehicle in a stack by sorting vehicles in a descending order of destinations. In Section 5.2.3, lane divisions are rounded up to largest integers less than or equal to those in the table. The lane division at the first entrance is decided by balancing loads of lanes. From the mean lane divisions and mean release rates at entrances, lane assignment rules established by the greedy method proposed in Section 5.2.3 and entrance capacities of slot assignment rules derived in Section 4.2 are verified.

<table>
<thead>
<tr>
<th>Entrance Number</th>
<th>GSRIM</th>
<th>SSRIM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>UNIF</td>
<td>EXPO10</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>11.8</td>
<td>4.7</td>
</tr>
<tr>
<td>3</td>
<td>12.2</td>
<td>6.3</td>
</tr>
<tr>
<td>4</td>
<td>12.6</td>
<td>8.2</td>
</tr>
<tr>
<td>5</td>
<td>13</td>
<td>10.3</td>
</tr>
<tr>
<td>6</td>
<td>13.4</td>
<td>12.3</td>
</tr>
<tr>
<td>7</td>
<td>13.9</td>
<td>14.2</td>
</tr>
<tr>
<td>8</td>
<td>14.3</td>
<td>15.9</td>
</tr>
<tr>
<td>9</td>
<td>14.8</td>
<td>17.3</td>
</tr>
<tr>
<td>10</td>
<td>15.2</td>
<td>18.3</td>
</tr>
</tbody>
</table>

Table 6.3.3 Mean lane divisions assigned dynamically

One noteworthy point is that the dynamic lane assignment rules increase the number of lane changes because the only restriction of lane change is the capacity constraint. Table 6.3.4 and Table 6.3.5 show comparisons of number of lane changes between “Fixed” and “Dynamic” cases in GSRIM and SSRIM, respectively. On average, the increase is 17.5% in UNIF, 19.4% in Bell-Shape, and smaller in EXPO10, 0.4%, and EXPO20, 1.5%.
In cases of short-trip length distributions, stacks have more space to execute lane changes due to more frequent access and egress of vehicles to the system. Hence, the increase is small. From the above observations, the dynamic lane assignment rule is best used in cases of short-trip length distributions, in which release rates increase while the number of lane changes also increases but insignificantly.

<table>
<thead>
<tr>
<th>Entrance Number</th>
<th>UNIF</th>
<th>EXPO10</th>
<th>EXPO20</th>
<th>BS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fixed</td>
<td>Dynamic</td>
<td>Fixed</td>
<td>Dynamic</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0.163</td>
<td>0.923</td>
<td>1.127</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0.273</td>
<td>1.249</td>
<td>1.215</td>
</tr>
<tr>
<td>4</td>
<td>0.455</td>
<td>0.363</td>
<td>1.457</td>
<td>1.153</td>
</tr>
<tr>
<td>5</td>
<td>0.187</td>
<td>0.428</td>
<td>0.936</td>
<td>1.017</td>
</tr>
<tr>
<td>6</td>
<td>0.616</td>
<td>0.487</td>
<td>1.015</td>
<td>0.863</td>
</tr>
<tr>
<td>7</td>
<td>0.231</td>
<td>0.52</td>
<td>0.699</td>
<td>0.72</td>
</tr>
<tr>
<td>8</td>
<td>0.752</td>
<td>0.563</td>
<td>0.517</td>
<td>0.588</td>
</tr>
<tr>
<td>9</td>
<td>0.267</td>
<td>0.603</td>
<td>0.425</td>
<td>0.46</td>
</tr>
<tr>
<td>10</td>
<td>0.926</td>
<td>0.641</td>
<td>0.234</td>
<td>0.345</td>
</tr>
</tbody>
</table>

Table 6.3.4 Mean number of LC12 with fixed and dynamic LDs in GSRIM

<table>
<thead>
<tr>
<th>Entrance Number</th>
<th>UNIF</th>
<th>EXPO10</th>
<th>EXPO20</th>
<th>BS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fixed</td>
<td>Dynamic</td>
<td>Fixed</td>
<td>Dynamic</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0.144</td>
<td>0.684</td>
<td>1.087</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0.244</td>
<td>1.296</td>
<td>1.128</td>
</tr>
<tr>
<td>4</td>
<td>0.451</td>
<td>0.328</td>
<td>1.093</td>
<td>1.084</td>
</tr>
<tr>
<td>5</td>
<td>0.157</td>
<td>0.382</td>
<td>1.075</td>
<td>0.938</td>
</tr>
<tr>
<td>6</td>
<td>0.583</td>
<td>0.432</td>
<td>0.602</td>
<td>0.813</td>
</tr>
<tr>
<td>7</td>
<td>0.192</td>
<td>0.452</td>
<td>0.61</td>
<td>0.692</td>
</tr>
<tr>
<td>8</td>
<td>0.667</td>
<td>0.487</td>
<td>0.628</td>
<td>0.581</td>
</tr>
<tr>
<td>9</td>
<td>0.23</td>
<td>0.515</td>
<td>0.425</td>
<td>0.488</td>
</tr>
<tr>
<td>10</td>
<td>0.762</td>
<td>0.535</td>
<td>0.323</td>
<td>0.397</td>
</tr>
</tbody>
</table>

Table 6.3.5 Mean number of LC12 with fixed and dynamic LDs in SSRIM
6.4 Number of Lane Changes

The ramp spacing in a one-lane AHS is determined by maneuvering processes that vary with slot assignment rules. In a multi-lane AHS, lane change is considered as a major factor in determining the ramp spacing. A lane change process consists of space adjustment processes on both slots in a stack and a leaving process in one slot, which is concurrently a joining process in another slot. The time of a lane change needs to consider the cooperation of both slots and space adjustment processes.

Lane change is subject to the capacity constraint and sequences of vehicle destinations in both slots. For example, a group of 3 vehicles with destinations of 4, 6, and 7 are assigned to the left lane. The target slot has enough space to admit these vehicles but has a vehicle with destination 5. Then, the vehicle with destination 4 has to move separately from the other two vehicles. Or the target slot has no vehicle with destinations within 4 to 7, but its space can only admit two vehicles. Then, one of the three vehicles has to wait until there is a space available in the slot. Due to this complexity, simulation is used to determine the number of lane changes. The ramp spacing can, therefore, be determined given the time of lane change. The simulation results of number of lane changes per stack are shown in Table 6.4.1 and Table 6.4.2. The system is operated in heavy traffic by producing full stacks in the system. Hence, in the tables, we don’t show the number of lane changes during the first pair of entrance and exit.

<table>
<thead>
<tr>
<th>Entrance Number</th>
<th>GSRIM</th>
<th>SSRIM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>UNIF</td>
<td>EXPO10</td>
</tr>
<tr>
<td>2</td>
<td>0.00</td>
<td>0.923</td>
</tr>
<tr>
<td>3</td>
<td>0.455</td>
<td>0.942</td>
</tr>
<tr>
<td>4</td>
<td>0.187</td>
<td>0.936</td>
</tr>
<tr>
<td>5</td>
<td>0.616</td>
<td>0.967</td>
</tr>
<tr>
<td>6</td>
<td>0.231</td>
<td>0.949</td>
</tr>
<tr>
<td>7</td>
<td>0.752</td>
<td>0.517</td>
</tr>
<tr>
<td>8</td>
<td>0.267</td>
<td>0.907</td>
</tr>
<tr>
<td>9</td>
<td>0.926</td>
<td>0.234</td>
</tr>
</tbody>
</table>

Table 6.4.1 Mean number of LC12 per stack in GSRIM and SSRIM under various OD patterns
Table 6.4.2 Mean number of LC21 per stack in GSRIM and SSRIM under various OD patterns

<table>
<thead>
<tr>
<th>Entrance Number</th>
<th>GSRIM</th>
<th></th>
<th>SSRIM</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>UNIF</td>
<td>EXPO10</td>
<td>EXPO20</td>
<td>BS</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0.807</td>
<td>0.86</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0.933</td>
<td>0.85</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0.586</td>
<td>1.471</td>
<td>0.807</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>0.194</td>
<td>0.712</td>
<td>0.785</td>
<td>0.444</td>
</tr>
<tr>
<td>6</td>
<td>0.707</td>
<td>1.01</td>
<td>0.769</td>
<td>0.732</td>
</tr>
<tr>
<td>7</td>
<td>0.193</td>
<td>0.734</td>
<td>0.752</td>
<td>0.764</td>
</tr>
<tr>
<td>8</td>
<td>0.796</td>
<td>0.482</td>
<td>0.738</td>
<td>0.852</td>
</tr>
<tr>
<td>9</td>
<td>0.165</td>
<td>0.455</td>
<td>0.727</td>
<td>0.995</td>
</tr>
<tr>
<td>10</td>
<td>0.89</td>
<td>0.047</td>
<td>1.2</td>
<td>0.158</td>
</tr>
</tbody>
</table>

No lane changes occur in the beginning sections under UNIF and BS because of full stacks. These lane change requests are fulfilled in the following sections once space is available. Not every release in SSRIM causes lane changes. A LC12 is initiated when there are vehicles joining at the front end of a slot. In GSRIM, vehicles with destination numbers greater than or equal to lane divisions can always be released and initiate LC12. Hence, in general, the mean number of LC12 in GSRIM is larger than that in SSRIM. However, in SSRIM, larger numbers of LC12 may occur in some sections due to completion of lane changes accumulated from upstream sections. In cases of short trip length distributions, lane changes are likely to take place more often, because stacks have more space for lane changes when passing ramps.

In general, LC21 has a similar pattern as LC12. For example, in UNIF, LC12 and LC21 both increase in sections of entrances 4, 6, 8, and 10 and decrease in sections of entrances 5, 7, and 9. Because LC12 frees space in lower slots for LC21 to take place, so does LC21. But the accumulation of unfulfilled lane changes may temporarily break the pattern. However, this pattern is not obvious in EXPO20, because the exiting probabilities increase slightly and, hence, space for lane changes in stacks also increases slightly.
In a Random Stack, the mean number of lane change per stack between a pair of entrance and exit in a homogeneous highway with the exiting probability of 0.1 is 1.7; it is 0.7 for UNIF of GSRIM with mean exiting probability of 0.7. In EXPO10 of GSRIM, the number is 1.4 per stack with mean exiting probability of 0.2, and 3.2 lane changes in Random Stack with the same exiting probability. Therefore, lane/slot assignment rules reduce the number of lane changes more in short trip length distributions.

The experiment uses fixed lane divisions from analytic models. The system throughput is lower than analytic solutions, because

1. Upper slots are not always full.
   In simulation models, vehicles with destination numbers smaller than lane divisions are assigned to lower slots. In some cases, this may leave upper slots unoccupied and reduces space for admitting vehicles in lower slots. However, in the analytic model, we assume vehicles with long trips fill upper slots first and then lower slots. Hence, lower slots have more space to admit vehicles from entrances to increase release rates, and in turn, throughput.

2. Vehicles with long trips stay in lower slots.
   Vehicles with large destination numbers in lower slots will reduce the number of vehicles released in both SSRIM and GSRIM. This occurs when upper slots are already full of vehicles with long trips.
   Simulation results of mean release rates are shown in Tables 6.4.3 and 6.4.4.
The above two situations occur more often in a short trip-length OD pattern and, hence, cause more capacity loss. In GSRIM, the mean flow rate drops 7.5% in EXPO10 and 6% in UNIF, while, in SSRIM, it drops 12.4% in EXPO20 and 7.7% in UNIF. The results tell us that in practical operation of a two-lane AHS, the mean release rate of an entrance drops, on average, 6% in GSRIM and 9.3% in SSRIM under fixed lane divisions compared to analytic models.
In this experiment, the system throughput is reduced under fixed lane divisions because vehicles have to follow the lane assignment rules and, thus, reduce space in lower slots for admitting vehicles.

6.5 Vehicle Waiting Space

In Chapter 4, the entrance capacity is derived under the assumption that there is no shortage of vehicles. In reality, slots may pass an entrance and starve because of limited ramp space for storing vehicles or a low arrival rate. Hence, the entrance capacity is related to vehicle waiting space.

In this section, the simulation system has an entrance with 10 downstream exits. Vehicles arrive by a Poisson process. Distributions of number of vehicles in a slot, vehicle destinations in a slot, and destinations of waiting vehicles are assumed to be uniform. The parameters are the mean arrival rate and slot assignment rules. Different slot assignment rules result in different maximum mean release rates at an entrance. To build a standard for comparison of waiting space required at an entrance under various slot assignment rules, the mean arrival rates are defined as ratios of the maximum release rates of slot assignment rules. Data collected are the mean release rates under various mean arrival rates and slot assignment rules.

In Figure 6.5.1, given a ratio of their maximum release rates, SS needs the minimal space while EJSSRIM needs the most. In Table 6.3.1, we present the waiting space required at an entrance to achieve 80% of capacity and to ensure 99% of arrival vehicles won’t be blocked. The waiting space is calculated by the number of vehicles rounded up to an integer, and assuming the mean length of a vehicle is 5 meters and inter-vehicle space is the intraplatoon distance of 1 meter. SS needs 17 meters while EJSSRIM requires 107 meters. If the space at an entrance can store 30 vehicles, then GSRIM and SS can achieve almost 95% of their maximum release rate while EJSS and EJSSRIM can only support around 85%.
Figure 6.5.1 Comparison of mean queue lengths among SARs

Table 6.5.1 Space to store vehicles at an entrance achieving 80% of maximum capacity under various slot assignment rules

<table>
<thead>
<tr>
<th></th>
<th>SS</th>
<th>SSRIM</th>
<th>EJSS</th>
<th>EJSSRIM</th>
<th>GSRIM</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean no. of vehicles</td>
<td>2.9</td>
<td>9.1</td>
<td>9.6</td>
<td>17.1</td>
<td>6.9</td>
</tr>
<tr>
<td>waiting space (meters)</td>
<td>17</td>
<td>59</td>
<td>59</td>
<td>107</td>
<td>41</td>
</tr>
<tr>
<td>release rate (vehi/hour)</td>
<td>420.184</td>
<td>884.92</td>
<td>317.968</td>
<td>820.704</td>
<td>1538.96</td>
</tr>
</tbody>
</table>

Since the space at an entrance is limited, an entrance can never achieve the theoretical maximum capacity. This study provides criteria in designing the entrance space for storing waiting vehicles and selecting slot assignment rules based on the requirement of practical capacity.

6.6 Location of Exit

The basic requirement for a lane change is to have available space in stacks. To obtain space earlier, we assume that a group of exiting vehicles leave the highway
immediately after the release process is completed in both analytic and simulation models. This assumption implies that an exit is downstream from an entrance with a distance only sufficient for a release process and a preparation for an exiting process to take place. However, the ramp spacing varies in practical highway configurations. Therefore, the location of an exit relative to its immediate upstream entrance and its impact on lane changes is an interesting issue.

In general, an exit can be located anywhere within two adjacent entrances. When the exit is very close to the downstream entrance, then exiting vehicles will occupy space in lower slots and reduce the probability of lane change and increase the number of unsuccessful lane changes. The simulation system is the same as that in Section 6.4 but the location of an exit is close to the downstream entrance.

Table 6.6.1 shows the mean numbers of lane changes from the right lane to the left lane per stack in GSRIM under various OD patterns. Entrance numbers begin from 2 because vehicles have been sorted at the first entrance; hence, there is no lane change. The columns titled “Up” and “Down” in the table represent the cases that an exit is close to the upstream and the downstream entrances, respectively.

<table>
<thead>
<tr>
<th>Entrance Number</th>
<th>UNIF</th>
<th>EXPO10</th>
<th>EXPO20</th>
<th>BS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Down</td>
<td>Up</td>
<td>Down</td>
<td>Up</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0.548</td>
<td>1.249</td>
</tr>
<tr>
<td>4</td>
<td>0.015</td>
<td>0.455</td>
<td>1.968</td>
<td>1.457</td>
</tr>
<tr>
<td>5</td>
<td>0.027</td>
<td>0.187</td>
<td>0.979</td>
<td>0.936</td>
</tr>
<tr>
<td>6</td>
<td>0.06</td>
<td>0.616</td>
<td>0.842</td>
<td>1.015</td>
</tr>
<tr>
<td>7</td>
<td>0.086</td>
<td>0.231</td>
<td>0.589</td>
<td>0.699</td>
</tr>
<tr>
<td>8</td>
<td>0.149</td>
<td>0.752</td>
<td>0.638</td>
<td>0.517</td>
</tr>
<tr>
<td>9</td>
<td>0.206</td>
<td>0.267</td>
<td>0.507</td>
<td>0.425</td>
</tr>
<tr>
<td>10</td>
<td>0.329</td>
<td>0.926</td>
<td>0.325</td>
<td>0.234</td>
</tr>
</tbody>
</table>

Table 6.6.1 Mean number of LC12 per stack in GSRIM
In some situations, such as UNIF and BS, there is no lane change at the first few entrances because stacks are assumed to be full initially. In general, the mean numbers of lane changes are greater in “Up” cases than “Down” cases. However, in “Down” cases, at some entrances such as the entrance 4 in EXPO10 and the entrance 6 in EXPO20, big numbers occur because accumulated unsuccessful lane changes take place once stacks have space. Hence, in “Down” cases, the mean numbers of lane changes may be in a cycle of being smaller over several sections but larger in one or two following sections. This phenomenon occurs more often when vehicles access and egress a highway more frequently.

Compared to GSRIM, slots, in SSRIM, are more likely becoming empty. Lane changes occur more frequently and the difference between “Up” and “Down” is smaller, because stacks try to complete all lane changes if space is available. The mean number of LC12 in SSRIM is shown in Table 6.6.2.

<table>
<thead>
<tr>
<th>Entrance Number</th>
<th>UNIF Down</th>
<th>UNIF Up</th>
<th>EXPO10 Down</th>
<th>EXPO10 Up</th>
<th>EXPO20 Down</th>
<th>EXPO20 Up</th>
<th>BS Down</th>
<th>BS Up</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0.243</td>
<td>0.684</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0.163</td>
<td>0.451</td>
<td>1.261</td>
<td>1.296</td>
<td>0.558</td>
<td>0.877</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0.151</td>
<td>0.157</td>
<td>1.046</td>
<td>1.075</td>
<td>0.979</td>
<td>0.887</td>
<td>0.041</td>
<td>0.231</td>
</tr>
<tr>
<td>5</td>
<td>0.356</td>
<td>0.583</td>
<td>0.792</td>
<td>0.602</td>
<td>1.025</td>
<td>1.048</td>
<td>0.163</td>
<td>0.127</td>
</tr>
<tr>
<td>6</td>
<td>0.303</td>
<td>0.192</td>
<td>0.663</td>
<td>0.61</td>
<td>0.792</td>
<td>0.673</td>
<td>0.956</td>
<td>1.159</td>
</tr>
<tr>
<td>7</td>
<td>0.525</td>
<td>0.667</td>
<td>0.596</td>
<td>0.628</td>
<td>0.738</td>
<td>0.684</td>
<td>0.956</td>
<td>0.803</td>
</tr>
<tr>
<td>8</td>
<td>0.419</td>
<td>0.23</td>
<td>0.455</td>
<td>0.425</td>
<td>0.703</td>
<td>0.683</td>
<td>0.758</td>
<td>0.797</td>
</tr>
<tr>
<td>9</td>
<td>0.663</td>
<td>0.762</td>
<td>0.341</td>
<td>0.323</td>
<td>0.668</td>
<td>0.669</td>
<td>0.516</td>
<td>0.351</td>
</tr>
</tbody>
</table>

Table 6.6.2 Mean number of LC12 per stack in SSRIM

In general, LC21 has a similar pattern as LC12. When vehicles can easily change lanes in some sections of a highway, space in a stack can be used for both directions of lane changes until one slot is full. Therefore, LC12 and LC21 both
increase or decrease in a section. Table 6.6.3 shows the mean number of LC21 per stack in GSRIM and SSRIM.

<table>
<thead>
<tr>
<th>Entrance Number</th>
<th>GSRIM</th>
<th></th>
<th></th>
<th></th>
<th>SSRIM</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>UNIF</td>
<td>EXPO10</td>
<td>EXPO20</td>
<td>BS</td>
<td>UNIF</td>
<td>EXPO10</td>
<td>EXPO20</td>
<td>BS</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0.807</td>
<td>0.86</td>
<td>0</td>
<td>0</td>
<td>0.798</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0.933</td>
<td>0.85</td>
<td>0</td>
<td>0</td>
<td>0.874</td>
<td>0.831</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0.586</td>
<td>1.471</td>
<td>0.807</td>
<td>0</td>
<td>0.615</td>
<td>0.797</td>
<td>0.776</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>0.194</td>
<td>0.712</td>
<td>0.785</td>
<td>0.444</td>
<td>0.149</td>
<td>0.946</td>
<td>0.732</td>
<td>0.447</td>
</tr>
<tr>
<td>6</td>
<td>0.707</td>
<td>1.01</td>
<td>0.769</td>
<td>0.732</td>
<td>0.695</td>
<td>0.589</td>
<td>0.976</td>
<td>0.049</td>
</tr>
<tr>
<td>7</td>
<td>0.193</td>
<td>0.734</td>
<td>0.752</td>
<td>0.764</td>
<td>0.133</td>
<td>0.516</td>
<td>0.651</td>
<td>1.001</td>
</tr>
<tr>
<td>8</td>
<td>0.796</td>
<td>0.482</td>
<td>0.738</td>
<td>0.852</td>
<td>0.745</td>
<td>0.661</td>
<td>0.625</td>
<td>0.747</td>
</tr>
<tr>
<td>9</td>
<td>0.165</td>
<td>0.455</td>
<td>0.727</td>
<td>0.995</td>
<td>0.103</td>
<td>0.486</td>
<td>0.61</td>
<td>0.804</td>
</tr>
<tr>
<td>10</td>
<td>0.89</td>
<td>0.047</td>
<td>1.2</td>
<td>0.158</td>
<td>0.796</td>
<td>0.412</td>
<td>0.597</td>
<td>0.057</td>
</tr>
</tbody>
</table>

Table 6.6.3 Mean number of LC21 per stack in GSRIM and SSRIM

In some cases, exiting vehicles may stay in the upper slot because the lower slot is full. Hence, in order to guarantee successful exits in the simulation model, one more LC21 will be executed for these vehicles to form a single exiting platoon in the lower slot. This operation can be implemented because slots are designed to contain two platoons of full size or three platoons of size $2N_{max}-2$. This causes larger LC21, especially in short trip-length distributions.

Having an exit close to a downstream entrance may be a drawback in terms of lane change. However, it increases the release rate of the entrance. The space occupied by exiting vehicles is freed immediately upstream from the entrance and the distance between ramps is not sufficient for a lane change. Therefore, the whole space is used for admitting vehicles from the entrance. If the distance allows lane changes to take place, then vehicles from upper slots will occupy part of the freed space and the release size (i.e. the number of vehicles released) is reduced.
Table 6.6.4 compares the mean number of vehicles released per stack in “Up” and “Down” cases with the rule GSRIM. Mean release sizes per stack at the first few entrances are the same due to full stacks. The release size increases 20% at entrance 5 in UNIF. However, the release size drops at some entrances because the limitation on lane changes breaks the lane assignment rules, which are designed to maximize release rates. For instance, the mean release size drops 20.3% at entrance 3 in EXPO10. Lower slots have more vehicle destinations or less space to admit vehicles while upper slots may release exiting vehicles and provide space for lane changes. Hence, unsuccessful lane changes in upstream sections can be completed and vehicles in stacks are arranged again by lane assignment rules. As a result, the sum of release rates, shown in the last row of the table, doesn’t increase significantly, especially in short trip-length distributions like EXPO10.

<table>
<thead>
<tr>
<th>Entrance Number</th>
<th>GSRIM-Mean Release Size</th>
<th>UNIF</th>
<th>EXPO10</th>
<th>EXPO20</th>
<th>BS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Down</td>
<td>Up</td>
<td>Down</td>
<td>Up</td>
<td>Down</td>
</tr>
<tr>
<td>2</td>
<td>0.978</td>
<td>0.978</td>
<td>5.398</td>
<td>5.398</td>
<td>3.121</td>
</tr>
<tr>
<td>3</td>
<td>1.048</td>
<td>1.048</td>
<td>4.428</td>
<td>4.568</td>
<td>3.057</td>
</tr>
<tr>
<td>4</td>
<td>1.102</td>
<td>1.102</td>
<td>3.592</td>
<td>4.509</td>
<td>2.953</td>
</tr>
<tr>
<td>5</td>
<td>1.161</td>
<td>0.968</td>
<td>4.478</td>
<td>3.899</td>
<td>2.939</td>
</tr>
<tr>
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<td>1.238</td>
<td>1.191</td>
<td>4.041</td>
<td>3.942</td>
<td>2.32</td>
</tr>
<tr>
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<td>1.111</td>
<td>3.734</td>
<td>3.357</td>
<td>3.058</td>
</tr>
<tr>
<td>8</td>
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<td>1.387</td>
<td>3.333</td>
<td>3.22</td>
<td>2.927</td>
</tr>
<tr>
<td>9</td>
<td>1.492</td>
<td>1.306</td>
<td>3.152</td>
<td>3.117</td>
<td>2.845</td>
</tr>
<tr>
<td>10</td>
<td>1.611</td>
<td>1.666</td>
<td>3.038</td>
<td>2.907</td>
<td>2.747</td>
</tr>
</tbody>
</table>

Table 6.6.4 Mean release size per stack in GSRIM

6.7 Conclusions

Simulations conducted in this chapter focus on three issues: verification of analytic models, derivation of number of lane changes, and comparison of system performance under different configurations.
To verify analytic models, a dynamic lane assignment rule is applied to maximize space for admitting vehicles and minimize number of destinations in lower slots. Since, in both simulation and analytic models, the same assumptions are used and results are comparable, the greedy method developed to establish lane assignment rules maximizing system throughput is verified. Mathematical representations of slot assignment rules are also verified.

Under the dynamic lane assignment rule, the number of lane changes increases by 17.5% in UNIF and 19.4% in BS under GSRIM and increases insignificantly in short trip-length distributions due to more available space for lane changes. Hence, the dynamic lane assignment rule can be used to increase the system throughput without requiring large ramp spacing in a system of vehicles with short trips.

A lane change process involving the cooperation of two slots and adjustment of space for vehicles moving to specific positions requires more time and space. Hence, it becomes a critical factor in determining ramp spacing given the time of lane change. The number of LC12 presents a similar pattern as the number of LC21 since space can be used for lane changes in both direction and a lane change in one direction creates space for lane changes in the opposite direction. Hence, in general, both LC12 and LC21 increase or decrease in a section. However, the accumulation of unsuccessful lane changes may break the pattern. Compared to the rule of Random Stack, lane/slot assignment rules reduce the number of lane changes, especially in short trip-length distributions.

Vehicle waiting space restricts the entrance capacity. SS can nearly achieve its theoretical maximum release rate with the minimal space while EJSSRIM requires the largest space. By this study, we can design the waiting space at an entrance based on traffic demand or we can estimate entrance capacities based on slot assignment rules, the current waiting space, and the specified service level. The
service level is represented by percentage of blocked vehicles from entering an entrance. For example, the waiting space in SS is 17 meters, GSRIM 41 meters, and EJSSRIM 107 meters for maintaining 80% of their maximum release rates and 95% of time that vehicles can enter the entrance. If the waiting space is fixed for storing 30 vehicles, then SSRIM can release 1029 vehicles per hour, EJSSRIM 872, and GSRIM 1828 vehicles per hour.

The location of an exit relative to its immediate upstream entrance affects number of lane changes completed given fixed ramp spacing. In the situation that an exit is close to its downstream entrance, unsuccessful lane changes increase because space is not available in stacks. As a result, fewer lane changes occur over several sections followed by one or two sections with more lane changes. Furthermore, release rates increase at some entrances and decrease at others while the system throughput has an insignificant change.

Analytic models provide theoretical maximum system performance, which declines when considering limited ramp space and varied ramp spacing. The amount that performance declines can be predicted in simulation. Microscopic vehicle maneuvers are not simulated in this chapter since they involve complicated vehicle control algorithms. These algorithms determine the timing of maneuvering processes, which are regarded as input parameters in simulation models. With the advance of these algorithms, these parameters also vary. Therefore, simulation models in this chapter provide a macroscopic view of system performance under various system configurations. Some highway configurations are mentioned in Chapter 7 for future research.
Chapter 7

Conclusions

The objective of this report is to optimize performance of Automated Highway Systems through management of space, accounting for interaction between entrance and exit processes. To accomplish this objective, we developed a comprehensive framework, including a new integrated highway model called the moving slot model, and operational strategies, called slot/lane assignment rules. We summarize this research in Section 7.1 and describe major contributions in Section 7.2. Section 7.3 provides related research topics, extending the framework.

7.1 Research Summary

Highway capacity is related to inter-vehicle spacing. On conventional highways, human drivers decide the space by their perception of safety. Hence, the space may be either short resulting in the reduction of safety or long resulting in the reduction of capacity. The concept of Automated Highway Systems is proposed to boost capacity while maintaining safety by transferring the control of vehicles from human drivers to automatic devices. In this concept, a vehicle must keep at either a long distance to avoid collision or a short distance to reduce the damage upon colliding with its preceding vehicle. However, this concept does not specifically state issues related to vehicle maneuvering spacing. These issues are critical in determining achievable system performance. Therefore, the objective of this report is to optimize highway performance through management of space.

To achieve this goal, we develop a comprehensive framework consisting of a new highway model, called the moving slot model, and corresponding operational strategies, called slot/lane assignment rules. The design of this framework is based on a heavy traffic condition in order to relieve congestion. The model, inheriting the
moving cell model and the platoon model, defines space for vehicles and vehicle maneuvers. Operational strategies, in cooperation with the model, minimize space requirements for vehicle maneuvers on/between ramps.

In the moving slot model, highway space is divided into slots. In a one-lane AHS, a slot is a basic operational unit containing a platoon to produce high capacity and space for vehicle maneuvers, without affecting other slots. This design provides independence among slots. The length of a slot is adaptable to meet specific requirements under various operational scenarios. In a multi-lane AHS, an operational unit, called a stack, comprises well-aligned slots, one on each lane. Independence also exists among stacks.

The selection of the slot speed is based on minimizing travel time while keeping an acceptable capacity. As a result, the speed of 30m/s is adopted, considering the speed limit of most highways passing urban areas. Since all lanes are operated at the same speed, no speed difference among highway lanes is concluded.

The design of slot assignment rules accounts for vehicle types (e.g. trucks or passenger cars) and vehicle destinations (e.g. range and sequence). In this research, we focus on passenger cars, and slots can serve vehicles destined to all downstream exits. However, vehicles in a slot must be arranged in special patterns of destinations. These patterns result in only one exiting group of vehicles in one slot when passing an exit. This minimizes the ramp space for vehicles to exit. We assume that a slot implements one maneuver at a time and vehicles are served by the rule of First-Come-First Serve. We propose five slot assignment rules: Sorted Slot (SS), Sorted Slot with Release Improvement Mechanism (SSRIM), End-Join Sorted Slot (EJSS), End-Join Sorted Slot with Release Improve Mechanism (EJSSRIM), and Grouped Slot with Release Improvement Release (GSRIM). The Release Improve Mechanism (RIM) is proposed to improve the release rate by sorting vehicles without breaking the First-Come-First-Serve rule.
The selection of slot assignment rules applied at an entrance depends on the number of entrance lanes and the distance to the immediate downstream ramp. SS and EJSS can only be applied at entrances with one lane, since the operation of RIM needs two lanes. Generally, GSRIM provides the highest release rate and the fastest recovery of highway flow but requires the longest ramp spacing to complete maneuvering processes. EJSS and EJSSRIM require the minimal ramp spacing. GSRIM must be a unique slot assignment rule applied on a highway, whereas the other rules belonging to a family of sorted slots can be applied alternatively along a highway.

Lane assignment rules specify the lane a vehicle should use in each section of an AHS, from entrance to exit, based on the destination or characteristics of the vehicle. In this research, lane assignment rules are developed based on concepts of workload (Hall, 1995) and path intercepts (Ramaswamy, 1995). These rules are established by specifying a destination number, called a lane division, for each entrance on a highway. Vehicles in a stack move to lanes by comparing their destinations and lane divisions when passing entrances. On a two-lane highway, vehicles with destinations up to the lane division are assigned to the right lane; otherwise there are assigned to the left lane. Vehicles destined to the lane division can stay on both lanes but we prefer to assign them to the left lane to reserve more space to admit vehicles. We propose a greedy method to derive lane divisions based on the maximization of throughput. This method concludes that, in steady state, a lane division is chosen such that vehicles in a stack fill the upper slot and lane divisions are determined sequentially from upstream to downstream entrances.

The moving slot model is integrated with slot assignment rules and lane assignment rules in a multi-lane AHS. Vehicles from entrances are assigned target lanes. Lane changes to target lanes may involve serial selections of slots. Hence, in this sense, vehicles executing lane assignment rules execute a series of slot assignment rules. Many results from a two-lane AHS can be applied on a multi-lane AHS such as the operational unit is also a stack. When there are more lanes on a
highway, SSRIM performs closer to GSRIM in terms of throughput. Because most vehicles move to upper slots, the lowest slot will more likely be empty to admit more vehicles released from entrances.

We verify the framework by simulation. We propose a dynamic lane assignment rule to maximize system throughput. This rule states that a stack determines the use of lanes based on current destinations of vehicles such that the upper slot is occupied first by vehicles with longer trips. Mean release rates and lane divisions from both analytic and simulation models are comparable. However, the number of lane changes generally increases. The amount of increase is insignificant in short-trip length distributions due to more available space for lane changes. This implies that a dynamic lane assignment rule increases the system throughput without requiring large ramp spacing when trip lengths are short.

We also use simulation to study the impacts on system performance from the limited vehicle waiting space and the location of an exit relative to its upstream entrance. Limited vehicle waiting space restricts the entrance capacity. For example, the waiting space of 17 meters in SS, 41 meters in GSRIM, and 107 meters in EJSSRIM enables 80% of entrance capacities assuming that, 95% of time, vehicles can enter the entrance. The location of exit affects lane changes and the system throughput. In the situation that an exit is located closer to its downstream entrance, lane changes become more infrequent because stacks are more likely to be full. This causes fewer lane changes over several sections, but one or two following sections have more lane changes. Mean release rates increase at some entrances and decrease at the others while the system throughput, the sum of total flow rates at ramps, has an insignificant increment.

7.2 Contributions

In this report, we present a comprehensive framework to optimize highway flow. This framework is designed under a heavy traffic condition and provides
double the capacity of conventional highways. Higher capacity is achievable by altering parameters such as increasing the number of vehicles in a slot or increasing acceleration/deceleration rates. Therefore, it is able to relieve congestion.

The management of space is essential to operate a fully automated highway system efficiently and effectively. In this framework, the management of space accounts for vehicles and vehicle maneuvers. An operational unit contains a platoon to increase capacity and space for vehicle maneuvers, which varies with operational scenarios, such that independence exists among operational units. Therefore, this framework not only simplifies the control of AHS but also adapts to various system requirements.

The installation of an AHS on a current highway seems more feasible as long as the modification of the highway configuration is minimal. This framework provides strategies to minimize the ramp space required for vehicles to exit and strategies to minimize the number of lane changes, which can be translated into the space required between ramps for lane changes. Therefore, this framework expedites the application of AHS by reducing the modification of current highway configurations.

We develop mathematical representations of distributions of number of vehicles released under five slot assignment rules (i.e. SS, SSRIM, EJSS, EJSSRIM, and GSRIM) by assuming that there is no shortage of vehicles at entrances. The entrance capacities can be represented by these distributions. We also develop a greedy method to establish lane assignment rules. These analytic models are verified by simulation.

We also conduct simulations on systems with limited vehicle waiting space and with the varied location of exits. The results provides us criteria to design the vehicle waiting space at an entrance and the location of an exit relative to its upstream entrance under specific requirements of system performance or to estimate the system performance given the waiting space or the location of an exit.
7.3 Future Research

The following research interests are to continuously refine space management techniques, to extend the proposed framework to various system configurations, and to deal with the flow balancing between automated highways and local streets to avoid spillback destructing highway operation.

Space Sharing

The $L_{SA}$ is 25 meters under a speed difference of 10m/s and a constant acceleration rate of $2m/s^2$. It increases with the speed difference and reduces capacity. Hence, we attempt to manage the speed adjustment space to enhance capacity.

Both $L_{SA}$ and $L_{inter}$ are reserved in a slot for safety. However, unlike $L_{inter}$, which must be present at all times in case of an emergency, $L_{SA}$ is needed only when there is a lane change between lanes operated at different speeds. Therefore, it may be efficient to share a $L_{SA}$ among a group of adjacent slots. Several contiguous slots sharing a $L_{SA}$ become an operational unit. To fully implement this idea, we need to identify the sharing mechanism in one unit and ranges of destinations for each slot in a unit to gain better system performance.

Varied Highway Configurations

The system we have discussed in this report is an isolated highway with dedicated ramps. However, highways intersect one another, especially in urban areas. In these intersection areas, interactions among slots from different highways draw attention. We need a sophisticated mechanism to merge slots from different highways in a safe way.

A partially automated highway consists of manual lanes, automated lanes, and one transition lane between them. A transition lane functions like ramps for
automated lanes. However, vehicles on the transition lane are different from those at ramps in the following ways:

1. They are operated at a specified speed.
2. Vehicles to adjacent manual or automated lanes may be alternately located. Hence, it may be difficult to implement slot assignment rules.
3. Vehicles on the transition lane may form platoons before entering the adjacent automated lane. This process may cause significant delay.
4. A platoon may separate on the transition lane to become individual vehicles before entering the adjacent manual lane. This process requires adjustment of space.

A partial AHS would allow all vehicles, automatic or manual, to use the highway, which seems more feasible at the early stage of application of AHS. Therefore, a detailed study is justified.

**Dynamic Destination Assignment**

In AHS, capacity and flow rates at ramps are expected to increase. A spillback may occur when an exit or the local system cannot absorb increasing flows. To avoid a spillback, vehicles destined to a congested destination may be guided to downstream exits of excessive capacity. These vehicles will travel farther. However, the travel time may decline due to bypassing congested areas. It is challenging to re-assign vehicles in real-time traffic because available capacities at exits and local streets vary. There is also a priority issue for assigning vehicles to exit when capacities are limited.

Therefore, the problem is to determine a set of priority rules for re-assigning destinations based on dynamic traffic conditions with the objective to optimize the system performance in terms of total travel time and additional miles traveled.
REFERENCES


