Adaptive Feedforward and Anti-windup Compensation for Amplitude and Rate Saturation with Application to Precision Control

by

Shiying Zhou

A dissertation submitted in partial satisfaction of the requirements for the degree of
Doctor of Philosophy

in

Mechanical Engineering

in the

Graduate Division

of the

University of California, Berkeley

Committee in charge:

Professor Masayoshi Tomizuka, Chair
Professor Andrew Packard
Professor Javad Lavaei

Spring 2017
Adaptive Feedforward and Anti-windup Compensation for Amplitude and Rate Saturation with Application to Precision Control

Copyright 2017
by
Shiying Zhou
Abstract

Adaptive Feedforward and Anti-windup Compensation for Amplitude and Rate Saturation with Application to Precision Control

by

Shiying Zhou

Doctor of Philosophy in Mechanical Engineering

University of California, Berkeley

Professor Masayoshi Tomizuka, Chair

Precision motion systems have stringent error tolerance and are subject to different sources of disturbances. These disturbances can be periodic or non-periodic, wide band or narrow band and concentrate at different frequency ranges. Moreover, actuators are subject to saturation problems, which can lead to degraded servo performance and even instability when the controllers are commanding over the physical limits of the actuators. In order to investigate the precision motion control strategies for disturbance rejection as well as saturation compensation, we study the vibration control and anti-windup compensation for the amplitude and rate saturations for the hard disk drive servo system, especially for the dual-stage system.

The decoupled sensitivity design is presented to separate the dual-stage system into two single-input-single-output systems. Then the baseline controllers are designed using the Linear Quadratic Gaussian/Loop Transfer Recovery to shape the overall sensitivity function. First, in order to reject periodic disturbances, two repetitive disturbance observers (RDOBs) are added. The additional RDOBs are proven to be add-on components to the overall sensitivity, thus extending repetitive narrow notches at the harmonic frequencies. As a result, the stability of the system is not influenced. And the two actuators are limited to operate at different frequencies for repetitive disturbance rejection.

Second, the adaptive feed-forward control is designed to reject the low frequency wide-band vibrations. Two structures are proposed, in which pre-identification of either the plant model or the sensitivity model is performed. The infinite impulse response (IIR) filter is adapted, which is different from the conventional method using a finite impulse response filter. Besides, the error convergence is proven rigorously with assumptions. Therefore, the adaptive feed-forward control can approximate more complex unknown vibration transfer dynamics.

Third, the saturation problem of the secondary actuator in the dual-stage hard disk drive system is discussed. To address this, the linear conditioning structure is exploited. The anti-windup controllers for the amplitude saturation are synthesized by formulating
linear matrix inequality (LMI) optimizations. Robustness of the system is also analyzed. We combine the existing theorems and mathematical tools such as integral quadratic constraints, Schur complement, S-procedure, etc. and present procedural steps to formulate the LMI optimization problem for control synthesis. It is different from the method that utilizing the Lyapunov theory, thus does not involve the difficulty to come up with a feasible Lyapunov function.

Finally, both amplitude and rate saturations are considered together. A generalized anti-windup controller is synthesized by formulating the saturation problem into a robust control problem. With additional loop transformation, the anti-windup controller is synthesized by $\mu$-synthesis. This technique is less conservative and generalizable to systems with multiple different saturations. Note that, all the proposed techniques are add-on controls, which means the original baseline controls remain unmodified. This brings flexibility into design and implementation.
To My Family and Friends
# Contents

## List of Figures

vi

## List of Tables

iv

## 1 Introduction

1.1 Background ................................................................. 1
1.2 Literature Review .......................................................... 3
1.3 Overview and Contributions of this Dissertation ......................... 7
1.4 Dissertation Outline ....................................................... 10
1.5 Notations ........................................................................... 11

## 2 Dual-stage Model and Baseline Control Design with Repetitive Disturbance Rejection

2.1 Chapter Overview ............................................................ 12
2.2 Dual-stage HDD Plant Model .............................................. 12
2.3 Decoupled Sensitivity Design .............................................. 15
2.4 Baseline Control by Discrete Linear Quadratic Gaussian/ Loop Transfer Recovery ........................................... 16
2.5 Disturbance Observer for Repetitive Disturbance Rejection ............ 21
2.6 Chapter Summary ............................................................. 31

## 3 Adaptive Feedforward Design with Infinite Impulse Response Filter

3.1 Chapter Overview ............................................................ 32
3.2 Filtered-x Least Mean Square Method ................................... 32
3.3 Adaptive Feedforward Control Design .................................. 35
3.4 Simulation .......................................................................... 45
3.5 Chapter Summary ............................................................. 50

## 4 Amplitude Saturation Compensation

4.1 Chapter Overview ............................................................ 51
4.2 Amplitude Saturation Model and Linear Conditioning Framework .... 51
4.3 Anti-windup Controller for the Dual-stage HDD .................................. 53
4.4 Robustness Analysis .............................................................................. 59
4.5 Simulations .......................................................................................... 64
4.6 Chapter Summary .................................................................................. 69

5 Amplitude and Rate Saturation Compensation ......................................... 70
  5.1 Chapter Overview .................................................................................. 70
  5.2 Saturation Problem in Robust Control Framework ................................. 70
  5.3 Simulation ........................................................................................... 79
  5.4 Chapter Summary .................................................................................. 86

6 Conclusions and Future Works ................................................................. 88
  6.1 Summary .............................................................................................. 88
  6.2 Future Work ........................................................................................ 90

Bibliography .............................................................................................. 92

A List of Abbreviations .............................................................................. 98
List of Figures

1.1 Disturbance sources in a typical HDD servo system ........................................ 2
1.2 Exemplar spectrum of repetitive error ............................................................ 2
1.3 Examples of audio vibration spectra ................................................................. 3
1.4 Conventional Disk Drive Assembly and PZT-actuated Suspension ..................... 4
1.5 Magnitude response of the sensitivity of a dual-stage HDD ............................... 4
1.6 Sensitivity Decoupling Design For Dual Stage Systems ................................. 8

2.1 Actuators on dual-stage HDDs ................................................................... 13
2.2 bode plot for nominal and full model of dual stage HDD ............................... 14
2.3 Sensitivity Decoupling Design For Dual Stage Systems ................................. 16
2.4 LQG/LTR Result For VCM and PZT Loop ...................................................... 20
2.5 Sensitivity functions of the VCM, PZT loops and the overall sensitivity ........... 21
2.6 Block diagram of discrete control system with DOB ....................................... 22
2.7 Equivalent diagram from $u^*(k)$ to $y(k)$ In Figure 2.6 .............................. 22
2.8 Magnitude response of $\frac{1}{1-z^{-m}}$ with sampling frequency $T_s = 1.$ ........ 23
2.9 Equivalent block diagram of system in Figure 2.6 .......................................... 24
2.10 magnitude response of $1 - z^{-m}Q$ and $Q$ filter with different values of $\alpha$. Credits of [53] .............................................................. 25
2.11 Decoupled dual stage system with two RDOBs ............................................. 26
2.12 Overall sensitivity function with Discrete LQG/LTR controllers and RDOBs .... 28
2.13 PZT loop sensitivity function ................................................................. 29
2.14 Position Error Signal (PES) Spectra with only repetitive disturbances ............. 30
2.15 Position Error Signal (PES) Spectra with all disturbances .......................... 31

3.1 Filtered-x Least Mean square method for vibration rejection ....................... 33
3.2 A general feedback control system ............................................................... 35
3.3 Feedback control system with input disturbances modeling ........................ 36
3.4 Adaptive feedforward control ................................................................ 36
3.5 Deterministic and Stochastic disturbance models ......................................... 37
3.6 Proposed adaptive feedforward control structure-AFF1 .............................. 38
3.7 Proposed adaptive feedforward control structure-AFF2 ............................ 39
3.8 Equivalent block diagram of the one in Figure 3.6 ....................................... 41
3.9 Equivalent feedback loop of the adaptive system ........................................ 44
3.10 Frequency response of $S$ and $\hat{S}$ ................................................................. 46
3.11 Frequency response of full model times the nominal model ............................... 47
3.12 Power spectrum and time sequence of error signal with proposed adaptive feed-
forward control ........................................................................................................ 48
3.13 Actual VTP dynamics and the steady state IIR filter in $AFF1$ and $AFF2$ ........... 49

4.1 A feedback system with saturation and anti-windup ........................................ 52
4.2 Equivalent structure of the one in Figure 4.1 ....................................................... 53
4.3 The decoupled dual-stage system with amplitude saturation ............................... 54
4.4 AW compensation for a dual-stage system with sensitivity decoupling design ...... 54
4.5 Equivalent representation of the conditioning configuration in Fig. 4.4 .............. 55
4.6 Upper LFT representation of the nonlinear loop from $u_{lin}$ to $y_d$ in Figure 4.5 .... 57
4.7 A feedback system with AW compensation and plant uncertainty ..................... 60
4.8 Equivalent structure of the one in Figure 4.7 ....................................................... 61
4.9 Nonlinear Loop for robustness in Figure 4.8 ....................................................... 62
4.10 Nonlinear Loop for performance in Figure 4.8 .................................................... 63
4.11 Magnitude response of the sensitivity functions of the dual-stage system .......... 65
4.12 Error signal spectrum with white noise vibration at $0 \sim 5000Hz$ ..................... 66
4.13 Zoom in plot of $700Hz \sim 2500Hz$ in Figure 4.12 ............................................. 67
4.14 Comparison for different vibration gain $1, 3, 5, 7, 9$ ....................................... 67
4.15 Transient performance with external sudden shocks different frequencies ......... 68

5.1 Dual-stage model with amplitude and rate saturations ....................................... 71
5.2 Generalized anti-windup scheme in dual-stage systems ................................... 72
5.3 Robust control design structure to solve $K_{2x2}$ in Figure 5.2 ............................ 73
5.4 Robust control design structure with loop transformation of $\Delta_\alpha$ and $\Delta_v$ .... 75
5.5 Enhanced AW design for the dual stage HDD system with a filter $N_{vcm}$ ........... 76
5.6 $\mu$ synthesis design structure in LFT form for amplitude saturation ................. 77
5.7 Magnitude plot of a peak filter ............................................................................. 77
5.8 Error signal spectrum with white noise vibration at $0 \sim 5000Hz$ ..................... 79
5.9 Zoom in plot of $1800Hz \sim 3800Hz$ in Figure 5.8 ............................................. 80
5.10 Comparison for different vibration gain $1, 3, 5, 7, 9$ ....................................... 81
5.11 Transient performance with external sudden shocks different frequencies ......... 82
5.12 Weighting filters $W_1$ and $W_2$ ..................................................................... 83
5.13 Error spectrum with audio vibration from a song (with $W_1$) ......................... 84
5.14 Error spectra from $500Hz$ to $1500Hz$ ............................................................. 84
5.15 Error signal with single sinusoid input at $1000Hz$ (with $W_2$) ....................... 85
5.16 PES time response with sudden shock at $1000Hz$ (with $W_1$) ...................... 86
5.17 PES time response with sudden shock at $1000Hz$ (with $W_2$) ....................... 87

6.1 Summary of the thesis contribution .................................................................. 89
List of Tables

2.1 Parameters of the Resonance Modes of the VCM and PZT Plants . . . . . . . . 14
2.2 Gain margin and phase margin for the VCM loop and PZT loop . . . . . . . . 20
3.1 Simulation results . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 48
Acknowledgments

This thesis marks the end of my journey as a PhD student and my four years of study at UC Berkeley. I was lucky to get to know the most professional professors and study the challenging and interesting courses, which improved my ability as a learner and a researcher. I was lucky to meet and work with the intelligent students and lab mates here at UC Berkeley, which was definitely an amazing experience in my life. At the end of this journey, there are many people that I want to thank.

First and foremost, I would like to express my special thanks to my advisor, Professor Masayoshi Tomizuka. He has been guiding me with patience and giving me many valuable advice on my research and my career. In my most bewildered moments, he encouraged me to pursue my favorite career and provided me with great supports through my years of PhD study. His open-mind to embrace the most advanced researches and knowledge inspired me to stay curious about the world and never stop learning for the rest of my life.

I want to thank Professor Andrew Packard for being an inspiring teacher. His ME231 and ME234 led me into the world of control theories. He always gave the lectures with the greatest passion and with rigorous and detailed explanations. His diligent attitudes towards work and positive attitudes towards life encouraged me to work harder and never relax the constraints on myself. I want to thank Professor Javad Lavaeiyanesi for serving as my committee member. His guidance and advice in the writing of this dissertation was very helpful.

I want to thank Computer Mechanics Lab for providing the fundings to support my research and study. I want to thank Western Digital for giving me great internship opportunities. Thank you to the colleagues, Guoxiao, Edward, Xi, Li, Haiming for their inspiring discussion and help. I also want to thank Professor Bogy for welcoming me and providing me a study desk in the CML lab.

I want to express my special thanks to Yizhou Wang, as a great friend, colleague and mentor. I am very grateful for his trust and guidance, which gave me new possibilities for my future careers. I want to thank Minghui Zheng for being a helpful friend and a productive partner in research. It was a very nice experience working with her and I benefited a lot from the inspiring discussion we had. I want to thank the Mechanical Systems Control labmates, Xu, Liting, Changliu, Wei, Cong, Wenlong, Junkai, Chung-Yen, Raechel, Chen-Yu, Kevin, Hsien-Chung, Te, Yongxiang, Dennis, Xiaowen, Yu, Cheng, Daisuke, Jianyu, Shuyang, Kiwoo, Zining, Yeping, Yujiao, Yu-Chu, Jiachen, Chen, Zhuo and Taohan.

Especially, I want to thank my parents, Fengying and Zhanwei, my brother Kairong for their love and supports throughout my long academic journey. Words can not express how grateful I am for their sacrifices. I want to thank my parents-in-law, Yi and Zepeng, for treating me as their own daughter with their unselfish love and supports. Finally, I want to thank my beloved husband, Haoyu Wu, who is always a wonderful friend and caring soul mate. He always has strong faith in me and I would not have pursued and survived this PhD journey without him.
Chapter 1

Introduction

1.1 Background

Nowadays, despite the emerging of solid state drive, hard disk drives (HDDs) are still used extensively as cost-effective and reliable solutions for data storage. In addition to its usual utilization in desktops and laptops, HDDs become the primary storage medium for the majority of cloud data storage centers currently. The control tasks in HDDs are generally categorized into two types. One is referred to as track seeking and the other is track following. Track following requires that the read/write head be positioned over the data track center at nano-scale accuracy, which requires an error tolerance around 7nms in a 2013 HDD. As the track density of data stored on the magnetic disk quickly approaches 200,000 tracks per inch, the servo control challenges are significantly amplified.

Meanwhile, the HDD servo system is subject to various sources of disturbances due to hardware imperfection, environmental disturbance and special errors due to task nature [1]. A typical disk drive with various internal or external sources of disturbances and errors is depicted in Figure 1.1 [2]. These disturbances influence the track seeking/following performance, which is measured by three times the head position variance of the true error signal (PES), i.e., $3\sigma_{PES}$. The PES is the deviation of the read/write head from the desired track center, which is composed of two major types of errors: repetitive error and non-repetitive error.

The repetitive error is usually caused by the eccentricity of the track, offset of the track center with respect to the spindle center or motor geometry etc [2]. An example of repetitive error spectrum is shown in Figure 1.2. They have definite temporal pattern and remain the same every time the disk is spun. This causes the so-called repeatable runout (RRO) during the track following control operation. The non-repetitive error comes from internal disk drive vibration or external audio vibration and shocks, combined with the electrical noise, cause the non-repeatable runout (NRRO). Examples of the external vibration spectra presented in a typical HDD are shown in Figure 1.3.

Besides, in order to satisfy the more and more stringent requirements on HDD servo
Figure 1.1: Disturbance sources in a typical HDD servo system

Figure 1.2: Exemplar spectrum of repetitive error
performance, a secondary actuator is introduced in addition to the conventional single stage actuation utilizing the voice coil motor (VCM). A piezoelectric actuator (PZT) is added to extend or retract the suspension to position the read/write head [3], see Figure 1.4. This dual actuation scheme improves the track seeking and following performance by increasing the achievable bandwidth and enhancing the vibration suppression. However, the inclusion of a secondary actuator transforms the system into a dual-input-single-output (DISO) system. Moreover, the VCM has a moving range about 1 inch, while the displacement range of the PZT is limited to $1 \sim 2\mu m$, which then sets a limit on the amplitude of the PZT actuation signal. Thus, during track seeking and track following under the external vibration, the PZT actuation signal saturates easily while the VCM control signal remains free from the saturation. Furthermore, the changing rate of the PZT input signal is also constrained to avoid possible damages. Therefore, the PZT input signal is subject to both the amplitude and the rate saturations, which leads to degraded vibration suppression performance and unacceptable bandwidth. More severely, it may cause stability issues. Figure 1.5 shows the magnitude frequency response of the sensitivity function of a dual-stage HDD system before and after PZT saturation.

1.2 Literature Review

According to the aforementioned servo challenges in HDD system, the goal of this dissertation is to design control techniques to address the following issues:

- Baseline control design for the DISO dual-stage HDD system;
• External non-repetitive vibration that is concentrated within low frequency range (around 0 ~ 2000Hz);
• Repetitive disturbances that are at the rotational frequency of the spindle and its multiples;
• Amplitude and rate saturations of the HDD secondary actuator.
For the baseline control design of the DISO dual-stage HDD system, various control design architectures have been proposed, see \cite{4, 5, 6, 7, 8} and others. They can be mainly classified into two categories. One is based on the classical single-input-single-output (SISO) design technique by properly design observers to decouple the control loop. Examples are the master-slave technique \cite{8}, the decoupled sensitivity design approach \cite{6}, the PQ method \cite{5} and a direct parallel design approaches. The other is based on modern state-space-based multi-input-multi-ouput (MIMO) control methodologies. Examples are optimal and robust control techniques, including but not limited to LQG/LTR, H-\infty, \mu-synthesis control \cite{7, 9, 10, 11}.

Vibration rejection methods include feedback and feed-forward controls. The conventional feedback methods use loop shaping to shape the sensitivity function of the system \cite{12, 3}. One limitation of feedback based methods is the trade-off between bandwidth and stability margin. The increase of the bandwidth is often accompanied by the decrease in stability margin. Another limitation is due to the waterbed effect in linear system by Bode's Integral Theorem \cite{13}, i.e., the sensitivity suppression in some frequency regions will inevitably lead to sensitivity amplification in other regions. So the compromise between vibration rejection and unwanted high frequency amplification is an issue in the conventional feedback design methods. Therefore, one should be aware of this limitation and minimize the detrimental effects when designing controllers for the linear servo systems.

Due to the aforementioned limitations in conventional linear feedback design, feed-forward design methods are investigated in this dissertation. Generally, the disturbances information is necessary for the feedforward control design. In some cases, the models of the disturbances are known in advance and can be directly modeled for vibration suppression, as addressed in \cite{14, 15}. In other cases, the disturbances entering the servo system can be measured by properly installed sensors. Then by offline or online design of additional filters, the feed-forward signal can be generated and injected into the loop to cancel the disturbances. However, the disturbances nature and the HDD dynamics are subject to change. Different units of HDDs will cause system variations and so do using and aging of the HDDs. Therefore, it will be advantageous to make the vibration compensation adaptive.

Among different adaptive feed-forward control methods for vibration suppression, the Filtered-x Least Mean Square (FxLMS) method \cite{16} has been widely used in HDD vibration suppression. The FxLMS method cancels the vibration effect with measured vibration signal by utilizing the output from an adaptive finite impulse response (FIR) filter. There are benefits for using adaptive FIR filter in feed-forward compensation. First, the dynamics of an FIR filter is straightforward that the output is only determined by the input signal, which means the signals needed for the adaptation can be obtained effortlessly; Second, due to simplicity of the FIR dynamics, no stability issue needs to be considered during the adaptation; Finally, simple gradient algorithm like least mean square (LMS) \cite{16} can be applied to update the FIR coefficients with a fast and well-understood convergence behavior for a simple unknown vibration transmission dynamics \cite{17}. For example, gradient algorithms were used to adapt an FIR filter in real time to approximate the vibration transmission dynamics in HDD \cite{18, 19}. 
For suppression of the repetitive vibration, repetitive control (RC) [20, 21] is a well-known control design technique for systems that are subject to periodic disturbances or references. An internal model \( \frac{1}{1-z^{-N}} \), where \( N \) is the period of the disturbance or reference, is incorporated into the feedback system such that errors in the previous repetition can be used to improve the current tracking or regulation performance. Repetitive control has been tested and verified in several applications such as track following in HDDs, industrial robot manipulators executing repetitive tasks, vehicle regulation control and so on [22, 23, 24]. Moreover, the internal model of the repetitive disturbance can be incorporated into the closed loop feedback system in the form of the disturbance observer (DOB), in which the repetitive error presented in the feedback system is extracted by the DOB and injected back to the plant directly to cancel the repetitive disturbance. A repetitive disturbance observer (RDOB) is proposed to suppress the repetitive disturbances without large gain amplification at other undesired frequencies [25].

To deal with the actuator amplitude saturation problem, various approaches have been proposed. Among these approaches, the gain-scheduling approach and the anti-windup (AW) approach are attractive in practice. In these techniques, the control design for the baseline performance and the constraint handling are decoupled [26, 27, 28]. In the gain-scheduling approach, the gains of the baseline controllers are adjusted when saturations are detected. It is straightforward to implement but the stability is difficult to guarantee. In the anti-windup approach, conditioning schemes are designed and optimization problems are formulated to synthesize the anti-windup filters [29, 30]. Among these methods, the linear matrix inequality (LMI) based approaches were developed to synthesize the AW compensator in a systematic way. AW techniques were applied to the secondary stage of the dual stage HDD system without exploiting the control capacity of the primary stage [28, 31]. Besides, the focuses are mainly on the track seeking process while the performance improvement of the track following process has not been addressed. An additional filter was designed in a patent to involve the primary stage for AW compensation [32]. However, the filter was designed in an ad-hoc way, which requires physical knowledge of the system and control design expertises to manually tune the filter parameters recursively. The stability of the overall system is still difficult to guarantee.

Anti-windup algorithms to address both amplitude and rate saturations have been studied recently [33, 34, 35]. However, these algorithms still have limitations in the practical applications. Design complexity is the first challenge. Secondly, the baseline feedback control has to be redesigned in some scenarios, which involve extensive trial-and-error effort. A practical AW scheme to address both amplitude and rate saturations in HDDs was discussed in [32]. Nevertheless, the stability of the overall system remains difficult to guarantee in these approaches.


1.3 Overview and Contributions of this Dissertation

In this dissertation, the goal is to address the aforementioned issues stated in Section 1.2. We draw extensively from the existing literature, exploit the strength and extend the techniques to solve the aforementioned control challenges. The proposed methods focus on schematic procedures that are simple in concepts and easy to design and implement. Besides, the proposed controls are all add-on structures, which means the original controls remain unchanged.

Baseline Control Design and Decoupled Repetitive Disturbance Observer

The sensitivity decoupling approach, as shown in Figure 1.6, provides the basic framework for designing the baseline controller for the dual-stage HDD system. In the figure, the discrete time z-domain index (z) is omitted for simplicity. \( P_v, P_m, \hat{P}_m, C_v \) and \( C_m \) are respectively the VCM plant, the PZT plant, the nominal PZT plant, the VCM controller and the PZT controller. The closed loop sensitivity function is calculated as

\[
S = \frac{1}{(1 + C_vP_v)(1 + C_mP_m) + C_vP_vC_m(\hat{P}_m - P_m)}.
\]

Design \( \hat{P}_m = P_m \) at the frequencies below the system bandwidth and let \( C_vP_v \) be small at the frequencies where \( \hat{P}_m \neq P_m \), then \( C_vP_vC_m(\hat{P}_m - P_m) \approx 0 \) at all frequencies. Therefore, the closed loop sensitivity function is decoupled into the VCM and PZT loop sensitivity functions respectively. The sequential SISO loop shaping techniques can then be applied to satisfy the stability and performance requirements. Therefore, the discrete Linear Quadratic Gaussian/ Loop Transfer Recovery (LQG/LTR) technique was applied to the two stages separately to design the baseline controllers \( C_m \) and \( C_v \).

Then two enhanced Repetitive Disturbance Observer (RDOB) are extended to the dual stage structure separately for the periodic disturbances rejection. Selective band Q filters are designed separately for both RDOBs. Therefore, the VCM actuator can operate primarily in the low frequency range (0 ~ 1000Hz) and the PZT actuator operates in the mid-frequency range (1000 ~ 2000Hz) to minimize saturation effects.

Adaptive Feed-forward Control With Infinite Impulse Response Filter For Vibration Suppression

Compared to the adaptive FIR, the infinite impulse response (IIR) based adaptive feed-forward control has two benefits:

1. It provides more accurate approximation to the actual vibration transmission path dynamics which is more closer to an IIR structure;
2. It is more efficient that it can approximate the unknown dynamics with fewer parameters than an FIR filter.

However, there are also some drawbacks that hinder the application of the adaptive IIR filters:

1. Since the IIR filters represent the coupling relationship between input and output, the signals needed for the parameter adaptation can not be obtained without proper modification of the closed-loop structure. So the modification should be feasible for implementation with stable and converging behavior;

2. The poles of the IIR filter may go beyond the unit circle during adaptation which causes instability;

3. The convergence speed for the adaptive IIR filter is generally not so desirable.

Therefore, despite that the vibration control using adaptive IIR filter has been investigated since at least the mid-1980s [36] and applications using adaptive IIR filter for noise cancellation exist in many literatures [37, 38], there are still some questions that are not answered explicitly. First, the general algorithm used for adaptive IIR filter is the recursive least square (RLS) algorithm in the output-error (OE) form, which is bimodal [37]; Second, the stability of the OE form algorithm is often difficult to prove and guarantee [39]. The alternative is to use the equation-error (EE) form with the RLS algorithm. However, it is much more complicated because of the need to generate estimates of the desired signals [40]. Besides, the error convergence of the algorithm was not proved rigorously, either. Furthermore, the application of the adaptive IIR filter for feed-forward compensation in HDDs is hardly found except that an IIR filter was designed offline in [41].

Concerning the above challenges with the adaptive IIR filter, this dissertation focuses on an adaptive feed-forward control for vibration suppression based on an IIR filter structure and its application in HDDs [42]. Two designs are proposed to extract the necessary signals
for parameter adaptation with additional filtering to guarantee the error convergence. Stability of the adaptive IIR is handled by projecting the poles into the unit circle. The RLS algorithm is exploited for the parameter adaptation with additional signal normalization and deadzone handling for algorithm stability and robustness. Besides, the error convergence of the resulting adaptation algorithm is rigorously proved. The relative convergence speeds of the IIR-RLS and FIR-LMS techniques are already studied in [37]. It was shown that the rates of convergence between these two algorithms are comparable. The proposed designs are verified in simulation. Superior compensation performance at steady state and faster error convergence with complicated vibration transmission dynamics are shown. The proposed schemes are also robust to the color noise as long as the bound for the noise is known and is not excessively large. In addition, it can be integrated into the existing baseline control structure trivially.

**Anti-windup Compensation For Amplitude and Rate Saturated System**

To deal with the amplitude saturation problem of the secondary actuator in the dual-stage HDD system, an AW compensation schemes is proposed which may be regarded as an enhanced version of the scheme in [28, 31]. Firstly, the AW controllers are added to the decoupled dual stage HDD system in a different linear conditioning framework [43] from the one in [28, 31]. It is shown that the proposed configuration can also be decoupled into a linear loop and a nonlinear loop, from which a linear-matrix-inequality (LMI) optimization problem can be formulated [44]. Besides, the LMI optimization problem is formulated via the Integral Quadratic Constraint (IQC) [45]. Secondly, an additional filter is designed to further enhance the AW compensation performance. By treating the saturation as a norm bounded uncertainty and introducing loop transformation, the anti-windup problem can be solved by formulating a robust control problem. Meanwhile, the stability and performance of the overall system can be guaranteed simultaneously. The proposed design technique is thus straightforward and less-time consuming.

To further address the slew-rate saturation of the control signal, a generalized anti-windup scheme to address both amplitude and rate saturations is proposed [46]. It is based on the linear conditioning scheme and is generalized from the proposed amplitude saturation compensation scheme. The baseline control does not need to be modified. The anti-windup filters are designed by optimizing the induced $l_2$ norm from the disturbance to the position error signal. Such optimization is transformed to an equivalent robust control problem which is solved by MATLAB robust control toolbox efficiently. Likewise, the stability of the overall system is guaranteed. Various types of weighting functions are also provided for different performance goals. In summary, the technique is simple in concept and effective in application. Besides, it can be readily extended to a general system with multiple different saturations.

The track following/seeking performance is investigated by the simulation of the dual
stage HDD system subject to amplitude/rate saturations. The results show performance improvement compared to an existing method for amplitude saturation. The effectiveness of the proposed anti-windup scheme for amplitude and rate saturations are also verified by comparing with the saturated and non-saturated performance.

The contributions of this dissertation can be summarized as follows:

- Baseline control for the dual-stage system is designed using the decoupled sensitivity technique and the LQG/LTR method;
- Adaptive feed-forward control for vibration suppression based on an IIR filter structure is designed. Two structures for parameter adaptation are proposed. Besides, the error convergence is proved explicitly;
- An enhanced anti-windup scheme is proposed for the dual-stage system to solve the amplitude saturation problem of the secondary actuator.
- A generalized anti-windup scheme considering both amplitude and rate saturations is proposed. It is an add-on linear conditioning structure which consists of a dual-input-dual-output filter. Loop transformations to reduce redundant constraints and weighting functions to accommodate different performances are also discussed.

1.4 Dissertation Outline

The remainder of this dissertation is organized as follows. In Chapter 2, the fundamental results of the decoupled sensitivity design, the discrete LQG/LTR control and the disturbance observer are reviewed and summarized. Combined with the HDD models, the baseline controllers for the dual-stage HDD are designed. We then formulate the repetitive DOB for the dual-stage system and design two selective band filters to limit the operation of the two actuators in the selective frequency ranges. In Chapter 3, the adaptive feed-forward control design with the IIR filter for vibration suppression is discussed in detail. In Chapter 4 and 5, we introduce the amplitude and rate saturation problems, provide control solutions and robustness analysis. Chapter 6 concludes the thesis work and provides some future works.
# 1.5 Notations

The following general notations are used in the thesis:

<table>
<thead>
<tr>
<th>Notations</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(k)$</td>
<td>discrete time index</td>
</tr>
<tr>
<td>$u(k)$</td>
<td>discrete time signal</td>
</tr>
<tr>
<td>$U(z^{-1})$</td>
<td>Z-transform of $u(k)$</td>
</tr>
<tr>
<td>$q^{-1}$</td>
<td>one-step delay operator, which satisfies $u(k) = q^{-1}u(k + 1)$</td>
</tr>
<tr>
<td>$G(z^{-1})$</td>
<td>discrete-time transfer function</td>
</tr>
<tr>
<td>$G(z)$</td>
<td>discrete-time transfer function by replacing $z^{-1}$ with $z$</td>
</tr>
<tr>
<td>$\omega$</td>
<td>frequency index</td>
</tr>
<tr>
<td>$(\hat{\cdot})$</td>
<td>estimate of some value</td>
</tr>
<tr>
<td>$S$</td>
<td>sensitivity function from disturbance to output error</td>
</tr>
<tr>
<td>$C$</td>
<td>controller</td>
</tr>
<tr>
<td>$P$</td>
<td>plant model</td>
</tr>
<tr>
<td>$P^{-1}$</td>
<td>plant inverse</td>
</tr>
<tr>
<td>$G_{u(k)\rightarrow y(k)}$</td>
<td>transfer function from $u(k)$ to $y(k)$</td>
</tr>
<tr>
<td>$rpm$</td>
<td>revolutions per minute</td>
</tr>
<tr>
<td>$sec$</td>
<td>second</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>standard deviation</td>
</tr>
<tr>
<td>$E(\cdot)$</td>
<td>expectation operator</td>
</tr>
<tr>
<td>$Sat(\cdot)$</td>
<td>saturation operator</td>
</tr>
<tr>
<td>$Dz(\cdot)$</td>
<td>deadzone operator</td>
</tr>
<tr>
<td>$sign(\cdot)$</td>
<td>sign of a signal</td>
</tr>
<tr>
<td>$|\cdot|_2$</td>
<td>$l_2$ norm of a square-summable sequences/signal or induced $l_2$ norm of a mapping</td>
</tr>
<tr>
<td>$(\cdot)^T$</td>
<td>transpose</td>
</tr>
<tr>
<td>$(\cdot)^{-1}$</td>
<td>inversion</td>
</tr>
</tbody>
</table>
Chapter 2

Dual-stage Model and Baseline Control Design with Repetitive Disturbance Rejection

2.1 Chapter Overview

In this chapter, the dual-stage HDD model is described in detail. Beside, we review some fundamental results of the decoupled sensitivity design, the discrete LQG/LTR control and the disturbance observer. We then apply and adapt the control techniques to design a baseline control for the dual-stage HDD system. The repetitive DOB for the dual-stage system are also designed with two selective band filters to limit the operation of the two actuators in the selective frequency ranges. The system achieved a good baseline loop shape with enhanced periodic disturbances rejection.

2.2 Dual-stage HDD Plant Model

Data in an hard disk drive (HDD) are arranged in concentric circles or tracks and are read or written with a read/write head. The two main functions of the head positioning servomechanism in disk drives are track seeking and track following. Track seeking moves the read/write head from the present track to a specified destination track in minimum time using a bounded control effort. Track following maintains the head as close as possible to the destination track center while information is being read from or written to the disk. The read/write head is actuated by Voice Coil Motor (VCM) in single-stage HDDs. However, with the ever increasing demand for larger storage capacity in HDDs, a secondary actuator, the piezoelectric-based (PZT) actuator is added to the VCM actuator to break the bottleneck of the single-stage HDDs. In this enhanced mechanical structure, the PZT is extended to the conventional VCM actuator for accurate positioning, as shown in Figure 2.1. In this so-called dual-stage scenario, the VCM provides coarse motion (typically \(\sim\) inches) for the
positioning servo due to its wide range of motion while the added PZT actuator provides faster and finer positioning with a limited range of motion (typically $\sim \mu m$).

A practical model for the VCM plant in a dual-stage HDD can be of more than twenty orders, as the magnitude frequency response is shown in the lower plot of Figure 2.2 from a HDD benchmark package [47]. The PZT plant has fewer resonances than the VCM plant, but the resonances appear at some of the same frequencies as those of the VCM plant. The transfer function for the VCM and PZT in this study are expressed as

$$G_{vcm}(s) = \frac{K_v}{s^2} + \sum_{i=1}^{5} \frac{g_{vi}w_{vi}^2}{s^2 + 2\zeta_{vi}w_{vi}s + w_{vi}^2}; \quad (2.1)$$

$$G_{pzt}(s) = K_p + \sum_{i=1}^{2} \frac{g_{pi}w_{pi}^2}{s^2 + 2\zeta_{pi}w_{pi}s + w_{pi}^2}. \quad (2.2)$$

$G$ represents the plant model, the subscript $vcm$ and $pzt$ are for VCM and PZT respectively. $K_v$ and $K_p$ are the gains of the nominal VCM and PZT models. The second order parts represent the resonance modes of the VCM and PZT models. The notations and parameters for the resonance modes are shown in Table. 2.1. These resonances are usually attenuated by notch filters which can be designed intuitively. Therefore, a nominal model is usually considered as the control design plant which captures the primary model properties. For the VCM, the discrete time nominal model can be described as

$$\begin{bmatrix} x_{v1}(k+1) \\ x_{v2}(k+1) \end{bmatrix} = A_v \begin{bmatrix} x_{v1}(k) \\ x_{v2}(k) \end{bmatrix} + B_v u_v, \quad (2.3)$$
where \( u_v \) is the VCM actuator input, \( x_{v1} \) is the position of the VCM head in the unit of tracks, \( x_{v2} \) is the velocity, and

\[
A_v = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}; \\
B_v = \begin{bmatrix} T^2 k_y k_v \\ T k_y k_v \end{bmatrix}.
\]  

(2.4)
T is the sampling time, \( k_y \) is the position measurement gain and \( k_v \) is the acceleration constant. For the PZT, the nominal model is a pure gain and can be described as

\[
x_p(k+1) = 0x_p(k) + k_p u_p,
\]

where \( x_p \) is the position of PZT head in the unit of tracks. \( k_p \) is a constant gain which is determined by the PZT plant. \( u_p \) is the PZT control signal. The overall output of the system is the combined displacement of the VCM plus PZT actuator, which is described as

\[
y(k) = y_v(k) + y_p(k) = C_v x_v(k) + C_p x_p(k). \tag{2.6}
\]

The system parameters are as follows:

- the rotation speed: 7200 \( rpm \);
- the number of servo sector: 220;
- the sampling time: \( T = 3.7879 \times 10^{-5} sec \);
- the acceleration constant: \( k_v = 951.2 \frac{m}{(s^2 A)} \);
- the position measurement gain: \( k_y = 3.937 \times 10^6 \) track \( \cdot m^{-1} \);
- the output gain: \( C_v = 1 \) and \( C_p = 1 \).

The magnitude frequency responses of the low-order nominal models are shown in the upper plot of Figure 2.2.

### 2.3 Decoupled Sensitivity Design

One consideration for the controller design in dual-stage HDDs is that, even though in the dual-stage HDDs, the VCM and PZT are actuated together to generate the motion of the read/write head, the only available measurement is the displacement of the read/write head slider, which is derived from the embedded servo sector pattern on the surface of the HDDs. Thus, the control design problem for the dual-stage HDDs becomes a control problem of a dual-input single-output (DISO) system. If the relative displacement of the secondary stage actuator with respect to the VCM can be measured or estimated, the control design for the DISO system becomes a single-input-single output (SISO) control problem. The decoupled sensitivity design in Figure 2.3 is one of such techniques to allow for the independent SISO control design, where \( P_v, P_m, \hat{P}_m, C_v \) and \( C_m \) are respectively the VCM plant, the PZT plant, the nominal PZT plant, the VCM controller and the PZT controller. Note the discrete time z-domain index \( z \) is omitted for simplicity.

The open loop transfer function for the structure is derived as

\[
L = C_v P_v (1 + C_m \hat{P}_m) + C_m P_m. \tag{2.7}
\]
So the sensitivity function from disturbance $d$ to output $y$ is

$$S = \frac{1}{(1 + C_v P_v)(1 + C_m P_m) + C_v P_v C_m (\hat{P}_m - P_m)}.$$  \hspace{1cm} (2.8)

For the term $C_v P_v C_m (\hat{P}_m - P_m)$, let $P_m = \hat{P}_m$ at the low frequencies. At the high frequencies, design the loop shape of $C_v P_v$ to roll off quickly, as it is the conventional requirement for a desirable loop shape. Thus, the term $C_v P_v C_m (\hat{P}_m - P_m)$ is small at all frequencies, and the sensitivity function in (2.8) is approximated as

$$S \approx \frac{1}{(1 + C_v P_v)(1 + C_m P_m)}.$$  \hspace{1cm} (2.9)

Therefore, as long as the individual controller for the VCM and PZT stages are stable and properly designed, the overall stability is guaranteed.

### 2.4 Baseline Control by Discrete Linear Quadratic Gaussian/ Loop Transfer Recovery

**Introduction**

The linear quadratic (LQ) optimal control ensures the asymptotically stability by simply designing the regulation matrices $Q$ and $R$. It has attractive robustness property of the optimal state feedback system under the assumptions of controllability/stabilizability and observability/detectability. Whilst, the steady state Kalman filter provides an asymptotically stable stochastic state observer by choosing noise covariance matrices $W$ and $V$. When there is noise in the control system, the linear quadratic Gaussian (LQG) method was proposed \[48\] to combine the optimal state feedback controller and the least square Kalman filter. The controllers and estimators can be machine computed. However, when the state estimator is
CHAPTER 2. DUAL-STAGE MODEL AND BASELINE CONTROL DESIGN WITH REPETITIVE DISTURBANCE REJECTION

Included, the nice properties of either LQ control or Kalman filter are no longer present. In order to exploit the design benefits of the LQ controller and Kalman filter, the loop transfer recovery (LTR) method is introduced for its usefulness in the sense of design.

Recover the transfer loop of the linear quadratic Gaussian control is thus called linear quadratic Gaussian/loop transfer recovery (LQG/LTR). It is aimed to recover the robustness and sensitivity properties of the optimal state feedback control through the output feedback. This technique simplifies the use of the LQG control and achieves the practical feedback control performance with a reasonable amount of effort.

In the LQG/LTR process, only one pair of the cost weighting matrices \( (Q \text{ and } R) \) or noise covariance matrices \( W \) and \( V \) need to be specified by the designer. Then the other pair is automatically assigned during the recovery process. This results in a tremendous reduction in the complexity of the design. While the continuous time LQG/LTR exists, we exploit the Discrete LQG/LTR method for the baseline controller design. The mechanism by which the recovery is achieved is essentially the same as that for the continuous time case: the controller cancels the plant zeros and possibly some of the stable poles, then inserts the additional zeros to properly shape the loop. So LQG/LTR does not apply to the plant that has zeros outside the unit circle, due to the internal stability requirement. The main theorem for the recovery result is provided.

**Theorem 1.** Let the open loop design plant model has a state space realization \((A, B, C, D)\). If the open loop transfer function of the design plant model has no finite zero in \( \{z : |z| > 1\} \) and \( \det(CB) \neq 0 \), then

\[
\lim_{R \downarrow 0} G_p G_{LQG} = G_{TFL} \tag{2.10}
\]

where \( R \) is the control weighting matrix, \( G_p = C(zI - A)^{-1}B \) is the open loop transfer function from control to output, and \( G_{LQG} \) is the LQG controller and \( G_{TFL} \) is the target feedback loop to be recovered by the LQG/LTR method.

**Design Procedures**

Since the VCM and PZT sensitivity functions are decoupled in the overall sensitivity, the design of VCM and PZT controllers are separated. Therefore, discrete SISO LQG/LTR is applied to design the individual VCM and PZT controllers. We provide the process for the VCM controller design, and the design steps are the same for the PZT controller except a plant model change.

1. The state space model for the VCM plant is

\[
x_v(k + 1) = A_v x_v(k) + B_v u_v(k) \\
y_v(k) = C_v x_v(k). \tag{2.11}
\]

\(^1z\) is the z-domain notation
Incorporate the desired controller property, which is a single integrator in this case,

\[ u_v = \frac{Tz}{z - 1} u, \quad \text{(2.12)} \]

where \( u \) is the real control input. So the design plant model is

\[
\begin{bmatrix}
    x_v(k + 1) \\
    x_c(k + 1)
\end{bmatrix} = \begin{bmatrix}
    A_v & B_v C_c \\
    0 & A_c
\end{bmatrix} \begin{bmatrix}
    x_v(k) \\
    x_c(k)
\end{bmatrix} + \begin{bmatrix}
    B_v D_c \\
    B_c
\end{bmatrix} u(k) \\
\triangleq A_e \begin{bmatrix}
    x_v(k) \\
    x_c(k)
\end{bmatrix} \triangleq B_e \quad \text{(2.13)}
\]

where \((A_c, B_c, C_c)\) is the state space realization from the real control input \( u \) to model control \( u_v \).

2. Design discrete time steady state Kalman filter. Denote \( x_e \triangleq \begin{bmatrix} x_v \\ x_c \end{bmatrix} \) as the extended state. The extended system of (2.13) with zero mean input noise \( w(k) \) and output noise \( v(k) \) is

\[
x_e(k + 1) = A_e x_e(k) + B_e u(k) + Lw(k) \\
y_e(k) = C_e x_e(k) + v(k), \quad \text{(2.14)}
\]

where \( E(w(k)w^T(k)) = 1 \) and \( E(v(k)v^T(k)) = \mu \). \( L \) and \( \mu \) are two design parameters in the discrete LQG/LTR design. Choose \( L \) and \( \mu \) properly, usually let \( L = B_e \), and solve the discrete algebraic Riccati equation (DARE) below

\[
M = A_e M A_e^T - A_e M C_e^T (C_e M C_e^T + \mu)^{-1} C_e M A_e^T + LL^T. \quad \text{(2.15)}
\]

The Kalman filter gain is obtained as

\[
F = M C_e^T (C_e M C_e^T + \mu)^{-1}. \quad \text{(2.16)}
\]

Therefore, the target feedback loop to be recovered is

\[
G_{TFL} = C_e (zI - A_e)^{-1} A_e F. \quad \text{(2.17)}
\]

3. Solve the discrete time cheap control problem where the cost function is

\[
J = \frac{1}{2} \sum_k \{x_e^T(k)Q x_e(k) + R u^2(k)\};
\]

\[
Q = C_e^T C_e. \quad \text{(2.18)}
\]
CHAPTER 2. DUAL-STAGE MODEL AND BASELINE CONTROL DESIGN WITH REPETITIVE DISTURBANCE REJECTION

The DARE for this ‘cheap’ control problem is

\[ P = A_e^T P A_e + Q - A_e^T P B_e (R + B_e^T P B_e)^{-1} B_e^T P A_e. \] (2.19)

So the feedback gain is then

\[ K = -(R + B_e^T P B_e)^{-1} B_e^T P A_e. \] (2.20)

In summary, the discrete LQG/LTR controller is calculated as

\[ G_{LQG} = z K \{ z I - (I - FC_e)(A_e - LK) \}^{-1} F, \] (2.21)

which is summarized in Proposition 1 with proof.

**Proposition 1.** The discrete time LQG/LTR controller for the design plant model

\[ x_e(k + 1) = A_e x_e(k) + B_e u(k) \]
\[ y_e(k) = C_e x_e(k) \]

can be written as

\[ G_{LQG} = z K \{ z I - (I - FC_e)(A_e - LK) \}^{-1} F. \] (2.22)

**Proof.** The state space representation for the discrete time LQG controller is

\[ \hat{x}_e(k + 1|k) = A_e \hat{x}_e(k|k) + B_e u(k) + F [y(k + 1) - C_e A_e \hat{x}_e(k|k) - C_e B_e u(k)] \] (2.23)
\[ u(k) = K \hat{x}_e(k|k). \] (2.24)

Substituting (2.24) into (2.23), and note that $B_e = L$, we obtain the transfer function from $y(k + 1)$ to $u(k)$

\[ G_{y(k+1)\to u(k)} = K [z I - (A_e - LK) + FC_e (A_e - LK)]^{-1} F. \] (2.25)

Advance for one step yields the transfer function from $y(k)$ to $u(k)$, as is in (2.22).

If it is impossible or not a good choice to use $\hat{x}_e(k|k)$ for state estimation, $\hat{x}_e(k|k - 1)$ can be used instead for the control implementation. The LQG/LTR controller in this case is

\[ G_{LQG} = K (I - FC_e) [z I - (A_e - B_e K)(I - FC_e)]^{-1} (A_e - B_e K) F + FK, \] (2.26)

where $(A_e, B_e, C_e)$ are the state space matrices of the design plant model. The control action is expressed as

\[ u(k) = K \hat{x}_e(k|k - 1) \] (2.27)

Proof can be seen in [49] and [50].
CHAPTER 2. DUAL-STAGE MODEL AND BASELINE CONTROL DESIGN WITH REPETITIVE DISTURBANCE REJECTION

<table>
<thead>
<tr>
<th></th>
<th>VCM</th>
<th>PZT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gain Margin (dB)</td>
<td>4.2766</td>
<td>9.0726</td>
</tr>
<tr>
<td>GM Frequency (Hz)</td>
<td>4386</td>
<td>13199</td>
</tr>
<tr>
<td>Phase Margin (Degree)</td>
<td>69.8</td>
<td>119.3</td>
</tr>
<tr>
<td>PM Frequency (Hz)</td>
<td>982</td>
<td>1156.5</td>
</tr>
</tbody>
</table>

Table 2.2: Gain margin and phase margin for the VCM loop and PZT loop

Loop Shaping Results by LQG/LTR

An integrator is included in the VCM plant for extra disturbance regulation at low frequencies and \( L = B_e \). The noise parameter is \( \mu = 10^8 \) and the cheap control weight is \( R = 1e^{-13} \). The final controller is the LTR control in series with the single integrator. A gain 5000 is added to the final controller for bandwidth extension. The final loop shape is in Figure 2.4a. The gain margin and phase margin are shown in Table 2.2.

For the PZT loop, the nominal model is a pure gain, so a lag compensator is extended to the nominal model. LTR is performed with \( L = B_e, \mu = 5e^{-5} \) and \( R = 2e^{-4} \). Likewise, an extra gain of 20 is added to the final controller. Note that due to the PZT design plant model being a pure gain, this LQG/LTR procedure is actually automatically achieving a proper loop shape for the PZT loop. The tuning parameters also include the two cornering frequencies of the lag compensator at the first step. The loop shapes are shown in Figure 2.4b. The gain margin and phase margin are in Table 2.2.

The individual sensitivity and overall sensitivity for the dual-stage HDDs are shown in Figure 2.5.

![LTR result for VCM loop](image1)

![LTR result for PZT loop](image2)

(a) LTR result for VCM loop (b) LTR result for PZT loop

Figure 2.4: LQG/LTR Result For VCM and PZT Loop
2.5 Disturbance Observer for Repetitive Disturbance Rejection

Disturbance Observer

Uncertainties exist in the control of mechanical systems, which include friction, varying load inertia, un-modeled dynamics, actuator saturation, backlash, sensor noise and so on. There are various classic control techniques to handle uncertainties. They can be generally classified into two categories, robust control and adaptive control. Disturbance Observer (DOB) is one of the robust control approaches for handling disturbance in motion control. It is introduced in [51] in 1987 and refined in [52] in 1991. The discrete time version of the DOB is shown in Figure 2.6. The structure in the red rectangle is the DOB. The plant model is $P(z^{-1}) \approx z^{-m} P_n(z^{-1})$, where $P_n(z^{-1})$ is the nominal plant model without delay. $m$ is the pure delay of $P(z^{-1})$. The $Q$ filter is the design parameter to ensure good servo performance and robustness. The signal coming into the $Q$ filter is essentially the delayed disturbance signal, contaminated by model mismatch between real plant $P(z^{-1})$ and nominal plant $P_n(z^{-1})$. Therefore, the $Q$ filter should be cautiously design for system robustness and stability.
By rearranging the block diagram, the forward transfer path from signal \( u^*(k) \) to \( y(k) \) in Figure 2.6 is transformed into the structure in Figure 2.7. The transfer function from \( d(k) \) to \( y(k) \) is thus

\[
G_{d(k)\rightarrow y(k)}(z^{-1}) = \frac{P(z^{-1})}{1 + \frac{1}{1-z^{-m}Q(z^{-1})} \frac{Q(z^{-1})}{P_n(z^{-1})} P(z^{-1})}
\]

\[
= \frac{(1 - z^{-m}Q(z^{-1}))P(z^{-1})P_n(z^{-1})}{(1 - z^{-m}Q(z^{-1}))P_n(z^{-1}) + Q(z^{-1})P(z^{-1})}.
\]

If \( Q(z^{-1}) = 0 \), \( G_{d(k)\rightarrow y(k)}(z^{-1}) = P(z^{-1}) \), which means disturbance is filtered by the plant dynamics without attenuation; if \( 1-z^{-m}Q(z^{-1}) = 0 \), then (2.28) is equivalent to \( G_{d(k)\rightarrow y(k)}(z^{-1}) = 0 \), which means disturbance is completely suppressed. Therefore, by designing \( Q(z^{-1}) \) properly, the desired disturbance rejection dynamics can be incorporated into the system.
CHAPTER 2. DUAL-STAGE MODEL AND BASELINE CONTROL DESIGN WITH REPETITIVE DISTURBANCE REJECTION

23

Recall that the sensitivity function for the linear feedback control systems is \( \frac{1}{1+G_{OL}} \), where \( G_{OL} \) represents the open loop transfer function of the feedback system. For the rest of the thesis, the \( z \)-domain notation (\( z^{-1} \)) will be omitted if no mis-understanding is caused. In presence of disturbances with known modes, internal model principle refers to that, a dynamic compensator having a factor of the known disturbance model is included into the open loop function to achieve error regulation. This is achieved by the high gain in the open loop \( G_{OL} \) induced by the internal model of the disturbance. To reject repetitive disturbances, repetitive control (RC) exploits the internal model principle. RC absorbs the internal model \( \frac{1}{1-z^{-N}} \) into the open loop transfer function \( G_{OL} \), thus setting the open loop gain to infinity or a large value at the known periodic disturbance frequencies depending on \( N \). As a result, comb-shape peaks are created by this internal model \( \frac{1}{1-z^{-N}} \) in the open loop magnitude response, as shown in Figure 2.8. However, these comb-shape peaks are not narrow enough that they also create gain amplifications in neighboring frequencies of the comb center frequencies. Therefore, this phenomena leads to unnecessary suppression in the sensitivity at the frequencies around disturbance frequencies. Due to limitations from Bode’s Integral Theorem, sensitivity amplifications at other frequencies where non-repetitive disturbance may concentrate is expected, which degrades the overall control performance for disturbance rejection. This problem becomes more severe if there are large non-periodic components in the disturbance source.

Figure 2.8: Magnitude response of \( \frac{1}{1-z^{-N}} \) with sampling frequency \( T_s = 1 \).
**Enhanced Repetitive Disturbance Observer**

Due to limitation in conventional repetitive control, a modified version of the internal model for the periodic disturbance is proposed in [53]. This method discussed an enhanced repetitive controller in the context of DOB. A tuning parameter $\alpha$ is embedded to adjust the gain amplification at surrounding frequencies of the periodic frequencies.

For regulation purpose, the reference signal in Figure 2.6 is set to zero. Then the structure in Figure 2.6 can be rearranged and transformed into the structure in Figure 2.9. Denote the transfer function from $e(k)$ to $u(k)$ as $C_{eq}$, i.e., equivalent controller, then

$$C_{eq} = \frac{C + QP_n^{-1}}{1 - z^{-m}Q}.$$  \hspace{1cm} (2.29)

So the open loop transfer function is

$$G_{OL} = \frac{CP + QP_n^{-1}P}{1 - z^{-m}Q}.$$  \hspace{1cm} (2.30)

Then the sensitivity function is just

$$S(z^{-1}) = \frac{1}{1 + G_{OL}} = \frac{1 - z^{-m}Q}{1 + PC + (PP_n^{-1} - z^{-m})Q}.$$  \hspace{1cm} (2.31)

In the frequency range where the plant $P$ is well modeled by $z^{-m}P_n$, it is true that $PP_n^{-1} - z^{-m} \approx 0$ and this condition generally applies at low frequencies up to slightly above the system bandwidth. Thus, the sensitivity function in (2.31) is rewritten as

$$S = \frac{1 - z^{-m}Q}{1 + PC}.$$  \hspace{1cm} (2.32)
Figure 2.10: magnitude response of $1 - z^{-m}Q$ and $Q$ filter with different values of $\alpha$. Credits of [53]

which is just the original sensitivity $\frac{1}{1+PC}$ with additional shaping from $1 - z^{-m}Q$.

To design repetitive narrow notches at periodic frequencies, [53] proposed $Q$ filter that satisfies

$$1 - z^{-m}Q = \frac{1 - z^{-N}}{1 - \alpha^N z^{-N}},\quad 0 \leq \alpha < 1.$$  \hspace{1cm} (2.33)

If there are only repetitive disturbances that satisfies

$$(1 - q^{-N})d(k) = 0,$$  \hspace{1cm} (2.34)

then from (2.28) or (2.31), it is straightforward that the periodic disturbance is attenuated at the repetitive frequencies. Besides, with the additional tuning parameter $\alpha$ in (2.33), the amplification in non-repetitive frequencies can be adjusted by tuning parameter $\alpha$. This is shown by the magnitude response corresponding to different $\alpha$ value in Figure 2.10.

**Enhanced Repetitive DOB for Dual-stage HDDs**

In this thesis, we apply the enhanced RDOB proposed in [53] to the dual-stage HDD with decoupled sensitivity structure. Two selective band $Q$ filters are designed separately for the
VCM loop and PZT loop to reject periodic disturbances. This is to ensure that the control capacities of both control loops can be fully exploited while at the same time not to command too much from the PZT actuator which saturates easily.

Generally, the VCM actuator works mainly at low frequencies (0 – 1000Hz) and the PZT actuator works in the middle frequency range (1000 ~ 2000Hz). Thus, two different frequency range RDOBs are added to the VCM loop and PZT loop, which are controlled as shown in Figure 2.3. To add the RDOB in the VCM loop, exploit the concept of the equivalent controller as derived in (2.29) and the structure in Figure 2.9, the RDOB for the VCM is shown in Figure 2.11. In the figure, the VCM model $P_V \approx z^{-m_1} \hat{P}_V$ below the VCM loop bandwidth. $\hat{P}_V$ is the nominal VCM plant model without the delay step $m_1$. Likewise, the RDOB for the PZT loop is denoted as RDOB2 in Figure 2.11, where $P_m = z^{-m_2} \hat{P}_m$ below the PZT loop bandwidth and $m_2$ is the delay.

According to the sensitivity decoupling result in (2.8) and (2.9), let the equivalent controller for the VCM loop and PZT loop be $C_{V(eq)}$ and $C_{m(eq)}$ respectively, then the overall sensitivity function with two add-on RDOBs is

$$S = \left( \frac{1}{C_{V(eq)} P_V + 1} \right) \left( \frac{1}{C_{m(eq)} P_m + 1} \right), \quad (2.35)$$
where the equivalent controllers are

\[ C_{V(eq)} = \frac{C_V + Q_V \hat{P}_V^{-1}}{1 - z^{-m_1}Q_V} \]

\[ C_{m(eq)} = \frac{C_m + Q_m \hat{P}_m^{-1}}{1 - z^{-m_2}Q_m} \]  

(2.36)

where \( Q_V \) and \( Q_m \) are the \( Q \) filters for the VCM loop and the PZT loop respectively. So the overall sensitivity function is rewritten into

\[ S = \left( \frac{1 - z^{-m_1}Q_V}{1 + P_V C_V + (P_V \hat{P}_V^{-1} - z^{-m_1})Q_V} \right) \left( \frac{1 - z^{-m_2}Q_m}{1 + P_m C_m + (P_m \hat{P}_m^{-1} - z^{-m_2})Q_m} \right). \]  

(2.37)

So the factors \( 1 - z^{-m_1}Q_V \) and \( 1 - z^{-m_2}Q_m \) are incorporated into the overall sensitivity as plotted in Figure 2.5. The two filters \( Q_V \) and \( Q_m \) remain to be designed. In addition to designing \( Q_V \) and \( Q_m \) to satisfy the relation in (2.33) for repetitive disturbance rejection, \( Q_V \) and \( Q_m \) filters should also satisfy that \( Q_V \approx 0 \) above the VCM bandwidth frequency and \( Q_m \approx 0 \) above the PZT bandwidth frequency. This is to ensure system robustness, as at high frequencies, \( P_V \hat{P}_V^{-1} = z^{-m_1} \) and \( P_m \hat{P}_m^{-1} = z^{-m_2} \) are not guaranteed generally.

**Design of \( Q_V \) and \( Q_m \)**

In discrete time, the periodic disturbances are at the frequencies of the roots of \( 1 - z^{-N} = 0 \). Solve this equation, we have

\[ e^{j\omega N} = e^{j2k\pi}, \]

(2.38)

where \( \omega \in [0, 2\pi) \) and is related to the continuous time frequency \( f_c \) as \( \omega = 2\pi f_c T_s \), where \( T_s \) is the sampling time. So \( f_c = k f_s \), where \( k = 0, 1, 2... \) and \( f_s = 1/T_s \) is the sampling frequency. \( f_s/N \) is thus the fundamental frequency of the repetitive disturbance and \( N \) is the parameter related to the frequency contents.

Solving (2.33), the \( Q \) filter for the periodic disturbances is

\[ Q = \frac{z^{-(N-m)} (1 - \alpha^N)}{1 - \alpha^N z^{-N}}. \]

(2.39)

As discussed, to ensure \( (P P_n^{-1} - z^{-m})Q \approx 0 \) at all frequencies, a fundamental low-pass filter is included in the \( Q \) filter. A zero-phase low-pass filter,

\[ q_0(z, z^{-1}) = \frac{(1 + z^{-1})_{n_0}(1 + z)^{n_0}}{4^{n_0}} \]

(2.40)

is exploited to make sure the influence to the final \( Q \) filter structure is as small as possible. Note that a non-causal forward step \( n_0 \) is in the zero-phase low-pass in (2.40). It is not a
problem for $Q$ filter implementation since there are enough delays ($N - m$) in the repetitive $Q$ filter in (2.39).

Therefore, the $Q$ filter for the VCM loop is designed as $Q_V = Qq_0(z, z^{-1})$, with the VCM plant delay $m = m_1$. An additional band-pass is added to the $Q$ filter for the PZT loop. Because it is desirable that the VCM actuates primarily at low frequencies and the PZT actuates mainly at mid frequencies. This can also limit the PZT actuator saturation. A zero-phase band-pass filter is designed from a butterworth filter. The butterworth filter has magnitude response which is maximally flat in the passband and is monotonic in the passband and stopband. Let the butterworth band-pass filter be $B(z^{-1})$, then the final zero-phase band-pass is just $q_b(z, z^{-1}) = B(z)B(z^{-1})$. So the $Q$ filter for the PZT is written as $Q_m = Qq_b(z, z^{-1})$ with the PZT plant delay $m_2$.

Simulation
The baseline control for the dual-stage HDD is designed with the decouple sensitivity structure as shown in Section 2.4. The sampling frequency is $f_s = 26400Hz$. The parameter $N$ is related to the rotation speed of the disk as $N = \frac{60f_s}{7200RPM} = 220$. So the repetitive dis-
turbances are at fundamental frequency 120Hz and its multiples. The cut-off frequency for the low-pass filter is 1000Hz and the band-pass has the selective band as $720Hz \sim 3600Hz$, which is

$$q_{b-sim}(z, z^{-1}) = \frac{0.07801 - 0.156z^{-2} + 0.07801z^{-4}}{1 - 2.838z^{-1} + 3.165z^{-2} - 1.694z^{-3} + 0.3816z^{-4} \times \frac{0.07801z^4 - 0.156z^2 + 0.07801}{1 - 2.838z + 3.165z^2 - 1.694z^3 + 0.3816z^4}}.$$  

The final sensitivity function with the two RDOBs is shown in Figure 2.12. The PZT sensitivity is re-plotted in Figure 2.13. It shows that due to the inclusion of the bandpass filter, the selective band is only restricted to $720Hz \sim 3600Hz$ for the RDOB of the PZT loop. Thus, the PZT actuator can mainly actuate for repetitive disturbance rejection in mid frequency range.

We simulate the structure in two scenarios, one is with only repetitive disturbance, the other is with both repetitive and non-repetitive disturbance. Besides, four combinations of the single-stage/dual-stage control and with/without RDOBs are compared. The simulation result with only repetitive disturbance is shown in Figure 2.14. From Figure 2.14b, one can see that the dual stage structure improves the overall $3\sigma$ by 11% compared to the single-stage structure. Beside, from Figure 2.14a, the inclusion of the RDOB indeed improves the repetitive disturbance rejection performance by 17.7%. Moreover, the dual-stage with both RDOBs improves the overall disturbance rejection by 36.8% compared to the single-stage without RDOB, as is shown in Figure 2.14c.
CHAPTER 2. DUAL-STAGE MODEL AND BASELINE CONTROL DESIGN WITH REPETITIVE DISTURBANCE REJECTION

The case with both repetitive and non-repetitive disturbances is shown in Figure 2.15. It can be seen that the RDOBs suppress the repetitive disturbances without sacrificing the performance at non-repetitive frequencies. The narrow notches created in the sensitivity function by the enhanced RDOBs are focused enough such the sensitivity amplification at other frequencies are small. In Figure 2.15b, with all disturbances injected, compared to the single-stage without RDOB, the overall performance of the dual-stage with both RDOBs improve the performance by 13% in terms of the PES $3\sigma$ value.
2.6 Chapter Summary

In this chapter, the full model of the dual-stage HDD was described and the two baseline controllers were designed by the LQG/LTR technique separately. The two controllers were then combined using the decoupled sensitivity structure, which ensured that the overall system stability was guaranteed by the stabilities of the two individual loops. The enhanced repetitive disturbance observers were added to the dual-stage HDD to reject the repetitive disturbances. The equivalent controller was formed by the baseline LQG/LTR controller and the RDOB. With properly-designed $Q$ filters for the VCM loop and the PZT loop, the overall sensitivity was still decoupled and the system stability and robustness were guaranteed. The simulation results showed that the dual-stage structure improved the performance compared to the single-stage in terms of the error signal $3\sigma$ value. Besides, with the inclusion of the enhanced RDOBs, the repetitive disturbances were rejected with minimal non-repetitive disturbance amplification.
Chapter 3

Adaptive Feedforward Design with Infinite Impulse Response Filter

3.1 Chapter Overview

In this chapter, the adaptive feedforward (AFF) control for rejecting the external non-repetitive vibration concentrated at low frequencies (0 \( \sim \) 2000Hz) is discussed. The feedforward control is based on an infinite impulse response (IIR) filter structure. The vibration signal and the output signal are available for the algorithm to adaptively update the parameters of the vibration transmission path dynamics. Two design structures for parameter adaptation are proposed. They provide different methods to get the necessary signals for parameter adaptation of the IIR filter which is different from the conventional finite impulse response (FIR) filter adaptation design. Performance of the proposed designs is compared with the conventional Filtered-x Least Mean Square (FxLMS) method on a HDD benchmark problem. The simulation results show that the proposed designs have smaller 3\( \sigma \) value and peak to peak value at steady state.

3.2 Filtered-x Least Mean Square Method

During the track-following process, HDDs are usually subject to external vibrations and shocks from surroundings, such as audio speakers nearby and shocks from human activities. These external disturbances are generally concentrated at low frequencies (0 \( \sim \) 2000Hz) and have large energy. They increase the position error signal (PES) and result in a large track mis-registration (TMR), i.e., 3\( \sigma \), which is defined as three times of the standard deviation of the PES. Therefore, the servo performance is degraded due to insufficient vibration rejection capability. Besides, the increasing density of data stored on the magnetic disk places more stringent requirements on the 3\( \sigma \) value. Therefore, the feedforward control methods are investigated for vibration rejection.
Generally, the disturbances information is necessary for the feedforward control design. In some cases, the models of the disturbances are known in advance and can be directly exploited for vibration suppression, as addressed in [14, 15]. In other cases, the disturbances entering the servo system can be measured by properly installed sensors. Then by offline or online design of the additional filters, the feedforward signal is generated and injected into the servo loop directly to cancel the disturbances. However, the disturbances nature and the HDD dynamics are subject to change. Manufacturing errors of different HDDs, wear and tear of the HDDs can cause system variations. Therefore, it is advantageous to make the feedforward vibration compensation adaptive.

Among different adaptive feedforward control schemes for vibration suppression, the FxLMS method [16] has been used widely for vibration suppression when the vibration injected to the system is available. Basic structure of FxLMS algorithm is shown in Figure 3.1, where $x(k)$ represents the sensor measured signal related to the input vibration $d(k)$ by an unknown vibration transmission path dynamics. $S^*$ is the auxiliary transmission path dynamics of the system and $S$ is the estimate of $S^*$. The filtered sensor signal $x_c(k)$ and the estimation error $e(k)$ between the filtered out $y_c(k)$ and the actual disturbance $d(k)$ are exploited by the adaptive algorithm, which then adaptively adjust the FIR coefficients $w(k)$. $y(k)$ is just the intermediate signals generated by the adaptive FIR filter. Assume there are $m + 1$ coefficients in the FIR filter, so

$$y(k) = w_0 x(k) + w_1 x(k - 1) + \ldots + w_m x(k - m) = w^T x(k), \quad (3.1)$$

where

$$w \triangleq \begin{bmatrix} w_0 \\ w_1 \\ \vdots \\ w_m \end{bmatrix} \quad \text{and} \quad x(k) \triangleq \begin{bmatrix} x(k) \\ x(k-1) \\ \vdots \\ x(k-m) \end{bmatrix}. \quad (3.2)$$
Then the expectation of the square residual errors at current time instance $k$ is minimized, i.e.,

$$
\min_{w_i^s} E \left[ e^2(k) \right].
$$

(3.3)

From Figure 3.1,

$$
e(k) = d(k) - y_c(k) = d(k) - S^T x(k).
$$

Swap the order of $S$ and $w$, and denote

$$
S x(k) = x_c(k) \triangleq \begin{bmatrix}
x_c(k) \\
x_c(k-1) \\
\vdots \\
x_c(k-m)
\end{bmatrix}.
$$

So $e(k) = d(k) - w^T x_c(k)$. Then the FIR coefficients are updated at every iteration in the direction of the steepest descend as

$$
w(k+1) = w(k) - \mu \frac{\partial}{\partial w(k)} \left( E \left[ e^2(k) \right] \right),
$$

(3.4)

where $\mu$ is the step size. The gradient is

$$
\frac{\partial}{\partial w(k)} \left( E \left[ e^2(k) \right] \right) = E \left[ -2e(k)x_c(k) \right].
$$

In the actual implementation, $x_c(k)$ is not measurable. So the estimate of $S$, which is $S^*$ in Figure 3.1, is used to filter the signal $x(k)$. In summary, the update law

$$
w(k+1) = w(k) + 2\mu e(k)x_c^*(k),
$$

(3.5)

is implemented, where

$$
x_c^*(k) \triangleq \begin{bmatrix}
x_c^*(k) \\
x_c^*(k-1) \\
\vdots \\
x_c^*(k-m)
\end{bmatrix} = S^* x(k).
$$

(3.6)

FxLMS has been applied in HDDs for disturbance rejection using acceleration or shock sensor information, as described in [54, 55]. The FIR filter is generally an 8-tap filter, with the error $e(k)$ being the position error signal and the input signal $x_c^*(k)$ being the sensor signal filtered by the sensitivity function of the HDD closed-loop system. Then the adaptation law in (3.5) is implemented. In [54], two feedforward controllers were designed to match the electromechanical impedance between the disturbance and the measured error. One
controller is an IIR filter designed offline and the other controller is an FIR filter adapted by the FxLMS algorithm online. In [55], the transfer function from the acceleration signal measured by a shock sensor to the error signal was identified by adapting an FIR filter coefficients with the gradient descend algorithm. However, the actual vibration transmission path dynamics is more complex than an FIR filter. Thus, the capacity of the FxLMS algorithm for adaptive vibration rejection is limited. In the following section, we design an adaptive feedforward control for vibration suppression based on an IIR filter structure and discuss its application in HDDs.

### 3.3 Adaptive Feedforward Control Design

A general feedback control system in Figure 3.2 is used for adaptive feedforward control derivation. In the figure, P and C represent the plant and controller respectively. \( d(k) \) is the input disturbance, \( e(k) \) is the measured error and \( n(k) \) is the output noise. \( r(k) \) is the reference signal and is set as zero for regulation under disturbances. The adaptive feedforward control design is applicable to the dual-stage structure in Figure 2.3 by a transformation.

#### Disturbance Model

The input disturbance \( d(k) \) in Figure 3.2 is further represented in Figure 3.3. The vibration source \( v(k) \) represents the sensor measurable signal. Then \( v(k) \) goes through additional vibration transmission paths, which include the mechanical based spindle actuator dynamics, the external chassis mechanics etc. in a typical HDD system. These disturbances enter the servo system as plant input disturbance or plant output disturbance, which can be lumped together as the input disturbance, as shown in Figure 3.4. \( G_{st}(z^{-1}) \) represents the overall disturbance model from \( v(k) \) to \( d(k) \). Therefore, by filtering the signal \( v(k) \) with an estimate of the vibration transmission path dynamic \( \hat{G}_{st} \), a feedforward control signal \( u_{ff}(k) \) is obtained and can be injected directly to cancel the input distance \( d(k) \). In this thesis, we assume that \( v(k) \) is measurable. This is not a restrictive assumption when vibration signal is not measurable. Because most of the disturbances in servo systems can be modeled as
CHAPTER 3. ADAPTIVE FEEDFORWARD DESIGN WITH INFINITE IMPULSE RESPONSE FILTER

Figure 3.3: Feedback control system with input disturbances modeling

Figure 3.4: Adaptive feedforward control
deterministic or stochastic disturbances with some forms of parameterization. To model the
deterministic disturbance, pass a deterministic fundamental generator signal, Dirac pulse,
through a disturbance filter as shown in Figure 3.5. Likewise, the majority of the stochastic
disturbances in the control system can be modeled as a discrete-time white noise passing
through a disturbance filter [56]. The white noise is just a random signal with uniform
energy over all frequencies between 0Hz and the Nyquist frequency. Generally, the discrete-
time Gaussian white noise is used as the fundamental generator signal. Then a disturbance
structure is assumed, which gives the parameterization of the disturbance model. Note that
this prior knowledge is necessary and usually is obtained based on knowledge of the physical
system.

Let the fundamental generator signal be \( x(k) \), then some parameterizations of the dis-
turbance models are provided as follows.

- **Moving Average:**
  \[
  d(k) = c_0 x(k) + c_1 x(k - 1) + ... + c_m x(k - m),
  \]  \( (3.7) \)
  where \( c_i \)'s are parameters and \( m \) is the largest delay step.

- **Auto-regressive Moving Average:**
  \[
  d(k) = -a_1 d(k - 1) - ... - a_n d(k - n) + c_0 x(k) + c_1 x(k - 1) + ... + c_m x(k - m),
  \]  \( (3.8) \)
  where \( a_i \)'s and \( c_i \)'s are parameters and \( n \) is the largest delay of output.
Proposed Adaptive Feedforward Design

The auto-regressive moving average model is used to parameterize the disturbance filter $G_{vtp}$ in Figure 3.4. Assume

$$G_{vtp} = \frac{B}{A}$$

and let

$$\hat{G}_{vtp} = \frac{\hat{B}}{\hat{A}} = \frac{\hat{b}_0 + \hat{b}_1 z^{-1} + \ldots + \hat{b}_m z^{-m}}{1 + \hat{a}_1 z^{-1} + \ldots + \hat{a}_n z^{-n}},$$

where $B$ and $A$ are the numerator and denominator of the transfer function to represent $G_{vtp}$. $b_i$’s and $a_i$’s are the filter parameters. $n$ and $m$ are the degrees, which need to be chosen at first. Note that since the goal is to suppress the external disturbances generated by the disturbance model $G_{vtp}$, the orders of $\hat{G}_{vtp}$ do not need to match the exact orders of the real disturbance model, which may have very high orders.

The first proposed adaptive feedforward control design is shown in Figure 3.6, denoted as AFF1. $\hat{S}(z^{-1})$ is the estimate of the auxiliary path dynamics, which is the dynamics between
Figure 3.7: Proposed adaptive feedforward control structure-\textit{AFF2}

The performance goal of the proposed adaptive feedforward control is to guarantee that the error $e(k)$ in Figure 3.6 is bounded by a small value. This can be proved with the following assumptions.

A1. With a proper feedback control design, i.e., the controller $C$ in Figure 3.6, all the errors present at output $e(k)$ are only from the external disturbance $d(k)$;

A2. $S(e^{-j\omega}) = \frac{P(e^{-j\omega})}{1 + P(e^{-j\omega})C(e^{-j\omega})} \approx \hat{S}(e^{-j\omega})$ for $\omega/(2\pi T_s) \in [0, f_c]$. $f_c$ represents the cutoff frequency below which the approximation is guaranteed.

The auxiliary path $S(z^{-1})$ is important as it aligns the dynamics mismatch between the vibration signal $v(k)$ and the PES. An offline identification of the auxiliary path dynamics is performed before designing the adaptive feedforward control. Therefore, the feedforward disturbance rejection performance depends on the identification accuracy of $S(z^{-1})$. It is
important that $\hat{S}(z^{-1}) \approx S$ at the frequencies that the external disturbance concentrates. To apply the structure in Figure 3.6 to the dual-stage HDD as shown in Figure 2.3, just identify the dynamics from the input disturbance to the overall output error and replace $\hat{S}$ with

$$
\hat{S} = G_{d\rightarrow y} = \frac{P_V}{1 + C_mP_m + C_VP_V(1 + C_m\hat{P}_m)} = \frac{P_V}{(1 + C_mP_m)(1 + C_VP_V) + C_mC_VP_V(\hat{P}_m - P_m)},
$$

which is just the VCM plant model combined with the overall sensitivity function for the decoupled dual-stage HDD.

The second structure for the adaptive feedforward disturbance rejection is shown in Figure 3.7, denoted as AFF2. $P = z^{-d}P_{an}$ is the real full-order plant model. $P_n$ is the nominal plant model for $P_{an}$ without the plant delay $d$. $Q$ is the additional filter, which is usually a lowpass/bandpass filter, to constrain the signal in the vibration frequency range. Other similar notations have the same definitions as in Figure 3.6. This structure is more practical when the auxiliary path dynamics $S$ in Figure 3.6 is too complicated to identify. Note that, the connections marked with number 1 are used to extract the signal $y_f(k)$ and are replaced with the dash connections marked with number 2 to extract the filtered error signal $e_f(k)$. If $P(z^{-1})\hat{P}_n^{-1} \approx z^{-d}$ in the frequency range where the vibration concentrates, the estimate of the disturbance is thus obtained as $\hat{d}(k - d)$.

The Parameter Adaptation Algorithm (PAA)

The parameter adaptation algorithm (PAA) in Figure 3.6 and Figure 3.7 are similar. Therefore, only the structure in Figure 3.6 is used for the PAA derivation. The block diagram in Figure 3.6 is rearranged to the structure in Figure 3.8. In the equivalent structure, the VTP dynamics and the adaptive filter are in a parallel structure. Without the auxiliary path $S = \frac{P}{1+P_C}$, the structure approximates the equation-error (EE) method system identification configuration [56]. The EE method, which is also referred to as Series-Parallel method, is generally used for identification of unknown systems with white or color noise. It is always stable with designated adaptation gain but the estimate of parameters can be biased. However, it is not a problem in this case since the error convergence is the main goal. To simplify the notations, $(z^{-1})$ is dropped in the following derivations. Follow the dynamic path from $v(k)$ to $e(k)$, we have

$$
P \left[ \frac{B}{A}v(k) - \frac{\hat{B}}{A}v(k) - Ce(k) \right] + n(k) = e(k). \quad (3.11)$$
Multiply $\frac{1}{1+PC}$ to the both sides of (3.11),

$$
\frac{P}{1+PC} \left[ \frac{B}{A} v(k) - \frac{\hat{B}}{A} v(k) - Ce(k) \right] + \frac{1}{1+PC} n(k) = \frac{1}{1+PC} e(k),
$$

which is simplified as

$$
\frac{P}{1+PC} \left[ \frac{B}{A} - \frac{\hat{B}}{A} \right] v(k) + w(k) = e(k).
$$

(3.13)

Denote $S \triangleq \frac{P}{1+PC}$ and $v'(k) \triangleq Sv(k)$, (3.13) is rewritten as

$$
\frac{B}{A} v'(k) = S \frac{\hat{B}}{A} v(k) + e(k) - w(k).
$$

(3.14)

Denote $y_f(k) \triangleq S \frac{\hat{B}}{A} v(k) + e(k)$, thus,

$$
\frac{B}{A} v'(k) = y_f(k) - w(k),
$$

(3.15)

where $w(k)$ is a color noise. With Assumption A1, $S$ is replaced with $\hat{S}$ without losing the accuracy in the low frequency range. The signals $v'(k)$ and $y_f(k)$ in Figure 3.6 are also
defined. Next, we want to show that \( e_f(k) \) can be used in the PAA to guarantee the stability of the algorithm.

At time instance \( k \), \( e_f(k) \) in Figure 3.6 is calculated as

\[
e_f(k) = \hat{A}(z^{-1})e(k)
\]

\[
= \hat{A}(z^{-1}, k-1)S\left( \frac{B}{A}v(k) - \frac{\hat{B}(z^{-1}, k-1)}{\hat{A}(z^{-1}, k-1)}v(k) \right),
\]

where \( z^{-1} \) is the one-step delay operator and \( \hat{A}(z^{-1}, k-1), \hat{B}(z^{-1}, k-1) \) represent the estimates at time instance \( k-1 \). If coefficients of \( \hat{A}(z^{-1}) \) and \( \hat{B}(z^{-1}) \) were updating slowly\(^1\) within specific time steps, the order of the operations on the signal \( v(k) \) by \( S \), \( \hat{A} \) and \( \hat{B} \) can be swapped [36]. Therefore, (3.16) is equivalent to

\[
e_f(k) = \hat{A}(z^{-1}, k-1)B \frac{v'(k)}{A} - \hat{B}(z^{-1}, k-1)v'(k)
\]

\[
= \hat{A}(z^{-1}, k-1)y_f(k) - \hat{B}(z^{-1}, k-1)v'(k).
\]

Use the model in (3.9), denote the regressor vector \( \phi_f \) and the parameter vector \( \theta \) as

\[
\phi_f(k-1) \triangleq \begin{bmatrix} -y_f(k-1) \\ \vdots \\ -y_f(k-n) \\ v'(k) \\ \vdots \\ v'(k-m) \end{bmatrix} \quad \text{and} \quad \hat{\theta}(k) \triangleq \begin{bmatrix} \hat{a}_1(k) \\ \vdots \\ \hat{a}_n(k) \\ \hat{b}_0(k) \\ \vdots \\ \hat{b}_m(k) \end{bmatrix},
\]

then (3.17) is rewritten as

\[
e_f(k) = y_f(k) - \hat{\theta}(k-1)^T \phi_f(k-1).
\]

Therefore, using the input, output and error signals derived, the recursive least square PAA with forgetting factor is applied [56]:

\[
\hat{\theta}(k) = \hat{\theta}(k-1) + \frac{F(k-1)\phi_f(k-1)e_f(k)}{1 + \phi_f^T(k-1)F(k-1)\phi_f(k-1)};
\]

\[
e_f(k) = y_f(k) - \hat{\theta}^T(k-1)\phi_f(k-1);
\]

\[
F(k) = \frac{1}{\lambda(k-1)}[F(k-1) - \frac{F(k-1)\phi_f(k-1)\phi_f^T(k-1)F(k-1)}{\lambda(k-1) + \phi_f^T(k-1)F(k-1)\phi_f(k-1)}];
\]

\[
\lambda(k) = 1 - 0.05 \times 0.995^k,
\]

where \( F(k) \) is the gain matrix that places different weights on the parameters \( \theta \). The forgetting factor \( \lambda(k) \) increases from 0.95 to 1 in order to accelerate the convergence [56].

\(^1\)It can be shown that \( S\hat{B}v(k) - \hat{B}Sv(k) = \sum_{j=0}^{n_s} s_j \{ \sum_{i=0}^{n_B} (b_i(k-j) - b_i(k))v(k-j-i) \} \), where \( n_s \) is order of \( S \) and \( n_B \) is order of \( B \). So if coefficients of \( B \) are changing slowly within the time step \( j \), which is smaller than the order of \( S \), assumption is met.
CHAPTER 3. ADAPTIVE FEEDFORWARD DESIGN WITH INFINITE IMPULSE RESPONSE FILTER

Stability Analysis

With the parameter adaptation algorithm in (3.20)~(3.23), the filtered error

\[ e^*_f(k) = \frac{e_f(k)}{(1 + \phi^T_f(k-1)F(k-1)\phi_f(k-1))} \]

can be proved to converge to zero in the frequency range where \( S = \hat{S} \). Note that, the noise \( w(k) \) in (3.15) is generally not white, so the adaptive filter \( \hat{B}/\hat{A} \) is just an underestimate of the true unknown disturbance filter which has higher order dynamics. As long as \( \frac{\hat{B}}{\hat{A}} \approx G_{vtp} \) at the low frequencies where external vibrations concentrate, the feedforward control is well-behaved. Besides, there are mismatches between \( S \) and \( \hat{S} \) at high frequencies, thus the parameters \( \hat{\theta} \) is difficult to converge to the true values. However, the main goal is vibration suppression, then the parameter convergence does not influence the performance.

Next, we prove the stability of the adaptive algorithm. Denote \( \tilde{\theta}(k) \triangleq \hat{\theta}(k) - \theta \) and combine this with (3.19), the error equation is obtained as

\[ e^*_f(k) = -\tilde{\theta}(k)^T\phi_f(k-1). \tag{3.24} \]

Then subtract \( \theta \) from (3.20) yields

\[ \tilde{\theta}(k) = \tilde{\theta}(k-1) + F(k-1)\phi_f(k-1)e^*_f(k). \tag{3.25} \]

Then combine (3.24) and (3.25), an equivalent feedback connection corresponding to the adaptive system is obtained in Figure 3.9. In the figure, \( \lambda^* = \max_k \lambda(k) \) and \( \lambda^* < 1 \). To prove \( e^*_f(k) \) converges to zero, we need to show that the overall adaptive system in Figure 3.9 is asymptotically hyperstable. To show this, first, the nonlinear block \( NL \) is passive and satisfies the Popov inequality [56]. Second, the block \( L2 \) is strictly positive real (SPR) since \( \lambda^* - \lambda(k-1) > 0 \). So the feedback path formed by the block \( L2 \) and the nonlinear block \( NL \) is still passive and satisfies Popov inequality. Finally, the feedforward path \( L1 \) is a parallel connection of the positive unity constant and the constant \( \lambda^*/2 < 1 \), thus it is passive. In conclusion, the overall structure is hyperstable, then \( \lim_{k \to \infty} e^*_f(k) \) is bounded.

The filtered error \( e_f(k) \) is related to \( e^*_f(k) \) by \( e_f(k) = \hat{A}(z^{-1}, k-1)e(k) \) and \( e^*_f(k) = e_f(k)/(1 + \phi^T_f(k-1)F(k-1)\phi_f(k-1)) \). Therefore, \( e^*_f(k) \) is a moving average of the error \( e(k) \), which is then normalized by a bounded signal vector \( \phi_f(k) \). So \( e_f(k) \) is bounded. When the adaptive parameters converge to the steady state value, \( e(k) \) converges.

Additional Considerations

For practical implementation of the algorithms, several heuristic adjustments are implemented.

- **Stability of the adaptive disturbance filter**
  The adaptive disturbance filter is an IIR filter, so it becomes unstable when the poles
shift outside the unit circle during adaptation. To guarantee that the filter stays stable during the whole process, the stability can be regulated by monitoring the geometric positions of the poles to the unit circle or parameterize the adaptive filters in an inherently stable structure[57, 58]. In this work, the stability of the adaptive filter is checked at every iteration. Assume the adaptive filter is parameterized into stable and unstable parts as

$$G(z^{-1}) = \frac{B}{A^+ A^-},$$

where $A^+$ is stable and $A^- = 1 + a_1 z^{-1} + \ldots + a_{nu} z^{-nu}$ contains all the poles outside the unit circle. The stability is preserved by projecting the unstable poles into the unit circle, which is replacing $A^-$ by $A^-_{proj} = z^{-nu} + a_1 z^{-nu+1} + \ldots + a_{nu}$. Note that, to implement this, $A(z^{-1}, k)$ is factored into the form $A^+(z^{-1}, k)\hat{A}^-(z^{-1}, k)$ at every iteration, which is then replaced by $A^+(z^{-1}, k)\hat{A}^-_{proj}(z^{-1}, k).$ This does add some real time computation if the order of $A(z^{-1})$ is high.

- **Signal Normalization**
  In order to avoid the instability due to unbounded input and output signals, the normalized signals $\phi_{fn}(k)$ and $e_{fn}(k)$ are used in the implementation instead of $\phi_f(k)$ and
CHAPTER 3. ADAPTIVE FEEDFORWARD DESIGN WITH INFINITE IMPULSE RESPONSE FILTER

$e_f(k)$. The signal normalization is

$$m(k) = g_1m(k - 1) + g_2 \max\{1, ||\phi_f(k)||\};$$

$$\phi_{fn}(k) = \frac{\phi_f(k)}{m(k)};$$

$$e_{fn}(k) = \frac{e_f(k)}{m(k)},$$

where $g_1 = 0.9, g_2 = 1$ is used [56].

- **Robustness**
  When noise and/or modeling error exist(s), the stability of the algorithm is not guaranteed. Thus, a deadzone is added for the robustness concern. Instead of $e_f(k)$, $Dz[e_f(k)]$ is used in the implementation:

$$Dz[e_f(k)] = \begin{cases} 
0 & \text{if } |e_f(k)| < \delta \\
 e_f(k) & \text{if } |e_f(k)| \geq \delta ,
\end{cases}$$

(3.26)

where $\delta$ is a bound for the uncertainties including noise. In the simulation, $\delta$ is chosen as the largest absolute value of noise magnitude which is assumed known. When the magnitude of the noise is excessively large, the signal-noise-ratio should be large enough for the algorithm to perform effectively.

In conclusion, the signal normalization and deadzone handling are added to the PAA algorithm in (3.20)–(3.23), the final implemented PAA algorithm becomes

$$\dot{\hat{\theta}}(k) = \dot{\hat{\theta}}(k - 1) + \frac{F(k - 1)\phi_{fn}(k - 1)e_{fnd}(k)}{1 + \phi_{fn}^T(k - 1)F(k - 1)\phi_{fn}(k - 1)};$$

(3.27)

$$e_f(k) = y_f(k) - \hat{\theta}^T(k - 1)\phi_f(k - 1);$$

(3.28)

$$F(k) = \frac{1}{\lambda(k - 1)}[F(k - 1) - \frac{F(k - 1)\phi_f(k - 1)\phi_f^T(k - 1)F(k - 1)}{\lambda(k - 1) + \phi_f^T(k - 1)F(k - 1)\phi_f(k - 1)}];$$

(3.29)

$$\lambda(k) = 1 - 0.05 \times 0.995^k,$$

(3.30)

where

$$e_{fnd}(k) = \begin{cases} 
0 & \text{if } |e_f(k)| < \delta \\
e_{fn}(k) & \text{if } |e_f(k)| \geq \delta .
\end{cases}$$

(3.31)

### 3.4 Simulation

The well formulated open-source HDD benchmark simulation package [47] is used in the simulation for verification. Note that only the single-stage system with the voice coil motor
is tested. A practical model for the single-stage HDD can be of orders higher than twenty and with several high frequency resonances, as discussed in Section 2.2. Therefore, the model resonances are attenuated by notch filters and the full order model is reduced to a nominal model for the baseline controller design. A Proportional-Integral-Derivative (PID) baseline feedback controller $C$ is cascaded with three notch filters. The baseline closed loop system has a gain margin of $5.81 \text{dB}$, a phase margin of $47.7$ degrees and a bandwidth of about $1000\text{Hz}$. Besides, the auxiliary path dynamics $S$ was identified using the System ID toolbox in MATLAB. The path identified is from the input $d(k)$ to the output $e(k)$ shown in Figure 3.6. The frequency response of the estimated $\hat{S}$ and the true $S$ are shown in Figure 3.10. From the figure, it can be seen that the magnitude mismatch starts at about $1800\text{Hz}$ and the phase mismatch starts at about $300\text{Hz}$.

Both two adaptive feedforward structures in Figure 3.6 (AFF1) and Figure 3.7 (AFF2) were implemented and compared with the FxLMS algorithm. The sampling time is $T_s = 3.7879 \times 10^{-5}\text{sec}$. Since in Figure 3.7, a stable inverse of the plant model is needed. Therefore, the frequency response of the stable inverse of the nominal model is also plotted in Figure 3.11, as well as the multiplications between this stable inverse and the full/nominal models. The blue line curve is the stable inverse of the nominal plant model, which is implemented in AFF2. Ideally, $P_n \times P_n^{-1} = 1$ is desired, as represented by the red dash curve. However, due to mismatch between the full model and the nominal model, $P \times P_n^{-1}$, which is the black line, is the best that can be achieved. Besides, it shows that the magnitude mismatch begins at about $2000\text{Hz}$ and the phase mismatch begins at about $2500\text{Hz}$. There-
fore, one can expect that the final adapted IIR filter in $AFF_2$ has better dynamics match compared to the adaptive IIR filter in $AFF_1$.

The vibration transmission path (VTP) dynamics generally includes mechanics based spindle actuation and some external chassis mechanics. In the simulation, the solid red line in Figure 3.13 is used as the VTP dynamics. The FxLMS has 7 unknown parameters with a learning rate $\mu = 2 \times 10^{-5}$. For comparison, the order of the adaptive IIR is chosen as 3 to make sure that the same number of unknown parameters are adapted. Additionally, to filter our high frequency noise, a low-pass filter with cut-off frequency at 3000Hz is added before all the signals are used for the adaptation algorithm.

Four cases are compared and the time/frequency domain results are shown in Figure 3.12. In the figure, Baseline corresponds to the case that only the feedback controller $C$ is designed for vibration rejection. As in Figure 3.12, the vibration has large energy concentrated at low frequencies, so the performance at low frequencies is still undesirable. Then $AFF_1$ and $AFF_2$ are the baseline feedback control with the two proposed adaptive feedforward controls.
Figure 3.12: Power spectrum and time sequence of error signal with proposed adaptive feedforward control

Table 3.1: Simulation results

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>FxLMS</th>
<th>AFF1</th>
<th>AFF2</th>
</tr>
</thead>
<tbody>
<tr>
<td>3σ value</td>
<td>4.1</td>
<td>1.71</td>
<td>0.34</td>
<td>0.42</td>
</tr>
<tr>
<td>Peak to peak</td>
<td>10.6</td>
<td>4.15</td>
<td>0.87</td>
<td>1.14</td>
</tr>
</tbody>
</table>

in Figure 3.6 and Figure 3.7. From the time domain result in Figure 3.12, it can be seen that both AFF1 and AFF2 have faster transient. The peak-to-peak value and 3σ value are listed in Table 3.1. It shows that both proposed designs achieve better performance than the FxLMS method. The steady state frequency response of the adaptive IIR and the VTP dynamics are shown in Figure 3.13. It shows that the adaptive IIR does not converge exactly to the actual dynamics. This is due to the adaptive IIR being an underestimate of the actual dynamics. For the AFF1 structure, there are significant model mismatches between $S(z^{-1})$ and $\hat{S}(z^{-1})$ at high frequencies. Therefore, at the low frequencies, the magnitude response and phase response match quite well. But at higher frequencies, the model mismatch becomes worse. This is also true for AFF2 as there are model mismatches between the full plant model and the nominal model at high frequencies. From the perspective of implementation, a selective-band filter can be added to guarantee that the high frequencies model uncertainties does not degrade the performance of the algorithm.
Figure 3.13: Actual VTP dynamics and the steady state IIR filter in $AFF1$ and $AFF2$. 
CHAPTER 3. ADAPTIVE FEEDFORWARD DESIGN WITH INFINITE IMPULSE RESPONSE FILTER

3.5 Chapter Summary

In this chapter, two adaptive feedforward controls were designed for low frequency vibration suppression. The auto-regressive moving average model, i.e., the infinite impulse response filter structure, was adapted using the extracted signals from the measurable signal. Simulations on a realistic open-source HDD benchmark problem showed that the proposed designs, compared to the conventional FxLMS algorithm, improved the vibration suppression performance in terms of the error $3\sigma$ value and had faster transient and smaller peak to peak value. The proposed designs are suitable for low frequency vibration suppression when the vibration signal can be measured by sensor or modeled by fundamental generator signals. The trade-off is that when the vibration transmission path becomes more complex, the stability monitoring of the adaptive filter itself requires more computation. Besides, initial guess of the adaptive filter influences the performance.
Chapter 4

Amplitude Saturation Compensation

4.1 Chapter Overview

In this chapter, the amplitude saturation problem of the actuator is considered. The secondary piezoelectric actuator (PZT) of the dual-stage HDD has a limited movement range about \(1 \sim 2\mu m\), which then sets a limit on the amplitude of the PZT actuation signal. Therefore, during track seeking and track following control with the external vibrations, the PZT actuation signal saturates easily while the primary actuator control signal remains free from the saturation. As a result, the saturation of the PZT leads to degraded vibration suppression performance and unacceptable bandwidth. Moreover, it may cause stability issues.

To solve the amplitude saturation of the PZT actuator, we design the add-on anti-windup (AW) controller in a linear conditioning framework. It can be regarded as an enhanced version of the scheme in [28, 31]. We proposed a different linear conditioning framework and show that the proposed configuration can be decoupled into a linear loop and a nonlinear loop, for which a linear matrix inequalities (LMI) optimization problem is formulated. The LMI optimization problem is formulated using the Integral Quadratic Constraint (IQC) [45], S-procedure, Kalman-Yakubovich-Popov (KYP) Lemma [59]. This procedure is also exploited to analyze the uncertainties in such an anti-windup compensation scheme. The proposed scheme and an existing AW scheme are evaluated and compared by simulation on a dual-stage HDD benchmark control problem with white noise disturbance.

4.2 Amplitude Saturation Model and Linear Conditioning Framework

A fundamental feedback system is shown in Figure 4.1. In the figure, \(u\) is the original control signal and \(u_m\) is the signal regulated by the saturation block. The saturation operator is
CHAPTER 4. AMPLITUDE SATURATION COMPENSATION

Defined as

$$Sat(u) = \begin{cases} u & |u| < u_{\text{max}} \\ \text{sign}(u) u_{\text{max}} & |u| \geq u_{\text{max}} \end{cases},$$

where $u_{\text{max}}$ is saturation limit. Originally, windup refers to integrator control action in $C$ keeps accumulating residual error, which leads to overshoot and the accumulated error is never wound. Now anti-windup represents the general techniques to avoid the detrimental effects from commanding the control action over the limit of the actuators. A linear conditioning framework for anti-windup compensation [43] is shown in Figure 4.1. It is an add-on structure such that the original controller does not need to be modified. This is beneficial in the practical implementations. In the structure, the difference between the desired control $u$ and the actual control $u_m$ is extracted as $\tilde{u} = u - u_m$, which is then filtered to obtain $u_d$ and $y_d$. These two signals are fed back to the overall output $y$ and the non-regulated control signal $u$. As a result, $u_{\text{lin}}$ and $y_{\text{lin}}$ are obtained to approximate the original unsaturated control signal and output signal. $\hat{P}$ is an estimate of the plant model and $I$ represents an identity matrix with appropriate dimension. $M$ is the transfer function to be designed for compensating saturation effects. Note that, when $M = I$, then $u_d = 0$ and $y_d = \hat{P}(u - u_m)$, which is just the estimate of the error that is cut-off by the saturation operator. This scheme is also referred to as the Internal model control (IMC) scheme. Then,

$$y_{\text{lin}} = y_d + y = \hat{P}(u - u_m) + P u_m \approx P u,$$

which is the original expected error resulted from the original unsaturated control. Perform a block diagram transformation, an equivalent structure is obtained in Figure 4.2, in which the relation between $u$ and $\tilde{u}$ is a deadzone operation

$$\tilde{u} = Dz(u) = \begin{cases} 0 & |u| \leq u_m \\ u - u_m & u > u_m \\ u + u_m & u < u_m \end{cases}.$$
CHAPTER 4. AMPLITUDE SATURATION COMPENSATION

The equivalence between Figure 4.1 and Figure 4.2 can be verified by checking the following equations:

\[
\begin{align*}
Dz(u) &= u - \text{Sat}(u) \\
u_{\text{lin}} - u_d &= u \\
(M - I)\bar{u} &= u_d \\
\hat{P}M\bar{u} &= y_d \\
y_{\text{lin}} &= y_d + y \\
C(r - y_{\text{lin}}) &= u_{\text{lin}}.
\end{align*}
\]

So the linear conditioning framework is decoupled into the nominal linear loop and the nonlinear loop that results from saturation and anti-windup design. By designing \( M \), the degradation to the intended performance without saturation can be regulated, as

\[
\underbrace{y_{\text{lin}}}_{\text{intended output}} - \underbrace{y_d}_{\text{influence from saturation}} = \underbrace{y}_{\text{actual output}}.
\]

If the actuator does not saturate, then \( \bar{u} = 0 \) and \( y_d = 0 \), so the nominal linear loop achieves desired performance with the original controller \( C \). If the actuator saturates, then \( \bar{u} \neq 0 \) and \( y_d \neq 0 \), so the nonlinear compensation loop should be designed such that the response of the saturated system \( y \) deviates as little as possible from the response of the nominal linear loop \( y_{\text{lin}} \) as \( y = y_{\text{lin}} - y_d \) [60].

4.3 Anti-windup Controller for the Dual-stage HDD

The dual-stage HDD system is a dual-input-single-output system since only the head position error signal is measurable. Recall the decoupled sensitivity control design in Figure 2.3,
 CHAPTER 4. AMPLITUDE SATURATION COMPENSATION

Figure 4.3: The decoupled dual-stage system with amplitude saturation

![Decoupled Dual-Stage System](image)

Figure 4.4: AW compensation for a dual-stage system with sensitivity decoupling design

![AW Compensation System](image)

A saturation operator, was added after the controller $C_m$ to model the limitation on the actuation signal into the PZT plant. This is redrawn in Figure 4.3. Without the saturation operator, the closed loop sensitivity function is still the same as in (2.8) and (2.9). We applied the linear conditioning framework for anti-windup compensation to the dual-stage HDD. The final compensation structure is shown in Figure 4.4. The notations are defined similarly as those defined in the aforementioned sections. Instead of connecting $y_d$ back to input of $C_m$ [28, 31], $y_d$ is added to the overall output $y$. It is motivated by the idea that the primary actuator VCM has a larger operating range and tolerates higher control gain. Therefore, the portion of error that is cut off due to amplitude saturation of the secondary actuator PZT, is compensated by both actuators.
As shown in Figure 4.4, \( u - u_m = \tilde{u} \), which is then filtered with two AW controllers \( M - I \) and \( M\hat{P}_m \). \( M \) is the design parameter. When \( M = I \), the controller \( M\hat{P}_m \) is aimed to make \( y_{lin} \) in the feedback loop remains close to the output without saturation. We assume that all signals belong to the space of vector-valued sequences, where, at each time instance, the vectors take value in an Euclidean space with proper dimensions. Thus, without confusion, all signals are treated as discrete time sequences throughout the thesis unless declared otherwise. For any discrete time signal \( x(k) \), if the \( l_2 \) norm of \( x(k) \) is finite, i.e., \( \|x\|_{l_2} := \sqrt{\sum_{k=0}^{\infty} \|x(k)\|^2} < \infty \), we say that \( x(k) \in l_2(\mathbb{N}_+) \), where \( \mathbb{N}_+ \) denotes the set of all non-negative integers. \( l_2(\mathbb{N}_+) \) is abbreviated as \( l_2 \) hereinafter.

The structure in Figure 4.4 is transformed into an equivalent representation in Figure 4.5, where the saturation operator is replaced by a deadzone operator since \( Dz(u) = u - Sat(u) \). The equivalence is verified by checking signal relationships at every summing junctions and the input/output signals for every block. In Figure 4.5, the nominal linear loop is also decoupled from the nonlinear compensation loop. If the PZT does not saturate, then \( \tilde{u} = 0 \) and \( y_{d} = 0 \), so the nominal linear loop dominates. If the PZT saturates, then \( \tilde{u} \neq 0 \) and \( y_{d} \neq 0 \), it is straightforward to see that the size of \( y_{d} \) directly influences how large the saturated response \( y \) deviates from the nominal linear response \( y_{lin} \). Since \( u_{tin} \) is generated from the nominal PZT controller \( C_m \), its magnitude is already prescribed. Therefore, the
performance of the AW control can be judged by the upper bound of
\[ \frac{\|y_d\|_{l_2}}{\|u_{lin}\|_{l_2}}. \]

Denote the nonlinear mapping from \(u_{lin}\) to \(y_d\) as \(\phi: l_2 \rightarrow l_2\). Then, the induced \(l_2\) norm of \(\phi\), defined as
\[ \|\phi\|_{l_2} := \sup_{0 \neq x \in l_2} \frac{\|\phi(x)\|_{l_2}}{\|x\|_{l_2}}, \]
is minimized over all possible \(M\) to optimize the AW performance. To guarantee the stability of the overall structure, the nominal closed loop in Figure 4.5 should be stable and mathematically well-posed, which is true. Besides, \(M - I\) and \(\hat{P}_m M\) should be internally stable. This is explained in the following derivations.

**Synthesizing M by Solving LMI Optimization**

We consider the case when the PZT plant model \(P_m\) is stabilizable and detectable. Let \(\hat{P}_m\) be the nominal model of \(P_m\), with a state space representation \(\hat{P}_m \sim (A_p, B_p, C_p, D_p)\). The state dimension is \(n_p\). Let \(M\) be a coprime factor of \(\hat{P}_m\), i.e., \(\hat{P}_m = M^{-1}N\). Then, let \(M - I\) and \(M\hat{P}_m\) have the coprime factorization \([61]\) characterized by
\[
\begin{bmatrix}
M - I \\
M\hat{P}_m
\end{bmatrix} \triangleq 
\begin{bmatrix}
M - I \\
N
\end{bmatrix} \sim 
\begin{bmatrix}
A_p + B_p F & B_p \\
F & 0 \\
C_p + D_p F & D_p
\end{bmatrix}.
\]

Therefore, given an \(F\), \(M - I\) and \(M\hat{P}_m\) are synthesized. The AW problem is thus transformed to an optimization problem,
\[
\min_F \|\phi\|_{l_2},
\]
which can be formulated into a linear matrix inequality (LMI) problem as stated in Theorem 2. To simplify the notation, the theorem is stated by treating the system in Figure 4.5 as a single-input-single-output system, which is easily extended to the multi-input-multi-output case.

**Theorem 2.** Assume that the input signal \(u\) and the output signal \(\tilde{u}\) of the deadzone operator in Figure 4.5 satisfy the quadratic constraint
\[
\begin{bmatrix}
u^T \\
\tilde{u}
\end{bmatrix} 
\begin{bmatrix}
0 & eI \\
eI & gI
\end{bmatrix} 
\begin{bmatrix}
u \\
\tilde{u}
\end{bmatrix} \geq 0.
\]
If there exist matrices $Q \succ 0$, $L$, scalars $x,y$ and $t \geq 0$ such that the following LMI optimization problem is solved

$$\min_{Q \succ 0, t \geq 0} t$$

subject to

$$\begin{bmatrix}
-Q & -L^T (QA_p^T + L^T B_p^T) (QC_p^T + L^T D_p^T) & 0 \\
-L & yI & xB_p^T \\
(A_pQ + B_pL)B_px & -Q & 0 \\
(C_pQ + D_pL)D_px & 0 & -I \\
0 & I & 0 \\
0 & 0 & -tI
\end{bmatrix} \preceq 0. \quad (4.4)$$

Then there exist dynamic compensators $M - I$ and $M\hat{P}_m$ of the order $n_p$ in (4.1) achieving $\|\phi\|_{l_2} \leq \sqrt{t}$ with $F$ given by $F = LQ^{-1}$.

Proof. The goal of minimizing $\|\phi\|_{l_2}$ is equivalent to ensuring that $\phi$ is dissipative with respect to the supply rate $S_0$, i.e.,

$$S_0(u_{lin}, y_d) = [u_{lin}^T \quad y_d^T] \begin{bmatrix} \gamma^2 I & 0 \\ 0 & -I \end{bmatrix} \begin{bmatrix} u_{lin} \\ y_d \end{bmatrix} \succeq 0, \quad (4.5)$$

where $\gamma$ is the upper bound of $\|\phi\|_{l_2}$ and will be minimized. Redraw the nonlinear relationship $\phi(u_{lin}) = y_d$ in Figure 4.5 to a linear fractional transformation (LFT) [62] in Figure 4.6, where $M_e$ represents the dynamics from $[\tilde{u} \quad u_{lin}]$ to $[u \quad y_{lin}]$ in Figure 4.5. Then,

$$M_e = \begin{bmatrix} -(M - I) & I \\ M\hat{P}_m & 0 \end{bmatrix}.$$
CHAPTER 4. AMPLITUDE SATURATION COMPENSATION

With the factorization in (4.1), we have
\[ M - I = F \left( e^{j\Omega} I - A_p - B_p F \right)^{-1} B_p \]
and
\[ M \hat{P}_m = (C_p + D_p F) \left( e^{j\Omega} I - A_p - B_p F \right)^{-1} B_p + D_p, \]
where \( \Omega \) is the frequency domain index. By using the relationship between signals \( u, \hat{u}, u_{lin} \) and \( y_d \) in Figure 4.6 and applying the S-procedure [63] to the quadratic forms in (4.3) and (4.5), a four-by-four block matrix inequality is obtained. It can be rewritten into the following frequency domain inequality (FDI)
\[
\begin{bmatrix}
\left( e^{j\Omega} I - A_p - B_p F \right)^{-1} B_p
\end{bmatrix}^T I \geq 0,
\]
for all \( \Omega \in [0, 2\pi] \). It is obtained by applying Schur complement subsequently. According to the KYP Lemma [59], if \( (A_p + B_p F, B_p) \) is controllable, (4.6) is true if and only if there exists a matrix \( W = W^T \) such that the following nonlinear matrix inequality holds.
\[
\begin{bmatrix}
(A_p + B_p F)^T W (A_p + B_p F) - W + (C_p + D_p F)^T (C_p + D_p F)
\end{bmatrix} \geq 0,
\]
Then apply the standard Schur complement to (4.7) for matrix expansion and use the relationships \( t = \gamma^2, Q = W^{-1}, x = e^{-1} \) and \( y = e^{-1} g e^{-1} \), the LMI in (4.4) is obtained.

Note that the optimization problem in (4.4) is always feasible since \( M = I \) is a feasible solution. It can be seen from Figure 4.5 that when \( M = I \), the nonlinear compensation loop reduces to the series connection of the deadzone operator and the PZT plant \( \hat{P}_m \), which are both \( l_2 \) norm bounded.

To guarantee the global stability of the overall system in Fig. 4.5, the following conditions should be satisfied,

1. The nominal linear loop is stable and and mathematically well-posed, i.e., \( \lim_{z \to \infty} (1 + C_m(z) P_m(z)) \) exists;
2. \( M - I \) and \( M \hat{P}_m \) are internally stable;
3. \( \phi \) is well-defined and finite gain \( l_2 \)-stable.

Condition 1 is guaranteed by the sensitivity decoupling design of \( C_v \) and \( C_m \). By solving the optimization problem in (4.4), \( \| \phi \|_2 \leq \sqrt{r} \) is guaranteed. Therefore, condition 3 is also satisfied. The internal stability of \( M - I \) and \( MP_m \) is proved by choosing a Lyapunov function candidate as \( V(k) = x(k)^T W x(k) \), where \( x(k) \) is the state. By observing from (4.7) that \( (A_p + B_p F)^TW(A_p + B_p F) - W < 0 \) is true, the Lyapunov difference \( V(k+1) - V(k) \) is shown to be less than zero. Therefore, \( A_p + B_p F \) is Schur stable and condition 2 is satisfied.

In conclusion, the structure in Figure 4.4 is globally stable.

Both saturation and deadzone operators belong to Sector\([0,1]\) nonlinearity, i.e., for a scalar \( x_s \),

\[
0 \leq x_s \text{Sat}(x_s) \leq x_s^2 \quad \text{and} \quad 0 \leq x_s \text{Dz}(x_s) \leq x_s^2.
\]

Thus, it follows that there exists a positive scalar \( h > 0 \) such that \( h (u - \text{Dz}(u)) \text{Dz}(u) \geq 0 \), which is equivalent to adding the constraint \( y = -0.5x \) to (4.4) when synthesizing the AW controllers for the system in Figure 4.4. To obtain a smaller upper bound on \( \| \phi \|_2 \), the new sector condition in [64] can be used, which is equivalent to adding additional constraint \( y < 0 \) to the optimization problem. Note that, in this case, only local stability is assured with a guaranteed region of attraction.

To summarize, the AW compensation scheme for the dual-stage HDD system is designed by solving an LMI optimization problem, without modifying the original nominal controllers. If there is no saturation, the dual-stage HDD system achieves desired servo performance and still retains good performance and stability if the PZT actuator saturates.

### 4.4 Robustness Analysis

The robustness of the AW structure lies on the assumption that, the nominal linear design is robust. Therefore, the goal of the AW structure is to retain the nominal linear performance and robustness when the actuator saturation happens. In Section 4.3, we have discussed how to retain the nominal linear performance during saturation by minimizing the \( l_2 \) gain of a nonlinear loop. In this section, how to retain the robustness of the system with the AW controllers is analyzed. The continuous time robustness analysis for such an AW compensation framework is provided in [65]. We provide a discrete time counterpart for the robustness analysis of the system in Figure 4.1, which also applies to the dual-stage system with AW compensation in Figure 4.4. First, an LMI optimization problem is formulated for the purpose of retaining system robustness. Then, an LMI optimization is formulated subsequently for performance. From these two optimizations, we found that setting \( M = I \) is the most robust solution. However, a weighted optimization is easily obtained by combining the optimizations to balance performance and robustness goals.

Recall the AW structure in Figure 4.1. The plant is expressed as \( P = \hat{P} + P_\Delta \), where \( P_\Delta \) is an additive plant uncertainty. It is redrawn in Figure 4.7. Perform a block diagram transformation, the structure in Figure 4.8 is obtained, where the design parameters are \( M \)
and $N$. The connections related to the AW design is already verified by letting $N = \hat{P}$ in Figure 4.8. To check the equivalence of the plant uncertainty part, note that in Figure 4.8,

$$(u_{lin} - M\tilde{u}) P_{\Delta} + u_{lin}\dot{\hat{P}} = y_{lin}.\)

This is also true from Figure 4.7 as

$$(u - \tilde{u}) \left( P_{\Delta} + \dot{\hat{P}} \right) = y_{lin} - y_d$$

$$(u - \tilde{u}) \left( P_{\Delta} + \dot{\hat{P}} \right) = y_{lin} - \dot{\hat{P}}M\tilde{u}$$

$$(u_{lin} - (M - I)\tilde{u} - \tilde{u}) \left( P_{\Delta} + \dot{\hat{P}} \right) = y_{lin} - \dot{\hat{P}}M\tilde{u}$$

$$(u_{lin} - M\tilde{u}) P_{\Delta} + \dot{\hat{P}}(u_{lin} - M\tilde{u}) = y_{lin} - \dot{\hat{P}}M\tilde{u}$$

$$(u_{lin} - M\tilde{u}) P_{\Delta} + u_{lin}\dot{\hat{P}} = y_{lin}.\)

**Optimization for Robustness**

In Figure 4.8, the influence of the plant uncertainty $P_{\Delta}$ to the overall robustness is reflected by the signal $y_{\Delta}$. The existence of $y_{\Delta}$ alters the original nominal response $y_{lin}$. Therefore, for the purpose of retaining the nominal system robustness, we want to minimize the signal $z$, which is related to the signal $u_{lin}$ by a nonlinear relationship. Redraw the nonlinear part from $u_{lin}$ to $z$ in Figure 4.9. A nonlinear optimization problem is formulated as

$$\min_{M} \max_{u_{lin} \neq 0} \{ \frac{\|z\|_{L_2}}{\|u_{lin}\|_{L_2}} \},$$
where the design parameter is $M$. Follow the steps for Theorem 2, an LMI optimization problem to synthesize $M$ for robustness is obtained.

**Theorem 3.** Assume that the input signal $u$ and the output signal $\tilde{u}$ of the deadzone operator in Figure 4.9 satisfy the quadratic constraint in (4.3) with $e = 1/2$ and $g = -1$. If there exist matrices $Q > 0$, $L$, scalars $\mu$ and $t \geq 0$ such that the following LMI optimization problem is solved

$$
\begin{align*}
\min_{\{Q > 0, t \geq 0, L, \mu\}} & \quad t \\
\text{subject to} & \quad \begin{bmatrix}
-Q & -\frac{1}{2}LT & QA_p^T + LB_p^T & LT & L^T \\
-\frac{1}{2}L & -\mu^{-1}I & \mu^{-1}B_p & \mu^{-1}I & \frac{1}{2}I \\
A_pQ + B_pL & \mu^{-1}B_p & -Q & 0 & 0 \\
L & \mu^{-1}I & 0 & -I & 0 \\
\frac{1}{2}I & 0 & 0 & -tI
\end{bmatrix} \preceq 0.
\end{align*}
$$

(4.8)

Then there exist dynamic compensators

$$
M \sim \begin{bmatrix}
A_p + B_pF_1 & B_p \\
F_1 & I
\end{bmatrix}
$$

of the order $n_p$ achieving $\|z\|_2 \leq \sqrt{t+1}$ with $F_1$ given by $F_1 = LQ^{-1}$.

**Proof.** The proof is similar to those for Theorem 2. Except for the following changes:

- (4.5) is replaced with

$$
S_0(u_{\text{in}}, z) = \begin{bmatrix} u_{\text{in}}^T & z^T \end{bmatrix} \begin{bmatrix} \gamma^2 I & 0 \\ 0 & -I \end{bmatrix} \begin{bmatrix} u_{\text{in}} \\ z \end{bmatrix} \succeq 0,
$$


- \( M_e \) in Figure 4.6 has the transfer matrix as

\[
M_e = \begin{bmatrix}
-(M - I) & I \\
-M & I
\end{bmatrix}
\]

- The FDI in (4.6) is replaced with

\[
\begin{bmatrix}
\left((e^{j\Omega}I - A_p - B_pF_1)^{-1}B_p\right)^T \\
I
\end{bmatrix}
\begin{bmatrix}
-F_1^TF_1 - \frac{1}{t}F_1^TF_1 \\
(1/2\mu - 1)F_1 - \frac{\mu}{2t}F_1
\end{bmatrix}
\begin{bmatrix}
(1/2\mu - 1)F_1 - \frac{\mu}{2}F_1 \\
-\frac{\mu^2}{4t}I + (\mu - 1)I
\end{bmatrix}
\]

\[
\times \begin{bmatrix}
\left((e^{j\Omega}I - A_p - B_pF_1)^{-1}B_p\right)
\end{bmatrix} \succeq 0
\]

- The nonlinear matrix inequality in (4.7) is replaced with

\[
\begin{bmatrix}
(A_p + B_pF_1)^TW(A_p + B_pF_1) - W + (1/t + 1)F_1^TF_1 \\
(A_p + B_pF_1)^TWB_p + (1 - \frac{\mu}{2})F_1^TF_1 + \frac{\mu}{2t}F_1^TF_1
\end{bmatrix}
\begin{bmatrix}
R \\
\pm R
\end{bmatrix}
\begin{bmatrix}
B_p^TWB_p + \frac{\mu^2}{4t}I + (1 - \mu)I
\end{bmatrix}
\succeq 0
\]

Then the LMI in (4.8) is obtained by Schur complement and additional matrix multiplication.

After using Theorem 3 to synthesize filter \( M \), the nonlinear gain from \( u_{lin} \) to \( z \) is minimized. Next, consider the performance in terms of minimizing the deviation from the actual response to the nominal linear response, a nonlinear optimization is obtained as

\[
\min_{N} \max_{u_{lin} \neq 0} \{ \frac{\|y_d\|_2}{\|u_{lin}\|_2} \}.
\]
Different from (4.2), $M$ is already synthesized by Theorem 3, the free parameter is $N$ in this case. Redraw the nonlinear loop to synthesize $N$ for performance in Figure 4.10, where $MN$ is combined to one single block $N$. Similarly, an LMI optimization for synthesizing $N$ is formulated.

**Theorem 4.** Assume that the input signal $u$ and the output signal $\tilde{u}$ of the deadzone operator in Figure 4.9 satisfy the quadratic constraint in (4.3) with $e = 1/2$ and $g = -1$. If there exist matrices $Q = Q^T > 0$, $L$, scalars $\mu$ and $t \geq 0$ such that the following LMI optimization problem is solved

$$
\begin{align*}
\min_{Q \succeq 0, t \geq 0, L, \mu > 0} t \\
\text{subject to} \\
\begin{bmatrix}
-Q & -\frac{1}{2}QF_1 & QA_p^T + QF_{1}^TB_p^T & QC_p^T + LT_Dp & 0 \\
-\frac{1}{2}F_1Q & -\mu^{-1}I & \mu^{-1}B_p & \mu^{-1}D_p & I \\
A_pQ + B_pF_1Q & \mu^{-1}B_p & -Q & 0 & 0 \\
C_pQ + D_pL & \mu^{-1}D_p & 0 & -I & 0 \\
0 & I & 0 & 0 & -4tI
\end{bmatrix} \preceq 0,
\end{align*}
$$

(4.9)

where $F_1$ is obtained from Theorem 3. Then there exist dynamic compensator

$$
N \sim \begin{bmatrix}
A_p + B_pF_1 & B_p \\
C_p + D_pF & D_p
\end{bmatrix}
$$

of the order $n_p$ achieving $\frac{\|y_d\|_2}{\|u_{lin}\|_2} \leq \sqrt{t}$ with $F$ given by $F = LQ^{-1}$. Note that since $F_1$ is already fixed by Theorem 3, the purpose of this optimization is mainly to adjust the gain, i.e., by adjusting $C_p + D_pF$ to optimize for performance.

**Proof.** The proof is similar to those for Theorem 2. Except for the following changes:
• $M_e$ in Figure 4.6 has the transfer matrix as

$$M_e = \begin{bmatrix} -(M - I) & I \\ N & 0 \end{bmatrix}$$

• The FDI in (4.6) is replaced with

$$\begin{bmatrix} ((e^{j\Omega}I - A_p - B_pF_1)^{-1} B_p) \end{bmatrix} \begin{bmatrix} I \\ (C_p + D_pF)^T(C_p + D_pF) \frac{\mu}{2} F_1^T - (C_p + D_pF)^T D_p \\ D_p^T D_p + \mu I - \frac{\mu^2}{4\gamma^2} I \end{bmatrix} \begin{bmatrix} ((e^{j\Omega}I - A_p - B_pF_1)^{-1} B_p) \end{bmatrix} \succeq 0,$$

where $F_1$ is fixed.

• The nonlinear matrix inequality in (4.7) is replaced with

$$\begin{bmatrix} (A_p + B_pF_1)^T W (A_p + B_pF_1) - W + (C_p + D_pF)^T(C_p + D_pF) \\ (A_p + B_pF_1)^T W B_p - \frac{\mu}{2} F_1^T + (C_p + D_pF)^T D_p \\ B_p^T W B_p + D_p^T D_p - \mu I + \frac{\mu^2}{4\gamma^2} I \end{bmatrix} \succeq 0$$

Then the LMI in (4.9) is obtained by Schur complement and additional matrix multiplication.

To summarize, combine Theorem 3 and Theorem 4, a sequential process to optimize for system robustness is obtained. Besides, to balance between good performance and good robustness, a weighted optimization by combining (4.4) and (4.8) can be obtained intuitively as $w(4.4) + (1 - w)(4.8)$, where $0 \leq w \leq 1$ is the weighting parameter.

As we discover in the simulation, the final synthesized robust controllers $M \approx I$ and $N \approx \hat{P}$, which is just the IMC scheme. And this IMC scheme is the one that preserves the robustness of the nominal control at the most. It is also found that using only Theorem 2 for performance yields a solution that is robust enough for the general uncertainties in the HDD system. This may be due to the reason that robustness degradation due to plant perturbation is small when optimizing for performance.

### 4.5 Simulations

In this section, the AW control schemes in Section 4.3 were applied to a dual-stage HDD system for vibration rejection and sudden shock disturbance response during track following.
And it was then compared with the scheme in [28]. The simulation study was performed on a single stage HDD benchmark system [47] augmented with the secondary PZT model from [28]. The PZT plant has an input saturation limit of ±1.54V. The baseline controllers were designed by classic SISO loop shaping methods and implemented using the sensitivity decoupling structure introduced in Section 2.3. The magnitude response of the sensitivity function of the VCM stage, the PZT stage and the overall system are in Figure 4.11. The disturbance signal $d$ in Figure 4.4 is chosen to be a white noise concentrated at 0 ~ 5000Hz such that the vibration suppression performance can be analyzed comprehensively. The sampling frequency is 26400Hz.

The PZT model is used to solve the optimization problem in (4.4) with the constraint $y = -0.5x$. The resulting $F$ is $[-1.2194 1.7339]$. Then the filters $M - I$ and $MP_m$ were obtained from (4.1). Figure 4.12 shows the error signal spectra of four cases with $3\sigma$ values. The Worst case is when the saturation limit is imposed on the PZT plant input but no AW compensation is implemented. The Ideal case is when no saturation limit is imposed. Therefore, the Ideal case represents the best performance the system can achieve.

Figure 4.11: Magnitude response of the sensitivity functions of the dual-stage system
with the nominal linear controllers and the *Worst* case represents the worst scenario when saturation happens. It shows that the *Proposed* technique provides larger attenuation than the *Literature* method at frequencies $700\,Hz \sim 2500\,Hz$ as shown in Figure 4.13 with some degradation at $0 \sim 600\,Hz$. This is explained as follows. The VCM loop is designed to primarily operate at low frequencies while the PZT loop to target high frequencies regulation. These two loops can be treated separately by the sensitivity decoupling design, where the VCM loop is a linear loop. In the *Proposed* technique in Figure 4.4, the AW signal $y_d$ is injected to both the VCM and the PZT loops. Thus, if the PZT saturates, which is generally at high frequencies, the VCM loop also provides the control effort for the AW compensation. This reduces the saturation of the PZT, hence improved the performance at high frequencies. But the low frequency performance mainly contributed by the VCM loop, is compromised due to the waterbed effect [13]. Therefore, there is a compromise between disturbance attenuation at low frequencies and at high frequencies in the *Literature* and *Proposed* methods. However, the attenuation at $0 \sim 600\,Hz$ in the *Proposed* method is already good enough to suppress low
frequency disturbance. Besides, a good AW compensation scheme for the dual-stage HDD system should provide enhanced performance at higher frequencies, especially at $1000\,Hz \sim 2000\,Hz$, since this is where the external disturbances concentrate if they are induced by audio vibrations. Therefore, the Proposed method is an enhanced version of the Literature method.

To analyze the performance of anti-windup compensation, original disturbance $d$ is multiplied by increasing vibration gains. The corresponding $3\sigma$ values and saturation percentage are plotted for five vibration gains in Figure 4.14. The saturation percentage is computed as
the percentage of the non-zero elements of signal $\tilde{u}$ in Figure 4.4 during the simulation time. From Figure 4.14, for a specific vibration gain, $3\sigma$ value decreases in the order of Worst, Literature, Proposed, Ideal and so are the saturation percentage; note the saturation percentage for the Ideal case is zero. As vibration gain increases, both $3\sigma$ value and saturation percentage increase but they always fall between the values of the Ideal and Worst cases. Moreover, the improvement of the Proposed compared to the Literature is more pronounced as the vibration gain increases.

To check the transient performance, the response to sudden shock was examined. The shock input is represented by a half cycle sinusoidal signal in simulation. Figure 4.15 shows the responses to four shocks with frequencies of the half cycle sinusoidal signals at 1000Hz, 1500Hz, 2000Hz and 2500Hz. It is seen that the Proposed method reduced the overshoot and the transient time compared to the Worst case and has a faster transient than the Ideal case and the Literature method. The Literature has a transient time that is slightly faster than the Worst case but overshoot is comparable to the Worst case.
CHAPTER 4. AMPLITUDE SATURATION COMPENSATION

4.6 Chapter Summary

In this chapter, we discussed the actuator amplitude saturation and the anti-windup control design by utilizing the linear conditioning framework. An LMI optimization problem for performance was formulated. Besides, the stability of the overall control structure was guaranteed. The robustness of the system was also analyzed by formulating a sequential LMI optimization problem from the perspective of retaining the nominal system robustness. The scheme was applied to a dual-stage HDD system with the secondary actuator subject to the amplitude saturation. This AW scheme was shown to be decoupled. The proposed technique was simulated with white noise vibration and half-cycle sinusoidal shocks at different frequencies. The frequency domain and time domain results showed that the proposed scheme was an enhanced version of the technique in [28].

In the next chapter, the rate saturation of the actuator is also considered and we discuss solving the amplitude and rate saturation problems in the robust control framework. Besides, regarding the amplitude saturation only, the AW controller in this chapter can also be further enhanced in the dual-stage HDD.
Chapter 5

Amplitude and Rate Saturation Compensation

5.1 Chapter Overview

In this chapter, the actuator amplitude and rate saturations are considered at the same time. The saturations could be treated as bounded nonlinear uncertainties. The AW controllers are constructed by the add-on linear conditioning structures. Afterwards, these AW controllers can be synthesized routinely in the robust control framework. Therefore, the stability and performance of the AW design is considered in a unified framework. However, a compromise of this technique is the conservative performance. To reduce the conservatism, we introduced a loop transformation to add additional free parameters in the $\mu$-synthesis process. Besides, the system uncertainties can be included as well. The proposed technique can be readily extended to a general system with multiple different saturations.

5.2 Saturation Problem in Robust Control Framework

Saturation Models

Consider a discrete-time dual-stage HDD model as shown in Figure 5.1, where the $z$-domain index ($z$) is omitted for simplicity and all signals are treated as discrete-time sequences unless when they are suffixed with $(k)$ to indicate the scalar sample at time step $k$. $C_v$ and $P_v$ are the controller and the plant of the VCM; $C_m$ and $P_m$ are the controller and the plant of the PZT; $\hat{P}_m$ is the nominal model of the PZT. $r$, $y$ and $d$ are the reference signal, the position signal and the disturbance signal respectively. $u$ is the nominal linear control signal without saturations and $u_m$ is the admissible control signal of the PZT with two constraints:

1. **Amplitude constraint:** $|u_m(k)| \leq \bar{u}_{\text{amp}}$, where $\bar{u}_{\text{amp}} > 0$ is the amplitude bound of the
CHAPTER 5. AMPLITUDE AND RATE SATURATION COMPENSATION

2. Rate constraint: \( |u_m(k) - u_m(k-1)| \leq \bar{u}_{\text{rate}} \), where \( \bar{u}_{\text{rate}} > 0 \) is the changing rate bound of the control signal.

\( \text{Sat}_a(u(k)) \) and \( \text{Sat}_v(u_v(k)) \) are defined as

\[
\text{Sat}_a(u(k)) = \text{sign}(u(k)) \min\{|u(k)|, \bar{u}_{\text{amp}}\};
\]
\[
\text{Sat}_v(u_v(k)) = \text{sign}(u_v(k)) \min\{|u_v(k)|, \bar{u}_{\text{rate}}\};
\]

where \( \bar{u}_{\text{amp}} \) and \( \bar{u}_{\text{rate}} \) are the amplitude and rate saturation bounds for the actuator.

According to the structure in the dashed square in Figure 5.1, the relationship between \( u \) and \( u_m \) is

\[
\text{Sat}_v[\text{Sat}_a(u(k)) - u_m(k-1)] + u_m(k-1) = u_m(k). \quad (5.1)
\]

From (5.1), \( |u_m(k) - u_m(k-1)| \leq \bar{u}_{\text{rate}} \). Besides, \( u_m(k) - u_m(k-1) \leq \text{Sat}_a(u(k)) - u_m(k-1) \), which leads to \( |u_m(k)| \leq |\text{Sat}_a(u(k))| \leq \bar{u}_{\text{amp}} \). Therefore, \( u_m \) satisfies the amplitude and rate constraints.

Due to amplitude and rate constraints on \( u_m \), the system in Figure 5.1 cannot behave linearly as designed by the nominal controllers when saturations occur. This leads to performance degradation and even instability.

**Generalized Anti-windup Scheme**

To reduce these catastrophic effects due to amplitude and rate saturations of the actuator, an anti-windup scheme is introduced in Figure 5.2. It is an add-on linear conditioning structure,
where a two-by-two filter $K_{2 \times 2}$ is introduced. Two differential signals, $u - \hat{u}$ and $\hat{u} - u_m$, are generated by the differences between the demanded control action before saturation and the achieved control action after saturation. The two differentials are filtered by the estimate of the secondary plant model $\hat{P}_m$ and then input to the two by two linear filter $K_{2 \times 2}$. $\tilde{u}_a$ and $\tilde{u}_v$ are the inputs of $K_{2 \times 2}$; $y_{k1}$ and $y_{k2}$ are the outputs of $K_{2 \times 2}$, which are added to the primary loop, the VCM loop and the secondary loop, the PZT loop respectively. Therefore, both the VCM stage and the PZT stage are involved for anti-windup compensation. When there is no amplitude or rate saturation in the system, $y_{k1} = 0$ and $y_{k2} = 0$, then the anti-windup scheme does not change the behavior of the original linear system. When saturations occur, $y_{k1} \neq 0$ and $y_{k2} \neq 0$, the anti-windup scheme compensates for the degraded linear performance.

The $K_{2 \times 2}$ design problem has two requirements. Define the nonlinear mapping from $d$ to $y$ in Figure 5.2 as $T_{yd}$. Firstly, the induced $l_2$ norm of $T_{yd}$, which is defined as

$$\|T_{yd}\|_{l_2} = \sup_{0 \neq d \in l_2} \frac{\|y\|_{l_2}}{\|d\|_{l_2}},$$  \hspace{1cm} (5.2)$$

should be minimized for performance. $\|\|_{l_2}$ represents the $l_2$ norm of a signal or a nonlinear mapping. Secondly, the stability of the system in Figure 5.2 should be guaranteed. With the existence of the two nonlinear saturation operators, the $K_{2 \times 2}$ design problem becomes an optimization problem with intractable nonlinear constraints.
Since the saturation operator satisfies \( \| \text{sat}(u) \|_{l_2} \leq \| u \|_{l_2} \), it can be treated as a set of linear uncertainties [66]. Therefore, the \( K_{2\times2} \) design problem can be cast into a linear robust control design problem for the system in Figure 5.3. In Figure 5.3, \( \Delta_a \) and \( \Delta_v \) are uncertainties with \( \| \Delta_{a,v} \|_{l_2} \leq 1 \). \( G_{\text{syn}} \) represents the generalized five-input-five-output dynamic system from

\[
\begin{bmatrix}
\hat{u} \\
\hat{u}_v \\
d \\
y_{k1} \\
y_{k2}
\end{bmatrix}
\rightarrow
\begin{bmatrix}
u \\
u_v \\
y \\
\tilde{u}_a \\
\tilde{u}_v
\end{bmatrix}
\]

according to the signal relationships in Figure 5.2. The \( K_{2\times2} \) design problem is summarized as follows.

Design a linear controller \( K_{2\times2} \) such that the following two conditions are satisfied for any \( \Delta_a \) and \( \Delta_v \) with \( \| \Delta_a \|_{l_2} \leq 1 \) and \( \| \Delta_v \|_{l_2} \leq 1 \).

1. Stability: when \( d = 0 \), the closed-loop system in Figure 5.2 is globally asymptotically stable in the sense of Lyapunov;
2. **Performance**: for all $d \in l_2$, there exists a finite positive $\gamma_p$ such that $\|y\|_{l_2} \leq \gamma_p \|d\|_{l_2}$ under zero initial conditions.

Therefore, an optimization problem can be formulated as

$$\begin{align*}
\min_{K_{2\times2}} \gamma_p \\
\text{stabilizing} \\
\text{subject to } \|T_{yd}\|_{l_2} \leq \gamma_p \\
\|\Delta_a\|_{l_2} \leq 1 \\
\|\Delta_v\|_{l_2} \leq 1.
\end{align*}$$

(5.3)

By the $\mu$-synthesis technique, (5.3) is transformed into another optimization problem which can be solved by DK iteration $[67]$. MATLAB robust control toolbox can be employed to solve this class of problems effectively and efficiently.

In summary, a robust control problem is formulated to design the anti-windup scheme for the systems with amplitude and rate saturations. The approach is straightforward and effective in practice. The stability of the overall system is guaranteed trivially. Moreover, this anti-windup scheme is generalizable to systems with multiple different saturations.

**Loop Transform**

To solve the anti-windup problem, it is not ideal to directly use $G_{\text{syn}}$ in Figure 5.3 for DK iteration. The reasons are twofolds. First, the controller obtained by DK iteration may not be globally optimal. Second, the $l_2$ norm bound on the saturation operators are not sharp enough which may include redundant uncertainties. Therefore, the DK iteration with $G_{\text{syn}}$ may lead to conservative or even infeasible $K_{2\times2}$.

To search for a $K_{2\times2}$ with a smaller $\gamma_p$ and to provide more flexibilities, two free parameters are introduced. Recall that in Figure 5.3, $\Delta_a$ with $\|\Delta_a\|_{l_2} \leq 1$ is used to replace the saturation operator since $\|\hat{u}\|_{l_2} \leq 1$.

Thus, a loop transformation between $[u \quad \hat{u}]$ and new signals $[u_{\psi_a} \quad y_{\psi_a}]$ can be found as

$$
\begin{bmatrix}
u_{\psi_a} \\
y_{\psi_a}
\end{bmatrix} =
\begin{bmatrix}
a & 0 \\
-a & 2a
\end{bmatrix}
\begin{bmatrix}
u \\
\hat{u}
\end{bmatrix},
$$

(5.4)

where $a > 0$ is the free parameter. It can be calculated from (5.4) that $u = u_{\psi_a}/a$ and $\hat{u} = (y_{\psi_a} + u_{\psi_a})/(2a)$. Since $u$ and $\hat{u}$ satisfy the sector constraint, i.e., $\hat{u}^T \hat{u} \leq u^T \hat{u}$, then

$$
\begin{bmatrix}
y_{\psi_a} + u_{\psi_a} \\
2a
\end{bmatrix}^T
\begin{bmatrix}
y_{\psi_a} + u_{\psi_a} \\
2a
\end{bmatrix} \leq
\begin{bmatrix}
u_{\psi_a} \\
\frac{a}{2a}
\end{bmatrix}^T
\begin{bmatrix}
y_{\psi_a} + u_{\psi_a} \\
2a
\end{bmatrix}
$$

(5.5)
which can be simplified to $y^T_{\psi_a} y_{\psi_a} \leq u^T_{\psi_a} u_{\psi_a}$. Therefore,

$$\frac{\|y_{\psi_a}\|_{l_2}}{\|u_{\psi_a}\|_{l_2}} \leq 1$$

is also satisfied.

Similarly, for $\hat{u}_v = \Delta_v(u_v)$,

$$\begin{bmatrix} u_{\psi_v} \\ y_{\psi_v} \end{bmatrix} = \begin{bmatrix} b & 0 \\ -b & 2b \end{bmatrix} \begin{bmatrix} u_v \\ \hat{u}_v \end{bmatrix},$$

where $b > 0$ is the free parameter. Denote the mapping from $u_{\psi_a}$ to $y_{\psi_a}$ as $\psi_a : l_2 \rightarrow l_2$ and the mapping from $u_{\psi_v}$ to $y_{\psi_v}$ as $\psi_v : l_2 \rightarrow l_2$. Then the structure in Figure 5.3 becomes the one in Figure 5.4. The DK iteration is now performed for the shaded five-input-five-output
Enhanced Amplitude Anti-windup

In Chapter 4, we already introduced the LMI optimization method to synthesize the controllers $M - I$ and $N$. Consider the AW scheme in Figure 5.5. If $N_{vcm} = 0$, this reduces to the AW scheme introduced in [28, 31]. Note that, the technique introduced in Chapter 4 corresponds to a special case of $N_{vcm} = 1$ in Figure 5.5. Therefore, we consider whether it is possible to further enhance the performance under actuator amplitude saturation if $N_{vcm} \neq 1$.

Then $N_{vcm}$ can be synthesized in the same technique as discussed in Section 5.2 except that only the amplitude saturation operator is replaced with the nonlinear uncertainty transform $\Psi$. Besides, a loop transform with $a$ satisfying (5.4) is also introduced. The synthesis structure is shown in Figure 5.6.
CHAPTER 5. AMPLITUDE AND RATE SATURATION COMPENSATION

Figure 5.6: $\mu$ synthesis design structure in LFT form for amplitude saturation

**Weighting Function**

![Magnitude plot of a peak filter](image)

Figure 5.7: Magnitude plot of a peak filter

As in the general robust control design procedures, weighting functions can be assigned to emphasize disturbance attenuation in different frequency ranges. Therefore, for the output signal $y$ in Figure 5.4 and Figure 5.6, a weighting function $W_y$ can be added when using DK
CHAPTER 5. AMPLITUDE AND RATE SATURATION COMPENSATION

iteration to synthesize $K_{2x2}$ or $N_{vcm}$, i.e.,

$$y \rightarrow W_y \rightarrow y_w$$

Some choices of parameterized weighting functions are provided as follows.

1. To achieve low frequency vibration suppression with an appropriate bandwidth, low-pass filters can be included. A choice of a typical low-pass filter is parameterized as

$$W_{lp}(s) = g_\infty + \frac{w_c(g_0 - g_\infty)}{s + w_c \sqrt{|g_\infty^2 - 1|}}, \quad (5.6)$$

where $s$ is the Laplace variable, $w_c$ is the desired bandwidth for the system, $g_0^{-1}$ is the upper bound of the low frequency static error and $g_\infty^{-1}$ is the upper bound of the maximum allowable value of $\gamma_p$ in (5.3).

2. To further improve servo performance at frequencies with large external disturbance, peak filters or band-pass filters can be included.

The peak filter is parameterized [68] as

$$W_{pp}(s) = Q_f(s) = \frac{s^2 + 2\zeta_1 w_d s + w_d^2}{s^2 + 2\zeta_2 w_d s + w_d^2} \quad (5.7)$$

where $w_d$ is the peak frequency $(rad/s)$, and $0 < \zeta_2 < \zeta_1 < 1$. The shape of the peak filter can be designed with parameters $M_p, M_b, \Delta$ as shown in Figure 5.7, where $M_p$ and $M_b$ are the peak height in decibel and $\Delta$ is the percentage of the disturbance frequency $w_d$ to denote the peak width. Then $\zeta_1$ and $\zeta_2$ can be calculated according to the desired shape of the peak filter by using

$$\zeta_1 = \frac{\Delta^2 + 2\Delta}{2\Delta + 2} \sqrt{10^{M_b/20} - 1}$$

and

$$\zeta_2 = \frac{\zeta_1}{10^{M_p/20}}.$$  

3. The band-pass filter is parameterized as

$$W_{bp}(s) = Q_f(s) = \frac{K}{1 + \tau_1 s} \frac{\tau_2 s}{1 + \tau_2 s} \quad (5.8)$$

where $\tau_2 > \tau_1$. $[\frac{1}{\tau_2}, \frac{1}{\tau_1}]$ is the pass band and $K$ determines the maximum amplitude of the filter.

The aforementioned weighting filters can be cascaded to form the final weighting filter $W_y$ to perform the DK iteration according to the performance requirements.
5.3 Simulation

The anti-windup control techniques introduced in Section 5.2 are implemented. Two cases are verified. The first is the anti-windup design with only amplitude saturation, which is an enhanced form of the anti-windup scheme introduced in Chapter 4. The second is the general anti-windup scheme for both amplitude and rate saturations.

Enhanced Amplitude Anti-windup

In this simulation, the structure in Figure 5.5 is implemented and verified. The simulation parameters are the same as described in Section 4.5. Two cases are compared, the original case with \( N_{vcm} = 1 \) in Figure 5.5, which is already introduced in Chapter 4 and the enhanced with \( N_{vcm} \) synthesized by the technique introduced in Section 5.2. The filter \( N_{vcm} \) is designed by applying DK iteration to the structure in Figure 5.6. The resulting filter is an 8th order
CHAPTER 5. AMPLITUDE AND RATE SATURATION COMPENSATION

Figure 5.9: Zoom in plot of $1800\text{Hz} \sim 3800\text{Hz}$ in Figure 5.8

infinite impulse response filter as

$$N_{vcm} = \frac{5.46z^8 - 6.3z^7 - 8.2z^6 + 7.289z^5 + 7.94z^4 - 3.97z^3 - 4.346z^2 + 2.126z + 0.012}{z^8 + 0.793z^7 - 2.33z^6 - 2.348z^5 + 1.394z^4 + 2.313z^3 + 0.237z^2 - 0.758z - 0.2976}$$

Besides, the ideal and worst cases are also shown for better comparison.

Figure 5.8 shows the error signal spectrum of the four cases with $3\sigma$ values. It shows that, the enhanced and the original method has comparable vibration suppression at all frequencies except that, the enhanced method further improves the performance of the original method at frequencies $1800 \sim 3800\text{Hz}$. This frequency range is also shown in Figure 5.9. Besides, the $3\sigma$ value of the enhanced technique is close to that of the ideal case.

Then the original disturbance $d$ is multiplied by increasing vibration gains. The corresponding $3\sigma$ values and saturation percentage are plotted for five vibration gains in Figure 5.10. It can be seen that for a specific vibration gain, the enhanced method has smaller $3\sigma$ value compared to the original method and so are the saturation percentage. Besides, as the vibration gain increases, both $3\sigma$ value and saturation percentage increase but they always fall between the values of the ideal and worst cases.

The transient performance is also verified by checking the response to the sudden shock. The shock input is represented by a half cycle sinusoidal signal. Figure 5.11 shows the responses to four shocks with frequencies of the half cycle sinusoidal signals at $1000\text{Hz}$, $1500\text{Hz}$, $2000\text{Hz}$ and $2500\text{Hz}$. Both original and enhanced methods reduce the overshoot and transient time compared to the worst case. Besides, the enhanced method has smaller oscillation during transient compared to the original method.

In summary, if only the amplitude saturation is considered, the enhanced method described in Figure 5.5 can be implemented to further improve the anti-windup performance
compared to the original method with $N_{vcm} = 1$. 

Figure 5.10: Comparison for different vibration gain 1, 3, 5, 7, 9.
Figure 5.11: Transient performance with external sudden shocks different frequencies.
Amplitude and Rate Saturation

In this simulation study, both amplitude and rate saturation problems are considered. The simulation setups are already introduced in Section 4.5. A low-pass filter $W_{lp}(s)$ as suggested in (5.6) is designed with $w_c = 2000\text{Hz}$, $g_0 = 1000$ and $g_\infty = 2/3$. A peak filter $W_{pp}(s)$ as suggested in (5.7) is designed with $\zeta_1 = 0.4$, $\zeta_2 = 0.1$, and $w_d = 1000\text{Hz}$. The two filters are then discretized to $W_{lp}(z)$ and $W_{pp}(z)$. Two weighting functions, $W_1 = W_{lp}(z)$ and $W_2 = 0.5W_{lp}(z)W_{pp}(z)$ are used in the DK iteration for synthesizing the two-by-two filter $K_{2\times2}$. The magnitude frequency responses of $W_1$ and $W_2$ are shown in Figure 5.12. It shows that $W_2$ has amplification around 1000Hz to suppress disturbance around 1000Hz.

Three kinds of disturbances are injected, which are audio vibrations, sudden shocks and sinusoidal signals at 1000Hz. The audio vibration signal is injected to the dual-stage HDD system as the signal $d$ in Figure 5.2 and the position error signal (PES) $y$ is measured. Three cases are compared. The ideal and worst cases still represent the best and worst performance the system can achieve without and with amplitude and rate saturations. The proposed is the proposed anti-windup scheme.

With the audio vibration and $W_1$ as the weighting function, the error spectra of the three cases are compared in Figure 5.13. It shows that with the proposed anti-windup scheme, the vibration below 2000Hz is attenuated significantly. Besides, the performance of the proposed method is very close to that of the ideal case. Since the majority of the vibration power concentrates around 1000Hz, $W_2$ is also used as the weighting function to further attenuate the disturbances around 1000Hz. The error spectra from 500Hz to 1500Hz with both $W_1$ and $W_2$ are shown in Figure 5.14. It shows that with the inclusion of $W_2$, the error spectrum around 1000Hz of the proposed method is almost the same as that of the ideal case. Thus,
the performance around 1000Hz is enhanced due to inclusion of the 1000Hz peak in the weighting function. Moreover, the responses to a sinusoidal signal disturbance injection at 1000Hz with $W_2$ are shown in Figure 5.15. The error magnitude of the proposed anti-windup
Figure 5.15: Error signal with single sinusoid input at 1000Hz (with $W_2$) scheme is just slightly larger than that of the ideal case.
To investigate the transient performance of the proposed scheme, a shock disturbance approximated by a half cycle sinusoidal signal at 1000Hz is injected. With weighting function $W_1$, the PES responses as well as the two signals $\tilde{u}_a$ and $\tilde{u}_v$ in Figure 5.2 are shown in Figure 5.16. The percentage values in the second and third rows are the saturation percentages. It shows that the amplitude and rate saturation percentages decrease by the proposed anti-windup scheme. Moreover, the overshoot is reduced slightly and the transient time is also reduced. To further improve the performance for 1000Hz shock, the response with $W_2$ is shown in Figure 5.17. It shows that the overshoot is further reduced compared to the response in Figure 5.16 and is close to the response of the ideal case.

### 5.4 Chapter Summary

In this chapter, we proposed a generalized anti-windup scheme in the robust control framework. By regarding the saturations as $l_2$ norm bounded uncertainties and performing loop
transformation, the anti-windup controls were synthesized by formulating a robust control design problem. In this framework, the induced $l_2$ norm from the disturbances to the position error was minimized and the stability of the overall system was guaranteed. Detailed discussions on weighting function selection were also provided to accommodate different design purposes. The proposed technique was verified on a dual-stage HDD system with the secondary actuator subject to the amplitude and rate saturations. The technique was first applied to the case that only amplitude saturation was considered. The additional synthesized controller was verified by simulation that it was actually an enhanced version of the anti-windup controller discussed in Chapter 4. Besides, the technique was then applied to the case that both amplitude and rate saturations were considered. Simulations with the disturbances from real audio vibrations and sudden shocks were performed to demonstrate the effectiveness of the proposed techniques.
Chapter 6  

Conclusions and Future Works

6.1 Summary

This dissertation presented the control strategies for the dual-stage system with a focus on the applications to the dual-stage hard disk drive systems. The dual-stage system is comprised of a primary actuator and a secondary actuator but only the combined output of the two actuators were available. Therefore, the overall system became a dual-input-single-output (DISO) system. The main control goal was to suppression various sources of disturbances, such as low frequency wide band audio vibration, repetitive narrow disturbances and sudden shocks. Besides, the amplitude and rate saturation problems were addressed with the linear conditioning framework. All the proposed control techniques are add-on structures. Therefore, the original controllers does not need to be modified, which brings flexibility for design and implementation. An overview of the control strategies discussed in the dissertation is summarized in Figure 6.1.

In Chapter 2, the decoupled sensitivity structure was exploited to separate the primary loop and the secondary loop. Therefore, the DISO system was transformed to a single-input-single-output (SISO) system. Then the SISO loop shaping method, linear quadratic Gaussian/ loop transfer recovery, was used to design the controllers for both loops separately. Then the repetitive disturbance observers (RDOBs) were designed for both two loops with different selective band filters. So the two actuators were limited to operate at different frequency ranges, which restricted the saturation of the secondary actuator. Besides, the two add-on RDOBs were shown to be decoupled from the original sensitivity function. Therefore, the overall sensitivity function was augmented with multiple narrow notches at the repetitive frequencies. The parameter $\alpha$ in the RDOB provided the extra freedom to shape the sensitivity amplifications at other non-repetitive frequencies. As a result, the repetitive disturbances were rejected with moderate and small amplifications of the non-repetitive disturbances.

In order to suppress the wide band audio vibration, two adaptive feed-forward control structures were proposed in Chapter 3. The audio vibration entered the HDD servo system
through the exterior harness, where sensors were installed to collect the vibration information. Then this measurable sensor signal was treated as the fundamental generator signal. By properly filtering the error signal by the pre-identified auxiliary transfer functions, necessary signals were extracted to adjust the parameters of the adaptive feed-forward filter. The adaptive feed-forward filter was an infinite impulse response (IIR) filter, which was different from the conventional filtered-x least mean square (FxLMS) method. The FxLMS method adapted a finite impulse response (FIR) filter, which did not have stability issues. However, its capacity to approximate complex vibration transmission path dynamics was also limited. Therefore, we used the IIR filter to achieve better performance with fewer parameters. The stability issue was addressed with unit-circle projection without changing the phase response. Besides, the error convergence was proven rigorously with assumptions.

However, the aforementioned control techniques were designed without considering the limits on the amplitude and changing rate of the actuator control signal. Therefore, in the extreme operating scenarios, the control system was not able to operate as nominally designed. This actuator saturation problem and the corresponding anti-windup compensation techniques were addressed in Chapter 4 and Chapter 5. The linear conditioning framework was the main structure for the saturation compensation.

In Chapter 4, the anti-windup controls for the amplitude saturation compensation were synthesized by solving a linear matrix inequality (LMI) optimization. We presented procedu-
eral steps to formulate the LMI optimization by combining existing theorems and mathematical tools. Therefore, this procedure was straightforward and did not require the experience to design a feasible Lyapunov function. Besides, the robustness of the system was also analyzed following the same procedure, by which another LMI optimization was obtained. Then weighted combination of these two LMI optimizations could be obtained to balance the requirements for system performance and robustness. The anti-windup controls were verified to have improved performance and guaranteed close-loop stability.

In Chapter 5, the amplitude and rate saturation problems were addressed in a unified framework. The saturations could be treated as norm-bounded nonlinear uncertainties. Thus, the anti-windup controls were synthesized by the robustness control methodologies. As a result, the system stability, performance and robustness were all considered in a single framework. Beside, the free parameters introduced by the loop transformation yield less conservative anti-windup controls. The proposed technique was generalizable to systems with multiple different saturations.

6.2 Future Work

The possible future works are summarized in the followings.

- The adaptive feed-forward filter in Chapter 3 is parameterized in the auto-regressive moving average form. Thus, poles of the filter is checked and adjusted every iteration such that the filter remains stable. Therefore, an inherently stable filter structure, such as the lattice form, can be used to parameterize the filter. The regressor may have a different form and the filter implementation and parameter adaptation algorithm should also be modified.

- The adaptive feed-forward control was applied to the hard disk drive for audio vibration rejection, in which information of the vibration was measured by the sensor. However, the vibration information was not always available in other applications. Therefore, it will be meaningful to investigate using only white noise as the fundamental generator signal.

- When synthesizing the anti-windup control in Chapter 4, we can use different quadratic constraints on the amplitude saturation operation to find improved performance anti-windup control. However, an obvious compromise is that the system will be locally stable. So characterization of the region of attraction should also be performed. Moreover, is it possible to express the rate saturation model with a quadratic constraint, and synthesize the two-input-two-output control $K_{2\times2}$ in Figure 5.2 by the LMI optimization? According to previous attempts, the one step delay in the rate saturation model were causing problems. It may be interesting to study how to overcome this problem.
• The LMI optimization for synthesizing the anti-windup controllers is only applicable when the plant is stable and does not include non-minimum phase zeros. Therefore, investigating how to synthesize the anti-windup controls for a non-minimum phase plant with guaranteed regions of attraction is challenging and a good future work direction.
Bibliography


Appendix A

List of Abbreviations

AFF adaptive feedforward
AW anti-windup
DARE discrete algebraic Riccati equation
DISO dual-input-single-output
DOB disturbance observer
EE equation error
FDI frequency domain inequality
FIR finite impulse response
FxLMS filtered-x least mean square
HDD hard disk drive
IIR infinite impulse response
IMC internal model control
IQC integral quadratic constraint
KYP Kalman-Yakubovich-Popov
LMI linear matrix inequality
LMS least mean square
<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>LQ</td>
<td>linear quadratic</td>
</tr>
<tr>
<td>LQG</td>
<td>linear quadratic Gaussian</td>
</tr>
<tr>
<td>LQG/LTR</td>
<td>linear quadratic gaussian/loop transfer recovery</td>
</tr>
<tr>
<td>LTR</td>
<td>loop transfer recovery</td>
</tr>
<tr>
<td>MIMO</td>
<td>multi- input-multi- output</td>
</tr>
<tr>
<td>NRRO</td>
<td>non-repetitive runout</td>
</tr>
<tr>
<td>OE</td>
<td>output error</td>
</tr>
<tr>
<td>PAA</td>
<td>parameter adaptation algorithm</td>
</tr>
<tr>
<td>PES</td>
<td>position error signal</td>
</tr>
<tr>
<td>PID</td>
<td>Proportional-Integral-Derivative</td>
</tr>
<tr>
<td>PZT</td>
<td>piezoelectric transistor</td>
</tr>
<tr>
<td>RC</td>
<td>repetitive control</td>
</tr>
<tr>
<td>RDOB</td>
<td>repetitive disturbance observer</td>
</tr>
<tr>
<td>RLS</td>
<td>recursive least square</td>
</tr>
<tr>
<td>RRO</td>
<td>repetitive runout</td>
</tr>
<tr>
<td>SISO</td>
<td>single- input-single- output</td>
</tr>
<tr>
<td>SPR</td>
<td>strictly positive real</td>
</tr>
<tr>
<td>TMR</td>
<td>track mis-registration</td>
</tr>
<tr>
<td>VCM</td>
<td>voice coil motor</td>
</tr>
<tr>
<td>VTP</td>
<td>vibration transmission path</td>
</tr>
</tbody>
</table>