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Publication Date
1954-10-08
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PHOTODISINTEGRATION OF CARBON-12
BY 300-MEV BREMSSTRAHLUNG

Sheldon Dorman Softky
(Part A of Thesis)

October 8, 1954
PHOTODISINTEGRATION OF CARBON-12
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Sheldon Dorman Softky

Radiation Laboratory, Department of Physics
University of California, Berkeley, California
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ABSTRACT

The yield of stars due to the reaction \( ^{12}\text{C} \left( \gamma, 3\alpha \right) \) from 330-Mev bremsstrahlung has been measured. Ilford C2 emulsions 600\( \mu \) thick were exposed to the beam of the Berkeley synchrotron and scanned for three-prong stars. Those stars satisfying conditions for conservation of momentum in the process \( \left( \gamma, 3\alpha \right) \) were accepted and an excitation function for this reaction was obtained. Since no stars were found from quanta above 42 Mev, an upper limit for the cross section above this energy was obtained. The results are in agreement with those obtained with lower-energy synchrotrons and show that there are no prominent resonances below 100 Mev other than those already known.
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INTRODUCTION

Carbon-12 can be regarded as three alpha particles relatively loosely bound to one another (binding energy 7.37 Mev); thus, next to Be<sup>8</sup>, it is the best example of the alpha-particle model for light nuclei. This fact has led to great interest in the disintegration of C<sub>12</sub> into these alpha particles, and because of the low threshold for C<sub>12</sub> disintegration by gamma rays, many workers with low-energy electron accelerators have studied this reaction.

The photodisintegration of C<sub>12</sub> into three alpha particles was first observed by Hanni, Telegdi, and Zunti in 1948<sup>1</sup>. They irradiated nuclear emulsions with the 17.6-Mev γ-ray from protons on Li and observed the (γ, 3α) reactions as three-pronged stars formed by disintegration of the C<sub>12</sub> in the emulsion. They measured the energies of the alpha particles by using known range-energy curves, and also used the measured angles to show that momentum was conserved in the three-particle disintegration. They were able to show that the disintegration proceeded in two steps,

\[ C^{12} + \gamma \rightarrow \alpha_{1} + Be^{8*}, \]

\[ Be^{8*} \rightarrow \alpha_{2} + \alpha_{3}, \]

with the second step being the break-up of Be<sup>8</sup> in its 3-Mev excited state. This was not difficult to show with monochromatic gamma rays, because one-third of all the alpha particles from the reaction were of the same energy, namely,

\[ E_{1} = \frac{2}{3} (h\nu - |B_{g}| - E_{Be8}^{*}), \quad (1) \]

where \( h\nu \) is the gamma-ray energy, \( |B_{g}| \) the binding energy of Be<sup>8</sup> to the other alpha particle, and \( E_{Be8}^{*} \) the excitation energy of Be<sup>8</sup>. The simplest way to demonstrate Eq. (1) is to look at the first step in the disintegration with the \( \alpha_{1} \) of mass M going one way and the Be<sup>8</sup> nucleus of mass 2M going the other. Then, by conservation of momentum,
\[ \sqrt{2M E_1} = \sqrt{2x2M E_{Be^8}} \]

or \( E_1 = 2E_{Be^8} \)

Another way of saying this is to write

\[ (h\nu - |B|) = E_T, \]

where \( |B| \) is the binding energy of three alphas together to form \( C^{12} \).

Because \( |B| - |Bg| \), the difference between this binding energy and that used in Eq. (1), is only 110 kev, it is neglected here and the two "B's" are treated as identical. \( E_T \) is the total kinetic energy of all the alphas in the disintegration. Since Eq. (2) states that \( E_1 = 2/3 (E_1 + E_{Be^8}) \), and \( E_1 + E_{Be^8} = E_T - E_{Be^8} \), we have Eq. (1): \( E_1 = 2/3 (E_T - E_{Be^8}) \), so that if \( h\nu \) has only one value, and \( E_{Be^8} \) only one value, \( E_1 \) is fixed. Now if it can be shown that the remaining two-thirds of the alpha particles have a fairly flat distribution, the two-step disintegration will be obvious from looking at the distribution in \( E \) of all the alphas.

If we assume that the \( Be^8 \) responsible for the remaining two-thirds of the alphas breaks up isotropically in its own center-of-mass system, we can show (below) that these alphas form a uniform distribution in \( E \) between the kinematically possible limits. It will be assumed in all calculations of kinematics that the \( Z \) component of momentum is neglected; i.e., \( \frac{h\nu}{c} \) is assumed small compared to the other momenta.

In the vector diagram for velocities, \( v_{2,3} \) = velocities of second and third alphas in the laboratory system, and \( v_{Be} \) = velocity of \( Be^8 \) before disintegration in the laboratory system. Likewise, \( V_{2,3} \) = velocities of second and third alphas in the c.m. system. Further, by using Eq. (1) to find velocities:
The law of cosines yields:

\[(v_{2,3})^2 = v_{Be}^2 + (V_{2,3})^2 - 2 v_{Be} V_{2,3} \cos \theta\]

or

\[\frac{ET - E^*}{3M} + \frac{E^*}{2M} - 2 \sqrt{\frac{E^*(ET - E^*)}{3}} \cos \theta\]

\[2E_{2,3} = \frac{ET + 2E^*}{3} - 2 \sqrt{\frac{E^*(ET - E^*)}{3}} \cos \theta\]

where \(E_{2,3}\) is the energy of the second and third alphas in the laboratory system. Thus, differentiating the above,

\[\frac{dE_{2,3}}{d\theta} = \text{Const.} \times \sin \theta\]

Assuming isotropy of break-up in the c.m. system, where \(d\Omega = \text{differential solid angle, } d\Omega = 2\pi \sin \theta d\theta\),

\[\frac{dN_{2,3}}{d\Omega} = \text{Const. or } \frac{dN_{2,3}}{d\theta} = \text{Const.} \times 2\pi \sin \theta\]

Thus

\[\frac{dN_{2,3}}{dE_{2,3}} = \frac{dN}{dE} \times \frac{\text{Const.} \times \sin \theta}{\text{Const.} \times \sin \theta} = \text{Const.}\]

i.e., the distribution in \(E\) of two-thirds of the alphas is a constant. This means that for a sharp excited state in \(\text{Be}^8\), the \(N(E)\) of the alphas consists of a line due to one-third of the particles plus a uniform distribution due to the other two-thirds. This is what was actually observed in the first experiment with 17.6-Mev gamma rays.

Subsequent experiments with the continuous gamma-ray spectra from betatrons and synchrotrons gave as results the excitation curve for the reaction \(C^{12} \{\gamma, 3\alpha\}\), since the gamma energy \(hv\) is calculable from the
alpha energies, by: \( h\nu = E_T + 7.4 \text{ Mev} \)-provided, of course, that the momentum \( \frac{h\nu}{c} \) is neglected. In the above \( E_T = E_1 + E_2 + E_3 \).

The foregoing method of establishing excited states of Be\(^8\) was extrapolated\(^2,^3\) to a continuous gamma spectrum by solving Eq. (1) for \( E_{\text{Be}^8}^* \),

\[
(E_{\text{Be}^8}^*) = E_T - \frac{3}{2} E_1, \tag{1'}
\]

and thus getting a value of \( E^* \) for each value of an alpha energy and \( E_T \). Thus a level in Be\(^8\) gives a fixed value of \( E^* \) for one-third of the alphas in the disintegration regardless of the gamma energies involved, and the distribution \( N(E^*) \) obtained by transforming \( N(E) \) with Eq. (1') should still give one-third of the values of \( E^* \) in a line superimposed on some kind of continuous spectrum. In fact, the yield of stars versus gamma energy, when corrected for the shape of the bremsstrahlung spectrum, showed resonances\(^4\) which presumably indicate levels for gamma absorption in C\(^{12}\). The work with 24-Mev and 70-Mev synchrotrons showed that up to thirty percent of the disintegrations go through the ground state\(^4\) of Be\(^8\), and gave no evidence of the 3-Mev level in Be\(^8\) in this region of quantum energy, but did establish that a large fraction of the transitions are through a 16.9-Mev level in Be\(^8\).\(^3\) The 17.6-Mev gamma rays were not capable of exciting the 16.9-Mev level; further, they showed a minimum production of the ground state. There is a range of quantum energy in which it is difficult\(^2\) to assign a value to the energy level in Be\(^8\), and it must be pointed out that the only way there is of showing that the \( (\gamma, 3\alpha) \) reaction goes in two discrete steps is to show that a value of \( E_{\text{Be}^8}^* \) in \( N(E_{\text{Be}^8}^*) \) is favored. This is because any three-alpha disintegration can be described as two successive two-body disintegrations over a continuously varying value of \( E_{\text{Be}^8}^* \), even though Be\(^8\) may never actually exist during the process. This is shown as follows: Take any three-alpha disintegration and resolve the velocity vectors along the direction of one of the alphas. Call this the ith alpha.
$V_j$ and $V_k$ are velocities of the jth and kth alphas in their own center-of-mass system, and must be equal and opposite to conserve momentum. If they should happen to arise from the breaking up of Be$^8$, it would give

$$V_k = V_j = \sqrt{\frac{2}{M} \frac{E_1^*}{Z}} = \sqrt{\frac{E_i^*}{M}},$$

where $E_i^*$ would be the excitation energy. Even if the disintegration were a single-step process, however, the velocities could be resolved in this way and a value for $E_i^*$ could be given that could be associated with the ith alpha and written in terms of the jth and kth alphas as follows:

$$v_j = \sqrt{\frac{2E_j}{M}}, \quad v_k = \sqrt{\frac{2E_k}{M}},$$

$$V_j + V_k = 2\sqrt{\frac{E_i^*}{M}}, \quad v_i = \sqrt{\frac{2E_i^*}{M}};$$

by cosine law,

$$(V_j + V_k)^2 = v_j^2 + v_k^2 - 2v_j v_k \cos \phi_{jk}$$

or

$$\frac{4E_i^*}{M} = \frac{2E_j}{M} + \frac{2E_k}{M} - 2 \cdot \frac{2}{M} \sqrt{E_j E_k} \cos \phi_{jk} \quad (4)$$

or

$$2E_i^* = E_j + E_k - 2\sqrt{E_j E_k} \cos \phi_{jk}.$$
Equation (1') can be obtained directly from the above, thus showing that the two expressions for $E_i^*$ represent the same thing and that a value of $E_i^*$ is always derivable from the ith alpha even though the process does not actually go in two steps. The basis for deriving Eq. (1') ($E_i^* = E_T - \frac{3}{2} E_i$) from Eq. (4) is conservation of momentum, $m\vec{v_i} = m\vec{v_j} + m\vec{v_k}$. Therefore, vectorially, the lengths are the same, thus

$$\vec{v_i} \cdot \vec{v_i} = (\vec{v_j} + \vec{v_k}) \cdot (\vec{v_j} + \vec{v_k}),$$

or

$$v_i^2 = |\vec{v_j} + \vec{v_k}|^2 = \frac{2E_i}{M},$$

$$|v_j + v_k|^2 = v_j^2 + v_k^2 - 2v_j v_k \cos(\pi - \phi_{jk}) = \frac{2E_j}{M} + \frac{2E_k}{M} + \frac{2x2}{M} \sqrt{E_j E_k} \cos \phi_{jk},$$

or

$$E_i = E_j + E_k + 2 \sqrt{E_j E_k} \cos \phi_{jk} \quad (5)$$

Now, using Eq. (5) and $E_j + E_k = E_T - E_i$ substituted in Eq. (5),

$$E_i = E_T - E_i + 2 \sqrt{E_j E_k} \cos \phi_{jk}$$

or

$$-2 \sqrt{E_j E_k} \cos \phi_{jk} = E_T - 2E_i,$$

or

$$2E_i^* = (E_T - E_i) + (E_T - 2E_i) \quad \text{from Eq. (4)}$$

or

$$E_i^* = E_T - \frac{3}{2} E_i,$$

which shows that a value of $E_i^*_{\text{Be}^8}$ is always obtainable from each alpha of the $(\gamma, 3\alpha)$ process, even though the disintegration may not necessarily go through a state of Be$^8$ at all.

The present work was undertaken to see if the $(\gamma, 3\alpha)$ process exhibits a measurable cross section at considerably higher quantum energies, such as are obtainable with the beam from the 300-Mev Berkeley synchrotron. It seemed possible that there might be higher resonances than those obtained by Goward and Wilkins$^4$ with a 70-Mev synchrotron beam, or that there might be a region of continuous absorption above 70-Mev quantum energy.
METHOD

The experiment consisted of exposing 600μ Ilford C2 nuclear emulsions directly to the beam of the Berkeley synchrotron and scanning the developed plates for (γ, 3α) stars with a Bausch and Lomb microscope. Plates 600μ thick were used so that stars of high ET would be contained completely within the emulsion, and it was hoped that C2 emulsions would allow the relatively heavy tracks of alphas to stand out well against background fog.

To expose the plates, the bremsstrahlung beam of maximum energy 330 Mev was first collimated with a one-eighth-inch collimator with a tapering hole such that at a distance of twelve feet from the collimator the beam was approximately one centimeter in diameter. The beam was passed through a three-foot-long evacuated pipe with 0.005-inch duralumin entrance and exit foils, this pipe being in a magnetic field of 9.2 kilogauss. Thus the only electrons that hit the emulsion in its exposed region must have been scattered at least once after being bent by the sweeping field, or have been generated in the exit foil, the paper wrapping, or the emulsion itself. The plate was fastened, emulsion side toward the beam, on the exit foil of the evacuated pipe about twelve feet from the original beam collimator, so that the exposed spot on the plate was about one centimeter in diameter. The beam then passed through a monitor ionization chamber which had been calibrated in terms of total energy in the beam. Six plates were exposed in this manner, successive exposures increasing by a factor of two to guarantee that a correct exposure would be obtained. The arrangement is shown schematically in Fig. 1.

The plates were developed by the "cold development" method to hold distortion of the emulsions to a minimum. The developer consisted of

- borac acid 35 g
- sodium sulfite 15 g
- 10% potassium bromide 8 ml
- amidol 4.5 g
- distilled water 1 liter;
Fig. 1. Plate exposure arrangement
the short stop bath of

- acetic acid 2 ml
- distilled water 1 liter

the fixing bath of

- sodium bisulfite 7.5 g
- sodium thiosulfate 400 g
- distilled water 1 liter

The plates were presoaked in distilled water at 25°C for three hours, soaked in developer at 5°C for two hours, warmed to 22°C in developer for 45 minutes, soaked in short-stop at 8°C for two hours, soaked in fixer at 5°C for two days, washed in tap water at 5°C for three days. After drying, the plates exhibited considerable stain, and the exposed areas were too badly fogged to scan except for the two minimum exposures. Therefore they were bleached for three hours in a solution of one ml concentrated HCl to 1 liter distilled water at 5°C and then refixed for one day at 5°C to bleach out the fog. After being washed for two days, the plates were soaked in 5% glycerine solution for one hour to help prevent the emulsion from curling, and then were dried.

A previous test exposure had indicated the exposure which would yield the minimum usable star density, so the plate was scanned first that would guarantee somewhere between thirty and a hundred stars. The background fog was so bad that stars could not be distinguished deeper than 400μ (after correction of 2.55 for shrinkage) in the emulsion. This "extinction depth" was found to be well defined over the whole plate to within 10μ, and also constant, so the plate was scanned only to this depth and regarded as a 400μ-thick emulsion. The scanning was done with a Bausch and Lomb binocular microscope fitted with a Brower micrometer stage, using a Leitz 53X oil-immersion objective and 5X Bausch and Lomb eyepieces. A goniometer was used to measure the horizontal projections of angles between prongs.

Any three-prong star that looked as if momentum might be conserved between the three prongs if they were assumed to be alphas was measured. A calibrated reticle was used to measure horizontal
components of range, and the calibrated Z micrometer of the micro-
scope was used to measure depth-range components. Range-energy and
range-momentum curves were made up from the table given by Ilford
(see Figs. 2 and 3) and using these, the horizontal components of
momentum of the prongs were calculated and a vector diagram drawn
for each star. If the vector diagram for horizontal components of mo-
mentum closed to within 20 percent of the length of the shortest vector,
the star was accepted. If one prong was too short for its angle to be
measured accurately (because a vector diagram could not be drawn),
conservation of momentum for that prong and its opposite pair were
checked as follows: $E_{1*}$ was calculated for that prong by both Eqs. (1')
and (4), using the measured prong lengths to get the energies. If the
same value of $E_{1*}$ was obtained within appropriate limits of error for
the two equations, this indicated conservation of momentum and the star
was accepted. In all cases, as in all previous experiments done on this
reaction, the conservation of momentum in the Z direction (that of the
gamma ray) was neglected, because of the inaccuracies of Z measure-
ments and the small momentum contribution of the gamma ray. Stars
with $hv$ lower than 18 Mev were ignored.
Fig. 2. Energy and momentum vs. range of alphas in emulsion
Fig. 3. Energy and momentum vs. range of alphas in emulsion.
ANALYSIS OF RESULTS

All of one plate and most of another with twice its exposure were scanned, and it was discovered that relatively fewer stars were found on the darker plate than should be due to its exposure. This indicated that stars were certainly being missed on the darker plate and made it seem probable that stars were missed also on the lighter one. There was no reason to believe, however, that the heavy fog on the dark plate should have any effect on the energy of the stars overlooked, so all the stars found on both plates were used to determine the shape of the excitation curve, and the best that could be done to get a value of the average cross section was to use only the stars on the first plate. Because the rate of finding stars was only two or three a week with full-time scanning, the thirty-three found on the first plate and the thirteen found on the second provided all of the data in the experiment.

On the basis of the relation \( E_{1}^* = E_{T} - \frac{3}{2} E_{1} \) (Eq. 1'), three values of \( E_{Be}^* \) were calculated for each star with \( h\nu > 26 \text{ Mev} \), so as to compare with Ref. 3. The distribution in \( E^* \) of the sum of all these values is plotted in Fig. 4 for 1-Mev intervals up to 22 Mev. The shape of the distribution in Fig. 4 does not agree well with the peak obtained by Goward and Wilkins at 16.9 Mev, but the statistical fluctuations for such small numbers of stars per unit energy interval could easily be large enough to spoil the true shape. If the numbers were larger, one might speculate on the meaning of the broad peak centered at 7 Mev, but one must always keep in mind that only one-third of the values of \( E^* \) can, at best, contribute to a peak, and this makes the numbers considered in the \( N(E^*) \) look even smaller, so far as significance goes.

In order to calculate the excitation function, it was desirable to have a value of the cross section; therefore, the cross section was calculated in 4-Mev-wide intervals from 22 to 42 Mev in order to permit making the energy-dependent corrections, and the results were averaged to get a cross section over this region. Since no stars were found with \( h\nu > 42 \text{ Mev} \), this average cross section was then used to normalize the results to those of Ref. 4, wherein the cross section is based upon 1700 stars, and place an upper limit upon the cross section for higher energies.
Fig. 4. Distribution in $E^*$ of alphas from stars with $hv > 26$ Mev
The ordinate of Fig. 5 is proportional to the theoretical bremsstrahlung spectrum $k \frac{dN_k}{dk}$ which is the amount of energy per unit energy interval in the beam of the Berkeley synchrotron. The ordinate is for the energy loss of a single electron as radiation when it hits the internal target. The area under the curve is 51 Mev, so the expression for the energy in an interval $\Delta k$ can be written

$$\Delta k \cdot k \frac{dN_k}{dk} = Y_o \Delta k \cdot \frac{Qk_{\text{max}}}{51 \text{ Mev}}$$  \hspace{1cm} (6)$$

$Y_o$ is the ordinate of Fig. 5; $k_{\text{max}}$ is the maximum energy of the beam, 330 Mev; and $Q$ is the number of equivalent quanta in the beam, defined so that $Qk_{\text{max}}$ is the total energy in the beam in Mev; therefore the number of electrons is $\frac{Qk_{\text{max}}}{51 \text{ Mev}} \cdot Q$ is known in terms of the ionization in a calibrated chamber,

$$Q = 1.34 \times 10^{10} \text{ equiv. quanta/microcoulomb.}$$

For the first plate, whose exposure was for 0.16 $\mu$Coul of beam, Eq. (6) and Fig. 5 yield:

<table>
<thead>
<tr>
<th>Interval</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>18-22</td>
<td>$6.25 \times 10^8$ quanta</td>
</tr>
<tr>
<td>22-26</td>
<td>$5.10 \times 10^8$</td>
</tr>
<tr>
<td>26-30</td>
<td>$4.28 \times 10^8$</td>
</tr>
<tr>
<td>30-34</td>
<td>$3.69 \times 10^8$</td>
</tr>
<tr>
<td>34-38</td>
<td>$3.25 \times 10^8$</td>
</tr>
<tr>
<td>38-42</td>
<td>$2.87 \times 10^8$</td>
</tr>
<tr>
<td>42-50</td>
<td>$4.9 \times 10^8$</td>
</tr>
<tr>
<td>50-70</td>
<td>$8.8 \times 10^8$</td>
</tr>
<tr>
<td>70-100</td>
<td>$8.6 \times 10^8$</td>
</tr>
</tbody>
</table>

For the calculation of the average cross section $\sigma_o$ in the intervals $\Delta k$, for each interval the following holds:

$$n(\text{stars}) = J(\text{quanta}) \times N_0 t \left( \frac{\text{atoms C}^{12}}{\text{cm}^2} \right) \times \sigma_o \left( \text{cm}^2 \right)$$

$$N_0 t = \frac{0.27 \text{ gm cm}^{-3} \text{ of C}^{12} \text{ in emulsion}}{12 \text{ gm mol of C}} \times 0.602 \times 10^{24} \frac{\text{atoms C}^{12}}{\text{mol}} \times 0.04 \text{ cm thickness}$$

$$N_0 t = 5.42 \times 10^{20} \frac{\text{atoms C}^{12}}{\text{cm}^2 \text{ of emulsion}}$$
Fig. 5

THEORETICAL BREMSSTRAHLUNG ENERGY DISTRIBUTION FOR BERKELEY SYNCHROTRON
For the first plate, using J's calculated above for the intervals:

<table>
<thead>
<tr>
<th>E interval (Mev)</th>
<th>No. of stars</th>
<th>( \sigma_0 ) (uncorrected)</th>
</tr>
</thead>
<tbody>
<tr>
<td>22-26</td>
<td>5</td>
<td>( 0.18 \times 10^{-28} \text{ cm}^2 )</td>
</tr>
<tr>
<td>26-30</td>
<td>13</td>
<td>( 0.56 \times 10^{-28} \text{ cm}^2 )</td>
</tr>
<tr>
<td>30-34</td>
<td>8</td>
<td>( 0.40 \times 10^{-28} \text{ cm}^2 )</td>
</tr>
<tr>
<td>34-42</td>
<td>4</td>
<td>( 0.12 \times 10^{-28} \text{ cm}^2 )</td>
</tr>
</tbody>
</table>

Before these can be averaged, they must be corrected for losses caused by tracks leaving the top and bottom of the emulsion, since this is an energy-dependent correction. To derive this correction to a first approximation, consider all the tracks that lie a distance \( x \) from the surface of the emulsion and extend in all directions. The ends of these tracks all lie randomly on the surface of a sphere with radius \( L \), where \( L \) is the track length. The interesting case is where \( 2L < t \), the emulsion thickness. The probability that a track starting at \( x \) will leave the emulsion is simply the ratio of the area of the part of the sphere formed by the track ends that leave the emulsion to the total area of this sphere. This first area is:

\[
A_1 = \int_0^{\cos \frac{1x}{L}} 2\pi L^2 \sin \theta \, d\theta = 2\pi L^2 \left[ -\cos \theta \right]_0^{\frac{x}{L}} = \frac{2\pi L^2}{4\pi L^2} \left[ 1 - \frac{x}{L} \right]
\]

or

\[
A_1 = 2\pi L^2 \left[ 1 - \frac{x}{L} \right]
\]

so the fraction of tracks that leave is

\[
f = \frac{A_1}{4\pi L^2} = \frac{2\pi L^2}{4\pi L^2} \left[ 1 - \frac{x}{L} \right] = \frac{1}{2} \left[ 1 - \frac{x}{L} \right]
\]

for depth \( x \).
If the density of track beginnings per unit thickness of the emulsion is \( \rho \), then \( \rho \, dx \) tracks begin at depth \( x \), and the number of tracks missed is \( f_0 \rho \, dx \), so the total number missed as \( x \) goes from 0 to \( L \) is

\[
\int_0^L f_0 \rho \, dx = \int_0^L \frac{1}{2} \rho \left[ 1 - \frac{x}{L} \right] \, dx = \frac{1}{2} \rho \left[ x - \frac{x^2}{2L} \right]_0^L = \frac{1}{2} \rho \left[ L - \frac{L^2}{2L} \right],
\]

so \( \text{number missed} = \frac{1}{4} \rho \, L \) for each surface of the emulsion.

Now the approximation is made that the only time a star has to be discarded because prongs leave the emulsion is when only one prong leaves. Thus if for each star we label the prongs \( L_1, L_2, L_3 \), the total number of stars missed of all these is

\[
\frac{1}{4} \rho L_1 + \frac{1}{4} \rho L_2 + \frac{1}{4} \rho L_3 = \frac{1}{4} \rho (L_1 + L_2 + L_3),
\]

and if \( t \) is the emulsion thickness, the total number of stars is \( \rho \, t \); thus, for both surfaces,

\[
\rho \, t = N_{\text{obs}} + \frac{1}{2} \rho (L_1 + L_2 + L_3).
\]

The correction factor is therefore defined as

\[
C = \frac{\rho \, t}{N_{\text{obs}}} = \rho \, t \, \frac{\rho \, t}{\frac{1}{2} \rho (L_1 + L_2 + L_3)} = 1 - \frac{1}{2t} \left( \frac{L_1 + L_2 + L_3}{t} \right)
\]

as long as \( t > 2 \, L_1 \).

For the purposes of this approximation let \( L_1 = L_2 = L_3 = \text{range of alpha of } \frac{E_T}{3} \), so

\[
C = \frac{1}{1 - \frac{3}{2} \left[ R\left( \frac{\text{v}}{3} \right) \right]/400 \mu}
\]

For the average value of \( \text{h} \) for the energy intervals:

<table>
<thead>
<tr>
<th>( \text{h} )</th>
<th>( C )</th>
<th>( R\left( \frac{E_T}{3} \right) ) in ( \mu )</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>1.1</td>
<td>24</td>
</tr>
<tr>
<td>28</td>
<td>1.15</td>
<td>33</td>
</tr>
<tr>
<td>32</td>
<td>1.20</td>
<td>43</td>
</tr>
<tr>
<td>38</td>
<td>1.30</td>
<td>60</td>
</tr>
<tr>
<td>46</td>
<td>1.50</td>
<td>86</td>
</tr>
<tr>
<td>60</td>
<td>2.10</td>
<td>140</td>
</tr>
<tr>
<td>72</td>
<td>4</td>
<td>200</td>
</tr>
</tbody>
</table>
Applying this correction to the previously calculated cross section:

<table>
<thead>
<tr>
<th>$h$</th>
<th>$\sigma_0$</th>
<th>$C$</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>22-26</td>
<td>$0.18 \times 10^{-28}$ cm$^2$</td>
<td>1.1</td>
<td>$0.2 \times 10^{-28}$ cm$^2$</td>
</tr>
<tr>
<td>26-30</td>
<td>$0.56 \times 10^{-28}$ cm$^2$</td>
<td>1.15</td>
<td>$0.65 \times 10^{-28}$ cm$^2$</td>
</tr>
<tr>
<td>30-34</td>
<td>$0.40 \times 10^{-28}$ cm$^2$</td>
<td>1.20</td>
<td>$0.48 \times 10^{-28}$ cm$^2$</td>
</tr>
<tr>
<td>34-42</td>
<td>$0.12 \times 10^{-28}$ cm$^2$</td>
<td>1.30</td>
<td>$0.16 \times 10^{-28}$ cm$^2$</td>
</tr>
</tbody>
</table>

The average $\sigma$ over the 22- to 42-Mev interval is $0.33 \times 10^{-28}$ cm$^2$. From Ref. 4, in which the cross section is based upon 1700 stars, the cross sections for the same intervals as above are $1.0 \times 10^{-28}$ cm$^2$, $2.6 \times 10^{-28}$ cm$^2$, $1.5 \times 10^{-28}$ cm$^2$, and $0.6 \times 10^{-28}$ cm$^2$ in order of increasing energy. The average is $1.26 \times 10^{-28}$ cm$^2$, so the ratio of average $\sigma$'s is $0.33/1.26 = 26$ percent. Goward and Wilkins state in Ref. 4 that they have increased their value of $\sigma$ by a factor of three over the course of their work since they did their first experiment on carbon stars by improvements in scanning, "geometrical corrections", and beam monitoring; therefore this small value of $\sigma$ is perhaps not too startling. The excitation curve for this experiment will therefore be normalized to Goward and Wilkins's cross section. The resultant histogram is shown in Fig. 6 for 2-Mev intervals from 18 to 42 Mev, corrected both for losses from the emulsion surfaces and for shape of the bremsstrahlung spectrum. The agreement with the shape of Goward and Wilkins's curve, which is shown dotted, is not unreasonable, considering the statistical fluctuation. The number of stars in each interval is written in for clarity.
Fig. 6. Excitation function for $^{12}\text{C} \; (\gamma, 3\alpha)$
CONCLUSIONS

Since the fundamental purpose of the work was to investigate the $(\gamma, 3\alpha)$ cross section at higher energies than had previously been used, and no stars were found at these energies, the most that can be done is to estimate an upper limit for the average cross section at high energies. The cross section can be written

$$\sigma(E) = \frac{n(\text{stars}) \times C(E)}{J(E) \text{quanta} \times N_o} \text{ Mev}$$

where $C(E)$ is the correction factor. Therefore, for two different intervals of widths $\Delta E_1$ and $\Delta E_2$ and energies $E_1$ and $E_2$,

$$\frac{\sigma_1}{\sigma_2} = \frac{n_1 \text{ stars}}{\Delta E_1 \text{ Mev}} \times \frac{\Delta E_2 \text{ Mev}}{n_2 \text{ stars}} \times \frac{J_2 \text{ quanta}}{\Delta E_2 \text{ Mev}} \times \frac{\Delta E_1 \text{ Mev}}{J_1 \text{ quanta}} \times \frac{C_1}{C_2}$$

where $n_1$ and $n_2$ are the numbers of stars in Intervals 1 and 2 and $J_1$ and $J_2$ are the total numbers of quanta in these intervals. To estimate an upper limit, one star is assumed found in Interval 1, and Interval 2, in which ten stars were found, is 28 to 30 Mev. Let Interval 1 be, respectively, 42 to 50 Mev, 50 to 70 Mev, and 70 to 100 Mev. Then

$$\frac{\sigma_1}{\sigma_2} = \frac{n_1}{n_2} \times \frac{J_2}{J_1} \times \frac{C_1}{C_2} \quad \sigma_2 = 2.8 \times 10^{-28} \text{ cm}^2 \quad \text{(from Ref. 4)}$$

$$\overline{\sigma}_{42-50} \leq 0.32 \times 10^{-28} \text{ cm}^2$$

$$\overline{\sigma}_{50-70} \leq 0.25 \times 10^{-28} \text{ cm}^2$$

$$\overline{\sigma}_{70-100} \leq 0.49 \times 10^{-28} \text{ cm}^2$$

The value of the correction for the highest energy interval above is bad, but a better approximation is probably not worth the trouble.

The large value of the cross section and the resonances observed at approximately 30 Mev quantum energy have been explained by Gell-Mann and Telegdi on the basis that levels in $C_{12}$ become accessible for
dipole absorption of gamma radiation that were not accessible to lower-energy quanta. It would be expected that at energies considerably above 30 Mev, the levels even in a nucleus as light as $^12C$ would be so broadened and overlapping as to be a continuum, and no resonances would be observed. The lack of higher-energy resonances observed in the $(\gamma, 3\alpha)$ process is in accordance with experimental evidence on other photo-nuclear processes such as $(\gamma, n)$, $(\gamma, pn)$, which exhibit absorption in levels at lower energies. These processes also exhibit a smooth fall-off of cross section at high energies, which is consistent with absorption in a continuum of energy levels rather than in single levels.
ACKNOWLEDGMENTS

I am grateful to Professor A. C. Helmholtz for continued advice and assistance, to the synchrotron crew for the exposures, and to Mr. R. P. Michaelis for development of the plates.

This work was performed under the auspices of the U. S. Atomic Energy Commission.
REFERENCES