Shift Reference Manual

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SHIFT
Reference Manual

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1 Overview

SHIFT is a description language for dynamic networks of hybrid systems. Such systems consist of components which can be created, interconnected and destroyed as the system evolves in time. Components exhibit hybrid behavior, consisting of continuous-time phases separated by discrete-event transitions. They may evolve independently or jointly with each other. They interact through input-output connections and synchronization of events. The interaction network may evolve in time.

We believe this model offers the proper level of abstraction for describing complex applications such as highway and air traffic control systems, robotic shopfloors, and other systems whose operation cannot be captured by conventional models, which typically support only static interconnections.

SHIFT is extensible. We plan to use SHIFT as a mechanism for defining the Automated Highway System Tool Interchange Format (AHSTIF). An automated highway system is a hybrid system with specific characteristics and restrictions. SHIFT supports the definition and enforcement of AHSTIF-specific semantics.

This is a reference manual for SHIFT. It presents its material in a logical order, but it aims to be concise and has no pretense to be a SHIFT tutorial. Section 2 describes the model, and section 3 the language.

The name SHIFT is a permutation of HSTIF (Hybrid System Tool Interchange Format).

2 The Model

2.1 Components and Worlds

A component is a part of a system, or a world. The behavior of a component (its time evolution) depends on its state, and its environment (the visible part of other components).

A world is an evolving set of components. During the evolution of the world new components may be created, and the manner in which they interact may change. At all times the behavior of the world is self-contained, that is, it depends only on the state of its components.
2.2 Component Model

A component is defined by its state, inputs, outputs, and exported events; its continuous time evolution; and its discrete event evolution.

At time \( t \), the component’s state \( s(t) \) is the tuple \((x(t), q(t), l(t))\). The vector \( x(t) \in \mathbb{R}^n \) is the continuous state. The discrete state \( q(t) \) is a state in a finite state machine. The link state \( l(t) \) is a vector of references to other components.

Links describe both physical (e.g., wires and mechanical actuators) and logical (e.g., sensors and communication channels) interconnections between components. Links are dynamic: the components they reference may change in time.

A component’s inputs, outputs, and exported events define the component’s interface to the rest of the world. Outputs are variables whose values are accessible (for reading) by other components. Exported events are state machine transitions which can be synchronized to those of other components. Inputs are variables whose values (during both continuous and discrete phases) are defined externally to the component.

The continuous time evolution of the continuous state \( x \) is defined by differential equations and algebraic expressions. The differential equations are in the form

\[
\dot{x}_i = f^x_i(x(t), u(t), w(t))
\]

where \( u(t) \) is the input of the component, and \( w(t) \) the output of linked components. The algebraic definitions are in the form

\[
x_j = g^x_j(x(t), u(t), w(t)),
\]

with the restriction that algebraic equations are not allowed—that is, there may not be loops in the dependency graph of algebraically-defined states. In a given discrete state \( q \), a state variable must be defined either algebraically or differentially. However, the mode of definition for a variable may change with the discrete state.

The discrete event evolution of a component is defined by a finite state machine. Edges in the state machine are labeled with guards, events and actions. A guard is a predicate on the state of the component, its inputs, and the outputs of other components. A transition on an edge may be taken only when its guard is true. Events are points of synchronizations between components. State machines in linked components synchronize according to event correspondence rules, discussed in section 3.12. A transition may trigger the execution of actions, which change the state and output of the component, and create new components.

Associated to each discrete state is an invariant: a predicate on the states of a component and the output of linked components, which constrains their region of feasibility. The behavior of the system is undefined outside this region.

2.3 World Model

The world is a directed graph of components, where the labeled edges are links. The graph is encoded by the links \( l(t) \) of components. Let \( a, b \) be components with \( b \in l_a(t) \), then there is an edge from \( a \) to \( b \).

The \( l(t) \) evolves in time. A component may change \( l(t) \) through link or unlink actions associated with a transition.
3 The Language

SHIFT is a textual notation for the abstractions in section 2.

3.1 Notation

Non-terminals are in *italics*. Keywords and other literal tokens are in *typewriter*. Braces indicate repetition: \{ X \}^* means zero or more repetitions of X, \{ X \}^+ means one or more repetitions. Brackets indicate optional parts, that is \[ X \] stands for zero or one instances of X. The vertical bar (’|’) denotes alternation.

3.2 Lexical conventions

A SHIFT specification is a sequence of printable ASCII characters, including space, tab, and newline. The characters are separated into *tokens* according to the rules given below (the rules are similar, but not identical, to those of the C programming language).

- An *identifier* is a sequence of characters from the set \{'a'...'z', 'A'...'Z', '0'...'9', '_'\}. The sequence must start with an alphabetic character or '_'. Certain identifiers, called *keywords*, are reserved by SHIFT, meaning they are not available as user-defined names. Table 1 lists all the keywords.

<table>
<thead>
<tr>
<th>all</th>
<th>do</th>
<th>if</th>
<th>minel</th>
<th>state</th>
</tr>
</thead>
<tbody>
<tr>
<td>array</td>
<td>else</td>
<td>in</td>
<td>number</td>
<td>symbol</td>
</tr>
<tr>
<td>continuous</td>
<td>exists</td>
<td>input</td>
<td>one</td>
<td>then</td>
</tr>
<tr>
<td>create</td>
<td>export</td>
<td>invariant</td>
<td>output</td>
<td>transition</td>
</tr>
<tr>
<td>define</td>
<td>flow</td>
<td>logical</td>
<td>set</td>
<td>type</td>
</tr>
<tr>
<td>discrete</td>
<td>function</td>
<td>maxel</td>
<td>setup</td>
<td>when</td>
</tr>
</tbody>
</table>

Table 1: List of SHIFT keywords.

- An *operator* is a sequence of characters from the set \{ '+' , '*' , '-' , '/' , '<' , '>' , '=' , '&' , '|' , '!' , '!' \}, possibly enclosed in a balanced pair of parentheses, '(' and ')', '{' and '}', or [' and ']. For instance: (1/2). (This is different from C: for instance, the character sequence ‘x+3’ contains the tokens ‘x’, ‘+-’, and ‘3’.)

- A *numeric-constant* consists of a mantissa followed by an optional exponent. The mantissa consists of one or more digits and optionally a single ‘.’ (period) in any position. The exponent is in the form ‘e[|E[|*|{digit}]^+’.  

- A *symbolic-constant* is a sequence of characters starting with $ followed by an identifier (examples: $ONE, $TWO).

- A sequence of characters starting with ‘/*’ and ending at the first occurrence of the pair ‘*/’ (included) is a comment.
A sequence of characters starting with ‘/’ and ending at the first occurrence of the end-of-line character is a comment.

- Tabs, spaces, newlines, and comments terminate tokens but otherwise have no meaning.
- The characters ‘“’, ‘!’, ‘\’, ‘~’, and ‘?’ are reserved and may be used internally by SHIFT.
- The characters ‘@’, ‘#’, ‘?’ are guaranteed never to be used by SHIFT (thus they may be given special meanings by a preprocessor).
- The remaining characters are used as specified by the grammar rules in the rest of this document.

3.3 Scopes

Several SHIFT constructs establish a relationship between an identifier (for instance, ‘x’) and an entity (for instance, a variable, or a type). The section of text for which the relationship is valid is called the scope of the identifier. SHIFT has nested scopes; a scope always starts and ends within another scope. When referring to two such scopes, the former is called the inner scope, the latter the outer scope. The outermost scope is also called the global scope.

This manual defines scoping rules along with each construct which defines a scope. Some rules are valid for all scopes.

- It is illegal to define an identifier twice in the same scope.
- It is permitted to redefine an identifier in an inner scope.
- Scopes are transparent: all meanings of identifiers from the outer scope are also valid in an inner scope, except those of identifiers which are redefined.

Most constructs associating identifiers to entities are called declarations. Unlike other languages, SHIFT has no ‘declaration-before-use’ rule. A declaration is valid for its entire scope, no matter where it appears.

3.4 Preprocessing

SHIFT uses the same preprocessing as the ANSI C language, for the purpose of macro substitution (#define), file inclusion (#include), and all other available facilities.

3.5 Overall structure

A SHIFT specification is a sequence of definitions.

\[
\text{specification} \Rightarrow \{ \text{definition} \}^+ \\
\text{definition} \Rightarrow \text{component-type-definition} \\
\text{component-type-definition} \Rightarrow \text{global-variable-decl} \\
\text{component-type-definition} \Rightarrow \text{external-function-decl}
\]
3.6 Component types

A component type definition describes a set of components with common behavior.

\[
\text{component-type-definition} \Rightarrow \text{type type-name [: parent]} \{ \{ \text{type-clause } \}^+ \} [;]
\]

\[
\text{state-declarations} \Rightarrow \text{declaration-list}
\]

\[
\text{action-clause} \Rightarrow \text{local-event } \{ \text{local-event } \}^*
\]

\[
\text{parent} \Rightarrow \text{identifier}
\]

\[
\text{type-name} \Rightarrow \text{identifier}
\]

\[
\text{type-clause} \Rightarrow \text{state state-declarations}
\]

\[
\text{input input-declarations}
\]

\[
\text{output output-declarations}
\]

\[
\text{export export-list}
\]

\[
\text{setup action-clause}
\]

\[
\text{setup } \{ \text{action } ; \}^*
\]

\[
\text{flow flow-list}
\]

\[
\text{discrete discrete-state-list}
\]

\[
\text{transition transition-list}
\]

\[
\text{input-declarations} \Rightarrow \text{declaration-list}
\]

\[
\text{output-declarations} \Rightarrow \text{declaration-list}
\]

\[
\text{export-list} \Rightarrow \text{local-event } \{ \text{local-event } \}^*
\]

\[
\text{local-event} \Rightarrow \text{identifier}
\]

The definition ‘type X : Y { ... }’ defines a component type named X in the global scope. The body of the definition (all text between { and }) is the local scope for X.

Y, if present, must be the name of another component type called the parent type of X. The function P maps a type to its parent: in this case, \(Y = P(X)\). Type A is a supertype of B when \(A = P(B)\) or A is a supertype of \(P(B)\). In such a case, B is a subtype of A.

A type represents a set of components. The statements ‘component x has type X’ and ‘x \(\in\) X’ are equivalent.

The parent/child relationship implies certain constraints between the input, output, and export lists of the involved types. These are described in section 3.16.

In a type definition there can be multiple clauses of each kind. The items in all clauses of like kinds are concatenated in the order in which they appear, as if there were a single clause containing all of them.
3.7 Declaration lists

\[
\begin{align*}
\text{declaration-list} & \Rightarrow \text{declaration} \{ ; \text{declaration} \}^* \\
\text{declaration} & \Rightarrow \text{type variable-clause} \{ , \text{variable-clause} \}^* \\
\text{variable-clause} & \Rightarrow \text{variable-name} [ \text{init} ] \\
\text{type} & \Rightarrow \text{simple-type} \\
& \quad | \text{set} (\text{type}) \\
& \quad | \text{array} (\text{type}) \\
\text{variable-name} & \Rightarrow \text{identifier} \\
\text{init} & \Rightarrow : = \text{expression} \\
\text{simple-type} & \Rightarrow \text{number} \\
& \quad | \text{continuous number} \\
& \quad | \text{symbol} \\
& \quad | \text{type-name}
\end{align*}
\]

Input, output, and state declarations declare local variables in the scope of a component type. Variable-name identifies a variable, and type is its type. The initializing expression is evaluated at component creation time, and must be of a compatible type. If a variable is defined algebraically, its initialization is ignored. Variables are initialized to zero or nil unless otherwise specified.

3.8 Types

**Numbers.** Variables of number type hold real numbers, such as 1 or 2.72. Scalar variables whose type is continuous number may be used in the definition of the continuous evolution of a component, as shown in section 3.9.

**Symbols.** Variables of type symbol hold symbolic constants (section 3.2), such as $GO$ or $STOP$. Every different symbolic constant represents a distinct value. The only operations available for symbols are assignment and comparison.

**Component types.** A variable of type $X$, where $X$ is a component type, is a reference to an element in $T_X$, that is a component whose type is $X$ or any of its descendants.

**Sets.** A variable of type set($E$) contains a set of elements of type $E$. The set might be empty.

**Arrays.** A variable of type array($E$) contains a one-dimensional array of elements of type $E$. The same variable may hold arrays of different lengths at different times. The elements of the array are numbered consecutively starting from 0. An array might be empty.

3.8.1 Links

When the type in a declaration is a component type, or a set or array whose element is a component type, the declared variable is a reference to another component (or a set of components). Such variable, called link variable (or simply link), identifies edges in the link graph.

Let type $X$ define a link variable $c$ with type $Y$. Then each $x \in X$ has an edge to some $y \in Y$, or to the special component nil. The edge is identified by the pair $(x, c)$. In the local scope of $X$, $c$ is a valid expression, and it refers to $y$. This is a single-valued link.
If \( c \) has type `set(Y)` or `array(Y, ...)`, each \( x \in X \) has a set of edges to distinct components in \( Y \). In the case of a set the edges cannot be individually named. This is a *multi-valued link*.

For simplicity, this manual often uses \( c \) to refer to the element, or set of elements, currently linked by \((x, c)\).

### 3.9 Flows

```
flow-list \Rightarrow flow \{ , flow \} *
flow \Rightarrow flow-name \{ equation-list \}
flow-name \Rightarrow identifier
equation-list \Rightarrow flow-or-equation \{ , flow-or-equation \} *
flow-or-equation \Rightarrow differential-equation
                | algebraic-definition
                | flow-name
differential-equation \Rightarrow lhs \ '= expression
algebraic-definition \Rightarrow lhs = expression
lhs \Rightarrow continuous-number-state
| continuous-number-output
continuous-number-state \Rightarrow variable-name
continuous-number-output \Rightarrow variable-name
```

A *flow* is a named set of differential equations and algebraic definitions, used to define the continuous evolution of one or more variables in the component. *Flow-name* refers to the set of equations in *equation-list*. Its scope is the body of the enclosing component type definition.

Flows define the behavior of state or output variables of type `number`. The left-hand side of an algebraic definition, or a differential equation, is, respectively, one such variable or its derivative. (For convenience, these rules are more relaxed than those of the standard input-output-state model. The only difference between states and outputs is that outputs are visible on the outside, states are not.)

The right-hand sides of both kinds of equations are expressions of all variables of this component (states, inputs, and outputs) and the outputs of linked components.

There may be no circular dependencies in algebraic definitions.

When a *flow-name* appears in place of an equation, it stands for all equations in the corresponding *equation-list*. If a flow defines variable \( x \) more than once, all definitions of \( x \) except the last one are ignored.

Flows are used in the definition of discrete states, as shown in the next section. Two flow names have special meanings.

- The flow named `default` is the default flow for all states that do not explicitly specify one.
- The flow named `stop` sets the derivative of all variables to zero.

### 3.10 Discrete states

```
discrete-state-list \Rightarrow discrete-state-clause \{ , discrete-state-clause \} *
discrete-state-clause \Rightarrow state-name \{ { equation-list } | { invariant expression } \}
state-name \Rightarrow identifier
```

7
A *state-name* is the name of the state. There must be at least one state in each machine. The first state in the list is the initial state for a newly-created component.

*Flow* specifies the continuous behavior at the corresponding discrete state, as given in section 3.9. The optional *invariant* expression is expected to be always true.

### 3.11 Transitions

Transition defines one or more edges in the finite state machine of a component type: one edge from each state in the *from* set to the *to-state* state. *Set-of-states* is a constant expression which evaluates to a set of states, or a single state. In this context, the identifier *all* is the set of all states for this machine. See section 3.17 for a complete list of set functions and constructors.

*Event* is the event associated with this transition. It must appear in the *export-list* of the component. *Exported-event* is a local event in a linked component. The link variable in an exported event may not be algebraically defined. When a transition contains exported events, the state machine of this component potentially synchronizes with the state machines in other components. The synchronization rules are given in section 3.12.

The optional *guard-clause* contains a logical algebraic expression called the *guard*. A transition may be taken only if the guard is true.

If the guard contains an expression with the quantifiers *exists*, *min1*, or *max1*, the quantified variable defines a temporary link whose scope is the action list for the transition (see section 3.17).

The *action-clause* specifies actions which are taken concurrently with the transition, as described in section 3.14.

### 3.12 Synchronization rules

A component synchronizes its state machine with other state machines by labeling its own edges with *local-events* and *external-events*. Local events are exported; they can be used as external events by other components, and they can appear in *setup-only actions* (section 3.14.3). Each label of an edge $E$ establishes conditions under which a transition may be taken along $E$. 
When all conditions are satisfied, and the guard, if present, evaluates to true, and the component is in a state that has \( E \) as an outgoing edge, then the transition along \( E \) is taken simultaneously with other transitions as required by the conditions.

The conditions associated with each label are as follows. Let \( x \) and \( y \) be components, and \( Z \) a set of components. Let \( c \) be a single-valued link, and \( C \) a set-valued link. Let \( e_y \) be a local event for \( y \), and \( e_z \) a local event for all components in \( Z \).

- If \( c \) evaluates to `nil`, an edge labeled \( c:e_y \) may not be taken.
- If \( c \) evaluates to \( y \), an edge \( E \) labeled \( c:e_y \) must be taken simultaneously with an edge \( E' \) labeled \( e_y \) in \( y \); and, conversely, \( E' \) must be taken simultaneously with \( E \).
- If \( C \) evaluates to the empty set, the edge labeled \( C:e_z \) may not be taken if `set-sync-rule` is \( \text{on-} \). Otherwise it may be taken.
- If \( C \) evaluates to \( Z \) then an edge labeled \( e_z \) in any \( z \in Z \) may only be taken simultaneously with an edge labeled \( C:e_z \). The following also applies.
  - If the synchronization rule is \( \text{on-} \), then an edge labeled \( C:e_z \) may only be taken simultaneously with an edge labeled \( e_z \) in a single component \( z \in Z \). If a temporary link is specified, it is assigned the component \( z \). The scope of the temporary link is the action list for the transition.
  - Otherwise, if the rule is \( \text{all} \), an edge labeled \( C:e_z \) must be taken simultaneously with an edge labeled \( e_z \) in every \( z \in Z \).

3.13 Evolution of a Shift system

A Shift system starts by executing all initializations of global variables, at time \( t = 0 \). Then the system evolves by alternating discrete and continuous phases, starting with a discrete phase.

In the discrete mode, all possible transitions are taken, in some serial order unless explicitly synchronized. Time does not flow in the discrete mode. The system switches to continuous mode when no more transitions are possible.

The system evolves in continuous mode according to the flow associated to the discrete state of each component. As soon as it becomes possible for one or more components to execute a transition, time stops again.

3.14 Actions

\[
\begin{align*}
\text{action-clause} & \Rightarrow \text{define-clause} \text{ do-clause} \\
\text{define-clause} & \Rightarrow \text{define} \{ \text{local-definition} ; \}^* \\
\text{local-definition} & \Rightarrow \text{temp-var} := \text{expression} \\
\text{temp-var} & \Rightarrow \text{identifier} \\
\text{do-clause} & \Rightarrow \text{do} \{ \text{action} ; \}^* \\
\text{action} & \Rightarrow \text{reset-action} \\
& | \text{create-action} \\
& | \text{setup-only-action}
\end{align*}
\]
Actions are used to change the continuous state, create or destroy components, and change the way in which components are linked. **Setup-only actions** may only be used in a `setup` clause and establish I/O connections and synchronization of exported events.

A **local-definition** declares and initializes a variable whose scope is the following local definitions, and the actions in the `do-clause`.

### 3.14.1 Resets

\[
\text{reset-action} \Rightarrow \text{selector} := \text{expression}
\]

*Selector* refers to a state or output variable of this component, or an input variable of a linked component (it is defined in section 3.17). *Expression* is an expression of the old values (that is, before they are reset) of all variables (input, state, and output) of this component, and output variables of linked components. Resets have no effect on variables which are algebraically defined in the final state of the associated transition.

**Note on ‘:=’ vs. ‘=’**. The notation ‘\(l := r\)’ represents a one-time assignment which occurs during the discrete phase. The notation ‘\(l = r\)’ in certain contexts establishes equality of the left and right-hand sides as time flows. In other contexts, ‘\(x = y\)’ is a logical expression which evaluates to `true` when \(x\) is equal to \(y\).

### 3.14.2 Creation

\[
\begin{align*}
\text{create-action} & \Rightarrow \text{create-expression} \\
\text{create-expression} & \Rightarrow \text{create} ( \text{type-name} \{, \text{initializer}\}^*) \\
\text{initializer} & \Rightarrow \text{variable} := \text{expression} \\
\text{variable} & \Rightarrow \text{identifier}
\end{align*}
\]

The action `create(C, ...)` creates a new component of type `C`. The left-hand side of an `initializer` is a variable (not just an input) in the new component. The initializer sets this variable to the value of the right-hand side, overriding the initial value of the variable in the declaration of the new component.

The scopes of the left and right-hand sides of an initializer differ. Thus the action `create(C, v := v)` sets variable \(v\) in the new component to the value of variable \(v\) in the creating component.

The setup actions of the new component are executed after completion of the transition that created it.

### 3.14.3 Setup-only actions

\[
\begin{align*}
\text{setup-only-action} & \Rightarrow \text{external-event} \leftrightarrow \text{external-event} \{ \leftrightarrow \text{external-event} \}^* \\
\text{connection} & \Rightarrow \text{input} (\text{link-var}) = \text{expression} \\
\text{input} & \Rightarrow \text{identifier} \\
\text{link-var} & \Rightarrow \text{identifier}
\end{align*}
\]

Setup-only actions are only allowed in the `setup` phase of a component’s life. They establish static event synchronization and I/O connections for components without their direct involvement.
A connection makes the value of the left-hand side be that of the right-hand side at all times. If the right-hand side of a connection is not a continuously-varying number, the left-hand side may not contain continuously-varying numbers. In the input definition \( u(a) = E \), \( u \) must be an input of component \( a \). The connection is static: if \( a \) later changes, the input definition refers to the value of \( a \) at setup time. [ What happens when a flow conflicts with a connection? ]

The event synchronization

\[
a : e \leftrightarrow b : f
\]

specifies that event \( e \) in \( a \) and event \( f \) in \( b \) may only occur simultaneously. Event synchronizations are also static, and the synchronized components forever remain those reached through \( a \) and \( b \) at setup time.

Some useful forms of connections are I/O connections \((u(a) = y(b))\), where \( y \) is an output of linked component \( b \), I/I connections \((u(a) = v)\), where \( v \) is an input of this component), O/I and O/O connections.

There may not be circular dependencies in discrete definitions.

### 3.14.4 Linking and unlinking

Links are established and removed by resets (section 3.14.1) of link variables.

In a link statement of the form \( X := Y \), \( X \) refers to a single or multi-valued link in this component. The action modifies the edge, or edges, named by \( X \).

When \( X \) is a single-valued link, the action removes the existing link and adds a new one, from this component to the component obtained by evaluating \( Y \) (possibly to the nil component).

When \( X \) is a multi-valued link, the action can add or remove edges from the set, or leave it unchanged.

### 3.14.5 Execution of actions

The order in which actions are specified is inconsequential. Actions are executed in phases as follows.

1. All components specified by create-expressions are created.
2. The right-hand sides and the destinations of resets are evaluated, and so are the component initializers.
3. The previously computed values for resets and link actions and component initial values are assigned to their destinations.
4. Setup-only actions are executed.

Definitions in the define-clause are evaluated in the order in which they are given. The execution sequence for the create-expression is: initial values, passed in arguments, define/do actions.
3.15 Setup clause

If a setup clause is present, its actions are executed before the finite-state machine enters the initial state. If the initial state is $S_0$, this is equivalent to creating an additional state $S_{-1}$, making it the initial state, and placing an unguarded edge from $S_{-1}$ to $S_0$ with the setup actions. Section 3.14 lists all possible actions. Some actions (the setup-only actions of section 3.14.3) may only appear in the setup clause.

3.16 Input, output, and export declarations

Variables declared in an output list may be used outside a component. They may not be reset or defined outside the component.

The input list declares local variables which can only be defined and reset from outside the component.

The export declares local events of a type. They are all visible to other components.

If $X$ is the parent of $Y$, the export, input, and output lists of $Y$ must be supersets of, respectively, the export, input, and output lists of $X$. The same name in two input or output lists must refer to the same kind of object.
3.17 Expressions

\[
\text{expression} \Rightarrow \text{selector} \\
| \text{numeric-constant} \\
| \text{symbolic-constant} \\
| \text{true} \\
| \text{false} \\
| \text{nil} \\
| \text{expression} \ \text{binary-operator} \ \text{expression} \\
| \text{prefix-operator} \ \text{expression} \\
| \text{expression} \ \text{postfix-operator} \\
| \text{expression} \ \left[ \ \text{expression-list} \ \right] \\
| \text{expression} \ \left[ \ \text{expression} \ \right] \\
| \left( \ \text{expression} \ \right) \\
| \text{type} \\
| \{ \ \text{expression-list} \ \} \\
| \left[ \ \text{expression-list} \ \right] \\
| \text{all} \\
| \text{self} \\
| \text{state-name} \\
| \text{if} \ \text{expression} \ \text{then} \ \text{expression} \ \text{else} \ \text{expression} \\
| \text{exists} \ \text{identifier} \ \text{in} \ \text{expression} : \ \text{expression} \\
| \text{minel} \ \text{identifier} \ \text{in} \ \text{expression} : \ \text{expression} \\
| \text{maxel} \ \text{identifier} \ \text{in} \ \text{expression} : \ \text{expression} \\
\text{selector} \Rightarrow \text{continuous-selector} \\
| \text{link-selector} \\
\text{continuous-selector} \Rightarrow \text{number-var} \\
| \text{number-var} \ \left( \ \text{link-selector} \ \right) \\
\text{link-selector} \Rightarrow \text{link-var} \\
| \text{link-var} \ \left( \ \text{link-selector} \ \right) \\
\text{expression-list} \Rightarrow \text{expression} \ \left[ \ , \ \text{expression-list} \ \right] \\
\text{prefix-operator} \Rightarrow \text{operator} \\
\text{postfix-operator} \Rightarrow \text{operator}
\]

The indexing expression \( a [i] \) accesses the \((i + 1)\)-th element of array \( a \).

The expression \( \{ e_1, \ldots, e_n \} \) evaluates to a set containing the elements \( e_1, \ldots, e_n \), which must all be of the same type. Similarly, the expression \( [ e_0, \ldots, e_{n-1} ] \) evaluates to an array of length \( n \).

The expression \text{all} evaluates to the set of all states in the state machine of this type.

The expression \text{self} is a self-link, that is a reference to the component containing the expression.

The expression \text{nil} is a special component whose behavior is absolutely boring: it has no inputs, outputs, or exported events. It may be assigned to links of any type.

The logical expression \text{if} \ \text{x} \ \text{then} \ \text{y} \ \text{else} \ \text{z} \text{ evaluates to } \text{y} \text{ if } \text{x} \text { is true, else it evaluates to } \text{z}. Grammar ambiguities are resolved by operator precedence. [need precedence table here]
3.18 Predefined functions and operators

The predefined functions and operators are listed and explained in tables 2—4. Table 2 gives the standard functions on sets and logical values. Table 3 gives the arithmetic and mathematical operators. Table 4 lists miscellaneous operators.

\[
\begin{array}{|c|c|}
\hline
\text{Expression} & \text{Meaning} \\
\hline
\text{exists } x \text{ in } S : E & \text{A logical expression which evaluates to true if and only if the set } S \text{ contains at least one element which, when bound to } x, \text{ causes the expression } E \text{ to be true. If an exists expression appears in a guard, the scope of } x \text{ includes the actions for that transition, where it is bound to one of the components which satisfy the guard.} \\
\hline
\text{maxel } x \text{ in } S : E & \text{The element of } S \text{ which, when bound to } x, \text{ respectively maximizes or minimizes expression } E. \\
\hline
\text{minel } x \text{ in } S : E & \text{Respectively the union, intersection, and difference of sets } S_1 \text{ and } S_2. \\
\hline
x \text{ in } S & \text{A logical expression which is true if } x \text{ is a member of } S. \\
\hline
\text{reduce}(S, f) & \text{If } S = \{e_1, \ldots, e_n\}, \text{reduce}(S, f) \text{ returns } f(e_n, f(\ldots, f(e_2, e_1)\ldots)), \text{ and reduce}(S, f, e_0) \text{ returns } f(e_n, f(\ldots, f(e_2, e_1)\ldots)). f \text{ is a binary function or a binary operator enclosed in double quotes (e.g., "+").} \\
\hline
\text{size}(S) & \text{The number of elements in set } S. \\
\hline
\text{and, or, xor, not} & \text{The standard logical connectives.} \\
\hline
\end{array}
\]

Table 2: Set and logical operators.

3.19 External functions

\[
\begin{align*}
\text{external-function-decl} & \Rightarrow \text{function function-name ([arg-list]) -> return-type} \\
\text{arg-list} & \Rightarrow \text{declaration-list} \\
\text{return-type} & \Rightarrow \text{type}
\end{align*}
\]

SHIFT does not have functions, but a SHIFT program can refer to external functions, whose implementation must be provided for the purpose of simulation. External functions are written in C. The implementation of the simulator may impose further restrictions (for instance, it may require that the type of all arguments and return values be number), and also defines a correspondence between SHIFT types and C types (for instance, it may specify that the SHIFT number type corresponds to the C double type).

3.20 Global variables

\[
\begin{align*}
\text{global-variable-decl} & \Rightarrow \text{global declaration-list}
\end{align*}
\]

Global variables have global scope, can be used in expressions, and can be set by any component.
<table>
<thead>
<tr>
<th>Expression</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>+, -, *, /, **</td>
<td>The standard arithmetic and relational operators (/= is &quot;not equal.&quot;)</td>
</tr>
<tr>
<td>&lt;, &gt;, &lt;=, &gt;=, =, /=</td>
<td></td>
</tr>
<tr>
<td>exp(x), ln(x), log10(x)</td>
<td>The standard elementary mathematical functions.</td>
</tr>
<tr>
<td>sin(x), cos(x), tan(x)</td>
<td></td>
</tr>
<tr>
<td>sqrt(x)</td>
<td></td>
</tr>
<tr>
<td>atan(x), atan2(x, y)</td>
<td></td>
</tr>
<tr>
<td>abs(x)</td>
<td>Absolute value.</td>
</tr>
<tr>
<td>floor(x), trunc(x), round(x),</td>
<td>Coercion to integers. Floor, trunc, and round produce results</td>
</tr>
<tr>
<td></td>
<td>rounded toward (-\infty), toward 0, and toward nearest.</td>
</tr>
<tr>
<td>max(x_1, ..., x_n)</td>
<td>Maximum and minimum.</td>
</tr>
<tr>
<td>min(x_1, ..., x_n)</td>
<td></td>
</tr>
<tr>
<td>signum(x)</td>
<td>Returns -1, 0, or 1, depending on whether (x) is negative, zero, or positive.</td>
</tr>
<tr>
<td>random()</td>
<td>random requires no arguments and returns a random number</td>
</tr>
<tr>
<td></td>
<td>uniformly distributed in ([0, 1]).</td>
</tr>
</tbody>
</table>

Table 3: Arithmetic Operators and Elementary Functions

<table>
<thead>
<tr>
<th>Expression</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>narrow(X, y)</td>
<td>The compile-time type of narrow(X, y) is (X). Let (Y) be the compile-time type of (y). If (Y) is (X) or a supertype of (X), and the run-time value of (y) has type (X) or a subtype of (X), then narrow(X, y) returns (y), otherwise it is an error.</td>
</tr>
</tbody>
</table>

Table 4: Miscellaneous Operators

4 Acknowledgements

Most of the ideas in this manual are the result of discussions involving the authors and other PATH members. In particular we have greatly benefited from discussions with Datta Godbole, Raja Sengupta and Pravin Varaiya.