Sell the Plant? The Impact of Contract Manufacturing on Innovation, Capacity, and Profitability

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In the electronics industry and others, original equipment manufacturers (OEMs) are selling their production facilities to contract manufacturers (CMs). The CMs achieve high capacity utilization through pooling (supplying many different OEMs). Meanwhile, the OEMs focus on innovation: research and development, product design, and marketing. We examine how this change in industry structure affects investment in innovation and capacity, and thus profitability. In particular, innovation is noncontractible, so OEMs will invest less in innovation than is ideal for the industry as a whole. Hence, although contract manufacturing improves capacity utilization, it may reduce the profitability of the industry as a whole by weakening the incentives for innovation. Contract manufacturing is not the only means to achieve capacity pooling. Alternatively, the OEMs can pool capacity with one another through supply contracts or a joint venture. This may result in underinvestment or overinvestment in innovation and capacity, but always increases profitability. We find that the sale of production facilities to a CM improves profitability for the industry as a whole if and only if OEMs are subsequently in a strong bargaining position vis-à-vis the CM. If the OEMs are indeed very strong, the gain from pooling capacity via contract manufacturing is maximized in industries with moderate cost of capacity.

Key words: biform games; contract manufacturing; capacity pooling

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1. Introduction

The use of contract manufacturing is important and growing in a range of industries, including electronics, pharmaceuticals, automotive, and food and beverage production (Tully 1994). Increasingly, firms that traditionally manufactured their own products are outsourcing production and focusing instead on product design, development, and marketing. In the electronics industry, contract manufacturing grew at a compounded average growth rate of roughly 25% from 1989 to 1998. Contract manufacturing’s share of total electronics production grew from 9% in 1994 to 17% in 1998 (Francois 1999). Original equipment manufacturers (OEMs) outsourced $75 billion to contract manufacturers (CMs) in 2000, representing 10% of total electronics production. The bulk of electronics outsourcing is in computers (personal computers, workstations, peripherals) and communications (network and telecommunication equipment). Despite pessimism about the near term, the long-term growth rate for electronics contract manufacturing is projected to be 25% per year (Boase 2001). In pharmaceuticals, outsourcing accounted for 20% of production in 1988. In 1998, the figure was 50%–60%, and it is projected to grow to 60%–70% by 2005 (van Arnnum 2000).

Historically, firms have done both innovation (research and development, product design, and marketing) and production in-house. Under such vertical integration, because each firm fills its demand from its own production capacity, inefficiency in the use of capacity can result. For example, the pharmaceutical industry is characterized by long development cycles (roughly 12 years) and intense time-to-market pressure. Consequently, a company that wants to manufacture its own product must make a large capital investment in a plant before the drug has completed regulatory trials. If the drug fails, the plant may have little value (Tully 1994). The prospect of inefficient capacity use has implications for how firms invest in innovation and production capacity.

In industries where production asset specificity is low, contract manufacturing offers the potential for improved capacity utilization because CMs can pool demand from a diverse set of OEMs. A common belief is that contract manufacturing increases the rate of innovation by reducing the cost of production
capacity and allowing OEMs to focus their financial and managerial resources on product development and marketing. Surprisingly, we find that outsourcing may reduce innovation. We characterize the sensitivity of capacity and innovation to industry structure and the bargaining confidence (i.e., the anticipated bargaining strength) of OEMs.

The first step in our analysis is to compare the traditional firm model with a pooled model in which firms share their production capacity and act to maximize total system profit. The effect of such ideal pooling on innovation depends on how innovation affects demand: If innovation increases the potential market size, then pooling increases innovation; if innovation increases the probability that a product is successful, then pooling can either increase or reduce innovation. Second, we show how innovation and capacity investments deviate from their ideal levels when pooling is achieved by outsourcing to a CM. The basic problem is that OEMs will invest in innovation only what they expect to recoup by negotiating a favorable supply contract with the CM.

We evaluate the effect of outsourcing production to a CM on the profitability of the firms. Morgan Stanley reports that in the electronics industry, the set of OEM assets that are candidates for divestiture is substantial, with potential for more than $10 billion per year in sales (Fleck and Craig 2001). Managers of such assets must decide whether to sell the plant and outsource production or keep manufacturing in-house. While outsourcing manufacturing can increase profit by improving capacity utilization, it may instead reduce profit by weakening the incentives for innovation.

For an OEM, an alternative to outsourcing production to a CM is to retain production and pool capacity with other OEMs through supply contracts or a joint venture. Although it has received less attention in the business press, outsourcing among OEMs is widespread. In electronics, the OEM outsourcing market, at $115 billion in 2000, is over 50% larger than the CM outsourcing market (Boase 2001). For example, in addition to manufacturing its own computer products, Taiwan’s Acer manufactures for Compaq and IBM. Pharmaceutical companies use excess capacity to manufacture for competitors (Tully 1994). AMD and Fujitsu have been highly successful in their joint venture to produce flash memory chips (Mahon and Decker 2001). We prove that OEM outsourcing can result in excessive innovation relative to the ideal pooled case. If the bargaining power of an OEM vis-à-vis the CM is large, then total system profit is greater under CM outsourcing than OEM outsourcing. However, if OEMs anticipate being in a weak bargaining position vis-à-vis the CM, the OEMs would do better to outsource among themselves than to sell their production facilities to a CM.

**Literature Survey**

While OEMs may contract for capacity, they are unlikely to make these contracts contingent on their early investments in innovation or on the resulting market conditions. In the economics literature on incomplete contracts, Klein et al. (1978) and Williamson (1979) were the first to argue that if a buyer and a supplier cannot write complete, contingent contracts and if the values of their assets depend on collaboration, then they will make inefficient investments. Grossman and Hart (1986) investigate how changes in asset ownership affect the incentives for investment. In this spirit, our paper shows how industry structure—who owns the production facilities—influences the incentives for investment in innovation and capacity. Hart and Moore (1990) further illustrate how ownership influences incentives for employees as well as owner-managers. An implication is that consolidating our OEMs and CM into a single firm would not necessarily achieve the optimal levels of innovation and capacity because of difficulties in providing incentives to employees within a firm.

The economics literature on incomplete contracts is based on cooperative game theory. However, the use of cooperative game theory to analyze problems in operations and supply chain management is very new. Several papers assume that capacity/inventory is noncontractible and examine how firms invest in capacity and subsequently bargain cooperatively over its use. In contrast to the literature (all the papers referenced below assume that demand is exogenous), we assume that demand is endogenous, and that demand-stimulating innovation is noncontractible. Anupindi et al. (2001) and Granot and Sosic (2003) consider a network of retailers with independent stochastic demands: Each chooses his inventory level; then demand is realized; and the retailers bargain cooperatively over the transshipment of excess inventory to meet excess demand. Anupindi et al. (2001) propose an allocation mechanism (a rule for shipping inventory and sharing the gains from trade) that is in the core and creates incentives for retailers to choose system-optimal inventory levels in the initial stage. Granot and Sosic (2003) point out that this allocation mechanism will fail to achieve the first best if retailers can hold back some of their residual inventory. They propose two alternative allocation mechanisms that induce the retailers to share their residual inventory efficiently, but may not be in the core. In general, there are no core allocation rules that induce efficient sharing of residual inventories. Van Mieghem (1999) considers a setting in which one OEM can purchase capacity from a second OEM after demand occurs. To induce first-best capacity investments, the OEMs must write contracts with payments contingent on the state of demand. In Chod and Rudi (2003), two OEMs
invest in capacity, update their demand forecasts, and then trade capacity. With demand uncertainty formulated as a multiplicative shock to the price, the potential for trade increases the initial capacity investment.

A simplifying assumption in our analysis is that the OEMs do not compete for customers. Other researchers (Parlar 1988, Lippman and McCardle 1997, Mahajan and van Ryzin 2001) consider the inventory-stocking decisions made by competitive retailers. A common insight is that this competition causes the retailers to carry excess stock. Anupindi and Bassok (1999) and Müller et al. (2002) consider the case where retailers instead cooperate in ordering and pooling inventories. Lee and Whang (2002) and Rudi et al. (2001) consider settings in which retailers trade stock after experiencing demand.

Another key assumption in our analysis, and in the aforementioned OM and economics papers, is that all parties have common information. Spulber (1993) considers a monopolist manufacturer that must allocate its scarce capacity across a set of buyers, but does not know their individual or aggregate demand precisely. He derives optimal pricing schemes for the manufacturer, and proves that information asymmetry results in underinvestment in capacity by the manufacturer and inefficient allocation among the buyers. Cachon and Lariviere (1999) also consider capacity choice and allocation under information asymmetry, assuming a constant wholesale price. Tunca and Mendelson (2001) model multiple suppliers and OEMs that contract for supply and then, after obtaining private demand information, trade in an exchange.

Recent economics and industrial organization papers on outsourcing are surveyed by Sturgeon (2002) and Moorkerjee (2003).

This paper is organized as follows. Section 2 evaluates the impact of pooling capacity. Section 3 explores the setting where OEMs outsource production to a CM. Section 4 explores the issues in the previous two sections using a distinct innovation-demand model. Section 5 provides concluding remarks.

2. The Impact of Ideal Pooling on Investment and Profit

This section compares the traditional model in which each firm builds its own capacity to meet its own demand with a pooled model in which firms share their production capacity and chose their investments to maximize joint profits. This ideal pooled scenario provides a benchmark for the system with contract manufacturing considered in §3.

Consider an OEM that is developing a new product. Assume that the price per unit when q units are sold is \( M - q \). With probability \( e \), the product is successful and \( M = H \); otherwise, \( M = L \) where \( L \leq H \).

The OEM must invest in production capacity \( c \) at a cost of \( k > 0 \) per unit before the demand for the new product is realized. After the capacity cost is incurred, the marginal cost of production is, for simplicity, assumed to be negligible. The expected operational profit for the OEM is

\[
\Pi^0 = \max_{c \geq 0} \left\{ e(H - c) + (1 - e) \max_{q \in [0, c]} [(L - q)q] - kc \right\}.
\] (1)

Padmanabhan and Png (1997) employ a similar demand model, where \( e, H, \) and \( L \) are exogenous. We extend this model to consider the role of investment in innovation to stimulate demand.

Early investments by the OEM in innovation (market research and product development) may influence demand through either the potential market size \( H \) or the success probability \( e \). For example, in the pharmaceutical industry, basic research and clinical trials aimed at expanding a drug’s therapeutic range (i.e., the range of medical conditions for which the drug is proven to be efficacious) increase the potential market size. Drug development investments aimed at increasing the probability that a drug passes clinical trials increase the success probability. For any specific drug candidate, innovation investments may be directed primarily at one of these two objectives.

We consider the case where innovation influences the potential market size first and examine the case where innovation influences the success probability in §4. Assume that the total cost of innovation \( f(H) \) is an increasing function of the market size \( H \), twice differentiable and convex, and satisfies \( f(H) \to \infty \) as \( H \to \bar{H} < \infty \). The OEM chooses a market size that maximizes his total expected profit, which is given by

\[
V^0 = \max_{H \in [L, \bar{H}]} [\Pi^0(H) - f(H)].
\] (2)

Consider two identical OEMs that pool their production capacity. The maximum expected profit that they can achieve jointly is

\[
V^p = \max_{H_1, H_2 \in [L, \bar{H}]} [\Pi^p(H_1, H_2) - f(H_1) - f(H_2)],
\] (3)

where

\[
\Pi^p(H_1, H_2) = \max_{c \geq 0} \left\{ R^p(c, H_1, H_2) - 2kc \right\},
\] (4)

\[
R^p(c, H_1, H_2)
\]

\[
= e^2 \max_{q_1, q_2 \geq 0} \left\{ (H_1 - q_1)q_1 + (H_2 - q_2)q_2 \right\}
\]

\[
+ e(1 - e) \sum_{i=1,2} \max_{q_{ii}, q_{ii} \geq 0} \left\{ (H_i - q_{ii})q_{ii} + (L - q_{ii})q_{ii} \right\}
\]

\[
+ (1 - e)^2 \max_{q \in [0, c]} [(L - q)q].
\]

However, this model requires some simplifying assumptions.

\[
\tilde{q} = \frac{(H_1 + H_2) - 2\min(H_1, H_2)}{2}
\]

\[
\tilde{q} = \frac{(L - q)q}{2}\].

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Here, \( c \) denotes the production capacity per OEM, and \( H_i \) is the potential market size for OEM \( i \), for \( i = 1, 2 \). Henceforth, we assume the OEMs should exert some effort to innovate, i.e., that the optimal solution to (3) satisfies \( H_i^p > L; i = 1, 2 \).

Subsequent sections evaluate the effect of pooling on the optimal levels of capacity, market size, success probability, and expected profit. Throughout, the superscripts \( P \) and \( 0 \) indicate pooling or the absence of it. For example, the optimal capacity investment per OEM in the system with pooling is denoted \( c^p \), whereas optimal capacity for the isolated OEM is \( c^0 \).

The profit function \( \Pi^p(H) \) is convex, so problem (2) may fail to have a unique optimal solution. Similarly, problem (3) may fail to have a unique solution. Furthermore, an optimal solution to problem (3) is not necessarily symmetric because \( (\partial^2/\partial H_1 \partial H_2) \Pi^p(H_1, H_2) \) takes both positive and negative values. However, imposing a restriction on the innovation cost function ensures uniqueness and symmetry. In particular, assume for the remainder of this paper that

\[
\frac{d^2 f(H)}{dH^2} > \frac{e}{2}
\]

for \( H \in [L, \bar{H}] \). In many contexts, investments in innovation (research and development, product design, marketing) yield diminishing returns in terms of their impact on the market size (equivalently, the marginal cost of innovation is increasing). Condition (5) simply requires that the marginal cost of effort be increasing sufficiently rapidly. While this need not hold in every context, focusing on the case of unique, symmetric solutions greatly facilitates analytical comparisons. The following proposition compares the solutions to problems (2) and (3). Define \( \bar{H} = L + 2k/c^2 \).

**Proposition 1.** The optimization problem (2) has a unique solution \( H^p \). The optimization problem (3) has a unique solution, and it is symmetric:

\[
H_1^p = H_2^p = H^p.
\]

Furthermore,

\[
H^0 \leq H^p,
\]

where the inequality is strict if and only if

\[
H^0 < \bar{H}.
\]

Thus, innovation is larger when capacity is pooled. Two different insights underlie the proof of this result. First, for any fixed level of capacity, an increase in market size (either \( H \) or \( L \)) is more beneficial when capacity is pooled. Therefore, pooling drives the OEMs to invest more in innovation. Second, we have made a structural assumption that innovation increases not only the expected market size, but also its variability. Variability is highly detrimental to the isolated OEM. However, in the pooled scenario, the OEMs are better equipped to handle variability, so they exert more innovation effort. If \( L = 0 \), then the decision of whether to set \( H > 0 \) can be interpreted as a market entry decision. One scenario that may occur is \( H^p > H^0 = 0 \). That is, by improving the efficiency with which capacity is used, pooling stimulates entry into a market that would otherwise be unserved.

Although pooling increases innovation, pooling may either increase or decrease the level of capacity. The proof of this result relies on the observation that when OEMs are faced with the same potential market sizes, capacity in the pooled case takes on more moderate values. To see this, consider the case where \( H \) (and consequently the optimal capacity) is very large. Then, the incremental value of additional capacity is larger for the isolated OEM because the probability that capacity will be used is greater (\( \epsilon > \epsilon^p \)). In contrast, if \( H \) and (consequently the optimal capacity) is small, then the incremental value of additional capacity is greater in the pooled case because the probability that capacity will be used is greater (\( \epsilon^2 < 2\epsilon^p(1 - \epsilon) > \epsilon \)). The precise result is that

\[
\text{if } H \leq H, \quad \text{then } c^p(H, H) \geq c^0(H),
\]

\[
\text{if } H > H, \quad \text{then } c^p(H, H) < c^0(H),
\]

where \( H = \max[L + k/\epsilon, 3k/(2\epsilon)] \). This result, in conjunction with Proposition 1, yields:

**Proposition 2.**

\[
\begin{align*}
\text{if } H^0 \lessgtr H, & \quad \text{then } c^0 \lessgtr c^p; \\
\text{if } H^0 \lessgtr \bar{H}, & \quad \text{then } c^0 \lessgtr c^p.
\end{align*}
\]

An implication of Proposition 2 is that if innovation and capacity are costly (cheap), then pooling increases (reduces) the optimal capacity. For example, innovation and capacity may be relatively costly for pharmaceutical companies, especially firms developing highly novel drugs (e.g., biopharmaceuticals). Propositions 1 and 2 suggest that in this context, companies that develop and produce their own drugs underinvest (relative to the pooled ideal) in both innovation and capacity. In recent years the pharmaceutical industry has been marked by considerable merger activity. Mergers of drug companies may not achieve the ideal pooled outcome due to incentive problems within the combined firm. However, to the extent that such problems are overcome and that mergers do result in the ideal pooled outcome, our results suggest that mergers may lead drug companies to make larger investments in innovation and capacity.

Finally, we are concerned with the effect of pooling on expected profit for the system as a whole. Define the gain from pooling as

\[
\Delta = \Pi^p(H^p, H^p) - 2f(H^p) - 2[\Pi^0(H^0) - f(H^0)].
\]
Combining this with Proposition 2 yields

\[
\frac{\partial \Delta}{\partial k} = 2[c^0(H^0) - c^p(H^0, H^0)].
\]

Combining this with Proposition 2 yields

\[
\frac{\partial \Delta}{\partial k} \begin{cases} > 0 & \text{if } H^0 > \bar{H}, \\ < 0 & \text{if } H^0 < \bar{H}. \end{cases}
\]

Surprisingly, the gain from pooling is maximized at an intermediate value of the cost of capacity \(k\). If \(k\) is small, then \(H^0\) is large and capacity is smaller in the pooled case; hence, the gain from pooling is increasing in the capacity cost. Conversely, if \(k\) is large, then \(H^0\) is small, and the gain from pooling is decreasing in the capacity cost. By a similar analysis, we find that

\[
\frac{\partial \Delta}{\partial e} \begin{cases} > 0 & \text{if } e \leq \frac{1}{2} \text{ and } H^0 < \bar{H}, \\ < 0 & \text{if } e \geq \frac{1}{2} \text{ and } H^0 > \bar{H}. \end{cases}
\]

The gain from pooling is maximized at an intermediate value of the success probability \(e\), which corresponds to high variability in the market sizes.

### 3. Impact of Contract Manufacturing on Investment and Profit

To address the question of who should own the plant (i.e., the capability to produce), we first, in §3.1, compare the traditional model of §2 with a system in which an independent CM exclusively possesses the capability to produce. As before, both OEMs make early investments in innovation. Then, they negotiate supply contracts with the CM. The CM invests in capacity at a cost of \(k\) per unit. Finally, demand is realized, and the CM allocates capacity between the OEMs. In making their investment decisions, all parties seek to maximize their own expected profits. This model illustrates the incentive problems that arise due to an organizational split between innovation and production. These problems arise because innovation is not contractible: Each OEM invests in innovation only insofar as he expects to recoup this investment later by negotiating a favorable supply contract. Hence, joint profit is lower than in the ideal pooled model of §2. We characterize conditions under which the system with contract manufacturing achieves greater innovation and profit than the traditional model.

Second, in §3.2, we explore how the OEMs can retain plant ownership, yet pool capacity through supply contracts. We find that this yields greater profit than the traditional model without pooling. However, total profit may be either lower or higher than in the system with contract manufacturing. Presumably, if the introduction of a CM will increase the total system profit, OEMs should sell their production facilities to a CM at a price that reflects the future operational profits of the CM. However, we will not model the sale of the production facilities explicitly. Instead, we will focus on the individual product-based innovation and quantity decisions, which occur relatively frequently and set the stage for the asset divestiture decision.

To model the negotiations over supply contracts and capacity allocation, we first introduce some concepts from cooperative game theory. A cooperative game consists of a finite set of \(N\) players, and a characteristic value function \(v: 2^{[N]} \rightarrow \mathbb{R}\), which specifies the maximum value that can be created by each subset of players. An allocation for the cooperative game is a vector \(x \in \mathbb{R}^N\) specifying how this value is divided among the \(N\) players. An allocation satisfies the added value principle if each player realizes a fraction of the value he individually adds to the group

\[
x_i \leq v(N) - v(N \setminus \{i\}) \quad \text{for each player } i \in N,
\]

and if the allocation is efficient

\[
\sum_{i \in N} x_i = v(N).
\]

Osborne and Rubinstein (1994) provide an excellent introduction to cooperative game theory. Stuart (2001) proposes the biform game, in which each player \(i\) has a set of possible strategies \(A_i\) and associated cost function \(f_i: A_i \rightarrow \mathbb{R}\). The profile of strategies chosen by the players \(a \in \times_{i \in N} A_i\) determine the characteristic value function \(v^a: 2^{[N]} \rightarrow \mathbb{R}\) for a cooperative game. As in a strategic form noncooperative game, the players simultaneously choose strategies. However, in addition to the immediate cost associated with his own strategy, each player takes into account the cooperative game that results from the strategy profile. From the added value principle and conditional on the strategy profile \(a\), player \(i\) expects a net payoff of

\[
V_i(a) = \alpha_i [v^*(N) - v^*(N \setminus \{i\})] - f_i(a_i),
\]

where \(\alpha_i \in [0, 1]\) is his bargaining confidence index. That is, he believes that he will appropriate the fraction \(\alpha_i\) of his added value in the ensuing cooperative
game. In the simple case with \( N = 2 \), the Nash bargaining solution implies that \( \alpha = 1/2 \). The two parties will split the gain from cooperation (Nash 1953). The noncooperative solution concept of Nash equilibrium extends naturally to the biform game: Each player chooses a strategy that maximizes his own net payoff \( V_i \), given the strategic choices of the other players.

### 3.1. Pooling Capacity with a Contract Manufacturer

The two OEMs play the following biform game. First, they strategically choose market sizes \( H_i^{N} \in [L, H] \) and incur the associated costs \( f(H_i^{N}) \). All parties observe the demand distribution and cost of production, and then a cooperative game ensues as the OEMs and CM negotiate over supply contracts. OEM \( i \) has an added value of \( \Pi^P(H_i, H_j) - \Pi^P(H_i) \), where \( \Pi^P(H_i, H_j) \) is the expected operational profit for the integrated system with pooling and \( \Pi^P(H_i) \) is the expected operational profit that the CM and OEM \( j \) can achieve alone; \( i \neq j \). Each OEM has a bargaining confidence of \( \alpha \in [0, 1] \). Therefore, the OEMs’ strategic market sizes \((H_1^{N}, H_2^{N})\) constitute a Nash equilibrium if

\[
H_i^{N} = \arg\max_{H_i \in [L, H]} \{ \alpha[\Pi^P(H_i, H_j^{N}) - \Pi^P(H_i^{N})] - f(H) \}, \tag{8}
\]

\[
H_j^{N} = \arg\max_{H_j \in [L, H]} \{ \alpha[\Pi^P(H_j, H_i^{N}) - \Pi^P(H_j^{N})] - f(H) \}. \tag{9}
\]

We have assumed that the three parties bargain over transfer payments, but ultimately adopt supply contracts that will induce the CM to make the capacity investment that is optimal for the pooled system and allocate that capacity optimally between the OEMs after demand is realized. We now justify this assumption by considering tradable capacity options\(^2\) as a supply contract.

After observing the demand distributions, the CM sells \( c^P(H_i, H_j) \) tradable options to each OEM with an exercise price of zero. Each OEM is guaranteed \( c^P(H_i, H_j) \) units of production capacity and the legal right to sell this production capacity to the other OEM after demand is realized. The investment of the CM in production capacity, \( c \), must be at least \( 2c^P(H_i, H_j) \) because he faces severe penalties for failure to fulfill the capacity options. After the CM chooses his capacity investment and demand is realized, the capacity is traded in a cooperative game. The outcome is a capacity allocation that maximizes total revenue. The CM will participate in this trading only if he builds excess, speculative capacity; i.e., \( c > 2c^P(H_i, H_j) \). In the capacity allocation, the CM expects to obtain \( \alpha_m \) of his value-added. In the event that both OEMs have high demand, this is

\[
\max_{q_1, q_2 \geq 0 \atop q_1 + q_2 \leq c} \left[ (H_1 - q_1)q_1 + (H_2 - q_2)q_2 \right] - \max_{q_1, q_2 \geq 0 \atop q_1 + q_2 \leq c} \left[ (H_1 - q_1)q_1 + (H_2 - q_2)q_2 \right].
\]

In the event that OEM \( i \) has high demand and OEM \( j \) has low demand, the CM’s value-added is

\[
\max_{q_i, q_j \geq 0 \atop q_i + q_j \leq c} \left[ (H_i - q_i)q_i + (L - q_j)q_j \right] - \max_{q_i, q_j \geq 0 \atop q_i + q_j \leq c} \left[ (H_i - q_i)q_i + (L - q_j)q_j \right].
\]

In the event that both OEMs have low demand, the CM’s value-added is

\[
2 \max_{q \in [0, c/2]} [(L - q)q] - \sum_{i=1, 2} \max_{q \in [0, c^P(H_i, H_j)]} [(L - q)q].
\]

Therefore, the CM chooses the production capacity that maximizes his expected profit by solving the following problem:

\[
\max_{c \in [2c^P(H_i, H_j), \infty]} \{ \alpha_m R^P(c/2, H_i, H_j) - kc \}.
\]

Because \( R^P(\cdot, H_i, H_j) \) is concave and \( \alpha_m \leq 1 \), his optimal production capacity is \( 2c^P(H_i, H_j) \). That is, the CM will not build excess, speculative capacity. Thus, tradable options induce optimal capacity investment and allocation.

We proceed to investigate how the Nash equilibrium levels of innovation (8)–(9) differ from the ideal derived in §2. Consider the optimization problem

\[
\max_{H_i, H_j \in [L, H]} \{ \alpha[\Pi^P(H_i, H_j) - f(H_i) - f(H_j)] \}. \tag{10}
\]

From Proposition 1 we know that it has an optimal solution, which shall be denoted \((H_i^{N}(\alpha), H_j^{N}(\alpha))\). This is a Nash equilibrium because it satisfies conditions (8)–(9). Furthermore, when the OEMs have complete bargaining confidence (i.e., each anticipates capturing his entire value-added),

\[
H_i^{N}(1) = H_j^{N}(1) = H^P.
\]

This follows from the observation that when \( \alpha=1 \), problems (10) and (3) are identical. On the contrary, if the OEMs have zero bargaining confidence, then

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1 The set of all players’ confidence indices \( \{\alpha_i\}_{i=1}^{N} \) may not be consistent with an efficient allocation. The confidence indices represent the players’ beliefs, not the actual outcome of the cooperative game.

2 The world’s largest semiconductor contract manufacturer, Taiwan Semiconductor Manufacturing Corporation, sells tradable capacity options to OEMs. Each OEM is guaranteed the legal right to sell his reserved production capacity to another OEM after demand is realized (Economist 1996).
the OEMs have no incentive to invest in innovation because \( f \) is strictly increasing,

\[ H_1^N(0) = H_2^N(0) = L. \]

Although in principle the Nash equilibrium \((H_1^N(\alpha), H_2^N(\alpha))\) need not be either unique or symmetric, Proposition 3 establishes that this Nash equilibrium is unique and is symmetric (the proof relies on condition (5)).

Our main result is that market sizes increase with the bargaining confidence of the OEMs. This is highly intuitive: As the bargaining confidence of an OEM increases, he has a stronger incentive to invest in innovation.

**PROPOSITION 3.** \((H_1^N(\alpha), H_2^N(\alpha))\) is the unique Nash equilibrium, and it is symmetric:

\[ H_1^N(\alpha) = H_2^N(\alpha) = H_N(\alpha). \]

Furthermore, there exists \( \hat{\alpha} \in (0, 1) \) such that

\[ H_N(\alpha) = L \quad \text{for } \alpha \in [0, \hat{\alpha}], \quad (11) \]
\[ H_N(\alpha) \text{ is strictly increasing in } \alpha \quad \text{for } \alpha \in (\hat{\alpha}, 1) \text{ with } H_N(1) = H_P. \quad (12) \]

By comparing Propositions 1 and 3, we conclude that if the OEMs’ bargaining confidence is sufficiently high, the level of innovation effort (and resulting market size) will increase due to outsourcing. On the contrary, if the OEMs have little bargaining confidence, then each OEM—anticipating that much of the value created by his investment in innovation will be expropriated by the CM in the later contracting stage—will invest less in innovation.

Let \( c_N^i(\alpha) = c_P^i(H_1^N(\alpha), H_2^N(\alpha)) \) denote the optimal capacity in the Nash equilibrium. Because the optimal pooled capacity is monotone in the market size, (11)–(12) imply that

\[ c_N^i(\alpha) = [L - k]^+/2 \quad \text{for } \alpha \in [0, \hat{\alpha}], \]
\[ c_N^i(\alpha) \text{ is strictly increasing in } \alpha \quad \text{for } \alpha \in (\hat{\alpha}, 1) \text{ with } c_N^i(1) = c_P. \]

Intuitively, as the OEMs become more confident in their bargaining position and the expected market size grows, production capacity also increases. Recall that if \( H_P \) is sufficiently large, then \( c_P < c_0 \), and therefore \( c_N(\alpha) < c_0 \) (i.e., outsourcing reduces the level of capacity investment). This tends to occur when the cost of capacity is low or the success probability is high. Otherwise, if \( c_P > c_0 \) and the OEMs’ bargaining confidence is high, then outsourcing increases the level of capacity investment.

Outsourcing manufacturing can increase profit by improving the efficiency with which capacity is used. However, as demonstrated above, outsourcing can weaken the incentives for innovation, eroding system profit. On balance, which factor dominates depends on how much of his added value the OEM expects to capture in the contracting stage. Let \( \Delta(\alpha) \) denote the total system gain in expected profit due to outsourcing:

\[ \Delta(\alpha) = \Pi^P(H_1^N(\alpha), H_2^N(\alpha)) - f(H_1^N(\alpha)) - f(H_2^N(\alpha)) - 2[\Pi^P(H_P) - f(H_P)]. \quad (13) \]

It is straightforward to show that \( \Delta(1) > 0 \) and \( \Delta(0) \leq 0 \). Further, \( \Delta(\alpha) \) is increasing in \( \alpha \). Hence, if the OEMs’ bargaining confidence is sufficiently high, then sufficient incentives for innovation exist that the gain from outsourcing is positive. If the OEMs’ bargaining confidence is low, then outsourcing reduces system profit. Hence, OEMs that anticipate being in a weak bargaining position vis-à-vis a CM should retain their production facilities rather than outsource.

The gain due to outsourcing is also sensitive to the cost of capacity. The cost of capacity varies by industry. For example, the cost of capacity for software media replication is small relative to the cost of innovation (software development) and the anticipated sales revenue. This is not the case for semiconductor manufacturing, where fabrication capacity is very expensive. Because the cost of capacity varies by industry, the relative attractiveness of outsourcing varies by industry. Consider capacity costs sufficiently small that contract manufacturing is viable (i.e., it results in nonzero profit). For small, fixed \( \alpha \), the gain from contract manufacturing \( \Delta(\alpha) \) is increasing in \( k \). (When \( \alpha \) is small, contract manufacturing reduces the total system capacity, \( c_N(\alpha) < c_0 \), and thus reduces the impact on profit of an increase in the unit cost of capacity.) However, if \( \alpha \) is sufficiently large, the gain from contract manufacturing \( \Delta(\alpha) \) is maximized at an intermediate value of \( k \), as is the gain from ideal pooling. In conclusion, outsourcing is most attractive in industries where the capacity cost is moderate and OEMs are in a strong bargaining position vis-à-vis the CM.

We have demonstrated how contract manufacturing may fail to achieve ideal pooling. Alternatively, the OEMs may retain ownership of their production facilities and pool capacity among themselves, eliminating the CM. The next subsection investigates how this shifts the incentives for innovation and capacity investment.

### 3.2. Pooling Capacity Among Original Equipment Manufacturers

The two OEMs first strategically choose market sizes \( H_i^* \in [L, H] \) and incur the associated costs \( f(H_i^*) \). As before, a cooperative game ensues as the OEMs
bargain over the level of capacity investment. We assume that they achieve an optimal capacity investment and allocation of that capacity between the OEMs after demand is realized, either through supply contracts or a joint venture. The gain from cooperation \( \Pi'(H_1', H_2') - [\Pi'(H_1) + \Pi'(H_2)] \) will be evenly divided, according to the Nash bargaining solution. Therefore, the OEMs' strategic market sizes \((H_1^n, H_2^n)\) constitute a Nash equilibrium if

\[
H_1^n = \arg\max_{H_1 \in [L, H]} \left[ \Pi'(H) + \frac{1}{2} \left( \Pi'(H_1) + \Pi'(H_2) \right) - f(H) \right], \tag{14}
\]

\[
H_2^n = \arg\max_{H_2 \in [L, H]} \left[ \Pi'(H) + \frac{1}{2} \left( \Pi'(H_1) + \Pi'(H_2) \right) - f(H) \right]. \tag{15}
\]

We proceed to investigate how the Nash equilibrium level of innovation differs from the ideal derived in §2 and the level of innovation with contract manufacturing. Consider the optimization problem

\[
\max_{H_1, H_2 \in [L, H]} \left[ \frac{1}{2} \left( \Pi'(H_1, H_2) + \Pi'(H_1) + \Pi'(H_2) \right) - f(H_1) - f(H_2) \right],
\]

and let \((H_1^n, H_2^n)\) denote the optimal solution. This is a Nash equilibrium because it satisfies (14)–(15). Proposition 4 establishes that this is the unique Nash equilibrium and it is symmetric (the proof relies on condition (5)). Our main result is that when the OEMs outsource among themselves, the resulting market size is greater than would be adopted by an isolated OEM, but lower than in the ideal pooling scenario. That is, the OEMs underinvest in innovation.

**Proposition 4.** \((H_1^n, H_2^n)\) is the unique Nash equilibrium, and it is symmetric:

\[H_1^n = H_2^n = H^n.\]

Furthermore,

\[H^0 \leq H^n \leq H^P, \tag{16}\]

where the second inequality is strict if and only if (6) holds.

The insight behind the proof of Proposition 4 is that by strengthening his alternative option, each OEM can improve his bargaining outcome. Therefore, in deciding how much to invest in innovation, each OEM places weight on the expected operational profit \( \Pi'(H) \) in the scenario without pooling (where the optimal level of innovation is lower than in the pooling scenario), although pooling is inevitable. This qualitative insight remains true even if the OEMs fail to anticipate the Nash bargaining outcome; insofar as the OEM anticipates that his profit allocation will increase with his “go-it-alone” alternative \( \Pi^0 \), he will underinvest in innovation.

Proposition 4 implies that OEMs that own their production facilities increase their profit (relative to the traditional model) by outsourcing among themselves. To see this, observe that

\[
\frac{1}{2} \Pi'(H^n, H^n) - f(H^n) \geq \frac{1}{2} \Pi'(H^0, H^0) - f(H^0) \geq \Pi'(H^0) - f(H^0),
\]

where the first inequality follows by Proposition 4 and the observation that \((1/2)\Pi'(H, H) - f(H)\) is concave in \(H\) (which is established in the proof of Proposition 1). By comparing Propositions 3 and 4, we conclude that if the OEMs anticipate being in a weak bargaining position vis-à-vis the CM, then outsourcing among OEMs results in greater innovation and expected OEM profit than outsourcing to a CM. However, when the OEMs’ bargaining confidence is high (\(\alpha\) is close to unity), contract manufacturing results in greater system profit.

### 4. The Success Probability Innovation Model

Section 2 points out that in some industry contexts innovation investments may be directed primarily at increasing the probability that a product is successful rather than increasing the size of the potential market. This section explores the setting where the OEM’s early investment in innovation influences the product’s success probability \(e\), rather than its market size \(H\). We demonstrate that the effect of either pooling or outsourcing on innovation depends importantly on how innovation affects demand. When innovation influences the market size, pooling increases innovation, and outsourcing leads to underinvestment in innovation. In contrast, when innovation influences the success probability, pooling may reduce innovation, and outsourcing (among OEMs) can result in overinvestment in innovation.

Nonetheless, the primary qualitative results regarding the effect of outsourcing on system and OEM profit are similar under both innovation models: OEMs that own their production facilities increase their profits by outsourcing among themselves. Outsourcing to a CM increases system profit (relative to either setting in which the OEMs own their production facilities) if and only if the OEMs’ bargaining confidence is high. If the OEMs’ bargaining confidence is low, then outsourcing to a CM reduces OEM profit.

---

3 AMD and Fujitsu have a joint venture to produce flash memory chips. The firms cooperatively choose the capacity level of the joint venture's production facilities. Each firm has the nominal right to purchase 50% of the output at cost, and is penalized for taking less than 40%. However, as the demand for flash memory fluctuates, the two parties frequently renegotiate the contract, which specifies "cost" and "penalty" to ensure an efficient allocation of capacity among themselves (Doran 2001).
4.1. The Impact of Ideal Pooling on Investment and Profit

This subsection compares the traditional model—in which each firm builds its own capacity to meet its own demand—with a pooled model in which firms share their production capacity and choose their investments to maximize joint profits. The formulation is analogous to the case where innovation influences the market size. For any given success probability, the OEM’s expected operational profit $\Pi^0$ is given by (1). Assume that the total cost of innovation $g(e)$ is twice differentiable, strictly increasing, convex, and satisfies $g(e) \to \infty$ as $e \to 1$. The OEM chooses a success probability $e$ that maximizes his total expected profit, which is given by

$$V^0 = \max_{e \in [0, 1]} [\Pi^0(e) - g(e)].$$  \hspace{1cm} (17)

Consider two identical OEMs that pool their production capacity. The maximum expected profit that they can achieve jointly is

$$V^p = \max_{e_1, e_2 \in [0, 1]} [\Pi^0(e_1, e_2) - g(e_1) - g(e_2)],$$  \hspace{1cm} (18)

where

$$\Pi^0(e_1, e_2) = \max_{c \geq 0} [R^0(c, e_1, e_2) - 2kc],$$

$$R^0(c, e_1, e_2) = e_1 e_2 (H - c) + (e_1 + e_2 - 2e_1 e_2) c\max\{H - q_i, H - q_j\} + \frac{(1 - e_1)(1 - e_2) 2L}{e_2},$$

Section 2 demonstrates that when innovation influences the size of the market, pooling increases innovation. One might conjecture that a similar result holds when innovation influences the success probability. The main result of this subsection is that this conjecture need not hold: When innovation influences the success probability, the optimal innovation level in the pooled system may be larger or smaller than the innovation level without pooling.

In general, problem (17) may fail to have a unique solution. Similarly, problem (18) may fail to have a unique solution, and further, a solution need not be symmetric. The following proposition gives sufficient conditions for uniqueness and symmetry. Thereafter, we simply assume that problem (17) has a unique solution $e^0$ and that problem (18) has a unique and symmetric solution $e^p$.

**Proposition 5.** The optimization problem (17) has a solution $e^0$. A sufficient condition for $e^0$ to be the unique solution is that

$$\frac{d^2 g(e)}{de^2} > \max \left\{ \frac{(H - L)^2}{2}, \frac{(H - L)^3}{2k} \right\} \text{ for } e \in [0, 1].$$  \hspace{1cm} (19)

The optimization problem (18) has a solution $(e^p_1, e^p_2)$. Suppose that

$$\frac{d^2 g(e)}{de^2} > \frac{(H - L)^2}{4} \text{ for } e \in [0, 1].$$  \hspace{1cm} (20)

Then, the solution is symmetric, and furthermore, if

$$\frac{d^2 g(e)}{de^2} > \frac{1}{2} \frac{\partial^2 \Pi^0(e, e)}{\partial e_1^2} \text{ for } e \in [0, 1],$$  \hspace{1cm} (21)

then the solution is also unique:

$$e^p_1 = e^p_2 = e^p.$$

The functional form of $(\partial^2 / \partial e_1^2) \Pi^0(e, e)$ is given in the proof of Proposition 5 in the appendix.

We now proceed to characterize the impact of pooling on innovation (Proposition 6) and capacity (Proposition 7). The following lemma evaluating the effect of pooling on capacity for a fixed success probability is needed to prove Propositions 6 and 7. Define

$$e^p = 1 - \sqrt{(H - k)/(H - L)},$$

$$e^0 = \begin{cases} (k - L)^+ / (H - L) & \text{if } H < 3L, \\ [3k/(2H)]^+ & \text{if } H \geq 3L, \end{cases}$$

$$\bar{e} = \begin{cases} [k/(H - L)]^+ & \text{if } H < 3L, \\ [3k/(2H)]^+ & \text{if } H \geq 3L. \end{cases}$$

**Lemma 1.** For $e \in [0, 1]$, if $e < e^p$, then $e^p(e, e) = c^0(e) = 0$; if $e^p < e < e^0$, then $c^p(e, e) > c^0(e) = 0$; if $e^0 < e < e^\bar{e}$, then $c^p(e, e) > c^0(e) > 0$; if $e \leq e^\bar{e}$, then $c^p(e, e) = c^0(e) > 0$; if $e > e^\bar{e}$, then $0 < c^p(e, e) < c^0(e)$.

**Proposition 6.** If $e^p > e(k, H, L)$, then $e^p < e^0$; if $0 < e^0 < e(k, H, L)$, then $e^p > e^0$, where $0 < e(k, H, L) < 1$. Because market size is stochastically increasing in innovation, the OEM’s expected operational profit is increasing in $e$ whether capacity is pooled or not. However, variability in the market size is more detrimental to an isolated OEM, and this variability is a...
strictly concave function of $e$ maximized at $e = 1/2$. Hence, the isolated OEM tends to set his innovative effort at high or low rather than intermediate levels, whereas pooling motivates the OEM to adopt an intermediate level of innovation. This explains why the level of innovation may be greater or smaller under pooling. Propositions 1 and 6 highlight that the effect of pooling on innovation depends on how innovation affects the demand distribution. Although Propositions 1 and 6 report distinct results regarding the effect of pooling on innovation, both are consistent with the observation that the pooled OEM is more tolerant of market-size variability.

The proof of Proposition 6 is based on showing that $e(k, H, L)$ is the unique crossing point of $dV'/de$ and $dV'/de$, at which both functions are positive. The stated inequalities then follow from the first-order conditions for problems (17) and (18). In an extensive computational study, we observed that $e(k, H, L)$ is universally increasing in the cost of capacity $k$. Typically, when the cost of capacity $k$ is low, pooling reduces the optimal success probability (i.e., $e^p < e^0$), and for large $k$, pooling increases the optimal success probability. Figure 1 illustrates this phenomenon.

The next proposition characterizes the impact of pooling on the capacity level.

**Proposition 7.**

If $e^0 < \min(e(k, H, L), e)$, then $c^0 < c^p$; if $e^0 > \max(e(k, H, L), e)$, then $c^0 > c^p$.

The result is analogous to Proposition 2: Under both innovation models, if innovation and capacity are costly so that the level of innovation for an isolated OEM is small, then the optimal capacity investment is larger under pooling. On the contrary, if innovation and capacity are cheap, then pooling reduces the optimal capacity investment. Pooling may simultaneously lead to reduced innovation and increased capacity. For example, if $H = 3, L = 1, k = 1.2$, and $g(e) = 0.55 e^2 + 1_{[e>0.4]}(e - 0.4)/[10(1 - e)]$, then $e^0 = 0.55, c^0 = 0.45, c^p = 0.54, $ and $c^p = 0.46$. Here, the reduced innovation of the pooled OEM corresponds to more variability in the market size, and the pooled OEM increases his capacity to compensate.

Finally, we turn to the impact of pooling on expected profit. We define the gain from pooling $\Delta$ as in (7), but substitute the decision variable $e$ for $H$ and the cost function $g$ for $f$. As previously observed, $\Delta$ is increasing in the cost of capacity when $k$ is small, but decreasing when $k$ is large:

$$
\frac{\partial \Delta}{\partial k} > 0 \quad \text{if } e^0 < \min(e(k, H, L), e), \\
\frac{\partial \Delta}{\partial k} < 0 \quad \text{if } e^0 > \max(e(k, H, L), e).
$$

The result follows from Proposition 7 and the repeated application of the envelope theorem to $\partial \Delta/\partial k$.

One might expect the gain from pooling to increase with the market size $H$, as this introduces variability. Surprisingly, when the market size $H$ is large, the gain from pooling actually decreases with $H$. The precise result, obtained by similar analysis, is that

$$
\frac{\partial \Delta}{\partial H} > 0 \quad \text{if } e^0 < \min(e(k, H, L), e), \\
\frac{\partial \Delta}{\partial H} < 0 \quad \text{if } e^0 > \max(e(k, H, L), e) \text{ and } H > L + 2k/(e^0)^2.
$$

For large $H$, we have $e^0 > \max(e(k, H, L), e)$, which guarantees that an isolated OEM invests more in capacity ($c^p > c^0$) and more in innovation ($e^p > e^0$). In this state of affairs, increasing the market size $H$ adds more expected revenue for the isolated OEM than in the pooling scenario. We therefore conclude that the gain from pooling is maximized at some intermediate value of $H$.

4.2. Impact of Contract Manufacturing on Investment and Profit

Having considered the impact of pooling when the OEMs make decisions to maximize the profit of the total system, we now consider the impact of pooling when independent OEMs seek to maximize their own profit. The traditional model in which each OEM builds its own capacity to meet its own demand serves as the base case for comparison. Section 4.2.1 considers the case where OEMs outsource production to an independent CM and characterizes when outsourcing increases system profit. An alternative means of achieving pooling is for the OEMs to retain their production facilities and outsource among themselves through supply contracts or a joint venture; this is explored in §4.2.2.

4.2.1. Pooling Capacity with a Contract Manufacturer

Consider the setting in which the OEMs outsource production to a CM. The formulation is analogous to the case where innovation influences the market size. First, the OEMs strategically choose suc-
cess probabilities \( c_i^N \) and incur the associated costs \( g(c_i^N) \). Then, a cooperative game ensues as the OEMs and CM negotiate over supply contracts. As in §3.1, the CM and OEMs negotiate supply contracts that will induce the CM to make the capacity investment that is optimal for the pooled system and allocate that capacity optimally between the OEMs after demand is realized. In making his innovation investment, OEM \( i \) anticipates that in the subsequent contract negotiation stage he will capture \( \alpha \) of his value-added, \( \Pi^P(e_i, e_j) = \Pi^0(e_j) \); \( i \neq j \).

Hence, the OEMs’ strategic success probabilities \( (c_i^N, c_j^N) \) constitute a Nash equilibrium if
\[
e_i^N = \arg \max_{e_i \in [0, 1]} \alpha [\Pi^P(e_i, e_j^N) - \Pi^0(e_j^N)] - g(e_i),
\]
\[
e_j^N = \arg \max_{e_j \in [0, 1]} \alpha [\Pi^P(e_i^N, e_j) - \Pi^0(e_i^N)] - g(e_j).
\]

Consider the problem
\[
\max_{e_i, e_j \in [0, 1]} \alpha [\Pi^P(e_i, e_j) - g(e_i) - g(e_j)].
\]

From Proposition 5 we know that it has an optimal solution, which shall be denoted \( (c_i^N(\alpha), c_j^N(\alpha)) \). This is a Nash equilibrium in success probabilities because it satisfies (22)–(23). Results for the extreme cases where either the OEMs have complete or zero bargaining confidence are similar to the case where innovation influences market size. When the OEMs have complete bargaining confidence, they exert the level of effort that is optimal for the pooled system: \( c_i^N(1) = c_j^N(1) = e^0 \). When the OEMs have zero bargaining confidence, they have no incentive to invest in innovation, and consequently \( c_i^N(0) = c_j^N(0) = 0 \).

In general, the Nash equilibrium \( (c_i^N(\alpha), c_j^N(\alpha)) \) is not symmetric and is not the unique Nash equilibrium. This makes it difficult to perform comparative statics and analyze how outsourcing affects the levels of innovation effort and capacity investment. Techniques for comparative statics in systems with multiple Nash equilibria typically require monotonicity in the best-response function (cf. Lippman et al. 1987). Unfortunately, because \( \frac{\partial^2}{\partial e_i \partial e_j} \Pi^P(e_i, e_j) \) takes both positive and negative values, the optimal success probability for one OEM is not monotone as a function of the success probability of the other OEM. Fortunately, if the function \( g \) is sufficiently convex, the Nash equilibrium \( (c_i^N(\alpha), c_j^N(\alpha)) \) is unique and symmetric. Better yet, for a range of parameter values, the equilibrium is in dominant strategies: One OEM need not anticipate the success probability of the other. Hence, for a wide range of parameter values, we have comparative statics.

**Proposition 8.** Suppose that
\[
\frac{d^2 g(e)}{d e^2} > \frac{\alpha (H - L)^2}{4} \quad \text{for } e \in [0, 1],
\]
then, every Nash equilibrium is symmetric. If
\[
\frac{d^2 g(e)}{d e^2} > \frac{\alpha^2 \Pi^P(e, e)}{e_i^2} \quad \text{for } e \in [0, 1],
\]
then there exists a unique Nash equilibrium: \( c_i^N(\alpha) = c_j^N(\alpha) = c^N(\alpha) \), and for some \( \tilde{\alpha} \),
\[
e_N(\alpha) = \begin{cases} 0 & \text{for all } \alpha \in [0, \tilde{\alpha}], \\ \text{strictly increasing for } \alpha \in (\tilde{\alpha}, 1], & \end{cases}
\]

In particular, if
\[
k \in [H - L, (3L - H)/2],
\]
then (24) implies that \( e^N(\alpha) \) is a unique equilibrium in dominant strategies and satisfies (26).

The functional form of \((\partial^2/\partial e^2) \Pi^P(e, e)\) is given in the proof of Proposition 5 in the appendix. Proposition 8 shows that, just as in the case where innovation influences the market size, innovation is increasing in the OEMs’ bargaining confidence. The optimal capacity in the Nash equilibrium is \( c^N(\alpha) = c^N(\alpha)(\alpha, c^N_j(\alpha), c^N_i(\alpha)) \). Consistent with the previous model, because the optimal pooled capacity is monotone in the success probability, the level of capacity investment also increases with \( \alpha \).

Combining these results with Proposition 6, we conclude that if \( e^0 > \max(e(k, H, L), \tilde{e}) \), then innovation and capacity are always lower with outsourcing than for an isolated OEM; i.e.,
\[
e^N(\alpha) < e^0 \quad \text{and} \quad c^N(\alpha) < c^0 \quad \text{for all } \alpha \in [0, 1].
\]

This tends to occur when either innovation or capacity is cheap. On the contrary, if innovation and capacity are expensive, then \( e^0 \leq \min(e(k, H, L), \tilde{e}) \). In this case, there exist thresholds \( 0 < \tilde{\alpha} < \bar{\alpha} < 1 \) such that
\[
e^N(\alpha) < e^0 \quad \text{and} \quad c^N(\alpha) < c^0 \quad \text{if } \alpha < \tilde{\alpha},
\]
\[
e^N(\alpha) > e^0 \quad \text{and} \quad c^N(\alpha) > c^0 \quad \text{if } \alpha > \bar{\alpha},
\]
i.e., just as when innovation influences market size, outsourcing leads to higher innovation and capacity investment if and only if the OEMs’ bargaining confidence is high.

Finally, we characterize the impact of outsourcing manufacturing on system profit. Define \( \Delta(\alpha) \) as in (13), with the obvious substitution of decision variables \( e \) for \( H \). \( \Delta(\alpha) \) represents the change in expected system profit that results when OEMs outsource production to a CM rather than build capacity to meet their own demand. As in the previous section, \( \Delta(\alpha) \) is increasing in \( \alpha \) and becomes positive if and only if the bargaining confidence of the OEM is sufficiently large. Thus, regardless of how innovation
affects the demand distribution, if the OEMs’ bargaining confidence is sufficiently small, then outsourcing will reduce expected profit for the system as a whole; weakened incentives for innovation outweigh the benefits of efficient capacity utilization. The results in §3.1 regarding the impact of \(\alpha\) on \(\Delta(\alpha)\) when innovation influences the market size apply when in §3.2 how innovation affects the demand distribution.

These insights are best illustrated and extended with a numerical example. As in Figure 1, we consider the system with \(H = 15\), \(L = 10\), and \(g(e) = -7 \log(1 - e)\). Figure 2 shows how the capacity cost and the OEMs’ bargaining confidence affect the equilibrium success probability and subsequent capacity investment. The parameter values satisfy (27), so we have a unique equilibrium success probability in dominant strategies. Figure 3 shows the increase in expected profit due to contract manufacturing, as a proportion of the expected profit without pooling: \(\Delta(\alpha)/[2\Pi^0(e^0) - 2g(e^0)]\). The case with high production cost is motivated by observations from the pharmaceutical and semiconductor industries, whereas the case with low capacity cost is typical of the software industry, where outsourcing of media production is common. When the cost of capacity is large, the potential gain from outsourcing is great, but OEMs will not invest in innovation unless they expect success in bargaining over supply contracts (\(\alpha > 0.75\)). In contrast, when the cost of capacity is small, the OEMs invest in innovation over a wider range (\(\alpha > 0.45\)).

However, pooling adds little value; indeed, unless \(\alpha > 0.68\), outsourcing reduces expected profit for the system as a whole. The examples demonstrate that, consistent with the theoretical results, the success probability, capacity, and gain from outsourcing are all increasing in \(\alpha\).

Instead of outsourcing production to a CM, the OEMs may alternatively retain ownership of their production facilities and pool capacity among themselves. The next subsection investigates how this shifts the incentives for innovation and capacity investment.

### 4.2.2. Pooling Capacity Among Original Equipment Manufacturers

Consider the setting in which the OEMs outsource among themselves. The formulation is analogous to the case where innovation influences the market size in §3.2. First, the OEMs strategically choose success probabilities \(e_i\) and incur the associated costs \(g(e_i)\). Then, a cooperative game ensues as the OEMs bargain over the level of capacity investment. As in §3.2, the firms achieve the capacity investment that is optimal for the pooled system and allocate the capacity optimally. In making his innovation decision, OEM \(i\) anticipates capturing half of the subsequent gain from cooperation, so his expected revenue is

\[
\Pi^0(e_i) + \frac{1}{2}(\Pi^0(e_i, e_j) - [\Pi^0(e_i) + \Pi^0(e_j)]), \quad i \neq j.
\]

Thus, the OEMs’ strategic success probabilities \((e_i, e_j)\) constitute a Nash equilibrium if

\[
e_i^* = \arg\max_{e_i \in [0, 1]} \left[ \Pi^0(e) + \frac{1}{2}(\Pi^0(e, e_j^*)) - [\Pi^0(e) + \Pi^0(e_j^*)] - g(e) \right],
\]

\[
e_j^* = \arg\max_{e_j \in [0, 1]} \left[ \Pi^0(e) + \frac{1}{2}(\Pi^0(e_j^*, e) - [\Pi^0(e_i^*) + \Pi^0(e)]) - g(e) \right].
\]

This subsection demonstrates that the effect of OEM outsourcing on innovation depends importantly on how innovation affects the demand distribution. Section 3.2 demonstrates that when innovation influences the market size, OEMs that outsource among themselves underinvest in innovation. The main result of this subsection is that when innovation influences the success probability, the OEMs may overinvest in innovation.

Consider the problem

\[
\max_{e_i, e_j \in [0, 1]} \left[ \frac{1}{2}[\Pi^0(e_i, e_j) + \Pi^0(e_i) + \Pi^0(e_j)] - g(e_i) - g(e_j) \right],
\]

and let \((e_i^*, e_j^*)\) denote its optimal solution. This is a Nash equilibrium, but it is not necessarily unique or symmetric. Our main result is that when the two OEMs do make similar investments in innovation, the resulting success probabilities may be larger than in the ideal pooled case.
PROPOSITION 9. Suppose that

\[ \frac{d^2 g(e)}{de^2} > \frac{(H-L)^2}{8k} \left[ k + 2 \max(k, H-L) \right] \text{ for } e \in [0, 1]. \]

Then, every Nash equilibrium is symmetric, and furthermore, if

\[ \frac{d^2 g(e)}{de^2} > \frac{1}{2} \left[ \frac{\partial^2 \Pi^P(e, \bar{e})}{\partial e^2} + \frac{d^2 \Pi^P(e)}{de^2} \right] \text{ for } e \in [0, 1], \]

then there exists a unique Nash equilibrium

\[ e^1 = e^2 = e^0, \]

and it satisfies

\[ e^p < e^a < e^0 \text{ if } e^0 > e(k, H, L), \]

\[ e^0 < e^a < e^p \text{ if } 0 < e^0 < e(k, H, L). \] (30)

The functional form of \( (\partial^2/\partial e^2)\Pi^P(e, \bar{e}) \) and an upper bound on \( (d^2/de^2)\Pi^P(e) \) are given in the proof of Proposition 5 in the appendix. As in the setting where innovation increases the market size, the Nash equilibrium innovation level falls between the optimal innovation level for the ideal pooled system and the optimal innovation level for the isolated OEM. However, contrast with the setting in which innovation increases the market size emerges when capacity and innovation are cheap. Then, \( e^p < e^a < e^0 \). The OEMs overinvest in innovation (and hence capacity). Therefore, introduction of a CM will reduce innovation, regardless of the OEMs’ bargaining confidence. Alternatively, when capacity and innovation are expensive so that \( e^0 < e^a < e^p \), introduction of a CM will increase innovation if and only if the OEMs anticipate being in a strong bargaining position.

Although the two innovation models yield distinct results for the impact of outsourcing on innovation, both models yield similar results for the effect of outsourcing on system and OEM profit. Section 3.2 establishes that when innovation influences the market size, OEMs that own their production facilities increase their profit (relative to the traditional model) by outsourcing among themselves. Further, outsourcing to a CM increases system profit (relative to either setting in which the OEMs own their production facilities) if and only if the OEM’s bargaining confidence is high. Both results hold for the success probability model as well, provided that the relevant restrictions on the innovation cost function hold; the results follow from Propositions 8 and 9.

5. Discussion

Should an OEM sell the plant? Our analysis indicates that OEMs might do better to trade capacity among themselves rather than to outsource to a CM, particularly if the OEMs anticipate being in a weak bargaining position vis-a-vis the CM. Plant ownership gives the OEM a strategic alternative to supply contracts, and thus improves his bargaining position. Unfortunately, this leads the OEM to make investments in innovation that improve his strategic alternative (going it alone) at the expense of overall system efficiency. In particular, the OEM will overinvest to ensure that a new product will be successful when capacity and innovation are cheap, and will underinvest when such investments are costly. With contract manufacturing, the OEM will always underinvest in innovation, spending only what he expects to recoup by negotiating a favorable supply contract with the CM. From a system perspective, contract manufacturing performs well if and only if OEMs anticipate being in a strong bargaining position vis-a-vis the CM.

We have analyzed a stylized model with two symmetric OEMs and common information to obtain a tractable expression for the Nash equilibrium in innovation. However, our main insight, that with contract manufacturing an OEM will underinvest in innovation, is very robust. This insight relies only on the assumed sequence of events: innovation, contracting, then capacity investment. The OEM cannot generate revenue without the CM, and in negotiating a supply contract his innovation is a sunk cost. Therefore, the OEM cannot capture the full increase in expected contribution that flows from investment in innovation, and will spend less on innovation than would be optimal for the system as a whole. This is a classic hold-up problem.

Our assumption that OEMs make investments in innovation before they contract for capacity is motivated by examples from the electronics and automotive industries. However, an OEM with low bargaining confidence or a low value-added should consider purchasing capacity options before investing in innovation. A follow-on paper (Plambeck and Taylor 2004) describes how small biotechnology firms contract for manufacturing capacity three years in advance of production, before investing in clinical trials. Contracting for capacity in advance strengthens the incentive for innovation. However, advance contracts are renegotiated with high probability, which can weaken the incentive for investment. Contracting in advance and subsequent renegotiation impose high transaction costs, and may therefore be undesirable.

As an alternative to a formal contract, an OEM and CM may adopt an informal agreement, sustained by the potential for repeat business. Therefore, our single-period model may underestimate the efficiency of contract manufacturing. In practice, successful CMs seek collaborative, long-term partnerships with their OEM customers. Socoltron Vice President Eddie Maxie reports that Socoltron scrupulously
avoids “price gouging” of loyal OEM customers when total industry demand outstrips supply Maxie (2000). If our single-period game is extended to a repeated game, the OEMs and CM can adopt a subgame perfect Nash equilibrium that gives strict Pareto improvement over their single-period strategies. Each OEM invests more in innovation, and the CM, on observing auspicious demand conditions, offers a favorable supply contract (cf. Taylor and Plambeck 2003).

We have assumed that the OEMs operate in separate markets. This is appropriate, for example, when a pharmaceutical CM contracts with drug companies that serve separate markets, or an electronics CM produces subassemblies for PCs and networking equipment. However, CMs also supply competing OEMs. For example, Flextronics produces cellphones for Ericsson and Motorola (Thurm 2001). Competing OEMs may even pool capacity among themselves; the sales forces of AMD and Fujitsu compete vigorously in Europe to sell the flash memory produced in their joint venture. Modeling competition between OEMs that outsource to a CM or pool their own capacity may be a promising direction for future research. Because the benefit of pooling is closely related to the degree of demand correlation, explicitly modeling demand correlation may shed additional light on the impact of industry structures that involve pooling.

Another natural extension of our model would be to incorporate learning. To the extent that product research and development benefit from concurrent design for manufacturing, OEMs may prefer to keep manufacturing (and hence manufacturing expertise) in-house. On the contrary, insofar as the production cost per unit decreases with cumulative output, contract manufacturing is advantageous.

Although we do not model dynamics of the evolution of contract manufacturing over time, our results suggest that contract manufacturing might become a victim of its own success. Specifically, as OEMs sell off productive assets and transfer production to CMs, CMs will grow in size relative to their OEM customers. Daiwa Research Institute reports that in electronics, because of the emergence of a small number of large CMs, bargaining power has shifted to these CMs (Sarmah 2000). Further, as OEMs lose production expertise and capabilities, control may further shift to CMs. Our results suggest that an erosion in OEM power (bargaining confidence) will lead to less innovation, lower total industry profit, and thus, perhaps, a return to vertical integration. This hypothesis is consistent with the historical pattern in the computer, automobile, and bicycle industries, where industry structure has cycled from vertical integration to outsourcing and back again (Fine 1998).

A technical appendix (Plambeck and Taylor 2003) to this paper is available at http://mansci.pubs.informs.org/e companion.html.

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Appendix
Throughout, we use the notation

$$\frac{\partial F(x,y)}{\partial x} \text{ to denote } \left. \frac{\partial F(x,y)}{\partial x} \right|_{x=y=\hat{y}}.$$

Proof of Proposition 1. We begin by establishing existence of optimal solutions to (2). Because the function $f$ is continuous and $f(H) \to \infty$ as $H \to \tilde{H}$, the objective function in (2) is continuous and bounded above by $\tilde{H}^2/4$, and achieves its maximum in the region $[L, \tilde{H}]$. Note that

$$d \Pi^i(H) \frac{dH^2}{2} = \begin{cases} 0, & \text{if } H \in [L, L+(k-L)/e], \\
        e^2/2, & \text{if } H \in (L+(k-L)/e, L+k/e], \\
        e/2, & \text{if } H \in (L+k/e, \tilde{H}], \\
        e, & \text{if } H = \tilde{H}, \\
        e/2, & \text{if } H \in (\tilde{H}, \infty), \end{cases}$$

(A.1)

so (A.1) implies that (2) has a unique optimal solution because the objective is strictly concave. Existence of an optimal solution to (3), $(H^o_i, H^o_2)$, follows by an analogous argument.

Similarly, the objective function in (3) is continuous and bounded above by $\tilde{H}^2/2$, and achieves its maximum in $[L, \tilde{H}] \times [L, \tilde{H}]$. To prove symmetry, let us assume without loss of generality that $H^o_1 \geq H^o_2$. With a little effort one can show that

$$\frac{\partial \Pi^o(H^o_1, H^o_2)}{\partial H_1} - \frac{\partial \Pi^o(H^o_2, H^o_2)}{\partial H_2} \leq \frac{e}{2} (H^o_1 - H^o_2) \leq \frac{e}{2} (H^o_1 - H^o_2) \leq \frac{e}{2} (H^o_i - H^o_2),$$

(A.2)

(see Plambeck and Taylor 2003 for the details). We have assumed that $H^o_i > L$ $(i=1, 2)$, so from the first-order conditions for (3),

$$\frac{df(H^o_i)}{dH} = \frac{df(H^o_2)}{dH} = \frac{\partial \Pi^o(H^o_i, H^o_2)}{\partial H_1} - \frac{\partial \Pi^o(H^o_2, H^o_2)}{\partial H_2} \leq \frac{e}{2} (H^o_i - H^o_2).$$

If $H^o_1 > H^o_2$, this contradicts (5) so we must have $H^o_1 = H^o_2$. To establish uniqueness, we observe that

$$\frac{d^2 \Pi^o(H,H)}{dH^2} \leq \frac{(2-e)e^2}{1+2(1-e)e} \leq \frac{e(1+e)}{2} \leq \frac{e(4-(3-e)e)}{2(2-e)} \leq e.$$
so (5) implies concavity of the objective function \( \Pi^0(H, H) - 2f(H) \) and, hence, uniqueness of the optimal solution to (3).

To complete the proof of Proposition 1, we need to show that \( H^g > H^0 \) when \( H^0 < H \) and that \( H^g = H^0 \) when \( H^0 = H \). We have assumed that \( H^o > L \), so the desired result holds trivially when \( H^0 = L \). Suppose that \( H^0 > L \). We observe that

\[
H^0 \triangleq \inf[H: \Pi^0(H) > 0; H \geq L] = L + (k - L)^+ / c,
\]

and because \( f' > 0, H^0 > H^p \). Therefore, it will be sufficient to show that

\[
\frac{\partial \Pi^0(H, H)}{\partial H_2} = \begin{cases} \frac{d \Pi^0(H, H)}{dH} & \text{if } H \in (H^0, \bar{H}), \\ \frac{d \Pi^0(H)}{dH} & \text{if } H \in [\bar{H}, \infty) \end{cases} \tag{A.3}
\]

For the case \( H \leq 3L \), we have

\[
\frac{d \Pi^0(H, H)}{dH} - \frac{d \Pi^0(H)}{dH} = \begin{cases} (e - c^0)(3e - 3e^2)H + (1 - 5e + 3e^2)L + (2e - 1)k & \text{if } c^0(H) \in \left[0, \frac{H - L}{4}\right], \\ (e - c^0)(H - L) \frac{H - L}{4} & \text{if } c^0(H) \in \left[\frac{H - L}{4}, \frac{L + H - L}{2}\right], \\ (1 - e)(2k - e^2(H - L)) \frac{H - L}{8 - 4e} & \text{if } c^0(H) \in \left[\frac{L + H - L}{2}, \frac{L + H}{4}\right], \\ 0 & \text{if } c^0(H) \in \left[\frac{L + H}{4}, \frac{L - k}{4}\right], \end{cases}
\]

where \( c^0(H) \) is the optimal capacity for the pooled system, the maximizer in (4). From the first-order optimality conditions, \( c^0(H) \geq (H + L)/4 \) if and only if \( H < \bar{H} \). Thus, it remains to show that \( \partial (\partial \Pi^0(H, H)) / \partial H \Pi^0(H, H) > (d/dH) \Pi^0(H) \) if \( H < \bar{H} \). This is immediate if \( c^0(H) \in [(H - L)/4, (H + L)/4] \).

If \( \epsilon \leq 1/2 \) or \( k > L \), then \( H > H^0 \) implies that

\[
(3e - 3e^2)H + (1 - 5e + 3e^2)L + (2e - 1)k > 0. \tag{A.4}
\]

If \( c^0(H) < (H - L)/4 \), then

\[
c^0(H) = \frac{(e - c^0)(H - L) + L - k}{2 + 4e - 4e^2}.
\]

If \( \epsilon > 1/2 \), then \( c^0(H) < (H - L)/4 \) implies that \( H < [2k + (2e - 3)\epsilon]/(2e - 1) \), which in turn implies \( k > L \). Consequently, \( H > H^0 \) implies (A.4). Therefore, (A.3) is satisfied for \( H \leq 3L \). The proof for \( H > 3L \) is similar (see Plambeck and Taylor 2003).

**Proof of Proposition 3.** First, we will assume (5) and prove that all Nash equilibria are symmetric (by contradiction). Suppose that \((H, G)\) is a Nash equilibrium with \( H > G \). Then,

\[
\alpha \frac{d \Pi^0(H, G)}{dH_1} - \frac{d f(H)}{dH} = 0, \tag{A.5}
\]

\[
\alpha \frac{d \Pi^0(H, G)}{dH_2} - \frac{d f(G)}{dH} \begin{cases} = 0 & \text{if } G > L, \\ \leq 0 & \text{if } G = L. \end{cases} \tag{A.6}
\]

From (A.2),

\[
\alpha \left[ \frac{d \Pi^0(H, G)}{dH_1} - \frac{d \Pi^0(H, G)}{dH_2} \right] \leq \frac{\alpha c}{2} (H - G) \tag{A.7}
\]

Combining (A.5)–(A.7) we find that

\[
\frac{d f(H)}{dH} - \frac{d f(G)}{dH} \leq \frac{\alpha c}{2} (H - G),
\]

which contradicts (5).

To prove uniqueness, we observe that a symmetric Nash equilibrium \((H^N, H^N)\) must satisfy

\[
\alpha \frac{d \Pi^0(H^N, H^N)}{dH_1} - \frac{d f(H^N)}{dH} = 0 \quad \text{if } H^N > L,
\]

\[
\leq 0 \quad \text{if } H^N = L, \tag{A.8}
\]

and that if (5) is satisfied, (A.8) has a unique solution. Finally, the result (11)–(12) is obtained by applying the implicit function theorem to (A.8).

**Proof of Proposition 4.** First, we will prove (by contradiction) that (5) implies that all Nash equilibria are symmetric. Suppose that \((H_1, H_2)\) is a Nash equilibrium in market sizes with \( H_1 > H_2 \). Then, the following first-order conditions must be satisfied:

\[
\frac{1}{2} \left[ \frac{d \Pi^0(H_1, H_2)}{dH_1} + \frac{d \Pi^0(H_2)}{dH} \right] - \frac{d f(H_1)}{dH} = 0, \tag{A.9}
\]

\[
\frac{1}{2} \left[ \frac{d \Pi^0(H_1, H_2)}{dH_2} + \frac{d \Pi^0(H_2)}{dH} \right] - \frac{d f(H_2)}{dH} \begin{cases} = 0 & \text{if } H_2 > 0, \\ \leq 0 & \text{if } H_2 = 0. \end{cases} \tag{A.10}
\]

From (A.1),

\[
\frac{d \Pi^0(H_1)}{dH} - \frac{d \Pi^0(H_2)}{dH} \leq \frac{c}{2} (H_1 - H_2). \tag{A.11}
\]

Combining (A.9)–(A.11) with (A.2), we find that

\[
\frac{d f(H_1)}{dH} - \frac{d f(H_2)}{dH} \leq \frac{c}{2} (H_1 - H_2),
\]

which contradicts (5) if \( H_1 > H_2 \).

To establish uniqueness, we observe that any symmetric Nash equilibrium must satisfy the first-order condition

\[
\frac{1}{2} \left[ \frac{d^2 \Pi^0(H^N, H^N)}{dH_1^2} + \frac{d^2 \Pi^0(H^N)}{dH^2} \right] - \frac{d^2 f(H^N)}{dH^2} < 0,
\]

and condition (2) guarantees that for all \( H \geq 3L \),

\[
\frac{1}{2} \left[ \frac{d^2 \Pi^0(H, H)}{dH_1^2} + \frac{d^2 \Pi^0(H)}{dH^2} \right] - \frac{d^2 f(H)}{dH^2} < 0,
\]

so (A.12) has a unique solution. Finally, (16) follows immediately from (A.3) which was shown in the proof of Proposition 1.

**Proof of Proposition 5.** By arguments analogous to the proof of Proposition 1, problems (17) and (18) have optimal solutions. We also observe that

\[
\frac{d \Pi^0(e)}{de} \leq \max \left[ \frac{(H - L)^2}{2}, \frac{(H - L)^3}{2k} \right],
\]
and therefore (19) implies that the objective in (17) is strictly concave, so (17) has a unique optimal solution.

Next, we will prove that any optimal solution \((e_1^*, e_2^*)\) to (18) must be symmetric. Without loss of generality, suppose that \(e_1^* \geq e_2^*\). From the first-order conditions,

\[
\frac{dg(e_1^*)}{de} - \frac{dg(e_2^*)}{de} = \frac{\partial \Pi^P(e_1^*, e_2^*)}{\partial e_1} - \frac{\partial \Pi^P(e_1^*, e_2^*)}{\partial e_2} = 2 \max_{\eta_{i1} + \eta_{i2} \geq e^*} [(H - q_{i1})\eta_{i1} + (L - q_{i2})\eta_{i2}] - (H - c^p)^2 - \max_{c \in [0, e^*]} [(L - q)\eta] (e_1^* - e_2^*) \leq \frac{1}{2} (H - L)^2 (e_1^* - e_2^*)^2. \tag{A.13}
\]

(See Plambeck and Taylor 2003 for additional details as to why the last inequality holds.) Condition (20) implies that \(e_1^* = e_2^*\).

Finally, (21) implies that the objective function \(\Pi^P(e, e) - 2g(e)\) is a strictly concave function of \(e\), and problem (18) has a unique optimal solution. Using the envelope theorem, it is straightforward to obtain that \((\partial^2/\partial e_1^2)\Pi^P(e, e)\) has the following form:

\[
\frac{\partial^2 \Pi^P(e, e)}{\partial e_1^2} = \begin{cases} 
(1 + e^2 - 2c^p)H + k - 2ek - (2 - 3e + c^2)L \\
(2 - 3e - c^2)^3 \end{cases} \left[ \begin{array}{lc} 1 & \frac{(H - L)^2}{4} \frac{2k - 2e^2}{4} (2 - e)^3 \frac{k^2}{e^4} \\
\end{array} \right] \begin{array}{c} \text{if } c^p(e, e) \in \left(0, \min \left\{ \frac{L}{2}, \frac{H - L}{4} \right\} \right], \\
\text{if } c^p(e, e) \in \left(\frac{L}{2}, \frac{H - L}{4} \right], \\
\text{if } c^p(e, e) \in \left(\frac{H - L}{4}, \frac{H + L}{4} \right], \\
\text{if } c^p(e, e) \in \left(\frac{H + L}{4}, \frac{H - k}{2} \right]. \\
\end{array}
\]

\((\partial^2/\partial e_1^2)\Pi^P(e, e)\) is written as a function of \(e\) instead of \(c^p(e, e)\) in Plambeck and Taylor (2003).

**Proof of Lemma 1.** See Plambeck and Taylor (2003).

**Proof of Proposition 6.** It is helpful to define \(R^P(e, e) = e(H - c^p)e + (1 - e) \max_{c \in [0, e^*]} [(L - q)\eta]\). We will begin by showing that \(e(k, H, L) \in (0, 1)\). By the envelope theorem,

\[
\frac{\partial \Pi^P(1, 1)}{\partial e_1} = \frac{\partial}{\partial e_1} R^P(c^p(1, 1), 1, 1) < \frac{d}{de} R^P(c^p(1, 1)) = \frac{d \Pi^P(1)}{de},
\]

and therefore, \(e(k, H, L) < 1\). If \(L < k\), then \(e(k, H, L) \geq e^\phi > 0\). On the contrary, if \(L \geq k\), then

\[
\frac{\partial \Pi^P(0, 0)}{\partial e_1} = \frac{\partial}{\partial e_1} R^P(c^p(0, 0), 0, 0) > \frac{d \Pi^P(0)}{de},
\]

which implies that \(e(k, H, L)\) must be strictly positive.

To complete the proof, it is sufficient to establish the single crossing property

\[
\frac{\partial \Pi^P(e, e)}{\partial e_1} \begin{cases} > \frac{d \Pi^P(e)}{de} & \text{if } e < e(k, H, L), \\
< \frac{d \Pi^P(e)}{de} & \text{if } e > e(k, H, L). \tag{A.14}
\end{cases}
\]

We will need the following results. If \(e \geq \sqrt{2k/(H - L)}\), then

\[
\frac{d \Pi^P(e)}{de} - \frac{\partial \Pi^P(e, e)}{\partial e_1} = \frac{(2 - e)k^2}{4e^3} > 0.
\]

From the envelope theorem and the definition of \(e(k, H, L)\),

\[
\frac{\partial R^P(c^p(e, e), e(k, H, L), e(k, H, L))}{\partial e_1} = \frac{\partial R^P(c^p(e), e(k, H, L))}{\partial e}.
\]

We also observe that for any \(c\),

\[
\frac{\partial^2 R^P(c^p(e), e)}{\partial e_1 \partial e} > \frac{\partial^2 R^P(c^p(e), e, e)}{\partial e_1 \partial e} > 0, \quad \frac{dc^p(e, e)}{de} > \frac{dc^p(e, e)}{de} > 0.
\]

Suppose \(e(k, H, L) > 1/2\). Then, (A.15)–(A.17) imply \(c^p(e(k, H, L)) < c^p(e(k, H, L))\). An immediate corollary of Lemma 1 is that if \(c^p(e) < c^p(e, e)\), then \(e < e^\phi\); hence, \(e(k, H, L) < e^\phi\). The functions \((\partial/\partial e_1)\Pi^P(e, e)\) are differentiable with respect to \(e\) almost everywhere, and for every \(e \in [e(k, H, L), e^\phi]\),

\[
\frac{d}{de} \left[ \frac{d \Pi^P(e)}{de} - \frac{\partial \Pi^P(e, e)}{\partial e_1} \right] = \frac{\partial^2 R^P(c^p(e), e, e)}{\partial e_1^2} + \frac{\partial^2 R^P(c^p(e), e, e)}{\partial e_1 \partial e} \cdot \frac{dc^p(e)}{de} > 0.
\]

It is easy to verify \(e \leq \sqrt{2k/(H - L)} \wedge 1\) and that for all \(e \in [1/2 \vee e^\phi, e^\phi]\),

\[
\frac{\partial^2 R^P(c^p(e), e, e)}{\partial e_1 \partial e} \cdot \frac{dc^p(e)}{de} > \frac{\partial^2 R^P(c^p(e), e, e)}{\partial e_1 \partial e} > 0, \quad \frac{dc^p(e, e)}{de} > \frac{dc^p(e, e)}{de} > 0.
\]

Thus, we have shown (A.14) if \(e(k, H, L) > 1/2\). A related argument establishes (A.14) if \(e(k, H, L) \leq 1/2\) (see Plambeck and Taylor 2003).

**Proof of Proposition 7.** If \(e^\phi < \min(e(k, H, L), e)\), then \(c^p(e^\phi) \leq c^p(e^\phi, e^\phi) < c^p(e^\phi, e^\phi)\), where the first inequality
holds because $e^o < \varepsilon$ (by Lemma 1) and the second inequality holds because $c^o(\cdot, \cdot)$ is increasing and $e^o < e(k, H, L)$ implies $e^o < e^\beta$ (by Proposition 6). If $e^o > \max(e(k, H, L), \bar{e})$, then $c^o(e^o) > c^o(e^\beta) > c^o(e^o, e^o)$, where the first inequality holds because $e^\beta > \bar{e}$ and the second inequality holds because $c^o(\cdot, \cdot)$ is increasing and $e^o > e(k, H, L)$ implies $e^o > e^\beta$. □

**Proof of Proposition 8.** From (A.4),

$$\frac{\partial F(e, a)}{\partial e} \bigg|_{e = e^N(a)} = 0 \quad \text{(A.18)}$$

As in the proof of Proposition 3, symmetry follows from the bounds (24) and (A.18). Uniqueness follows from (25) by an argument analogous to that in the proof of Proposition 3. Define $F(e, a) = a^2 \gamma(x, e) - g(e)$. Given that $c^o(a)$ is the unique Nash equilibrium, application of the implicit function theorem to the OEM’s first-order condition.

establishes (26).

It remains to show that for the parametric region (27), $c^o(a)$ is an equilibrium in dominant strategies and (24) ensures that it is the unique equilibrium. (27) implies $H < 3L$. For $e \in [e^o, \bar{e}]$, with

$$\bar{e} = \frac{H + 2k - 3L}{H - L} - a \quad \text{and} \quad \bar{e} = \frac{2k}{H - L} - a,$$

the optimal capacity is

$$c^o(e, a) = \frac{(H-L)(e + a) + 2(L-k)}{4}. \quad \text{(A.19)}$$

Condition (27) implies that $\varepsilon \leq 0$ and $\bar{e} \geq 1$. Hence, $c^o(e, a)$ is given by (A.19) for every $e \in [0, 1]$. Restatement of (22) yields $c^o(a) = \max_{e \in [0, 1]} F(e, a)$. With a little effort one can show that the first-order condition is

$$\frac{\partial F(e, a)}{\partial e} = \frac{\alpha (H-L)^2}{4} + \frac{(H-L)(3L + H - 4k)}{8} - \frac{d g(e)}{d e} = 0. \quad \text{(A.20)}$$

Further,

$$\frac{\partial^2 F(e, a)}{\partial e^2} = \frac{\alpha (H-L)^2}{4} - \frac{d^2 g(e)}{d e^2} < 0, \quad \text{(A.21)}$$

where the inequality follows from (24). (A.21) ensures that the OEM’s optimal success probability is the unique solution to (A.20). Because (A.20) is independent of $a$, the OEM’s optimal success probability $e$ does not depend on the success probability of the other OEM, $a$. We conclude that (A.20) characterizes a unique equilibrium in dominant strategies. □

**Proof of Proposition 9.** The results on symmetry and uniqueness are obtained by arguments analogous to the proof of Proposition 4 (see Plambeck and Taylor 2003 for a detailed proof), and (30) is a straightforward extension of the proof of Proposition 6. □

**References**


