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PREFERRED STOCKS AND TAXES

By

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ABSTRACT

In this paper we show the possibility of existence of preferred stocks in a tax induced equilibrium. We show that the Miller equilibrium framework can accommodate more than two securities if different investor classes are taxed differently and the tax schedule is not flat. The introduction of uncertainty, bankruptcy, and loss of tax shelters allows for another component which can create equilibrium i.e. seniority. The equilibrium is obtained by equating the expected marginal tax benefit of all securities. Comparative statics with respect to various tax rates are derived as well.
I. Introduction and Initial Observations

In this paper we demonstrate how the interaction of corporate taxes, personal taxes, tax shelters and bankruptcy affects the choice among equity, debt and specifically preferred stocks as instruments of financing.

Preferred stocks have been used as a means of financing for a long time, yet the existing literature provides few convincing explanations for issuing preferreds by non-regulated firms. Preferred stocks resemble debt in that they promise predetermined levels of dividends. Unlike debt, however, preferred dividends are not tax deductible to the issuing firms, whereas interest payments are. Also, omission of dividends will not result in bankruptcy. Empirical studies (see for example Masulis (1983)) show that the impact of a change in the capital structure from equity to preferred stock is in the same direction but of lower magnitude as that resulting from a debt for equity swap.

There are two main reasons provided in the literature for issuing preferreds by utilities (see, e.g., Brealey and Myers (1986), Copeland and Weston (1988)). First, preferred stocks are considered as equity by the regulators, but provide a pattern of payments similar to that of debt. It is therefore more convenient for utilities to issue preferreds instead of debt, thus meeting regulators' equity constraints. Secondly, utilities can pass their costs to the consumers and hence are less affected by the tax disadvantages of preferreds.

Issuing preferreds by non-regulated firms can be justified by clientele effects. Eighty percent of dividends received by corporations (85 percent according to the former U.S. tax law) are tax exempt, thus
corporations will be better off receiving dividends of preferred stocks rather than interest on debt or capital gains.¹

Most of the earlier papers discussing preferred stocks have been fairly descriptive in nature, and, usually ignored taxation issues altogether. They include Fisher and Wilt (1968) and Bildersee (1973). A more sophisticated option type model advanced by David Emmanuel (1983) (see also a recent development examined by Winger, Chen, Martin, Petty and Hayden (1986)).

In a recent paper, Heinkel and Zechnor (1988) provide an informational equilibrium with preferred stocks. In their model, debt (as in Myers (1977)) creates incentives for underinvestment. Equity however, creates incentives for overinvestment and hence an equilibrium, in which the first best investment policy is pursued, is obtained. Preferred stocks, which allow omission of dividends when a good investment opportunity occurs, offer increased debt capacity. Some of Heinkel and Zechnor's predictions are similar to ours - but the structure of their model is very different. Titman (1984), in a paper relating capital structure to the firm's liquidation decision and to the value of its products, suggests that preferred stocks can be used to eliminate stockholders' incentive to liquidate in sub-optimal states of nature without causing bankruptcy.

The relevance of taxes to the existence of preferred stocks has received some attention only recently.² Brick and Fisher (1987) describe

¹For an analysis of the tax implications of dividends vs. capital gains and the optimality of corporate investment in other corporations, see De-Angelo and Masulis (1980) and Masulis and Trueman (1988).
²Although early as 1980 De-Angelo and Masulis provided an important analysis, which relates dividends taxes and capital structure. Preferred stocks were not mentioned directly, but the ideas and methods are very similar to the ones we use in section 2.
cash flows from various classes of debt and conclude that "risky claims should be as stratified as possible" (ibid. p. 392). Their model, which differs from ours also in its assumptions about tax losses and gains, can in principle include preferred stock. However, this form of financing is not explicitly addressed in their study.

Two other recent contributions (Fooladi and Roberts (1986) and Trigeorgis (1988)) discuss essentially only limited extensions of Miller's (1977) framework. These studies, however do provide useful empirical insights which relate preferred stocks and taxes: Fooladi and Roberts show that in Canada, where preferred stocks (specifically-dividends — see later discussion) enjoy a more favorable tax treatment, preferreds' share in the financing mix is four times higher than in the U.S.

Trigeorgis looks at a sample of utilities, and shows that tax rates and dividend payments are statistically related to the share of preferred in the capital structure.

It is the purpose of our paper to provide an integrative and more complete model discussing the tax role of preferred stocks. We analyze a Miller type model, and then extend the discussion to the more realistic case in which bankruptcy and loss of tax shelters are possible.

The model we propose is based upon two features which distinguish preferred stocks from equity and debt respectively: First, the effective tax rate on purchasers and issuers of preferreds is different from the effective tax rates on debt and equity. Secondly, the seniority structure of payments to investors is unique to each form of financing. As we shall show, these two distinguishing features are sufficient to justify the existence of preferred stocks in an equilibrium framework.
The paper is organized as follows. The next section offers a preliminary discussion of the differences between the three types of securities. In Section III, we discuss direct extensions of Miller's model. The advantages and disadvantages to the issuers and buyers of each type of security, as functions of their tax status, are addressed. In Section IV, the case where the firm may not be able to meet its obligations is considered. Section V contains conclusions.

II. ON DIFFERENCES BETWEEN TAXATION AND SENIORITY OF SECURITIES

Prior to the enactment of the 1986 tax law there was a clear differentiation between the taxation of capital gains on the one hand, and preferred income (dividends) and interest income on the other hand. Under the new law, the statutory rate is the same for all forms of income. However, these changes have not eliminated the tax differentiation between securities altogether. One can defer capital gains, but not interest or dividend income. Also, the equal taxation of capital gains, dividends and interest is a unique American phenomenon, whereas the tax code of most other industrialized nations still features a wedge between capital gains and other income. In fact, in some countries such as Japan, Belgium or Israel, capital gains are not taxed at all. Further, corporations are allowed to deduct 80% (85% according to the old law) of their dividends from corporate income for tax purposes. This differentiation leads to different effective taxation on preferred and common stock for different clienteles. Most income to preferred shareholders is in the form of dividends, whereas common stockholders obtain a large percentage of their income in the form of capital gains. This disbursement of income leads to different marginal expected tax rate for the two types of securities. Individuals may observe lower effective tax rates on total equity income,
whereas for corporations effective tax payments on total preferred income may be lower.

There is also a difference in the tax effects of each type of financing on the issuer of the securities. Dividend payments are not deductible, whereas interests payments are. The tax deductibility advantage of debt to the issuer, however, depends upon its tax status. This advantage will be more important for a profitable firm than for a non-profitable firm.

In addition to the different tax treatments, there are also differences in the seniority structure of claims that separate the three types of securities. Bondholders must receive their income first, or else they can force bankruptcy. Next come preferred shareholders, and finally equity holders, who are the residual claimants, get their share.

In Miller's original model, all tax rates were exogenously given (see also Talmor et al. (1986)). The firm could not change the marginal tax advantage of debt by changing its capital structure, and hence, no internal solution existed. We show in the next section that a Miller (1977) type model can accommodate only all three securities only under a graduated tax code. However, once real world bankruptcy and non-debt related tax shields are introduced, the expected marginal tax rate will change with the amount of debt (and/or preferred) taken, even if the statutory tax rate of the marginal investor is exogenously fixed. Given bankruptcy, non-debt related tax shields, debt and equity, a choice of preferred stock could affect the expected marginal tax rate on all three instruments, introducing non-linearities in the value function and hence a unique optimal level of all three securities may be obtained.
II. The Case of No Bankruptcy

In this section, we shall extend the Miller (1977) model to include preferred stocks. We assume that there is no bankruptcy and no non-debt related tax shelters, hence the only reason for the potential existence of preferred stock is the differential statutory marginal tax rates of investors in preferred stock, in common stock, and in debt and the marginal tax rates of the issuers of securities. To simplify the presentation, we assume a risk-neutral, one period framework.

We shall thus denote the tax rate on preferred shareholders as $t_p$, the tax rate on common stock as $t_g$, and the tax rate on interest income as $t_b$. Given a corporate tax rate of $t_c$, the cash flows to the firm claimants are as follows:

To bond holders : $D+DR(1-t_b)$
To preferred shareholders : $P+PR_p(1-t_p)$
To common shareholders : $((X-I-RD)(1-t_c)+I-P-D-PR_p)-S)(1-t_g)+S$

where $D$, $P$, and $S^3$ denote the levels of debt, preferred stocks, and initial investment of (common) stockholders, respectively. $R$ and $R_p$ denote the promised rates of return on debt and preferred stocks, respectively. The depreciation allowance is exogenous to the model and equal to the entire investment, $I = S+D+P$, and $X$ denotes the operating income.

$^3$Note that $S$ is not the market value of equity — but the residual portion of investment contributed by common stockholders. As in many models, the objective function in this partial equilibrium model is equity value (which is this framework is equivalent to firm value maximization).
The value of the levered (by debt and/or preferred stock) firm is given by:

\[ V_L = V_U + RD \left[ (1-t_b) - (1-t_c)(1-t_g) \right] + PR_p(t_g - t_p) \]

where \( V_U \) is the value of the unlevered firm. The last term in (2.1), \( PR_p(t_g - t_p) \), is the only term added to Miller's (1977) valuation formula.

From Eq. (2.1), one could see that a Miller type equilibrium will exist (i.e. the firm is indifferent among all forms of financing) if there is a marginal tax payer denoted by \( * \) for whom \( t_b^* \) and \( t_g^* \) are such that

\[ [(1 - t_b^*) - (1 - t_c)(1 - t_g^*)] = 0 \]

and also \( t_p^* = t_g^* \). Clearly, if for all investors \( i \), \( t_{pi} > t_{gi} \) or \( t_{gi} > t_{pi} \) such an equilibrium can never obtain.

If personal taxes on equity are lower, only equity (and debt) will be held whereas preferred stocks will become a redundant security; if, on the other hand, equity is taxed more heavily, only preferreds and debt will exist in equilibrium. (And then \( t_g^* \) is replaced by \( t_p^* \) in Eq. (2.2).)

The only exception is the case where \( t_{pi} = t_{gi} \), but then equity and preferreds are indistinguishable in this framework. Thus, the model presented so far will usually preclude the existence of two non-tax-deductible securities.

We shall now examine the case where for some investors \( t_{pi} > t_{gi} \), whereas for others \( t_{gi} > t_{pi} \). We shall also require

\[ t_{bi} \geq \max (t_{gi}, t_{pi}) \text{ for all } i. \]

\(^4\)This analysis extends and generalizes Fooladi and Roberts (1986) and Trigeorgis (1988) - but in our framework it is presented only as a preliminary step towards the more realistic, and more comprehensive model in the ensuing section. Also similar to Miller and all his followers, we implicitly assume no tax-arbitrage via short-sales.
In this case, strict clienteles will emerge and there will be two marginal investors: one, the marginal equity investor, $E$, who is taxed less on equity and for whom

\[(1 - t_{bE}) - (1 - t_c) (1 - t_{gE}) = 0\]

For the preferred stocks there will be a marginal investor $F$, for whom

\[(1 - t_{bF}) - (1 - t_c) (1 - t_{pF}) = 0\]

For an equilibrium to exist, it must also be that $t_{gE} = t_{pF}$.

The scenario then is as follows - all low tax investors from both clienteles invest in bonds. The higher tax investors for whom $t_{pI} > t_{gI}$ will invest in equity, whereas the others, for whom $t_{gI} > t_{pI}$ will invest in preferred shares. These observations are summarized in proposition (2.1).

**Proposition 2.1**

If for all investors $t_{pI} > t_{gI}$ only equity and debt will be issued in equilibrium. If $t_{gI} > t_{pI}$ only preferreds and debt will be held. Otherwise i.e. if for some investors, $i$, $t_{pI} > t_{gI}$ whereas for others, $j$, $t_{gj} > t_{pj}$, strict clienteles will form in equilibrium. All three securities will be held with two marginal investors who comply with the Miller conditions for equity and preferreds separately and such that $t^*_{gi} = t^*_{pj}$ ($t^*_{gi}$ is the tax rate of the marginal equity investor; $t^*_{gj}$ is the tax rate of the marginal preferred investor).

*Although in some cases our proposition implies that no equity should be issued, this should be interpreted in the same light as the Modigliani-Miller proposition - i.e. this is only a tax-perspective, purposely simplifying other factors, such as control, agency, etc. In this paper, the no-equity result is modified when bankruptcy and loss of tax shelters are considered.*
In the following section, we show how uncertainty about future income, which may lead to the loss of tax shelters, allows firms to influence the effective expected tax rate of the marginal individual, and thus, at equilibrium, each firm will have a unique choice of debt, preferred shares and equity. This choice will guarantee that, even under fairly general assumptions about prevailing statutory tax rates, all firms will reach equilibrium expected tax rates for all securities. This will of course include preferred stock.

III. Optimal Levels of Stocks, Preferreds and Debt When Bankruptcy is Possible

The model we present in this section is a one period, risk neutral\(^6\) model. Thus the value of the firm (with an exogenous investment decision) is equivalent to the expected after tax value of cash flows to all security holders. The firm receives a stochastic operating income, and then divides up the cash flows according to the seniority of claimholders. It can end up either solvent or bankrupt. We further assume, for simplicity sake, that all securities (i.e. debt and preferred stocks) are issued at par, and all market value adjustments are made through the promised rate of interest.

We now detail the ranges of income payments to claimants.\(^7\) The limits to the ranges are defined by realizations of \(X\), the stochastic operating income, which is defined over the support \([0, \infty]\) and are also

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\(^6\) State prices such as introduced by De-Angelo and Masulis (1980) could be incorporated into this model as well-but although that would render the model slightly more general, it would complicate the presentation a great deal without substantially altering the results.

\(^7\) Although the ranges are fairly realistic, one could, of course, imagine circumstances under which they would be somewhat different than those presented here. All we require for most of our results, however, is that in different ranges, different clienteles will be responsible for paying taxes. It is also true that we, like most, assume that strict seniority rules prevail.
functions of the decision variables: P, D and S, the interest rates, R, the preferred dividend $R_p$ and the tax rates. These limits are presented in Table 3.1 below.

Table 3.1

**First Range** $X \geq X_0$

Income is high enough so the firm pays all taxes and uses both depreciation and interest tax shields.

Stockholders income is

$$(1-t_c) \left[ (1-t_o) (X-I-RD) + I - PR_p P-D-S \right] + S$$

Bondholders income is

$$D + RD (1-t_b)$$

Preferred-holders income is

$$P + PR_p (1-t_p)$$

Note that $I = P + D + S$

**Second Range** $X_0 > X \geq X_1$

$X$ is somewhat lower. Now shareholders receive an amount lower than their initial investment, and hence pay no taxes. The firm still pays income tax.

Stockholders now receive

$$(1-t_c)(X-I-RD) + (I-P-D) - PR_p$$

Bondholders income is

$$D + RD (1-t_b)$$

Preferred-holders income is

$$P + PR_p (1-t_p)$$
Third Range  $X_2 \leq X < X_1$

Now the firm's income is less than the value of its tax shelters, $I + RD$. The firm and its shareholders pay no taxes. Preferred stockholders and debt holders are still paid in full.

Stockholders receive

$$(X - D - RD - P - PR_p)$$

Bondholders income is

$$D + RD (1-t_b)$$

Preferred-holders income is

$$P + PR_p (1-t_p)$$

Fourth Range  $X_3 < X \leq X_2$

The firm cannot pay its obligations to both preferred shareholders and bondholders in full. Shareholders receive no income, preferred shareholders get all the principal owed and some dividend. Bondholders are not affected.

Here Stockholders income is 0.

Preferred-holders receive $P + (X - D - RD - P) (1-t_p)$

Bondholders get $D + RD(1-t_b)$

Fifth Range  $X_4 \leq X < X_3$

Similar to the previous range, except that the firm cannot even pay the preferred principal. Here only bondholders pay any taxes.

Stockholders get 0.

Preferred-holders get $X - RD - D$

Bondholders get $D + RD (1-t_b)$
Sixth Range \( X_5 \leq X < X_4 \)

In this and the following range, only bondholders receive any income. Specifically, here principal is paid but interest cannot be repaid in full.

Stockholders get 0.
Preferred-holders get 0.
Bondholders get \( D + (X - D) (1-t_b) \)

Seventh (last) Range \( 0 = X_6 < X < X_5 \)

Bondholders take control of the firm and its residual cash flows. A bankruptcy cost \( (B) \) is paid.

Stockholders get 0.
Preferred-holders get 0.
Bondholders get \( X-B \).

We now use the above description to compute the value of the firm, (Equation (3.1) represents the expected value of future cash flows, which is an equivalent concept in a risk neutral world). We are assuming a density function \( f(X) \) of cash flows over the support \([0, \infty]\), and

\[
F(X_1) = \int_{-\infty}^{X_1} f(X)dx.
\]

\[ (3.1) \quad V_L = \int_{X_0}^{\infty} \left\{ [(X-I) (1-t_c) (1-t_g)+I]+RD[(1-t_b)-(1-t_c)(1-t_g)] \right\} \]

\[ + \frac{PR_p \cdot (t_g-t_p)}{X_0} \int_{X_0}^{X_1} f(X) dx \]

\[ + \int_{X_1}^{X_0} \left\{ (1-t_c) (X-I) + I + RD(t_c-t_b) - PR_p t_p \right\} f(X) dx \]
\[ x_1 + \int_{x_2} \left( x - pr_{r_p} - rdt_b \right) f(x) \, dx \]

\[ x_2 + \int_{x_3} \left( x(1-t_p) + d[r(t_p-t_b) + t_p] + pr_p \right) f(x) \, dx \]

\[ x_3 + \int_{x_4} \left( x - rdt_b \right) f(x) \, dx + \int_{x_5} \left[ x(1-t_b) + dt_b \right] f(x) \, dx \]

\[ x_4 + \int_{x_5} \left( x - b \right) f(x) \, dx \]

We note that the tax differentials play an important role in the determination of firm value since in different intervals, different claim-holders are responsible for the tax-liabilities relevant for the same cash flow. We now compute the required rate of interest on bonds and on preferred stocks respectively, which are the solutions of the following implicit equations:

\[ \sum_{x_4}^{\infty} \int_{x_4} \left[ d + rd(1-t_b) \right] f(x) \, dx + \int_{x_5} \left[ d + (x-d)(1-t_b) \right] f(x) \, dx \]

\[ x_5 + \int_{0}^{x_5} \left( x - b \right) f(x) \, dx = d + dr(1-t_b) \quad \text{(for debt)} \]
\[ (3.3) \quad \int_{X_2}^{\infty} \int_{X_2}^{X_3} \int_{X_3}^{X_4} [P + PR_p(1-t_p)] f(X) dX + \int_{X_2}^{\infty} [P + (X-D-RD-P)(1-t_p)] f(X) dX = P + PR_0(1-t_p) \]

\[ f(X) dX + \int_{X_4}^{(X-D-RD)} f(X) dX = P + PR_0(1-t_p) \quad \text{(for preferred)} \]

where \( R_0 \) is the interest rate on risk free taxable bonds. One notes from

(3.2) and (3.3) and the definition of the limits of integration that \( R \) (the promised rate of interest on bonds) is a function of \( D \), and \( R_p \) (the promised preferred dividend) is a function of \( D \) and \( P \).

The optimal amounts of debt and preferred stocks are obtained by
differentiating (3.1) with respect to \( D \) and \( P \), subject to (3.2) and (3.3)
and equating the derivatives to zero. We obtain:

(3.4) \[ \frac{\delta V_L}{\delta P} = \left[ t_g [1-F(X_0)] (R_pP)' - t_p [1-F(X_2)] \right] (R_pP)' \]
\[ + [F(X_2) - F(X_3)] t_p = 0 \]

(3.5) \[ \frac{\delta V_L}{\delta D} = (RD)' \left[ (1-t_b) - (1-t_c) (1-t_g) \right] \left( 1-F(X_0) \right) \]
\[ - [F(X_0) - F(X_4)] (RD)' t_b \]
\[ + (F(X_0) - F(X_1)) t_c + (F(X_2) \]
\[ - F(X_3)) (t_p + (RD)' t_g) + (F(X_4) - F(X_5)) t_b \]
\[ - (F(X_0) - F(X_2)) Pr \frac{\delta R_p}{\delta D} + F \frac{\delta R_p}{\delta D} (1-F(X_0)) (t_g - t_p) \]
\[ - B f(X_3) = 0. \]

Where \((RD)'\) is \( \left( \frac{\delta R}{\delta D} D + R \right) \) and \( R_p + \left( \frac{\delta R_p}{\delta P} P = [R_pP]' \right) \)
The first term in Eq. (3.4) represents the marginal expected decrease in tax payments by shareholders as more income is transferred to preferred stockholders in the states in which they are paid in full.

The second term in Eq. (3.4) represents the marginal expected increase in tax payments by preferred shareholders as principal (and interest) increase. Since \( R_p \) is non-linear in \( P \), (3.4) is, in general, non-linear and the sum of the first two terms depends on the tax rates, the limits and the distribution function.

The third term represents an additional expected tax benefit. An increase in principal will decrease the portion of payment considered interest in states in which preferred share holders are paid principal but only part of the interest due.

The above discussion leads to proposition 3.1.

**Proposition 3.1**

a) Even without progressive taxation (but when equity and preferred shares are taxed differentially) an interior optimal level of preferred shares may obtain.

b) A sufficient condition for this optimum to obtain is

\[
\begin{align*}
    t_g & > t_p \text{ and } \\
    \frac{(1-F(X_3))}{1-F(X_0)} & \left|_{P=0} < \frac{(1-F(X_1))}{1-F(X_0)} + \frac{F(X_1)-F(X_3)}{(1-F(X_0))(PR_p')} \right|_{P=I}
\end{align*}
\]

This condition will tend to hold for small values of \( (PR_p)'_P = I \).

c) Even if \( t_p < t_g \), it may not be optimal to issue any preferred stocks.

**Proof**

See Appendix.
The intuition of Proposition (3.1) is as follows: under certainty, if the tax rate on one security is lower (for all investors) than the tax rate on another security, (but the corporate tax treatment is the same) the high tax security will disappear. Under uncertainty, seniority matters, and the residual security has an "advantage" in the sense that its holders are the first to pay no taxes on their earnings. Thus, for example, if the firm is in the second range in our model, every dollar transferred from equity to preferred shareholders is transferred from essentially tax-free investors to a tax paying clientele, increasing the total tax liability. Thus for preferred stocks to be viable, we require not only $t_g > t_p$ (strictly so) but also other conditions on the distribution. On the other hand, in lower ranges, an increase in preferred principal lowers taxes owed. Therefore an equilibrium may obtain.

We should also note that in Eq. (3.5) one obtains positive and negative terms so that depending on the distribution, an interior solution for both debt and preferred shares may obtain.\footnote{We should also note that debt holders and preferred shareholders are assumed to be passive players in this game. In other words, as we change the amounts of debt and preferreds, the expected value of these securities does not change. It is guaranteed by Eqs. (3.2) and (3.3). However, changing the amount of debt and preferred stocks changes the total tax liability and the after tax expected cash flows to shareholders, thus affecting equity value.}

Proposition 3.1 illustrates the rationale for our results: although statutory tax rates may be the same, different priorities in cash flow distribution may cause, under uncertainty, the expected marginal tax rates to differ, and thus a change in the amount of security issues will change the marginal expected rates, creating room for several tax-deductible and several non-tax-deductible securities at equilibrium. This contrasts with
the certainty case where, if tax rates are the same for all investors, equilibrium allows only one taxable and one non-taxable security.

This is also the conclusion, for other reasons, of Heinkel-Zechner (1988). We now turn to some comparative statics. All derivatives will be with respect to a tax rate, and the proofs will utilize the following derivations.  

\[
(3.6) \quad \frac{dP^*}{dt_1} = - \left[ \frac{\delta^2 V}{\delta P \delta t_1} \cdot \frac{\delta^2 V}{\delta D^2} - \frac{\delta^2 V}{\delta P \delta D} \cdot \frac{\delta^2 V}{\delta D \delta t_1} \right] / H
\]

\[
(3.7) \quad \frac{dD^*}{dt_1} = - \left[ \frac{\delta^2 V}{\delta D \delta t_1} \cdot \frac{\delta^2 V}{\delta P^2} - \frac{\delta^2 V}{\delta P \delta D} \cdot \frac{\delta^2 V}{\delta P \delta t_1} \right] / H
\]

Where \( H \) is the determinant of the second order conditions which must be positive for a maximum to exist.

We shall also assume that 2\(^{nd}\) order conditions hold, which implies (Al) \( \frac{\delta^2 V}{\delta D^2} < 0; \frac{\delta^2 V}{\delta P^2} < 0. \)

We can now demonstrate the following claims:

**Claim 1**

If \( \frac{\delta^2 V}{\delta P \delta D} \geq 0 \) then \( \frac{dP^*}{dt_g} > 0; \frac{dD^*}{dt_g} > 0. \)

or - if the cross derivative between debt and preferreds is positive, then an increase in the rate of taxation on equity will increase both the optimal level of debt and the optimal level of preferred shares.

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\(^9\)We naturally assume in the ensuing discussion that an interior optimum exists for both preferred stocks and debt.
Proof

The results are easily obtained by Eqs. (3.6) and (3.7) and (A1).

\[
\frac{\delta^2 V}{\delta P \delta t_g} = (1-F(X_0)) (R_p + \frac{\delta R_p}{\delta P} P) > 0
\]

\[
\frac{2 \delta V}{\delta D \delta t_g} = (1-t_c) (RD)' (1-F(X_0)) + P \frac{\delta R_p}{\delta D} (1-F(X_0)) > 0
\]

The intuition is simple - an increase in the tax rate on equity makes both preferred stocks and debt more desirable. Note that if \(\frac{\delta^2 V}{\delta P \delta D} < 0\), then conclusions could be reversed, since the change in the optimal level of one non-equity security could inversely affect the benefits of the other one.

Claim 2

\[
\frac{dD^*}{dt_c} > 0 \quad ; \quad SGN \frac{dP^*}{dt_c} = SGN \frac{\delta^2 V}{\delta P \delta D}
\]

or - an increase in the corporate tax rate will increase the optimal level of debt; it will increase the optimal level of preferred stocks only if the cross-derivative between the two non-equity securities is positive.

Proof

Follow from equations (3.6) (3.7) and the following derivation

\[
\frac{\delta^2 V}{\delta D \delta t_c} = (RD)' t_c (1-F(X_0)) + F(X_0) - F(X_1) > 0.
\]
The intuition is this:

An increase in the corporate tax rate will increase the optimal level of debt - as expected. The change in the level of preferred stocks depends on the substitution between the two securities.\(^\text{10}\)

\[
\frac{\delta V}{\delta D_0 P} > 0, \text{ we shall obtain an impact of } t_c \text{ on the optimal level of preferreds similar to the impact on debt.}
\]

An increase in the corporate tax rate will thus raise the levels of preferred stocks as well as that of debt. This is in spite of the fact that preferreds, like equity, are not tax-deductible.

This point, and the pervasive influence of the cross derivative between the two securities are perhaps the most interesting issues raised by the comparative statics exercise.

We show why corporate taxes may affect the issuance of preferred shares when other financing alternatives are present. We also demonstrate that interactions between two forms of financing may affect firm value, comparative statics results and the optimal amount of each security.

We now turn to \( t_p \) and \( t_b \). Observing the ranges in Eq. (3.1) we note that for both debt and preferred stocks there are cases, i.e. ex-post levels of \( X \), for which a higher \( t_p \) or \( t_b \) will result in a higher cash flow. In other ranges, a higher \( t_b \) or \( t_p \) will lower cash flows.

The exact tradeoffs proved difficult to verify and hence comparative statics with respect to \( t_p \) and \( t_b \) are not presented here.

\(^\text{10}\)This claim should be interpreted carefully - although the absolute quantities of both preferreds and debt may increase, the relative use of debt vs. preferred may change in either direction (see Trigeorgis (1988) for an empirical investigation of this issue).
Conclusions and Empirical Implications

In this paper we have shown the possibility of existence of preferred stocks in a tax induced equilibrium. We have shown that the Miller equilibrium framework can accommodate more than two securities if different investor classes are taxed differently and the tax schedule is not flat. The introduction of uncertainty, bankruptcy, and loss of tax shelters allows for another component which can create equilibrium i.e. seniority. The equilibrium is obtained by equating the expected marginal tax benefit of all securities. Comparative statics with respect to various tax rates were derived as well.

An interesting ramification of our analysis is that since in the equilibrium under uncertainty the condition $t_g < t_p$ is not sufficient, many firms will not find issuing preferred stocks viable. This may explain the relative rarity of this form of financing.

Another testable hypothesis from the foregoing discussion is the correlation between tax rates and the share of preferred stocks in the financing mix.
APPENDIX

Proof of Proposition (3.1)

For parts (a) and b) we note the following –

A sufficient condition for an interior maximum to obtain is that at

\( P=0 \) \( \frac{\partial V}{\partial P} > 0 \) whereas at \( P=I \) \( \frac{\partial V}{\partial P} < 0 \). (These conditions replace the second order conditions which in our specific case are difficult to derive and yield no intuition.)

Using Eq. (3.4) the requirement \( \frac{\partial V}{\partial P} \bigg|_{P=0} > 0 \) translates into

\[ (A.1) \quad \frac{t_g}{t_p} > \frac{1-F(X_2)}{1-F(X_0)} \bigg|_{P=0} \]

This clearly requires \( t_g > 1 \).

For \( \frac{\partial V}{\partial P} \bigg|_{P=I} < 0 \), one needs to readjust the limits and cash flows (see Table 3.1) which leads to the following derivative for \( P > (1-D)/(1+R_p) \):

\[ (A.2) \quad \frac{\partial V}{\partial P} = \left[ t_g \left( 1-F(X_0) - t_p (1-F(X_1)) \right) (p p')' 
- t_p \left( F(X_1) - F(X_3) \right) \right] \]

For Eq. (A.2) to be negative at \( P=I \), we require:

\[ (A.3) \quad \frac{t_g}{t_p} < \frac{(1-F(X_1)) (p p')' + (F(X_1) - F(X_3))}{(1-F(X_0)) (p p')' \bigg|_{P=I}} \]

Combining (A.1) and (A.3)

We obtain

\[ (A.4) \quad \frac{(1-F(X_2))}{(1-F(X_0))} \bigg|_{P=0} < \frac{(1-F(X_1))}{(1-F(X_0))} \bigg|_{P=I} + \frac{(F(X_1)-F(X_3))}{(1-F(X_0))(p p')' \bigg|_{P=I}} \]

This will be true for example, for a very small \( (p p')' \).

If condition (A.4) holds with a strict inequality, we can find \( t_g > t_p \) such that (A.1) and (A.3) will hold.
We now turn to part (c).

Although, intuitively, part (c) seems fairly straightforward, an exact proof is not easy to construct in the general case. In order to simplify matters, and since all we need is one case, here it is:

Assume that for all ranges below \( X_1 \), firm income is zero. This implies a specific distribution. (However, as both the proof for this part and for parts (a) and (b) demonstrates, such an extreme case is certainly not necessary for part (c) to obtain.) Proceeding, we modify Eq. (3.4) to read

\[
\frac{\partial V}{\partial p} = t_g [1-F(X_0)] - t_p [1-F(X_1)]
\]

Since \((1-F(X_0)) < (1-F(X_1))\) then as long as \(1 < \frac{t_g}{t_p} \leq \frac{(1-F(X_1))}{(1-F(X_2))}\) the derivative will be identically negative. Assume that the smallest value for the RHS is \(1+\xi\) for all \(P\), \(0 \leq P \leq I\). Then for \(t_g\) such that \(t_p (1+\xi) > t_g > t_p\) no preferred stocks will be issued.
<table>
<thead>
<tr>
<th>$P &lt; (I-D)/(1+R_p)$</th>
<th>$P &gt; (I-D)/(1+R_p)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_0 = I + RD + PR_p/(1-t_c)$</td>
<td>$X_1 = P + D + RD + PR_p$</td>
</tr>
<tr>
<td>$X_1 = I + RD$</td>
<td>$X_2 = I + RD$</td>
</tr>
<tr>
<td>$X_2 = P + D + RD + PR_p$</td>
<td>$X_2 = I + RD$</td>
</tr>
<tr>
<td>$X_3 = P + D + RD$</td>
<td>$X_0$ and $X_3$ through</td>
</tr>
<tr>
<td>$X_4 = D + RD$</td>
<td>$X_6$ do not change</td>
</tr>
<tr>
<td>$X_5 = D$</td>
<td></td>
</tr>
<tr>
<td>$X_6 = B$</td>
<td></td>
</tr>
</tbody>
</table>

Note: If $P > (I-D)/(1+R_p)$ then $X_1$ and $X_2$ are switched and cash flows are somewhat different. Propositions 3.1 will usually hold however. Also, $P > (I-D)/(1+R_p)$ implies $S < PR_p$ which is usually not the case.
REFERENCES


