NCGIA
National Center for Geographic Information and Analysis

GIS Laboratory Exercises:
Volume 2, Technical Issues

Edited by
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Preface

This set of labs is designed to illustrate and reinforce principles presented in Volume II: Technical Issues in GIS of the NCGIA Core Curriculum in GIS (units 26 to 50). Since several of these labs were originally prepared in early 1989 for the test version of the Curriculum, they have been presented to students a number of times and have undergone extensive revision. We have also added some new labs to the original set.

While these labs are written for a specific software and hardware configuration, they are presented as models for generic lab development. Please modify them to suit your own configurations and curriculum. To assist with this task, we have included digital versions of the text. See Appendix A for more information.

Several people have contributed to the development of these labs. They were originally written by Michael Goodchild, Karen Kemp and Howard Veregin. We would like to thank the students and faculty at the University of California, Santa Barbara and other universities who participated in the evaluation of the Core Curriculum and provided valuable revision suggestions. The National Science Foundation is also thanked for its contribution through the establishment of the National Center for Geographic Information and Analysis.

Karen K Kemp
Santa Barbara, May 1991
Assignment 1
Coordinate Systems

Objectives: In this assignment you will be using some simple QuickBASIC programs to convert between different coordinate systems, make distance calculations, and draw maps on the screen.

Files: This assignment requires four QuickBASIC programs (LL.BAS, GCDIST.BAS, EUCLID.BAS and MAP.BAS) and a file containing coordinate data for Africa (AFRICA.DAT).

Coordinate System Conversions: Run the QuickBASIC program called LL.BAS to compute the latitude and longitude for each of the points listed in Table 1. Points A through F were derived from 1:25,000 US Geological Survey topographic maps of Maine. Points G and H were derived from a 1:100,000 topographic map of the area near Sydney, Australia. Write your answers (in degrees, minutes and seconds) in the appropriate columns of the Table. Be sure to designate latitudes with an N or an S, and longitudes with an E or a W.

1. a) Do the latitudes and longitudes you computed for the points seem to be correct? (You will need to refer to a map to answer this.) b) Can you account for any discrepancies you observe?
2. Compare your answers for points D and F. Can you account for the difference in UTM Eastings for these two points?

Table 1.

<table>
<thead>
<tr>
<th>Point</th>
<th>UTM Zone</th>
<th>UTM Easting</th>
<th>UTM Northing</th>
<th>Latitude</th>
<th>Longitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>19</td>
<td>416800</td>
<td>4627250</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>19</td>
<td>413390</td>
<td>4622340</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>19</td>
<td>254470</td>
<td>4672580</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>19</td>
<td>252025</td>
<td>4667800</td>
<td></td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>18</td>
<td>737650</td>
<td>4667450</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>18</td>
<td>747975</td>
<td>4667800</td>
<td></td>
<td></td>
</tr>
<tr>
<td>G</td>
<td>56</td>
<td>330650</td>
<td>6241950</td>
<td></td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>56</td>
<td>315300</td>
<td>6236050</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Distance Calculations: Run the program GCDIST.BAS, which calculates the distance between points on the earth’s surface using the great circle distance formula. The program assumes a value of 6371 km for the radius of the earth. Use the latitude and longitude values you entered into Table 1 to calculate the great circle distance between each pair of points in Table 2.
Table 2.

<table>
<thead>
<tr>
<th>Pair of points</th>
<th>Great circle distance (km)</th>
<th>Euclidean distance (km)</th>
<th>Difference (km)</th>
<th>Difference as a percentage of Euclidean distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-B</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C-D</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G-H</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D-F</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C-E</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Now examine the program and answer the following questions.

3. What computation is performed by lines 1700, 1900, 2200 and 2400?

4. a) Are all of the parentheses on line 2600 necessary?  b) Write out the line using the minimum number of parentheses needed to preserve the meaning of the equation.  c) Why would you want to use parentheses when you don't really need them?

5. If you were to increase the estimate of the earth’s radius by 1 percent, how would the great circle distance estimates be affected?

6. a) What happens if you enter latitude and longitude values incorrectly (e.g., enter characters instead of numbers, omit the commas, include too many commas, etc.)?  b) What happens if you enter invalid latitude or longitude values (e.g., latitudes greater than 90 or longitudes greater than 180)?

Load the program called EUCLID.BAS, which calculates distances between pairs of points based on the Euclidean distance formula. Before you run this program, you will have to make several changes, as follows:

a) Insert an message between the quotation marks on line 1900.

b) Add the appropriate line number to the GOTO statement in line 2100.

c) Find and correct the error in line 2300.

d) Add a line (number 2400) to convert the computed distance to kilometers. (UTM Northings and Eastings are given in meters, but your answer should be expressed in kilometers.)

Save the program once you have made these changes. Run the program to calculate the Euclidean distance between each pair of points listed in Table 2. Compute (by hand) the difference between the great circle and Euclidean distances, and the difference as a percentage of the Euclidean distance. Enter your answers in the appropriate column in Table 2.

7. Based on your knowledge of the UTM projection, how might you account for the differences in the values in the last column of Table 2?

Make a printout of EUCLID.BAS and hand it in along with the assignment.
Coordinate System Conversions: Now that you have some experience with QuickBASIC programming, open LL.BAS again. Examine the program to answer the following questions.

8. a) What are the # symbols for in line 1700 and others following it?
   b) Why are they used?

9. a) What are the ! symbols for in line 5300 and others following it?
   b) Why are they used?

10. Lines 5000 to 5200 contain a loop that assigns a value to the elements of an array called cmer. These elements are specified as cmer(zone), where zone varies between 1 and 60. Using the equation given on line 5100, calculate the value for cmer(22) and cmer(50).

\[
\begin{align*}
cmer(22) &= \\
cmer(50) &= 
\end{align*}
\]

11. After line 13100 you can observe occurrences of INT(LATITUDE), INT(MIN) and INT(SEC). What does the INT function do and why is it used here?

Graphics Programming: The final step in the assignment is to modify a QuickBASIC program (called MAP.BAS) that displays a map of Africa on the screen using the Mercator projection. Run this program. The coastline of Africa should appear in white. The dashed white horizontal lines are lines of latitude (parallels) ranging from 30 degrees N to 30 degrees S. The central dashed line is the equator.

12. a) Based on the output of the program, would you say that lines of latitude are evenly spaced on Mercator's projection?  b. If they are not evenly spaced, then do they get closer or farther apart as you move away from the equator?

Now examine MAP.BAS more closely. You need to modify this program so that it displays a second map on top of the Mercator map. This second map will be based on the Lambert cylindrical equal-area projection. Note that the procedure for displaying the Mercator map is broken into three steps in MAP.BAS. Step 1 (lines 3000 to 4000) involves converting the longitude and latitude values for Africa (read in from the file AFRICA.DAT) into x- and y-coordinates. Step 2 (lines 5000 to 5400) draws the Mercator map by connecting adjacent x-y coordinate pairs with straight lines. Step 3 (lines 6000 to 6700) draws the lines of latitude for Mercator's projection.

Modifying the program to display two maps is actually a very simple task. Basically, steps 1 through 3 must be reproduced between lines 6700 and 20000, making a few changes in order to display the Lambert map in a different color. There are only three important changes you need to make:

a) The equation for computing the y-coordinate for Mercator’s projection is

\[
y = \log(\tan(\frac{p}{4} + \frac{j}{2}))
\]

where j is the latitude (in radians). You can see this equation in action on lines 3900 and 6500 of the program. For the Lambert projection, the equation is

\[
y = \sin(j)
\]

b) The white color of the Mercator map is defined by the number 15 in lines 5300 and 6600. To get a different color, use a different number.
Any value between 0 and 15 is acceptable. Experiment or consult the QuickBASIC programming manual.

c) The value 8738 in line 6600 draws the lines of latitude as dashed lines on the Mercator map. You can also use this value for the Lambert map but, since the equator is at the same location on both maps, one of the equators will be completely hidden behind the other. Experiment with other values here or consult the QuickBASIC programming manual.

Once you have made the necessary changes to MAP.BAS, save the program as a text file. You may want to change the name of the file (e.g., MAP1.BAS) to preserve the original program. Run the program after saving it. Make a printout of your program and hand it in along with assignment.

13. Based on the output of your program, what can you say about the relative stretching or flattening of shapes on the two projections as you move away from the equator?

Optional: Modify the MAP.BAS program to display Tissot’s Indicatrix for the parallels 0, 10, 20 and 30 degrees S. Display the Indicatrix ellipses on the right side of the map. Superimpose the ellipses for the Lambert projection over those for the Mercator projection. Use the same colors for the ellipses that you used for the maps themselves.

For the Lambert projection, the equations for the major and minor axes of the Indicatrix (k and h, respectively) are

\[
\begin{align*}
  k &= \text{SEC}(j) = 1 / \text{COS}(j) \\
  h &= \text{COS}(j)
\end{align*}
\]

where \( j \) is the latitude (in radians). For the Mercator projection, \( k \) and \( h \) are equivalent:

\[
\begin{align*}
  k &= h = \text{SEC}(j) = 1 / \text{COS}(j)
\end{align*}
\]

You will need to use the CIRCLE function to draw the ellipses. The aspect ratio for the function should be defined as \( h/k \) and the "radius" of the ellipse should be defined as \( k/15 \) (the value of 15 is used as a simple scaling factor).

What does the Indicatrix tell you about relative amounts of angular deformation and areal exaggeration for the two projections?

Hand in a printout of your program.
'GCDIST.BAS

This program determines the great circle distance between two points. It assumes a spherical earth with a radius of 6371 kilometers.

radius = 6371  'radius of earth (km)
pi = 3.14159  'pi, the famous constant
degrad = (2 * pi) / 360  'to convert degrees to radians

CLS
DO

'Get latitude and longitude of points and convert to decimal degrees.

INPUT "Enter the latitude of the first point: ", deg, min, sec
lat1 = (deg + (min / 60) + (sec / 3600)) * degrad
INPUT "Enter the longitude of the first point: ", deg, min, sec
lon1 = (deg + (min / 60) + (sec / 3600)) * degrad
PRINT
INPUT "Enter the latitude of the second point: ", deg, min, sec
lat2 = (deg + (min / 60) + (sec / 3600)) * degrad
INPUT "Enter the longitude of the second point: ", deg, min, sec
lon2 = (deg + (min / 60) + (sec / 3600)) * degrad
PRINT

'Compute distance between the two points.

x = (SIN(lat1) * SIN(lat2)) + (COS(lat1) * COS(lat2) * COS(lon1 - lon2))
acosx = ATN(SQR(1 - (x ^ 2)) / x)
dist = radius * acosx

'Print distance.

PRINT "The great circle distance is"; dist; "km."
PRINT

'Get another pair of points?

PRINT "Calculate another distance? [y/n]"
IF INPUT$(1) = "n" THEN EXIT DO
LOOP
END
'LL.BAS

'This program converts UTM coordinates to latitude/longitude.
'The program handles only north latitude and west longitude.

'Dimension arrays.

DIM cmer(61), eprm(2), orgn(2), sphin(5), grco(2), spco(2)

'Enter the UTM zone number for a point.

CLS
INPUT "Enter UTM zone (or a 0 to quit): ", utmzone
IF utmzone = 0 THEN
    STOP
END IF

'Enter coordinates of the point.

INPUT "Enter UTM easting: ", utmx
INPUT "Enter UTM northing: ", utmy

'Some constants...

raddeg = 57.29578
k01 = 0.75
k02 = 0.703125
k03 = 0.68359375#
k04 = 0.67291259765625#
k05 = 0.6661631419939577#
k06 = 0.9375
k07 = 1.025390625#
k08 = 1.07666015625#
k09 = 1.110271903323263#
k10 = 0.234375
k11 = 0.41015625#
k12 = 0.538330078125#
k13 = 0.63446044921875#
k14 = 0.068359375#
k15 = 0.15380859375#
k16 = 0.2379226684570312#
k17 = 0.01922607421875#
k18 = 0.0528717041015625#
k19 = 5.28717041015625D-03
'Compute central meridians of UTM zones.

5000 FOR zone = 1 TO 60
5100 cmer(zone) = (zone * 6 - 180 - 3) / raddeg
5200 NEXT zone

'Constants for converting coordinates from meters.

5300 eprm(1) = 6378206!
5400 eprm(2) = 6356584!
5500 orgn(1) = 0!
5600 orgn(2) = 500000!
5700 scal = 0.9996
5800 grco(2) = utmx
5900 grco(1) = utmy
6000 utmz% = utmzone

'Calculate ellipsoid parameters.

6200 asem = eprm(1)
6300 bsem = eprm(2)
6400 asq = asem * asem
6500 bsq = bsem * bsem
6600 aux1 = asq - bsq
6700 eccsq = aux1 / asq
6800 eps = aux1 / bsq
6900 cprm = bsq / asem
7000 eccp4 = eccsq * eccsq
7100 eccp6 = eccp4 * eccsq
7200 eccp8 = eccp6 * eccsq
7300 eccp10 = eccp8 * eccsq
7400 a = 1! + k01 * eccsq + k02 * eccp4 + k03 * eccp6 + k04 * eccp8 + k05 * eccp10
7500 b = k01 * eccsq + k06 * eccp4 + k07 * eccp6 + k08 * eccp8 + k09 * eccp10
7600 c = k10 * eccp4 + k11 * eccp6 + k12 * eccp8 + k13 * eccp10
7700 d = k14 * eccp6 + k15 * eccp8 + k16 * eccp10
7800 e = k17 * eccp8 + k18 * eccp10
7900 f = k19 * eccp10
8000 gradm1 = 1! / (cprm * a)

'Return to original plane coordinates.

8200 ncco = (grco(1) - orgn(1)) / scal
8300 ecoo = (grco(2) - orgn(2)) / scal
8400 ecoosq = ecoo * ecoo
'Iterate for phipr (latitude of point on central meridian).

8700  phipr = gradm1 * ncoo
8800  cphipr = COS(phipr)
8900  sphipr = SIN(phipr)
9000  aux1 = sphipr
9100  twocos = 2! * cphipr
9200  aux2 = twocos * aux1
9300  sphin(1) = aux2
9400  even = 1
9500  FOR i = 3 TO 10
9600       aux3 = twocos * aux2 - aux1
9700       aux1 = aux2
9800       aux2 = aux3
9900       even = 1 - even
10000      IF even = 1 THEN
10100             sphin(i / 2) = aux3
10200      END IF
10300  NEXT i

10400  'Calculate meridian arc length.

10500  temp = a * phipr - b * sphin(1) / 2! + c * sphin(2) / 4! - d * sphin(3) / 6!
10600  mrdarc = cprm * (temp + e * sphin(4) / 8! - f * sphin(5) / 10!)

10700  'If difference is less than 0.1 mm, stop iterating.

10800  aux1 = ncoo - mrdarc
10900  IF ABS(aux1) < 0.0001 THEN
11000       GOTO 11500
11100  END IF
11200  phipr = phipr + aux1 * gradm1
11300  GOTO 8800

11400  'Calculate phipr- and eprm-dependent values.

11500  tpr = sphipr / cphipr
11600  tprsq = tpr * tpr
11800  etapsq = eps * cphipr * cphipr
11900  npr = asem / SQR(1! - eccsq * sphipr * sphipr)
12000  nprsq = npr * npr
12100  edvn = ecoo / npr
12200  edvnsq = ecoosq / nprsq
'Calculate spheroidal coordinates.

temp = 5! + 3! * tprsq + 6! * etapsq * (1! - tprsq)
temp = temp + edvnsq / 30! * (61! + 90! * tprsq * (1! + .5 * tprsq))
spco(1) = phipr + tpr * edvnsq *.5 * (-1! - etapsq + edvnsq / 12! * temp)
temp = -1 - 2 * tprsq - etapsq + (edvnsq / 20 * (5 + tprsq * (28 + 24 * tprsq)))
spco(2) = cmr(utmz%) + edvn / cphipr * (1! + edvnsq / 6! * temp)

'Calculate latitude and longitude of point.

latitude = ABS(spco(1) * raddeg)
longitude = ABS(spco(2) * raddeg)
deg = INT(latitude)
min = 60 * (latitude - deg)
sec = 60 * (min - INT(min))
PRINT
PRINT "Latitude = "; deg; "deg. "; INT(min); "min. "; INT(sec); "sec."
deg = INT(longitude)
min = 60 * (longitude - deg)
sec = 60 * (min - INT(min))
PRINT "Longitude = "; deg; "deg. "; INT(min); "min. "; INT(sec); "sec."
PRINT

'Return to beginning of program.

GOTO 800

END
'MAP.BAS

'This program draws a map of Africa using Mercator's projection.

'Hard-wire the screen display parameters and dimension arrays.

CLS
SCREEN 12
WINDOW (-1, -1)-(1.6667, 1)
DIM dlong(110), dlat(110), x(110), y(110)

pi = 3.14159 'pi, the famous constant

'REad the 110 longitude and latitude values for Africa.

OPEN "AFRICA.DAT" FOR INPUT AS #1
FOR i = 1 TO 110
   INPUT #1, dlong(i), dlat(i)
NEXT

'Mercator's projection (Step 1). Convert longitude and latitude from
degrees to radians. Calculate x- and y-coordinates for the projection.

FOR i = 1 TO 110
   rlong = dlong(i) * 2 * pi / 360
   rlat = dlat(i) * 2 * pi / 360
   x(i) = rlong
   y(i) = LOG(TAN(pi / 4 + rlat / 2))
NEXT

'Mercator's projection (Step 2). Draw the map in white (color = 15).

FOR i = 1 TO 109
   LINE (x(i), y(i))-(x(i + 1), y(i + 1)), 15
NEXT

'Mercator's projection (Step 3). Draw lines of latitude as dashed lines.

FOR dparallel = -30 TO 30 STEP 10
   rparallel = dparallel * 2 * pi / 360
   parallel = LOG(TAN(pi / 4 + rparallel / 2))
   LINE (-1, parallel)-(1.6667, parallel), 15, 8738
NEXT

END
Assignment 1
Answer Key

1. a) Yes, except for the Australian ones. b) The program cannot handle south latitudes or east longitudes.

2. The latitude and longitude of the two points are approximately the same. D and F are the same point. They have different UTM coordinates because they are in two adjacent UTM zones.

Table 1.

<table>
<thead>
<tr>
<th>Point</th>
<th>UTM Zone</th>
<th>UTM Easting</th>
<th>UTM Northing</th>
<th>Latitude</th>
<th>Longitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>19</td>
<td>416800</td>
<td>4627250</td>
<td>41°47'40&quot;N</td>
<td>70°0'4&quot;W</td>
</tr>
<tr>
<td>B</td>
<td>19</td>
<td>413390</td>
<td>4622340</td>
<td>41°45'0&quot;N</td>
<td>70°2'30&quot;W</td>
</tr>
<tr>
<td>C</td>
<td>19</td>
<td>254470</td>
<td>4672580</td>
<td>42°10'7&quot;N</td>
<td>71°58'20&quot;W</td>
</tr>
<tr>
<td>D</td>
<td>19</td>
<td>252025</td>
<td>4667800</td>
<td>42°7'30&quot;N</td>
<td>71°59'59&quot;W</td>
</tr>
<tr>
<td>E</td>
<td>18</td>
<td>737650</td>
<td>4667450</td>
<td>41°7'30&quot;N</td>
<td>72°7'30&quot;W</td>
</tr>
<tr>
<td>F</td>
<td>18</td>
<td>747975</td>
<td>4667800</td>
<td>42°7'30&quot;N</td>
<td>72°0'0&quot;W</td>
</tr>
<tr>
<td>G</td>
<td>56</td>
<td>330650</td>
<td>6241950</td>
<td>56°17'38&quot;S</td>
<td>150°15'48&quot;E</td>
</tr>
<tr>
<td>H</td>
<td>56</td>
<td>315300</td>
<td>6236050</td>
<td>56°14'7&quot;S</td>
<td>150°1'12&quot;E</td>
</tr>
</tbody>
</table>

Table 2.

<table>
<thead>
<tr>
<th>Pair of points</th>
<th>Great circle distance (km)</th>
<th>Euclidean distance (km)</th>
<th>Difference (km)</th>
<th>Difference as a percentage of Euclidean distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-B</td>
<td>5.820</td>
<td>5.978</td>
<td>0.158</td>
<td>-2.645</td>
</tr>
<tr>
<td>C-D</td>
<td>5.388</td>
<td>5.369</td>
<td>0.019</td>
<td>0.356</td>
</tr>
<tr>
<td>G-H</td>
<td>16.313</td>
<td>16.445</td>
<td>0.131</td>
<td>0.799</td>
</tr>
<tr>
<td>D-F</td>
<td>0</td>
<td>-- different UTM zone</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>C-E</td>
<td>13.560</td>
<td>-- different UTM zone</td>
<td>--</td>
<td>--</td>
</tr>
</tbody>
</table>
3. Conversion of degrees, minutes and seconds to decimal degrees.

4. a) No.
b) \( x = \sin(\text{lat1}) \cdot \sin(\text{lat2}) + \cos(\text{lat1}) \cdot \cos(\text{lat2}) \cdot \cos(\text{lon1} - \text{lon2}) \)
c) To make the program easier to read.

5. They would also increase by 1 percent. (See line 2800).

6. a) The program responds with "redo from start" and then prompts for new values.  b) The program computes and prints a value for distance, even though the value is invalid.

7. The degree and sign of the error varies over each UTM zone. The lowest error occurs along the standard lines parallel to the central meridian, and increases towards the central meridian and the edges of the zone.

8. a) They designate double precision variables.  b) They are used when high precision is required.

9. a) They designate single precision variables.  b) They are used to save memory or increase the speed of computation.

10. \( \text{cmer}(22) = -0.890 \)
    \( \text{cmer}(50) = 2.042 \)

11. The function returns the largest integer less than or equal to the argument. It is used here to convert decimal degrees to degrees, minutes and seconds.

12. a) Lines of latitude are not evenly spaced on Mercator's projection.
    b) They get farther apart as you move away from the equator.

13. The Mercator projection appears to stretch shapes more, while the Lambert projection seems to squash them. (This is only true in a relative sense, since the Mercator projection is actually conformal.)

Optional: Tissot's Indicatrix shows that angular deformation for the Lambert projection increases as you move away from the equator. However, as the ellipses always have the same area, there is no areal exaggeration (equal-area). For the Mercator projection the ellipses are always circular, indicating that there is no angular deformation (conformal). However, areal exaggeration is apparent since the ellipses get larger as you move away from the equator.
Assignment 1
Program Listing (Answer Key)

100 'EUCLID1.BAS

200 'This program determines the Euclidean distance between
300 'two points based on their UTM x- and y-coordinates.
400 'It assumes that the points are in the same UTM zone.
500 'Lines 1900, 2100 and 2300 have been modified. Line 2400 has been added.

550 CLS
600 DO

800 'Get data for a pair of points.

1000 INPUT "Enter UTM zone of first point: ", zone1
1100 INPUT "Enter UTM easting of first point: ", x1
1200 INPUT "Enter UTM northing of first point: ", y1
1300 PRINT
1400 INPUT "Enter UTM zone of second point: ", zone2
1500 INPUT "Enter UTM easting of second point: ", x2
1600 INPUT "Enter UTM northing of second point: ", y2
1700 PRINT

1750 'If points are in different zones, don't compute distance.

1800 IF zone1 <> zone2 THEN
1900 PRINT "Those points are in different zones."
2000 PRINT
2100 GOTO 2700
2200 END IF

2250 'Compute and print distance.

2300 dist = SQR(((x1 - x2)^2) + ((y1 - y2)^2))
2400 dist = dist / 1000
2500 PRINT "The Euclidean distance is"; dist; "km."
2600 PRINT

2650 'Get data for another pair of points?

2700 PRINT "Calculate another distance? [y/n]"
2800 IF INPUT$ (1) = "n" THEN EXIT DO

2900 LOOP
3000 END
'MAP1.BAS

'This program draws a map of Africa using Mercator's projection.
'Over this it superimposes a second map using Lambert's
'cylindrical equal-area projection.

'Hard-wire the screen display parameters and dimension arrays.

CLS
SCREEN 12
WINDOW (-1, -1)-(1.6667, 1)
DIM dlong(110), dlat(110), x(110), y(110)

pi = 3.14159
'pi, the famous constant

'REad the 110 longitude and latitude values for Africa.

OPEN "AFRICA.DAT" FOR INPUT AS #1
FOR i = 1 TO 110
  INPUT #1, dlong(i), dlat(i)
NEXT

'Mercator's projection (Step 1). Convert longitude and latitude from
degrees to radians. Calculate x- and y-coordinates for the projection.

FOR i = 1 TO 110
  rlong = dlong(i) * 2 * pi / 360
  rlat = dlat(i) * 2 * pi / 360
  x(i) = rlong
  y(i) = LOG(TAN(pi / 4 + rlat / 2))
NEXT

'Mercator's projection (Step 2). Draw the map in white (color = 15).

FOR i = 1 TO 109
  LINE (x(i), y(i))-(x(i + 1), y(i + 1)), 15
NEXT

'Mercator's projection (Step 3). Draw lines of latitude as dashed lines.

FOR dparallel = -30 TO 30 STEP 10
  rparallel = dparallel * 2 * pi / 360
  parallel = LOG(TAN(pi / 4 + rparallel / 2))
  LINE (-1, parallel)-(1.6667, parallel), 15, 8738
NEXT
'Lambert's projection (Step 1). Convert longitude and latitude from degrees to radians. Calculate x- and y-coordinates for the projection.

7000 FOR i = 1 TO 110
7100 rlong = dlong(i) * 2 * pi / 360
7200 rlat = dlat(i) * 2 * pi / 360
7300 x(i) = rlong
7400 y(i) = SIN(rlat)
7500 NEXT

9000 'Lambert's projection (Step 2).
9100 'Draw the map in red (color = 12) or some other color.

9200 FOR i = 1 TO 109
9300 LINE (x(i), y(i))-(x(i + 1), y(i + 1)), 12
9400 NEXT

10000 'Lambert's projection (Step 3). Draw lines of latitude as dashed lines.
10100 'Change the value at the end of the LINE statement to get a different dashed line than that used for the Mercator projection.

10300 FOR dparallel = -30 TO 30 STEP 10
10400 rparallel = dparallel * 2 * pi / 360
10500 parallel = SIN(rparallel)
10600 LINE (-1, parallel)-(1.6667, parallel), 12, , 21845
10700 NEXT

20000 END
'MAP2.BAS

'This program draws a map of Africa using Mercator's projection.
'Over this it superimposes a second map using Lambert's
cylindrical equal-area projection.
'Next it draws the ellipses for Tissot's Indicatrix for the two
'projections (in appropriate colors) for latitudes 0, 10, 20 and 30 S.

'Hard-wire the screen display parameters and dimension arrays.

CLS
SCREEN 12
WINDOW (-1, -1)-(1.6667, 1)
DIM dlong(110), dlat(110), x(110), y(110)
pi = 3.14159 'pi, the famous constant

'REad the 110 longitude and latitude values for Africa.

OPEN "AFRICA.DAT" FOR INPUT AS #1
FOR i = 1 TO 110
  INPUT #1, dlong(i), dlat(i)
NEXT

'Mercator's projection (Step 1). Convert longitude and latitude from
degrees to radians. Calculate x- and y-coordinates for the projection.

FOR i = 1 TO 110
  rlong = dlong(i) * 2 * pi / 360
  rlat = dlat(i) * 2 * pi / 360
  x(i) = rlong
  y(i) = LOG(TAN(pi / 4 + rlat / 2))
NEXT

'Mercator's projection (Step 2). Draw the map in white (color = 15).

FOR i = 1 TO 109
  LINE (x(i), y(i))-(x(i + 1), y(i + 1)), 15
NEXT
Mercator's projection (Step 3). Draw lines of latitude as dashed lines.

FOR dparallel = -30 TO 30 STEP 10
  rparallel = dparallel * 2 * pi / 360
  parallel = LOG(TAN(pi / 4 + rparallel / 2))
  LINE (-1, parallel)-(1.6667, parallel), 15, , 8738
NEXT

Mercator's projection (Step 4). Calculate Indicatrix axes
for the required parallels and then plot the ellipses.

Note that radius of circle must be scaled (in this case, divided by 15) to fit on the screen. Note that we use k (not h)
as the so-called radius of the ellipse since the CIRCLE function
will use this as the x-radius if aspect is less 1.

FOR dparallel = -30 TO 0 STEP 10
  rparallel = dparallel * 2 * pi / 360
  parallel = LOG(TAN(pi / 4 + rparallel / 2))
  h = 1 / COS(rparallel)
  k = h
  aspect = h / k
  CIRCLE (1.3333, parallel), (k / 15), 15, , , aspect
NEXT

Lambert's projection (Step 1). Convert longitude and latitude from
degrees to radians. Calculate x- and y-coordinates for the projection.

FOR i = 1 TO 110
  rlong = dlong(i) * 2 * pi / 360
  rlat = dlat(i) * 2 * pi / 360
  x(i) = rlong
  y(i) = SIN(rlat)
NEXT

Lambert's projection (Step 2).

Draw the map in red (color = 12) or some other color.

FOR i = 1 TO 109
  LINE (x(i), y(i))-(x(i + 1), y(i + 1)), 12
NEXT
10000 'Lambert's projection (Step 3). Draw lines of latitude as dashed lines.
10100 'Change the value at the end of the LINE statement to get a different
dashed line than that used for the Mercator projection.

10300 FOR dparallel = -30 TO 30 STEP 10
10400     rparallel = dparallel * 2 * pi / 360
10500     parallel = SIN(rparallel)
10600     LINE (-1, parallel)-(1.6667, parallel), 12, , 21845
10700 NEXT

10705 'Lambert's projection (Step 4). Calculate Indicatrix axes
10710 'for the required parallels and then plot the ellipses.

10750 FOR dparallel = -30 TO 0 STEP 10
10760     rparallel = dparallel * 2 * pi / 360
10770     parallel = SIN(rparallel)
10780     h = COS(rparallel)
10790     k = 1 / h
10800     aspect = h / k
10810     CIRCLE (1.3333, parallel), (k / 15), 12, , , aspect
10820 NEXT

20000 END
Assignment 2
Vector Data Structures (I)

Objectives: This assignment focuses on the manipulation of vector data. You will be using QuickBASIC to compute polygon areas, perform point-in-polygon tests and locate line intersections.

Files: This assignment requires four QuickBASIC programs (PIP.BAS, DARTS.BAS, INTER.BAS and FRACTAL.BAS) and two data files containing the vector representation of polygons (POLYGON.DAT and POLYGON2.DAT).

Point-in-Polygon Test: The program called PIP.BAS contains a point-in-polygon algorithm that determines whether a specified point falls within a given polygon. Before you can run this program you will have to make the following additions between lines 100 and 5000:

a) Use the CLS statement to clear the screen.
b) Open the polygon data file for reading. The statement to use is:

   OPEN "POLYGON.DAT" FOR INPUT AS #1

c) This data file defines a polygon as a set of x,y-coordinate pairs which, when joined by straight-line segments, describe a closed geometric figure. In order to read in the coordinates, you will first need to know the number of coordinates in the file. This number is given in the first line of the file. Read in this number and assign it to a variable called n. The statement to use is:

   INPUT #1, n

d) Use a DIM statement to dimension two arrays (called x and y) to store the x,y-coordinate pairs. The dimension of these arrays should be n+1 to allow the program to close the polygon (see step f).

e) Read in the x,y-coordinates from the data file and assign them to the arrays called x and y. Use the following "for loop" so that the subscript for these arrays (as defined by variable i) is automatically increased (or "incremented") by a value of 1 each time a new coordinate pair is read in.

   FOR i = 1 to n
       INPUT #1, x(i), y(i)
   NEXT

f) Close the polygon by making the last coordinate pair the same as the first, as follows:

   x(n+1) = x(1)
   y(n+1) = y(1)

g) Include a line containing a DO statement. This will begin a "do loop" that will be used when prompting the user to enter the x,y-coordinates of a point from the keyboard.

h) Read in the x,y-coordinates of a point from the keyboard. This will be the point for which the point-in-polygon test will be performed. Use a statement something like the following:
INPUT "Enter x-coordinate: ", xpt

This will read in the x-coordinate of the point and assign it to a variable called xpt. Include a similar line to read the y-coordinate and assign it to a variable called ypt.

Lines 5000 through 7700 perform the point-in-polygon test and print the results out to the screen. After line 7700, you will need to include a few more lines.

i) Include a mechanism for performing the point-in-polygon test on another point, should the user want to do that. Use a PRINT statement to print a message asking the user to enter an n (for "no") if another test is not desired. Then include the line:

IF INPUT$(1) = "n" THEN EXIT DO

This will cause the program to exit the do loop should the user enter an n. If any other character is entered, the program will return to the DO statement (see step g).

j) The last two lines of the program should contain LOOP and END statements, respectively. The first of these two lines ends the do loop and the second ends the program.

Now that you have made these modifications, save the program. Run it to determine whether each of the points listed in Table 1 is inside or outside of the polygon. Once the program is running to your satisfaction, make a printout to hand in along with the assignment.

Table 1.

<table>
<thead>
<tr>
<th>Point</th>
<th>x-coordinate</th>
<th>y-coordinate</th>
<th>Inside or outside?</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>25</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>101</td>
<td>299.7</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>631</td>
<td>246</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>387.5</td>
<td>224</td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>97</td>
<td>401</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>544.1</td>
<td>77</td>
<td></td>
</tr>
<tr>
<td>G</td>
<td>321.3</td>
<td>314</td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>111</td>
<td>49</td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>251</td>
<td>327</td>
<td></td>
</tr>
<tr>
<td>J</td>
<td>118</td>
<td>176</td>
<td></td>
</tr>
</tbody>
</table>

Area Estimation: A common method of estimating the area of a polygon is to overlay a grid of dots with a known average density and count the number of "hits" (i.e., the number of dots falling within the polygon). The area of the polygon is estimated as the number of hits divided by the average dot density. The area estimate improves as dot density increases.

The program called DARTS.BAS calculates the area of a polygon based on this approach. Open this program and examine it. Note that it reads in the same polygon data you used for the PIP.BAS program.
Lines 340 and 345 of the program are used to generate a point with random x,y-coordinates. The x-coordinate ranges from 0 to 639, and the y-coordinate ranges from 0 to 462. These values reflect the approximate size of the screen, measured in pixels. Lines 510 through 670 should look familiar. This is the same point-in-polygon algorithm used in PIP.BAS. The algorithm is used to determine whether the randomly-generated point is inside or outside the polygon. If it is outside the polygon, the program draws the point as a grey dot (line 820). If it is inside the polygon, the program draws the point as a red dot (line 930). This process is repeated until the user presses a key to temporarily suspend program execution (lines 2000 to the end of the program). After several thousand "trials" (one trial is equal to one random point), the program should give a fairly accurate estimate of polygon area.

Lines 1100 through 1170 are used to print out several statistics -- the number of trials, the number of hits, the number of hits as a percentage of the number of trials, and the estimate of polygon area. These statistics are printed out on the lower left of the screen. Run the program and monitor the statistics printed on the screen, with the goal of filling in Table 2. Press any key to make the program pause temporarily. Perform as many trials as you need in order to identify the shape of the polygon appearing in red on the screen.

1. a) What is the shape of the polygon? b) How many trials are needed to identify this shape?

On a piece of graph paper, graph the estimate of polygon area as a function of the number of trials.

2. a) Does the area estimate appear to be stabilizing as the number of trials increases? b) What is the best estimate of the area of the polygon?

Table 2.

<table>
<thead>
<tr>
<th>Number of trials</th>
<th>Number of hits</th>
<th>Number of hits as a percentage of number of trials</th>
<th>Polygon area (in pixels)</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1500</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>2500</td>
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<td></td>
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<tr>
<td>3000</td>
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<td></td>
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<tr>
<td>3500</td>
<td></td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

Other Area Estimates: The area of a polygon can also be calculated from the area of a set of trapezoids defined by the x,y-coordinate pairs of the polygon. Write a QuickBASIC program that implements this calculation. This program will be very similar to of PIP.BAS. Your program should:

a) clear the screen;

b) open the polygon data file called POLYGON.DAT for reading;

c) read in the number on the first line of the data file (the number of x,y-coordinate pairs) and assign it to a variable called n;
d) dimension two arrays (called x and y) for storing the x,y-coordinates from the file (the dimension of the arrays should be n+1);

e) read in the x- and y-coordinates from the file and store these in the two arrays using a for loop;

f) close the polygon;

g) initialize a variable called area (the polygon area estimate) to zero;

h) use the following FOR loop to calculate the area of the polygon

\[
\text{FOR } i = 1 \text{ to } n \\
\quad \text{area} = \text{area} + (x(i+1) - x(i)) \times (y(i+1) + y(i)) / 2 \\
\text{NEXT}
\]

i) print out the area of the polygon.

Be sure to include comments describing the function of each program section. Run the program to obtain the polygon area estimate. Make a printout of the program to hand in along with the assignment.

3. What is the area estimate you obtained with your program?

On the graph you constructed previously, draw a horizontal line representing the area estimate obtained with your program. Hand in this graph along with your assignment.

4. How does the area estimate you obtained with your program compare to the estimates you obtained with DARTS.BAS?

Modify the program to read data from POLYGON2.DAT rather than POLYGON.DAT. These two data files are identical except that the coordinate pairs in POLYGON.DAT are arranged in clockwise order, while those in POLYGON2.DAT are in counter-clockwise order.

5. a) What is the area estimate you obtained for POLYGON2.DAT? b) How does this estimate compare to that obtained for POLYGON.DAT?

Line Intersection: The program called INTER.BAS finds the intersection of two straight line segments. Run the program to compute the point of intersection for the examples listed in Table 3.
Table 3.

<table>
<thead>
<tr>
<th>Example</th>
<th>x</th>
<th>y</th>
<th>x</th>
<th>y</th>
<th>x</th>
<th>y</th>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>100</td>
<td>100</td>
<td>300</td>
<td>300</td>
<td>300</td>
<td>100</td>
<td>100</td>
<td>300</td>
</tr>
<tr>
<td>B</td>
<td>400</td>
<td>100</td>
<td>400</td>
<td>300</td>
<td>10</td>
<td>250</td>
<td>600</td>
<td>250</td>
</tr>
<tr>
<td>C</td>
<td>400</td>
<td>100</td>
<td>400</td>
<td>300</td>
<td>400</td>
<td>150</td>
<td>500</td>
<td>150</td>
</tr>
<tr>
<td>D</td>
<td>400</td>
<td>100</td>
<td>400</td>
<td>300</td>
<td>450</td>
<td>150</td>
<td>500</td>
<td>150</td>
</tr>
<tr>
<td>E</td>
<td>100</td>
<td>100</td>
<td>300</td>
<td>300</td>
<td>200</td>
<td>200</td>
<td>400</td>
<td>400</td>
</tr>
<tr>
<td>F</td>
<td>400</td>
<td>100</td>
<td>200</td>
<td>200</td>
<td>300</td>
<td>150</td>
<td>250</td>
<td>175</td>
</tr>
<tr>
<td>G</td>
<td>100</td>
<td>100</td>
<td>300</td>
<td>300</td>
<td>300</td>
<td>300</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

6. Examples E, F and G represent a special case of intersection that the program cannot handle. a) Explain what this special case is. b) Explain how E, F and G are each slightly different examples of this special case.

Now open the program called FRACTAL.BAS. You will be using this program to find intersection points for lines composed of multiple straight-line segments. The program first draws a straight line (in blue) across the screen. Then it draws a wiggly line (in green) using the fractal concept. The coordinates of the fractal line are determined randomly, so each time you run the program a different line will be drawn.

Note that the end points of the straight and fractal lines are the same. Also note that the arrays for storing the x,y-coordinates for the fractal line (i.e., x2 and y2) are now dimensioned at 65. This is because the fractal line is composed of 64 individual straight-line segments (65 coordinate pairs).

The variable called w (line 1000) defines the "wiggliness" of the fractal line. Run the program with different values of w and observe how the line changes.

7. a) What is the effect of increasing the value of w? b) What is the effect of decreasing w? c) What happens when w is zero?

Modify the program so that it calculates and displays the intersections between the straight and fractal lines. The easiest way to do this is to insert a modified version of the line intersection algorithm between lines 4100 and 20000 of the program. To do this, open INTER.BAS, highlight lines 4500 to 11200, and select the Copy option from the Edit menu. Now open FRACTAL.BAS, click on line 20000, and select the Paste option from the Edit menu. This procedure will copy the line intersection algorithm from INTER.BAS into FRACTAL.BAS.

Now make the following modifications to the algorithm:

a) Change all occurrences of x2(1) to x2(k), and all occurrences of x2(2) to x2(k+1). Likewise, change all occurrences of y2(1) to y2(k), and all occurrences of y2(2) to y2(k+1).

b) Replace line 10500 with the following:

CIRCLE ( xi, yi ), 5, 15

This will draw a white circle centered on each intersection point.
c) Delete the PRINT statements in lines 10200 and 11100, and the GOTO statement on line 10600. Do a bit of cleaning up between lines 10100 and 11200, since some of the IF statements are no longer required now that the associated PRINT statements have been deleted.

Save the program and run it several times to generate different fractal lines. Hand in a printout of the program along with your assignment.

8. Explain the rationale for modification a, above.

9. a) Does the program always manage to identify the intersections between the straight line and the fractal line? b) If not, can you explain why the program might be missing some intersections? c) Can you suggest how the program might be modified to fix this problem?
Assignment 2
Program Listing

100   'PIP.BAS

5000  'Perform point-in-polygon test.

5100  in = 1
5200  FOR i = 1 TO n
5300     IF x(i + 1) <> x(i) THEN
5400         IF (x(i + 1) - xpt) * (xpt - x(i)) >= 0 THEN
5500             IF x(i + 1) <> xpt OR x(i) >= xpt THEN
5600                 IF x(i) <> xpt OR x(i + 1) >= xpt THEN
5700                   b = (y(i + 1) - y(i)) / (x(i + 1) - x(i))
5800                   a = y(i) - b * x(i)
5900                   yi = a + b * xpt
6000                 IF yi > ypt THEN
6100                   in = in * -1
6200             END IF
6300         END IF
6400     END IF
6500  END IF
6600  END IF
6700  NEXT

7000  'Print results.

7100  PRINT
7200  IF in = -1 THEN
7300      PRINT "That point is INSIDE the polygon."
7400  ELSE
7500      PRINT "That point is OUTSIDE the polygon."
7600  END IF
7700  PRINT
'DARTS.BAS

'This program estimates the area of a polygon based on the
'probability of a randomly-generated point falling within it.

CLS
SCREEN 12

'Open the data file for reading.
OPEN "POLYGON.DAT" FOR INPUT AS #1

'Read in the number of points defining the outline of the polygon.
INPUT #1, n

'Dimension the arrays for the x,y-coordinates of the points.
'The dimension should be n+1 to allow the polygon to be closed.
DIM x(n + 1), y(n + 1)

'Read in the x,y-coordinates of the points.
FOR i = 1 TO n
    INPUT #1, x(i), y(i)
NEXT

'Close the polygon.
x(n + 1) = x(1)
y(n + 1) = y(1)

'Draw a rectangle on the screen. (The dimensions of the box define
'the maximum and minimum coordinates of the random points.)
LINE (0, 0)-(639, 462), 7, B

'num = the number of trials; nin is the number of hits.
'Random generation of points is based on internal clock.

num = 0
nin = 0
RANDOMIZE (TIMER)
LOCATE 30, 1
PRINT "Hit any key to pause. " ;
'Generate random x,y-coordinates for a point in the rectangle.'

xpt = RND * 639
ypt = RND * 462

'Increase the number of trials by one.'

num = num + 1

'Statistics to print on screen...'

LOCATE 25, 2
PRINT "Trials: ";
LOCATE 26, 2
PRINT "Hits: ";
LOCATE 27, 2
PRINT "Percent: ";
LOCATE 28, 2
PRINT "Area: ";

'Point-in-polygon algorithm determines whether random point is in the polygon (a hit) or not.'

in = 1
FOR i = 1 TO n
    IF x(i + 1) <> x(i) THEN
        IF (x(i + 1) - xpt) * (xpt - x(i)) >= 0 THEN
            IF x(i + 1) <> xpt OR x(i) >= xpt THEN
                IF x(i) <> xpt OR x(i + 1) >= xpt THEN
                    b = (y(i + 1) - y(i)) / (x(i + 1) - x(i))
                    a = y(i) - b * x(i)
                    yi = a + b * xpt
                    IF yi > ypt THEN
                        in = in * -1
                    END IF
                END IF
            END IF
        END IF
    END IF
END IF
END IF
END IF
END IF
NEXT

'If the point is NOT in the polygon, draw a grey dot.'

IF in = 1 THEN
    PSET (xpt, ypt), 7
END IF
'If the point IS in the polygon, draw a red dot
'and increase the number of hits (nin) by 1.

IF in = -1 THEN
  PSET(xpt, ypt), 12
  nin = nin + 1
END IF

'Print out statistics.

LOCATE 25, 11
PRINT num;
LOCATE 26, 11
PRINT nin;
LOCATE 27, 11
PRINT nin / num * 100;
LOCATE 28, 11
PRINT nin / num * 640 * 463;

'Monitor keyboard events to suspend or terminate program.
'If no key is pressed, get another random point...

a$ = INKEY$
IF LEN(a$) = 0 THEN 340

...otherwise, pause.

LOCATE 30, 1
PRINT "Hit q to quit or any other key to resume."

'If no other key is pressed, keep waiting...

a$ = INKEY$
IF LEN(a$) = 0 THEN 2700

...or, if a q is not pressed, get another random point...

IF a$ <> "q" THEN 320

...otherwise, terminate the program.

END
'INTER.BAS

'This program finds the point of intersection
'for two straight-line segments.

'Dimension the arrays to store the x,y-coordinates of the end
'points of the two lines. Since we are dealing with straight line
'segments, the arrays only need to contain two elements.
'The arrays called x1 and y1 store the x,y-coordinates
'for the end points of the first line segment.
'The arrays called x2 and y2 store the x,y-coordinates
'for the end points of the second line segment.

DIM x1(2), y1(2)
DIM x2(2), y2(2)

'Begin loop to read in coordinates of end points.

CLS
DO

PRINT
PRINT "Enter the x- and y-coordinates of the end points of each line."
PRINT "Separate the coordinates with a comma."
PRINT
PRINT "First end point of first line: ", x1(1), y1(1)
PRINT "Second end point of first line: ", x1(2), y1(2)
PRINT "First end point of second line: ", x2(1), y2(1)
PRINT "Second end point of second line: ", x2(2), y2(2)
PRINT

'Initialize flag that will tell us if intersection occurs.
intersect = 1

'Find intersection point. Variables xi and yi
'define x,y-coordinates of the intersection point.
IF x1(1) <> x1(2) THEN
    b1 = (y1(2) - y1(1)) / (x1(2) - x1(1))
ELSEIF x2(1) <> x2(2) THEN
    b2 = (y2(2) - y2(1)) / (x2(2) - x2(1))
    a1 = y1(1) - b1 * x1(1)
    a2 = y2(1) - b2 * x2(1)
    IF b1 = b2 THEN
        intersect = 0
ELSE
   \( xi = -1 \times (a1 - a2) / (b1 - b2) \)
   \( yi = a1 + b1 \times xi \)
END IF
ELSE
   \( xi = x2(1) \)
   \( a1 = y1(1) - b1 \times x1(1) \)
   \( yi = a1 + b1 \times xi \)
END IF
ELSE
   \( xi = x1(1) \)
   IF \( x2(1) <> x2(2) \) THEN
      \( b2 = (y2(2) - y2(1)) / (x2(2) - x2(1)) \)
   \( a2 = y2(1) - b2 \times x2(1) \)
   \( yi = a2 + b2 \times xi \)
ELSE
   intersect = 0
END IF
END IF

'Print results.'

IF intersect = 0 THEN
   PRINT "Lines do not intersect."
ELSE
   IF \( (x1(1) - xi) \times (xi - x1(2)) >= 0 \) THEN
      IF \( (x2(1) - xi) \times (xi - x2(2)) >= 0 \) THEN
         IF \( (y1(1) - yi) \times (yi - y1(2)) >= 0 \) THEN
            IF \( (y2(1) - yi) \times (yi - y2(2)) >= 0 \) THEN
               PRINT "Lines intersect at ("; xi; "; yi; ")"
               GOTO 14100
            END IF
         END IF
      END IF
   END IF
   PRINT "Lines do not intersect."
END IF

'Find intersection for another pair of line segments?'

PRINT "Find intersection for another pair of line segments? [y/n]"
IF INPUTS(1) = "n" THEN EXIT DO

LOOP

END
'FRACTAL.BAS

'This program draws a straight blue line
'and a wiggly green "fractal" line.

CLS
SCREEN 12

'Dimension the arrays x1 and y1 for storing the end points of the
'straight line (hence dimension of 2). Arrays x2 and y2 are for
'the end points of the fractal line. The dimension in this case is 65,
'as we want 64 segments in the fractal line. (Note that we ignore
'the 0th element of all arrays to avoid confusion.)

DIM x1(2), y1(2)
DIM x2(65), y2(65)

'Random number generation is based on internal clock.
RANDOMIZE (TIMER)

'Variable called w determines wiggliness of the fractal line.
w = 200

'Define coordinates of end points of straight line.
x1(1) = 20
y1(1) = 250
x1(2) = 620
y1(2) = 250

'Draw the line in blue.
LINE (x1(1), y1(1))-(x1(2), y1(2)), 1

'End points of fractal line are the same as those of straight line.
x2(1) = 20
y2(1) = 250
x2(65) = 620
y2(65) = 250
1990 'Generate fractal line.

2000 FOR k = 1 TO 6
2100   ka = 64 / 2 ^ k
2200   FOR j = (ka + 1) TO 65 STEP (2 * ka)
2300       x2(j) = (x2(j - ka) + x2(j + ka)) / 2
2310       x2(j) = x2(j) + (1 - 2 * RND) * w / 2 ^ k
2400       y2(j) = (y2(j - ka) + y2(j + ka)) / 2
2410       y2(j) = y2(j) + (1 - 2 * RND) * w / 2 ^ k
2500     NEXT
2600 NEXT

3990 'Draw fractal line in green.

4000 FOR k = 1 TO 64
4100       LINE (x2(k), y2(k))-(x2(k + 1), y2(k + 1)), 2
20000 NEXT

30000 END
Assignment 2
Answer Key

Table 1.

<table>
<thead>
<tr>
<th>Point</th>
<th>x-coordinate</th>
<th>y-coordinate</th>
<th>Inside or outside?</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>25</td>
<td>100</td>
<td>out</td>
</tr>
<tr>
<td>B</td>
<td>101</td>
<td>299.7</td>
<td>out</td>
</tr>
<tr>
<td>C</td>
<td>631</td>
<td>246</td>
<td>out</td>
</tr>
<tr>
<td>D</td>
<td>387.5</td>
<td>224</td>
<td>in</td>
</tr>
<tr>
<td>E</td>
<td>97</td>
<td>401</td>
<td>out</td>
</tr>
<tr>
<td>F</td>
<td>544.1</td>
<td>77</td>
<td>out</td>
</tr>
<tr>
<td>G</td>
<td>321.3</td>
<td>314</td>
<td>in</td>
</tr>
<tr>
<td>H</td>
<td>111</td>
<td>49</td>
<td>out</td>
</tr>
<tr>
<td>I</td>
<td>251</td>
<td>327</td>
<td>in</td>
</tr>
<tr>
<td>J</td>
<td>118</td>
<td>176</td>
<td>in</td>
</tr>
</tbody>
</table>

1. a) A maple leaf. b) Several thousand at minimum.

2. a) Depends on the maximum number of trials performed. b) The best estimate is the one for the largest number of trials.

3. Estimate is 90,919.13 pixels.

4. Estimate is very similar to the previous ones.

5. a) Estimate is -90,919.13 pixels. b) Negative of other estimate.

6. a) They are all instances of coincident lines. b) In E, the lines share a portion of the same line. In F, one line is a subset of the other line. In G, the lines are identical.

7. a) Increasing w makes the line more wiggly. b) Decreasing w makes the line less wiggly. c) When w is zero, the line is straight.
8. The program needs to find the intersections between the straight line and all 64 pieces of the fractal line. Each piece is referenced with a value of k, which is incremented in the loop.

9. 
   a) No. 
   b) Rounding error means that line 10400 sometimes evaluates as false when it is true. 
   c) Change the zeros in line 10400 to a smaller value to implement a fuzzy tolerance. (A value of -1 seems to work.)

<table>
<thead>
<tr>
<th>Number of trials</th>
<th>Number of hits</th>
<th>Number of hits as a percentage of number of trials</th>
<th>Polygon area estimate (in pixels)</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>137</td>
<td>27.35</td>
<td>81 029.62</td>
</tr>
<tr>
<td>1000</td>
<td>312</td>
<td>31.08</td>
<td>92 083.51</td>
</tr>
<tr>
<td>1500</td>
<td>468</td>
<td>30.97</td>
<td>91 778.80</td>
</tr>
<tr>
<td>2000</td>
<td>628</td>
<td>31.42</td>
<td>93 091.02</td>
</tr>
<tr>
<td>2500</td>
<td>785</td>
<td>31.38</td>
<td>92 858.77</td>
</tr>
<tr>
<td>3000</td>
<td>945</td>
<td>31.49</td>
<td>93 309.70</td>
</tr>
<tr>
<td>3500</td>
<td>1100</td>
<td>31.69</td>
<td>93 888.35</td>
</tr>
<tr>
<td>Example</td>
<td>First end point</td>
<td>Second end point</td>
<td>First end point</td>
</tr>
<tr>
<td>---------</td>
<td>----------------</td>
<td>------------------</td>
<td>----------------</td>
</tr>
<tr>
<td>A</td>
<td>100 100</td>
<td>300 300</td>
<td>300 100</td>
</tr>
<tr>
<td>B</td>
<td>400 100</td>
<td>400 300</td>
<td>10 250</td>
</tr>
<tr>
<td>C</td>
<td>400 100</td>
<td>400 300</td>
<td>400 150</td>
</tr>
<tr>
<td>D</td>
<td>400 100</td>
<td>400 300</td>
<td>450 150</td>
</tr>
<tr>
<td>E</td>
<td>100 100</td>
<td>300 300</td>
<td>200 200</td>
</tr>
<tr>
<td>F</td>
<td>400 100</td>
<td>200 200</td>
<td>300 150</td>
</tr>
<tr>
<td>G</td>
<td>100 100</td>
<td>300 300</td>
<td>300 300</td>
</tr>
</tbody>
</table>
Assignment 2
Program Listing (Answer Key)

100 'PIP1.BAS
200 'Point-in-polygon program.
300 CLS
1000 'Open the data file for reading.
1100 OPEN "POLYGON.DAT" FOR INPUT AS #1
1200 'Read in the number of points in the polygon.
1300 '(This is the first line of POLYGON.DAT).
1400 INPUT #1, n
2000 'Dimension the arrays for storing the x,y-coordinates
2100 'of the polygon. Dimension should be one greater than n
2200 'to allow the program to close the polygon.
2300 DIM x(n + 1), y(n + 1)
3100 'Read in the x,y-coordinates for the n points.
3200 FOR i = 1 TO n
3300 INPUT #1, x(i), y(i)
3400 NEXT
3500 'Close the polygon.
3600 x(n + 1) = x(1)
3700 y(n + 1) = y(1)
4000 'Begin loop for reading in x- and y-coordinates
4100 'of points and performing the test.
4200 DO
4300 'Read in the x- and y-coordinates of the point to test.
4400 INPUT "Enter x-coordinate of the point: ", xpt
4500 INPUT "Enter y-coordinate of the point: ", ypt
'Perform point-in-polygon test.

in = 1
FOR i = 1 TO n
    IF x(i + 1) <> x(i) THEN
        IF (x(i + 1) - xpt) * (xpt - x(i)) >= 0 THEN
            IF x(i) <> xpt OR x(i) >= xpt THEN
                b = (y(i + 1) - y(i)) / (x(i + 1) - x(i))
                a = y(i) - b * x(i)
                yi = a + b * xpt
                IF yi > ypt THEN
                    in = in * -1
                END IF
            END IF
        END IF
    END IF
END IF
END IF
END IF
END IF
NEXT

'Print results.

PRINT
IF in = -1 THEN
    PRINT "That point is INSIDE the polygon."
ELSE
    PRINT "That point is OUTSIDE the polygon."
END IF
PRINT

'Perform point in polygon test for another point?

PRINT "Perform the test on another point? [y/n]"
IF INPUT$(1) = "n" THEN EXIT DO

LOOP

END
'AREA.BAS

'Program to compute area of a polygon based on trapezoid method.

CLS

'Open the data file for reading.
OPEN "POLYGON.DAT" FOR INPUT AS #1

'Read in the number of points in the polygon.
'(This is the first line of POLYGON.DAT).

INPUT #1, n

'Dimension the arrays for storing the x,y-coordinates of the polygon. Dimension should be one greater than n to allow the program to close the polygon.

DIM x(n + 1), y(n + 1)

'REad in the x,y-coordinates for the n points.

FOR i = 1 TO n
    INPUT #1, x(i), y(i)
NEXT

'Close the polygon.

x(n + 1) = x(1)
y(n + 1) = y(1)

'Initialize area to zero.

area = 0

'Calculate and print out the area of the polygon.

FOR i = 1 to n
    area = area + (x(i + 1) - x(i)) * (y(i + 1) + y(i)) / 2
NEXT

PRINT "The polygon has an area of ", area, "pixels."

END
'FRACTAL1.BAS

200 'This program draws a straight blue line
300 'and a wiggly green "fractal" line.
400 'Circles are drawn at points where the two lines intersect.

500 CLS
600 SCREEN 12

700 'Dimension the arrays x1 and y1 for storing the end points of the
720 'straight line (hence dimension of 2). Arrays x2 and y2 are for
730 'the end points of the fractal line. The dimension in this case is 65,
740 'as we want 64 segments in the fractal line. (Note that we ignore
750 'the 0th element of all arrays to avoid confusion.)

800 DIM x1(2), y1(2)
810 DIM x2(65), y2(65)

890 'Random number generation is based on internal clock.

900 RANDOMIZE (TIMER)

990 'Variable called w determines wiggliness of the fractal line.

1000 w = 200

1090 'Define coordinates of end points of straight line.

1100 x1(1) = 20
1110 y1(1) = 250
1120 x1(2) = 620
1130 y1(2) = 250

1140 'Draw the line in blue.
1150 LINE (x1(1), y1(1))-(x1(2), y1(2)), 1

1190 'End points of fractal line are the same as those of straight line.

1200 x2(1) = 20
1210 y2(1) = 250
1220 x2(65) = 620
1230 y2(65) = 250
'Generate fractal line.

FOR k = 1 TO 6
  ka = 64 / 2 ^ k
  FOR j = (ka + 1) TO 65 STEP (2 * ka)
    x2(j) = (x2(j - ka) + x2(j + ka)) / 2
    y2(j) = (y2(j - ka) + y2(j + ka)) / 2
  NEXT
NEXT

'Draw fractal line in green.

FOR k = 1 TO 64
  LINE (x2(k), y2(k))-(x2(k + 1), y2(k + 1)), 2
NEXT

'Initialize a flag that will tell us when intersection occurs.

intersect = 1

'Find intersection point. Variables xi and yi define x,y-coordinates of the intersection point.

IF x1(1) <> x1(2) THEN
  b1 = (y1(2) - y1(1)) / (x1(2) - x1(1))
  IF x2(k) <> x2(k + 1) THEN
    b2 = (y2(k + 1) - y2(k)) / (x2(k + 1) - x2(k))
    a1 = y1(1) - b1 * x1(1)
    a2 = y2(k) - b2 * x2(k)
    IF b1 = b2 THEN
      intersect = 0
      xi = -1 * (a1 - a2) / (b1 - b2)
      yi = a1 + b1 * xi
    END IF
  ELSE
    xi = x2(k)
  END IF
ELSE
  xi = x2(k)
  a1 = y1(1) - b1 * x1(1)
  yi = a1 + b1 * xi
END IF
ELSE
  xi = x1(1)
  IF x2(k) <> x2(k + 1) THEN
    b2 = (y2(k + 1) - y2(k)) / (x2(k + 1) - x2(k))
    a2 = y2(k) - b2 * x2(k)
  END IF
END ELSE
yi = a2 + b2 * xi
ELSE
    intersect = 0
END IF
END IF

'Draw circles at intersection points.'

IF intersect <> 0 THEN
    IF (x1(1) - xi) * (xi - x1(2)) >= 0 THEN
        IF (x2(1) - xi) * (xi - x2(2)) >= 0 THEN
            IF (y1(1) - yi) * (yi - y1(2)) >= 0 THEN
                CIRCLE (xi, yi), 5, 15
            END IF
        END IF
    END IF
END IF
END IF
END IF

NEXT

END
Assignment 3
Vector Data Structures (II)

Objectives: In this assignment you will be using pcARC/INFO to examine different types of topological overlay.

Files: This assignment requires part of the pcARC/INFO Green River database. You will need the following coverages:

- DEVELOP  a polygon coverage of areas selected for development
- SITES  a polygon coverage of ecologically sensitive areas

Examining the Database: Use ARCPLOT to display the two coverages on the screen and examine the features that they contain.

1. Draw a sketch map of the polygons displayed on the screen. Using the IDENTIFY command, label each polygon on your map with the appropriate SITES-ID or DEVELOP-ID code.

In TABLES, select the PAT file for each coverage in turn.

2. What items are contained in SITES.PAT?

3. List the area of each polygon in the SITES coverage along with its SITES-ID code. (Ignore the external polygon.)

4. What items are contained in DEVELOP.PAT?

5. List the area of each polygon in the DEVELOP coverage along with its DEVELOP-ID code.

Topological Overlay: pcARC/INFO provides five different types of topological overlay. The commands are listed below. Each command corresponds to a different combination of Boolean (or logical) operators (i.e., AND, OR and NOT). Each command creates a new output coverage from two existing input coverages.

- UNION
- INTERSECT
- IDENTITY
- CLIP
- ERASECOV

6. Draw a set of sketch maps showing the polygons that would be produced if SITES and DEVELOP were used as the input coverages to each of the five overlay commands.

Perform all five types of overlay using SITES as the first input coverage (i.e., the "in_cover") and DEVELOP as the second input coverage (i.e., the "union_cover", "intersect_cover", etc). Give the output coverage a different name in each case.

Display each of the new coverages in ARCPLOT using the POLYGONSHADES and ARCS commands.

7. Draw a sketch map showing the polygons in each of the coverages. How do these compare to the sketch maps you produced in question 6?

Return to TABLES and select the PAT file for each of the new coverages in turn.

8. List the items contained in the PAT file for each coverage.

9. How are the PAT files for the coverages produced by CLIP and ERASECOV different from the files created with the other three
10. What items would be contained in these PAT files if you reversed the order of the first and second input coverages for CLIP and ERASECOV (i.e., you used DEVELOP as the "in_cover" and SITES as the "clip_cover" or "erase_cover").

11. Draw a sketch map showing the polygons that would be produced by CLIP and ERASECOV if you reversed the order of the input coverages.

12. Select the PAT file for the coverage produced by the UNION command. What is the total area of the polygons a) common to both SITES and DEVELOP b) found only in SITES c) found only in DEVELOP? (Assume that a value of 0 for any coverage-ID code indicates the external polygon for the associated coverage.)

13. Select the PAT file for the coverage produced by the INTERSECT command. a) What is the total area of the polygons in this coverage? b) How does this correspond to your answer for question 12?

14. Select the PAT file for the coverage produced by the IDENTITY command. What are the similarities and differences between this coverage the SITES coverage?

15. Select the PAT file for the coverage produced by the CLIP command. What are the similarities and differences between this coverage and the coverage produced by the INTERSECT command?

16. Select the PAT file for the coverage produced by the ERASECOV command. What are the similarities and differences between this coverage and the coverage produced by the CLIP command?

17. On each of the sketch maps you drew above (question 7), label the polygons in each of the five new coverages using their appropriate coverage-ID values.

18. Write out a Boolean expression that describes the polygons contained in each of the new coverages (e.g., SITES AND DEVELOP, SITES OR DEVELOP, SITES AND NOT DEVELOP, etc).
Assignment 3
Answer Key

1. 

2. AREA, PERIMETER, SITES#, SITES-ID, NUMBER

3. SITES-ID  AREA  
   1  134.522  
   2  164.198  
   3  141.235

4. AREA, PERIMETER, DEVELOP#, DEVELOP-ID, NAME

5. DEVELOP-ID  AREA  
   1  134.364  
   2  146.973  
   3  150.599

6. Answers will vary from student to student.
8. UNION: AREA, PERIMETER, COV#, COV-ID, SITES#, SITES-ID, NUMBER, DEVELOP#, DEVELOP-ID, NAME

INTERSECT: AREA, PERIMETER, COV#, COV-ID, SITES#, SITES-ID, NUMBER, DEVELOP#, DEVELOP-ID, NAME

IDENTITY: AREA, PERIMETER, COV#, COV-ID, SITES#, SITES-ID, NUMBER, DEVELOP#, DEVELOP-ID, NAME

CLIP: AREA, PERIMETER, COV#, COV-ID, NUMBER

ERASECOV: AREA, PERIMETER, COV#, COV-ID, NUMBER

9. They contain no items from DEVELOP.

10. CLIP: AREA, PERIMETER, COV#, COV-ID, NAME

ERASECOV: AREA, PERIMETER, COV#, COV-ID, NAME
12. a) 33.828 + 69.791 = 103.619
    b) 87.094 + 7.313 + 141.235 + 100.694 = 336.336
    c) 16.132 + 113.144 + 48.440 + 150.599 = 328.315

13. a) 33.828 + 69.791 = 103.619
    b) It is the same as 12 a). These are the polygons that SITES and DEVELO have in common.

14. They have the same total area (i.e., the area of the polygons in SITES). However, in the coverage produced by IDENTITY, the SITES polygons intersecting DEVELO polygons are now new polygons with attributes from DEVELO.

15. They both contain the same set of two polygons (i.e., the polygons common to SITES and DEVELO), but the coverage produced with CLIP has no attributes from DEVELO.

16. Neither contains any attributes from DEVELO, but in terms of the polygons retained the two coverages are mirror images of each other.

17. (See question 7.)

18. UNION: SITES OR DEVELOP

   INTERSECT: SITES AND DEVELOP

   IDENTITY: SITES OR (SITES AND DEVELOP)

   CLIP: SITES AND DEVELOP

   ERASECOV: SITES AND (NOT DEVELOP)
Assignment 4
Raster Data Structures

Objectives: In this assignment you will be writing a QuickBASIC program to perform run-length encoding on raster data.

Files: This assignment requires two QuickBASIC programs (MAP.BAS and MAPRUN.BAS) and three data files (ELEV.DAT, LAND.DAT and HAWAII.DAT).

Run-Length Encoding: The file called ELEV.DAT is a raster data file of elevations for Africa. Elevation values range from 2 (lowest elevation) to 6 (highest elevation). A value of 1 is used to designate water. The file called LAND.DAT is a raster data file of the same area in which water has been assigned a value of 0 and land a value of 1.

Each file contains 109 rows by 120 columns of cells. The cells are stored in "scan-line" order (i.e., beginning in the upper left corner and proceeding left to right along each row of cells).

Run the QuickBASIC program called MAP.BAS to display each file as a map.

Write a program to run-length encode each file in scan-line order. Each line in the output file should represent a "run" of cell values and should contain two numbers:

a) the length of the run, measured in cells, and

b) the cell value for the run.

The following example shows the output file that would be obtained by performing run-length encoding on an input file containing 4 rows by 5 columns of cells.

| 2 2 2 2 2 | 5 2 |
| 1 1 1 1 1 | 8 1 |
| 1 1 1 3 3 | 6 3 |
| 3 3 3 3 2 | 1 2 |

Use your program to perform run-length encoding on the ELEV.DAT data file. Run the QuickBASIC program called MAPRUN.BAS to display a map of the output file. If your program is written correctly, the map should look the same as the one you produced earlier using MAP.BAS. Be sure that the last run of cells is displayed on your map.

Also perform run-length encoding on the LAND.DAT data file.

Hand in a copy of your run-length encoding program once you have it working to your satisfaction. Answer the following questions.

1. In DOS, use DIR to calculate the "compaction ratio" for each of the two output files. This ratio is calculated by dividing the size of the output file in bytes by the size of the corresponding original data file in bytes.

2. Which of the two files has a better compaction ratio? Why?
3. Under what circumstances might you get a compaction ratio greater than 1?

Optional: As the attached article indicates, it is possible to make maps from poems using the Morton cell ordering system. Write a program to perform this task. The data file called HAWAII.DAT contains the data necessary to make the map of Hawaii referred to in the article.
Introduction: Recent theoretical work suggests a strong link between geography and poetry. A simple computation allows any line of poetry to be positioned in two-dimensional space. The entire poem may be mapped as a set of points joined by straight-line segments. By implication, maps are poems and poems are maps.

Terminology: The link between geography and poetry may be explained with reference to prosody, the inexact science of linguistic rhythms. Essentially, a poem is a collection of lines, which are collections of words, which are in turn collections of syllables (Preminger 1986). Prosody shows that these syllables may be differentiated according to the presence or absence of stress. In poems, stress tends to recur in cyclical patterns. Analysis of these patterns is known as scansion, and the taxonomical characterization of patterns is based on meter. Meter is measured in terms of a unit known as a foot. Feet are defined by particular patterns of stressed and unstressed syllables. For instance, an iambic foot contains two syllables, the first unstressed and the second stressed, as in the following example taken from Chaucer:

```
That slip / 'ry sci / ence stripped / me down / so bare
```

Following conventional symbology, a stressed syllable is denoted with a diacritical dash (-), an unstressed syllable with a diacritical cusp (H), a division between two feet with a virgule (/), and a division between two syllables with a hyphen (-) or with a blank space if the division occurs between two words.

In addition to iambic, there are three other common types of feet encountered in English-language poetry (Nims 1974). Trochaic foot is the transpose of iambic. Anapestic foot is characterized by two unstressed syllables followed by a stressed syllable, while in dactylic foot, a stressed syllable is followed by two unstressed syllables.

Meter also depends on the number of feet per line. In the iambic example presented above, there are 5 feet per line. This means that the line is denoted as I. pentameter. Similarly, a trochaic foot with 3 feet per line would be denoted as T. trimeter, an anapestic foot with 1 foot per line would be denoted as A. monometer, etc. Although the number of feet per line is always an integer, it is not uncommon for a poem to contain several feet that lack one or more syllables. Such fractional feet are called catalectic (Bain et al, 1981).

Mathematical Representation: The particular diacritical symbols used to denote whether a syllable is stressed or unstressed are wholly arbitrary. Thus one can safely replace these symbols with any others, so long as they are capable of differentiating between two discrete states. If one assigns a "1" to a stressed syllable and a "0" to an unstressed syllable, then any line of poetry can be represented by a string of bits (binary digits).

For example, the string for I. pentameter is

```
0101010101
```

This bit string is the binary (i.e., base 2) representation of a number whose digital (i.e., base 10) representation happens to be 341. Any line of poetry may be represented mathematically in this manner.

The Link with Geography: These observations are closely linked to developments in modern geography pertaining to the tessellation of two-dimensional space. Tessellation refers to the partitioning of space into a set of regular, non-overlapping, spatially exhaustive cells. Typically these cells are square in shape, by other shapes may also be used.

While tessellation itself is a relatively straightforward concept, the optimal method of assigning index numbers to the cells is the subject of debate (Goodchild 1989). A host of different indexing systems have been proposed, but most of these
have not been widely applied. The number of potential indexing systems is enormous. If there are $r$ rows and $c$ columns of cells, then there are $r!c!$ unique ways to arrange index numbers.

One indexing system that has proved useful, however, is that proposed by G. M. Morton (Morton, no date). In Morton’s system, there is an implicit relationship between the location of a cell and its index number. More specifically, the row and column positions of the cell are embedded as interleaved bit strings in the binary representation of the cell index number. The figure below illustrates this relationship for cell number 7 in Morton two-space, for which the row and column positions are 3 and 1 respectively.

A corollary of this relationship is that any bit string is associated with one and only one cell in Morton two-space. Since any line of poetry can be represented as a bit string, it follows that any such line can be uniquely located in Morton two-space. Furthermore, since a poem is composed of one or more lines, it is possible to locate each line in Morton two-space, join these locations with straight-line segments, and call the resulting product a map.
An Illustration: As an illustration, consider J. Diefenbaker’s poem, "The Scientist’s Lament", the first six lines of which are reproduced below.

I am a / sci-en-tist, / not a Frank / en-stein, Ed.
I’m not an / old sewn-up / corpse with a / re-claimed head,
A flat-top / type hair-do, / bolts pro-trud / ing, skin sort
Of green-grey / (or is it / blue?), a big / hair-y wart,
And a pet / dog named Boo / Boo. But my / brain, it longs
For a cold, / old, dark, dank / grave. Hey, where / are my thongs?

The entire poem has been mapped into Morton two-space based on the principles described above. The result, as shown in the figure below, is an accurate map of Oahu, Hawaii. The first line of the poem is Kaena Point, the most westerly point on the island. The poem then proceeds in a clockwise direction.
References


Assignment 4
Program Listing

100  'MAP.BAS -- a program to display a raster image on the screen.
190  'Choose data file, set up screen.
200  CLS
300  INPUT "Enter the name of the data file: ", filename$
400  SCREEN 12
500  WINDOW SCREEN (0, 0)-(160, 120)
600  'Input data from file and draw the map.
700  OPEN filename$ FOR INPUT AS #1
800  FOR y = 1 TO 109
810      FOR x = 1 TO 120
820          INPUT #1, z
830      LINE (x, y)-(x + 1, y + 1), z, BF
840    NEXT
850  NEXT
1000 END
'MAPRUN.BAS -- a program to display a run-encoded raster image.

'Choose data file, set up screen.

CLS
INPUT "Enter the name of the data file: ", filename$

SCREEN 12
WINDOW SCREEN (0, 0)-(160, 120)

'Input data from file.

OPEN filename$ FOR INPUT AS #1

'Hard-wire these guys.

nrows = 109
ncols = 120
ncells = nrows * ncols
DIM z(ncells)

'Unzip the run-encoded data.

endi = 0
DO UNTIL EOF(1)
  starti = endi + 1
  INPUT #1, frequency, value
  endi = endi + frequency
  FOR i = starti TO endi
    z(i) = value
  NEXT
LOOP

'Display the image.

FOR y = 1 TO nrows
  FOR x = 1 TO ncols
    cellno = (y - 1) * ncols + x
    LINE (x, y)-(x + 1, y + 1), z(cellno), BF
  NEXT
NEXT

END
Assignment 4
Answer Key

1. Different ratios are possible, depending on the format of the run-length encoded file. LAND.DAT and ELEV.DAT each contain 26,488 bytes. If a single space is left between the frequency and the cell value, the run-length encoded versions of these files contain 2300 bytes and 10,481 bytes, respectively. The corresponding compaction ratios are 0.087 and 0.396.

2. LAND.DAT yields a better compaction ratio since it has longer runs of the same value.

3. You might get a compaction ratio greater than 1 when there is high-frequency variation over space (i.e., short runs).
Assignment 4
Program Listing (Answer Key)

100 'RUN.BAS -- program to perform run-length encoding of raster data.
110 'NOTE: Students' actual programs may differ from this example.
200 'Choose data file names, open files.

210 CLS
220 INPUT "Enter the name of the input data file: ", infile$ 
230 INPUT "Enter the name of the output data file: ", outfile$
240 OPEN infile$ FOR INPUT AS #1
250 OPEN outfile$ FOR OUTPUT AS #2

300 'Hard-wired dimensions...

310 nrows = 109
320 ncols = 120
330 ncells = nrows * ncols
340 DIM z(ncells)

400 'Read data.

410 FOR i = 1 TO ncells
420 INPUT #1, z(i)
430 NEXT

500 'Perform run-length encoding.

510 count = 1
520 FOR i = 1 TO ncells - 1
530 IF z(i) = z(i + 1) THEN
540 count = count + 1
550 ELSE
560 PRINT #2, count; z(i)
570 count = 1
580 END IF
590 NEXT

600 PRINT #2, count; z(i)

700 END
'POEM.BAS -- program to turn a poem into a map.

'NOTE: Students' actual programs may differ from this example.

CLS
PRINT "This program turns poems into maps."
PRINT "The poem is entered as a set of strings of 0s and 1s."
PRINT "A 0 is an unstressed syllable and a 1 is a stressed syllable."
PRINT "Each 0 and 1 represents a binary digit (bit)."
PRINT "Each line of the poem is entered as a separate string of bits."
PRINT "You should leave a space between each bit."
PRINT "Input data include the number of lines in the poem (nlines)"
PRINT "and the maximum number of syllables in a line (nsylls)."
PRINT "For those lines that have less than nsylls syllables,"
PRINT "add extra 0s on the left-hand side of the bit string."
PRINT
INPUT "How many lines does the poem have? ", nlines
INPUT "What's the maximum number of syllables per line? ", nsylls
INPUT "What is the name of the input data file? ", filename$

'Dimension some arrays and open file for reading.
DIM zamboni(nsylls), x(nlines + 1), y(nlines + 1)
OPEN filename$ FOR INPUT AS #1

'Initialize max and min coordinates.
minx = 99999
miny = 99999
maxx = -99999
maxy = -99999

'Calculate x and y coordinates for each line of the poem.
FOR i = 1 TO nlines
    x(i) = 0
    y(i) = 0
    FOR j = nsylls - 1 TO 0 STEP -1
        INPUT #1, zamboni(j)
    NEXT
    FOR j = nsylls - 1 TO 0 STEP -1
        factor = 1
490     basex = j \ 2
500     FOR k = 0 TO basex - 1
510         factor = factor * 2
520     NEXT
530     IF j MOD 2 = 1 THEN x(i) = x(i) + zamboni(j) * factor
540     IF j MOD 2 = 0 THEN y(i) = y(i) + zamboni(j) * factor
550     NEXT
560     IF x(i) < minx THEN minx = x(i)
570     IF y(i) < miny THEN miny = y(i)
580     IF x(i) > maxx THEN maxx = x(i)
590     IF y(i) > maxy THEN maxy = y(i)
600     NEXT

640     'Last coordinate is the same as the first (if the poem is a polygon).
650     x(nlines + 1) = x(1)
660     y(nlines + 1) = y(1)

690     'Ready to draw the map!
700     SCREEN 12
710     WINDOW (minx - 5, miny - 5)-(maxx + 5, maxy + 5)
800     FOR i = 1 TO nlines
810         LINE (x(i), y(i))-(x(i + 1), y(i + 1))
820     NEXT
1000     END
Assignment 5
Surface Modeling with DEMs and TINs

Objectives: In this assignment you will be examining different types of surface models. You will be using QuickBASIC to interpolate and display gridded digital elevation models (DEMs), and pcARC/INFO to examine how surfaces can be modeled using triangulated irregular networks (TINs).

Files: The QuickBASIC part of this assignment requires four QuickBASIC programs (RELIEF.BAS, ILLUMIN.BAS, DRAIN.BAS and INTERPOL.BAS) and two data files (DEM.DAT and SAMPLE.DAT). For the ARC/INFO part of the assignment you will need two coverages:

- FACETS a polygon coverage of triangular "facets"
- EDGES a coverage containing the arcs defining these facets

Elevation Mapping with Gridded DEMs: Examine the QuickBASIC program called RELIEF.BAS. This program draws a six-color elevation map for a grid, using the elevation data stored in DEM.DAT. The program divides the range of elevation into six classes, and each class is assigned a different color. These colors are defined in the DATA statement on line 540 of the program. The first number defines the color of the lowest elevation class, the second number defines the color of the second-lowest elevation class, and so on.

Run the program several times, changing the values in line 540 to obtain different colors for the elevation ranges. Try to obtain a sequence of colors that conveys a sense of increasing elevation. The colors available to you are:

- 0 = black   1 = blue   2 = green   3 = cyan
- 4 = red     5 = magenta 6 = brown   7 = white
- 8 = dark gray 9 = light blue 10 = light green 11 = light cyan
- 12 = light red 13 = light magenta 14 = light yellow 15 = white

After running the program, answer the following questions. (You will need to monitor the statistics printed to the screen to answer some of them.)

1. What is the best color sequence you obtained?

2. a) How many cells are in the gridded DEM? b) Assuming that the grid is square (i.e., has the same number of rows and columns), how many rows and columns are in the grid? c) Does your answer agree with the number of elevation values read in by the program from DEM.DAT?

3. a) What are the maximum and minimum cell elevations (z-values)? b) Why does the program need to compute these values?

4. How does the program divide elevations into classes for coloring?

Interpolating to a Grid: Examine the QuickBASIC program called INTERPOL.BAS. This program interpolates the elevations (z-values) for the cells in a gridded DEM using the x,y,z-values for a sample of randomly-selected points. The program uses a distance-weighted interpolation method. For each cell in the grid, the elevation of the cell zc is calculated as:
The program uses a set of random points stored in a file called SAMPLE.DAT to perform the interpolation. This file contains a total of 200 points. When you run the program, you can select any number of points between 1 and 200 as the size of your sample. The program will read this many points from the file.

The program uses only a portion of this sample of points to interpolate the elevation for a given cell. This number of points it uses is defined by the size of the "neighborhood." You can define the neighborhood to contain any number of points between 1 and the total size of your sample.

To illustrate how the program works, imagine that you selected a sample size of 20 points and a neighborhood size of 5 points. The program would read in 20 points from SAMPLE.DAT. For each cell in the grid, it would compute the distance between the cell and each of these points. It would then select the 5 closest points to the cell to interpolate the cell’s elevation.

Run the program and create an output file called DEM1.DAT (or anything except DEM.DAT so that you don’t destroy the original DEM). To minimize execution time, choose a relatively small value for the sample size (< 25) and the neighborhood size (< 5). Now run RELIEF.BAS after changing the input file name in the program to match the output file you just created.

5. a) In general, how does the appearance of the resulting relief map differ from the maps you created previously? b) How might you account for this difference?

Run INTERPOL.BAS several times, experimenting with different sample and neighborhood sizes. Note that execution time rises dramatically as you increase the values of these variables.

6. How do changes in these variables affect the appearance of the map?

7. a) How do zmin and zmax compare to the values you obtained originally (question 3)? b) How might you account for any differences?

**Shaded Relief Mapping with Gridded DEMs:** Examine the QuickBASIC program called ILLUMIN.BAS. This program draws a shaded relief map for the same grid you used above (i.e., DEM.DAT). The program creates this map by calculating the slope and aspect of every cell in the grid based on the elevations of the cell’s eight neighbors. The illumination of the cell is calculated assuming a light source (i.e., the sun) located 45 degrees above the horizon in the south-west sky. Before computing the slope and aspect, cell elevations are scaled by a vertical exaggeration factor, which effectively stretches the range of illumination values for the grid.

Run the program several times, experimenting with different values for the illumination angle and vertical exaggeration factor.

8. What is the effect on the map of changing these values?
**Deriving Drainage Networks from Gridded DEMs:** Examine the QuickBASIC program called DRAIN.BAS. This program extracts and maps the drainage network for the same grid you used above (i.e., DEM.DAT). It does this by assuming that surface water can flow into any of the eight neighbors of a given cell. After determining the drainage patterns between cells, the program draws the drainage features for those cells that drain, at minimum, the number of cells defined by a threshold value. For example, if the threshold value was 4, then the drainage patterns would be drawn for every cell that drained at least 4 other cells, but would not be drawn for cells draining 3 or fewer cells.

Run the program several times, experimenting with different threshold values.

9. What effect does changing the threshold value have on the appearance of the drainage network?

10. List at least two problems with the appearance of the drainage network.

**The TIN Model:** Switch to the directory containing the FACETS and EDGES coverages. Use the ARCS command in ARCPLOT to display EDGES. Create a simple slope map by typing SHADESET COLOR to choose a shadeset file, and POLYGONS FACETS DEGREE_SLOPE SLOPE.LUT to shade in the triangular facets with different shades of grey based on their slopes. (The brighter the shade of grey, the steeper the slope.) Observe that the areas being filled with grey are the same triangular patches defined by the arcs in the EDGES coverage. You can use ARCS to display these arcs again.

Leave ARCPLOT and enter TABLES. Examine the arc attribute file for the EDGES coverage and the polygon attribute file for the FACETS coverage.

11. a) List the items contained in each of these files. b) Based on the names of these items and your knowledge of TINs, what do you think each of these items refers to?

Select the arc attribute file for the EDGES coverage. Now use the RESELECT command to extract only those arcs surrounding polygon # 457 (i.e., only those arcs for which the right polygon # is 457 or the left polygon # is 457). If you use the command correctly, exactly 3 arcs should be reselected.

12. List the command you used to reselect these arcs.

13. Fill in the following data for these arcs.

```
FNODE#   TNODE#   LPOLY#   RPOLY#   EDGES#   ZFROM ZTO
```

14. Draw a diagram illustrating these three arcs and showing:

   a) the polygon # of the triangular facet enclosed by the arcs
   b) the polygon # of each of the three adjacent triangular facets
   c) the node # of each node
   d) the elevation (z-value) of each node
   e) the arc # of each of the three arcs
   f) the direction in which each arc was digitized

Return to ARCPLOT and again display the arcs in the EDGES coverage. Use the RESELECT command as you did before to extract only those arcs surrounding polygon # 457. (Remember that the syntax of the RESELECT command in ARCPLOT is different from that in TABLES.)

Change the line color and display the arcs in EDGES again. (There will of course only be three arcs after you do the reselection.) This will highlight polygon # 457. Now use the IDENTIFY command to obtain the polygon # of each triangular facet adjacent to polygon # 457. Redraw the above diagram, if necessary, correcting for any misorientation.
Use the IDENTIFY command to obtain the slope and aspect of polygon # 457.

15. What slope and aspect values did you obtain?

16. Does the aspect value you obtained make sense in terms of the z-values of the nodes in your diagram? (Aspect varies between 0 and 360 degrees, and is measured in a clockwise direction from the 12 o’clock position.)

Optional: Revise INTERPOL.BAS to perform interpolation based on:

\[
Z_c = \frac{\sum_{i=1}^{n} \frac{z_i}{d_{i\alpha}}}{\sum_{i=1}^{n} \frac{1}{d_{i\alpha}}}
\]

where \( \alpha \) is a variable entered by the user. Run the program for values of \( \alpha \) between 1 and 3. How does changing this variable affect the appearance of the relief map? Hand in a copy of the revised program.
Assignment 5
Program Listing

100 'RELIEF.BAS -- A program to display a relief map in 6 colors.

400 DIM z(90, 90)
410 SCREEN 12
420 CLS
430 WINDOW (-10, -10)-(134, 100)

500 'Assign colors for elevation ranges.

510 FOR k = 1 TO 6
520 READ col(k)
530 NEXT
540 DATA 1,2,3,4,5,6

600 'Input elevation raster from file and determine zmax and zmin.

605 OPEN "DEM.DAT" FOR INPUT AS #1
610 zmax = -10000
615 zmin = 100000
630 FOR i = 1 TO 90
635 FOR j = 1 TO 90
640 INPUT #1, z(i, j)
645 IF z(i, j) > zmax THEN zmax = z(i, j)
650 IF z(i, j) < zmin THEN zmin = z(i, j)
655 NEXT
660 LOCATE 21, 60: PRINT "zmin = "; zmin;
670 LOCATE 23, 60: PRINT "zmax = "; zmax
680 LOCATE 25, 60: PRINT "no. of cells = "; i * 90;
690 NEXT

800 'Determine color for each elevation and fill pixel.

820 FOR i = 1 TO 90
830 FOR j = 1 TO 90
840 IF z(i, j) = zmax THEN
850 c = 6
860 ELSE
870 c = INT((z(i, j) - zmin) / (zmax - zmin) * 6) + 1
880 END IF
890 LINE (i, j)-(i + 1, j + 1), col(c), BF
900 NEXT
910 NEXT

1000 END
'INTERPOL.BAS

'This program interpolates a regular 90x90 grid of elevations
'from the x,y,z-coordinates of a set of random points.
'The program uses an weighted-distance method (inverse of
'square of distance) for a neighborhood of points around each
'grid cell. The size of the neighborhood is defined by the user.
'The user also defines the number of random points.

Input number of random points (npts) and
'number of points in neighborhood (neigh).

CLS
INPUT "How many random points do you want to use? ", npts
IF npts < 1 THEN
    PRINT
    PRINT "You must use at least one point."
    PRINT
    GOTO 200
END IF
IF npts > 200 THEN
    PRINT
    PRINT "You cannot use more than 200 points."
    PRINT
    GOTO 200
END IF

INPUT "How many points do you want in the neighborhood? ", neighs
IF neighs > npts THEN
    PRINT
    PRINT "Cannot be greater than the number of random points."
    PRINT
    GOTO 300
END IF

'Dimension necessary arrays.
DIM prow(npts), pcol(npts), pz(npts), dist(npts), s(npts)

'Open data files.
OPEN "SAMPLE.DAT" FOR INPUT AS #1
OPEN "DEM1.DAT" FOR OUTPUT AS #2
'Read in x,y,z-coordinates for random points.

FOR i = 1 TO npts
  INPUT #1, prow(i), pcol(i), pz(i)
NEXT

'Initialize a vector that will hold the numbers 1 through npts, sorted
'by the distance between random point i and the current grid cell.

FOR i = 1 TO npts
  s(i) = i
NEXT

'Main loop to interpolate value for each grid cell.

FOR r = 1 TO 90
  FOR c = 1 TO 90
    'Calculate distance between grid cell and each random point.
    FOR i = 1 TO npts
      distr = r - prow(s(i))
      distc = c - pcol(s(i))
      dist2(s(i)) = distr * distr + distc * distc
    NEXT

    'Sort random points by distance from current grid cell.
    sorted = 0
    DO WHILE sorted = 0
      sorted = 1
      FOR i = 1 TO (npts - 1)
        IF dist2(s(i)) > dist2(s(i + 1)) THEN
          temp = s(i)
          s(i) = s(i + 1)
          s(i + 1) = temp
          sorted = 0
        END IF
      NEXT
    END DO
  NEXT
NEXT

LOOP
5000   'Interpolate the value for the current grid cell.

5200   numer = 0
5250   denom = 0
5300   FOR i = 1 TO neighs
5350       numer = numer + (p(l(s(i))) / dist2(s(i)))
5400       denom = denom + (1 / dist2(s(i)))
5450   NEXT
5500   PRINT #2, numer / denom
5550   NEXT
5600   NEXT

10000   END
'ILLUMIN.BAS -- a program to create a shaded relief map.

'exag = vertical exaggeration; sun = illumination angle.

exag = 20
sun = 45
tansun = TAN(sun * 3.14159 / 180)

DIM z(90, 90)
SCREEN 12
CLS
WINDOW (-10, -10)-(134, 100)

'Assign grey scale for illumination.

FOR k = 1 TO 4
READ col(k)
NEXT
DATA 0,8,7,15

'Input elevation raster from file.

OPEN "DEM.DAT" FOR INPUT AS #1
FOR i = 1 TO 90
    FOR j = 1 TO 90
        INPUT #1, z(i, j)
        z(i, j) = z(i, j) * exag
        NEXT
    NEXT

'Step through each cell, omitting boundary cells.

FOR i = 2 TO 89
    FOR j = 2 TO 89
        b = -z(i - 1, j - 1) - z(i - 1, j) - z(i - 1, j + 1)
        b = (b + z(i + 1, j - 1) + z(i + 1, j) + z(i + 1, j + 1)) / 9
        c = -z(i - 1, j - 1) - z(i, j - 1) - z(i + 1, j - 1)
        c = (c + z(i - 1, j + 1) + z(i, j + 1) + z(i + 1, j + 1)) / 9
        il = (b + c + tansun)
        il = il / SQR(b^2 + c^2 + 1) / SQR(2 + tansun^2)
        IF il < 0 THEN il = 0
        c = INT(il * 4) + 1
        LINE (i, j)-(i + 1, j + 1), col(c), BF
        NEXT
    NEXT

END
'DRAIN.BAS -- a program to develop a reduced drainage network.

'Drainage net will be drawn for all cells that drain at minimum the
'number of cells specified by threshold value. (e.g., if threshold = 2
'then drainage net will be drawn for cells that drain 2 or more cells.)

threshold = 2

DIM z(90, 90), toi(90, 90), toj(90, 90), counter(90, 90)
SCREEN 12
CLS
WINDOW (-10, -10)-(134, 100)

'Input elevation raster from file.

OPEN "DEM.DAT" FOR INPUT AS #1
FOR i = 1 TO 90
    FOR j = 1 TO 90
        INPUT #1, z(i, j)
    NEXT
NEXT

'For each cell, determine which cell it drains into.

FOR i = 2 TO 89
    FOR j = 2 TO 89
        zmin = 1000
        FOR n = -1 TO 1
            FOR m = -1 TO 1
                IF z(i + n, j + m) < zmin THEN
                    zmin = z(i + n, j + m)
                    toi(i, j) = i + n
                    toj(i, j) = j + m
                END IF
            NEXT
        NEXT
        IF toi(i, j) = i AND toj(i, j) = j THEN
            toi(i, j) = 0
            toj(i, j) = 0
        END IF
    NEXT
NEXT

NEXT
NEXT
'Initialize matrix used to accumulate flow into each cell.'

1910 FOR i = 1 TO 90
1920    FOR j = 1 TO 90
1930        counter(i, j) = 0
1940    NEXT
1950 NEXT

11000 'Follow drainage route of each cell.'

11010 FOR i = 1 TO 90
11030    FOR j = 1 TO 90
11050        ix = i
11070        jx = j
11090        DO WHILE ix <> 0 AND jx <> 0
11110            counter(ix, jx) = counter(ix, jx) + 1
11130            ix = toi(ix, jx)
11150            jx = toj(ix, jx)
11170        LOOP
11190    NEXT
11210 NEXT

11400 'Draw drainage net.'

11410 FOR i = 1 TO 90
11420    FOR j = 1 TO 90
11430        IF counter(i, j) >= threshold THEN
11440            IF toi(i, j) <> 0 AND toj(i, j) <> 0 THEN
11450                LINE (i, j)-(toi(i, j), toj(i, j))
11460            END IF
11470        END IF
11480    NEXT
11490 NEXT

12000 END
Assignment 5
Answer Key

1. (Results will vary here.)

2. a) 8100  b) 90 rows and 90 columns  c) Yes, since the program contains a nested loop with i = 1 to 90 and j = 1 to 90.

3. a) 100 and 0  b) To divide the range of elevations into classes for coloring.

4. Equal interval slicing. Basically, the range of values is divided into six equally-sized classes. (See line 870.)

5. a) They are more generalized and contain some artifacts.  b) The program is estimating the values in 8100 cells from only a few points.

6. Increasing either variable reduces amount of generalization and number of artifacts.

7. a) New zmin is larger and new zmax is smaller.  b) The map has been generalized, so the maximum and minimum values are not as extreme.

8. Changing the illumination angle changes the amount of shadow. The higher the sun, the brighter the map. As the vertical exaggeration factor is increased, surface shape becomes more apparent.

9. An increase in the threshold results in a decrease in the number of drainage features displayed on the map.

10. Many features are unconnected to the network. The network only allows streams to flow in eight directions. In areas of low relief, there are sets of parallel streams very close to each other. (Others...)

11. FACETS,PAT
    AREA                  polygon area
    PERIMETER             polygon perimeter
    FACETS#               coverage#
    FACETS-ID             coverage-id
    DEGREE_SLOPE          slope in degrees
    ASPECT                aspect
    SAREA                 surface area of polygon
EDGES.AAT
FNODE# from node #
TNODE# to node #
LPOLY# left polygon #
RPOLY# right polygon #
LENGTH arc length
EDGES# coverage#
EDGES-ID coverage-id
DEGREE_SLOPE slope in degrees
LSLOPE slope of left polygon
RSLOPE slope of right polygon
LASCPECT aspect of left polygon
RASCPECT aspect of right polygon
ZFROM z-value of from node
ZTO z-value of to node

12. RESELECT RPOLY# = 457 OR LPOLY# = 457

13. FNODE# TNODE# LPOLY# RPOLY# EDGES# ZFROM ZTO

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>167</td>
<td>107</td>
<td>457</td>
<td>440</td>
<td>671</td>
<td>1240</td>
<td>1140</td>
</tr>
<tr>
<td>124</td>
<td>167</td>
<td>457</td>
<td>456</td>
<td>693</td>
<td>1180</td>
<td>1240</td>
</tr>
<tr>
<td>124</td>
<td>107</td>
<td>458</td>
<td>457</td>
<td>694</td>
<td>1180</td>
<td>1140</td>
</tr>
</tbody>
</table>

14. node 124
   (z = 1180)
   
   polygon 456
   arc 693
   arc 694
   polygon 457
   
   node 167
   (z = 1240)
   arc 671
   polygon 440
   
   node 107
   (z = 1140)
   arc 694
   polygon 458

15. SLOPE = 7.261 ASPECT = 38.293

16. Yes. (Explain in terms of the direction in which the facet is facing.)
Assignment 5
Program Listing (Answer Key)

'INTERPOL.BAS

'This program interpolates a regular 90x90 grid of elevations
'from the x,y,z-coordinates of a set of random points.
The program uses an weighted-distance method (as defined by
'the value of alpha) for a neighborhood of points around each
'grid cell. The size of the neighborhood is defined by the user.
The user also defines the number of random points.

'Input number of random points (npts) and
'number of points in neighborhood (neigh).

CLS
INPUT "How many random points do you want to use? ", npts
IF npts < 1 THEN
  PRINT
  PRINT "You must use at least one point."
  PRINT
  GOTO 200
END IF
IF npts > 200 THEN
  PRINT
  PRINT "You cannot use more than 200 points."
  PRINT
  GOTO 200
END IF

INPUT "How many points do you want in the neighborhood? ", neighs
IF neighs > npts THEN
  PRINT
  PRINT "Cannot be greater than the number of random points."
  PRINT
  GOTO 300
END IF

'Input value of alpha.

INPUT "What value of alpha do you want? ", alpha
IF alpha > 3 or alpha < 1 THEN
  PRINT
  PRINT "Alpha should be between 1 and 3."
  PRINT
  GOTO 345
END IF
'Divide alpha by 2 rather than taking square root of distance in line 3500.

alpha = alpha / 2

'Dimension arrays.

DIM prow(npts), pcol(npts), pz(npts), dist(npts), s(npts)

'Open data files.

OPEN "SAMPLE.DAT" FOR INPUT AS #1
OPEN "DEM1.DAT" FOR OUTPUT AS #2

'Read in x,y,z for random points.

FOR i = 1 TO npts
    INPUT #1, prow(i), pcol(i), pz(i)
NEXT

'Initialize a vector that will hold the numbers 1 through npts, sorted
'by the distance between random point i and the current grid cell.

FOR i = 1 TO npts
    s(i) = i
NEXT

'Main loop to interpolate value for each grid cell.

FOR r = 1 TO 90
    FOR c = 1 TO 90
        'Calculate distance between grid cell and each random point.

        FOR i = 1 TO npts
            distr = r - prow(s(i))
            distc = c - pcol(s(i))
            dist2(s(i)) = distr * distr + distc * distc
        NEXT
'Sort random points by distance from current grid cell.

sorted = 0
DO WHILE sorted = 0
    sorted = 1
    FOR i = 1 TO (npts - 1)
        IF dist2(s(i)) > dist2(s(i + 1)) THEN
            temp = s(i)
            s(i) = s(i + 1)
            s(i + 1) = temp
            sorted = 0
        END IF
    NEXT
LOOP

'Interpolate the value for the current grid cell.

numer = 0
denom = 0
FOR i = 1 TO neighs
    numer = numer + (pz(s(i)) / dist2(s(i))^alpha)
    denom = denom + (1 / dist2(s(i))^alpha)
NEXT
PRINT #2, numer / denom
NEXT
END
Assignment 6  
Kriging and the Semivariogram

Objectives: In this assignment you will be using QuickBASIC to construct a semivariogram and perform kriging.

Files: This assignment requires one QuickBASIC program (SEMIVAR.BAS) and two data files (ELEVPTS.DAT and PRECPTS.DAT).

The Semivariogram: Each of the two data files contains the x,y,z-coordinates of 100 random points. In ELEVPTS.DAT the z-coordinates refer to elevations (in meters), and in PRECPTS.DAT they refer to annual precipitation (in millimeters). Both data files were constructed from raster data files of Africa.

The QuickBASIC program called SEMIVAR.BAS computes the information necessary to construct the semivariogram. Each line of the program’s output contains two values. The first is the distance (or spatial lag), and the second is the computed semivariance for that spatial lag.

Run the program for each of the two data files and make a printout of each of the resulting output files. Using these printouts, draw the semivariogram for each of the two data files, putting the spatial lag on the x-axis and the semivariance on the y-axis. Hand in a copy of your semivariograms.

1. What is the approximate value of the sill, range and nugget for each of the data files?
2. Is the general form of the semivariogram the same or different for the two data files? What does this say about spatial variation of elevation versus precipitation?

Kriging: The semivariogram can be used to perform kriging, a type of interpolation. The objective of this part of the assignment is to use the semivariogram derived from the elevation data file to interpolate the elevation at a selected point based on the elevations at three neighboring points. In this case the interpolated elevation or z-value \((z_0)\) is estimated as

\[
z_0 = \lambda_1 z_1 + \lambda_2 z_2 + \lambda_3 z_3
\]

where \(z_1\) through \(z_3\) are the observed z-values at points 1 through 3, and \(\lambda_1\) through \(\lambda_3\) are a set of weights derived from the semivariogram. The observed z-values of the points are as follows:

\[
\begin{align*}
z_1 &= 1000 \\
z_2 &= 1500 \\
z_3 &= 2000
\end{align*}
\]

Computation of the weights in the above equation is most easily achieved using matrix algebra. Most of this computation has been done for you:

\[
\begin{align*}
\lambda_1 &= -2.91\times10^{-7} \times \gamma(0,1) + 1.64\times10^{-7} \times \gamma(0,2) + 1.27\times10^{-7} \times \gamma(0,3) + 0.125 \\
\lambda_2 &= 1.64\times10^{-7} \times \gamma(0,1) - 1.88\times10^{-7} \times \gamma(0,2) + 2.38\times10^{-8} \times \gamma(0,3) + 0.430 \\
\lambda_3 &= 1.27\times10^{-7} \times \gamma(0,1) + 2.38\times10^{-8} \times \gamma(0,2) - 1.51\times10^{-7} \times \gamma(0,3) + 0.446
\end{align*}
\]

In these equations, \(\gamma(0,1)\) is the semi-variance for the interpolated point (point 0) and neighboring point 1, \(\gamma(0,2)\) is the semi-variance for the interpolated point and neighboring point 2, and \(\gamma(0,3)\) is the semi-variance for the interpolated point.
and neighboring point 3. These values can be obtained directly from your semivariogram data if you know the distance between point 0 and each of the three neighboring points. Use the following distance values:

- distance between point 0 and neighboring point 1: 5
- distance between point 0 and neighboring point 2: 4
- distance between point 0 and neighboring point 3: 3

You're on the right track if you got a value of 5,628,776 for $\gamma(0,1)$.

3. Use equations (2), (3) and (4) to compute the values of $\lambda_1$, $\lambda_2$ and $\lambda_3$. (If the three computed values do not sum to approximately 1.0, something went wrong in your calculations.)

4. Use equation (1) to compute the value of $z_0$. 
Assignment 6
Program Listing

100 'SEMIVAR.BAS

150 'This program computes the semivariance from a set of points.

200 CLS
210 INPUT "What is the name of the input file? ", infile$
220 INPUT "What do you want to call the output file? ", outfile$

250 'Open data files for input and output.

260 OPEN infile$ FOR INPUT AS #1
270 OPEN outfile$ FOR OUTPUT AS #2

300 'Hard-wire values for some key variables.

310 npts = 100 'Number of points in input file.
320 maxd = 20 'Approx maximum distance between cells.

400 'Dimension a bunch of arrays.

410 DIM x(npts), y(npts), z(npts)
420 DIM variance(maxd), count(maxd)

500 'Input the x, y and z coordinates for the points.

510 FOR i = 1 TO npts
520 INPUT #1, x(i), y(i), z(i)
530 NEXT

600 'Initialize variance and count to zero for all spatial lags.

610 FOR k = 0 TO maxd
620 variance(k) = 0
630 count(k) = 0
640 NEXT
'Compute variance for all points separated by a given spatial lag.

FOR i = 1 TO npts
    PRINT "Working on point ", i
    FOR j = 1 TO npts
        xdist = x(i) - x(j)
        ydist = y(i) - y(j)
        zdiff = z(i) - z(j)
        dist = SQR(xdist ^ 2 + ydist ^ 2)
        disti = CINT(dist)
        variance(disti) = variance(disti) + zdiff ^ 2
        count(disti) = count(disti) + 1
    NEXT
NEXT

'Print semivariance and spatial lag.

FOR k = 0 TO maxd
    IF count(k) <> 0 THEN
        PRINT #2, k; .5 * variance(k) / count(k)
    END IF
NEXT

END
Assignment 6  
Answer Key

1. Elevation: sill = 11,876,000  range = 13  nugget = 385,000  
Precipitation: sill = 578,000  range = 5  nugget = 7,600

2. The form is different. Elevation has a greater range than precipitation, indicating that, over short distances, precipitation is more variable.

3. \( \lambda_1 = -0.501 \quad \lambda_2 = 0.719 \quad \lambda_3 = 0.782 \)

4. \( z_0 = -0.501 (1000) + 0.719 (1500) + 0.782 (2000) = 2141.5 \)
Assignment 7
Uncertainty, Fuzzy Logic and Relational Databases

Objectives: In this assignment you will be using fuzzy logic in a relational database schema to manipulate uncertainty in spatial data.

Representing Uncertainty in a Database: Uncertainty in spatial databases occurs because the features contained in these databases are abstractions of real-world phenomena. For example, a database containing the digitized outline of the Devereaux Slough contains uncertainty because the boundaries of the Slough change from season to season. A map of vegetation types in the Los Padres Forest contains uncertainty because it is necessary to generalize vegetation types in order to produce a legible map. A set of polygons representing different soil types in the Goleta Valley contains uncertainty because the boundary between adjacent soil types may actually be a fairly wide zone in which the soil properties of one soil type gradually merge with those of another soil type.

These example illustrate that the spatial databases you work with in a GIS are not accurate "snapshots" of the real world, but instead are imprecise "models" in which the degree and nature of uncertainty depends on the type of data represented and the purposes for which the database was designed.

Fuzzy logic can be used to represent uncertainty in spatial databases and manipulate this uncertainty as data are transformed by GIS functions. One way to implement fuzzy logic is to define a "membership function" for the features in the database. The membership function, which is usually denoted by the letter m, varies between 0.0 and 1.0 for any feature. This value indicates the degree of uncertainty in the feature. For example, assume that you assigned a value of m to each of the polygons in a soil map. A value of m near 1.0 would indicate little uncertainty (i.e., you are reasonable confident that the soil type of the polygon is correct), while a value near 0.0 would indicate a great deal of uncertainty (i.e., you are not very confident that the soil type is correct).

Your first task in this assignment is to examine how the membership function can be used to represent uncertainty. Assume that you have digitized the two maps depicted below. The first (VEG) shows vegetation types and the second (SLOPE) shows slope classes. Both maps cover the same geographical area. You can think of each digitized map as being equivalent to an ARC/INFO coverage.

The polygon attribute tables for the two coverages are also shown below. These tables are named VEG.PAT and SLOPE.PAT respectively. The items called "veg-id" and "slope-id" refer to the polygon numbers on the maps.
In order to use these membership function values to represent uncertainty, you will have to perform a relational database operation known as a relational join. This operation involves merging the attributes from two different tables based on a common item (sometimes referred to as the "key" item). For example, if you performed a relational join on VEG.PAT and VEG.MEM, you would obtain a veg-m value of 0.8 for the polygon with a veg-id value of 1, based on the common veg-type value of "chaparral."

Using a relational join, modify the polygon attribute tables for VEG and SLOPE by inserting the appropriate membership function values for each polygon. Record your answers in the table below.

VEG.PAT

<table>
<thead>
<tr>
<th>veg-id</th>
<th>veg-type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>chaparral</td>
</tr>
<tr>
<td>2</td>
<td>sage brush</td>
</tr>
<tr>
<td>3</td>
<td>grass</td>
</tr>
<tr>
<td>4</td>
<td>sage brush</td>
</tr>
<tr>
<td>5</td>
<td>barren</td>
</tr>
</tbody>
</table>

SLOPE.PAT

<table>
<thead>
<tr>
<th>slope-id</th>
<th>slope-class</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>steep</td>
</tr>
<tr>
<td>2</td>
<td>gentle</td>
</tr>
<tr>
<td>3</td>
<td>flat</td>
</tr>
<tr>
<td>4</td>
<td>gentle</td>
</tr>
<tr>
<td>5</td>
<td>steep</td>
</tr>
</tbody>
</table>

Now assume that you have available the following tables (called VEG.MEM and SLOPE.MEM), which indicate the membership function values for each veg-type and slope-class value.

VEG.MEM

<table>
<thead>
<tr>
<th>veg-type</th>
<th>veg-μ</th>
</tr>
</thead>
<tbody>
<tr>
<td>chaparral</td>
<td>0.8</td>
</tr>
<tr>
<td>sage brush</td>
<td>0.7</td>
</tr>
<tr>
<td>grass</td>
<td>0.8</td>
</tr>
<tr>
<td>barren</td>
<td>0.5</td>
</tr>
</tbody>
</table>

SLOPE.MEM

<table>
<thead>
<tr>
<th>slope-class</th>
<th>slope-μ</th>
</tr>
</thead>
<tbody>
<tr>
<td>steep</td>
<td>0.9</td>
</tr>
<tr>
<td>gentle</td>
<td>0.8</td>
</tr>
<tr>
<td>flat</td>
<td>0.6</td>
</tr>
</tbody>
</table>

In order to use these membership function values to represent uncertainty, you will have to perform a relational database operation known as a relational join. This operation involves merging the attributes from two different tables based on a common item (sometimes referred to as the "key" item). For example, if you performed a relational join on VEG.PAT and VEG.MEM, you would obtain a veg-m value of 0.8 for the polygon with a veg-id value of 1, based on the common veg-type value of "chaparral."

Using a relational join, modify the polygon attribute tables for VEG and SLOPE by inserting the appropriate membership function values for each polygon. Record your answers in the table below.
Manipulating Uncertainty in GIS Operations: Imagine that the two coverages are merged to create the following coverage (called "MERGED").

```
<table>
<thead>
<tr>
<th>veg-id</th>
<th>veg-type</th>
<th>veg-μ</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>chaparral</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>sage brush</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>grass</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>sage brush</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>barren</td>
<td></td>
</tr>
</tbody>
</table>
```

```
<table>
<thead>
<tr>
<th>slope-id</th>
<th>slope-class</th>
<th>slope-μ</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>steep</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>gentle</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>flat</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>gentle</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>steep</td>
<td></td>
</tr>
</tbody>
</table>
```

How is the membership function manipulated as the coverages are merged in this way? In order to answer that question, you first need to fill in the following polygon attribute table for MERGED based on the modified polygon attribute tables for VEG and SLOPE.
As you can see from this table, each polygon in MERGED has two membership function values, one from each of the two input coverages. The way in which these membership function values are combined depends on the way in which the attributes from the two input coverages are manipulated. Consider the set of polygons in MERGED for which veg-type = "chaparral" and slope-class = "steep". There is one polygon in this set (i.e., merged-id = 1).

The question that now arises is, "How certain are you that this polygon actually has a veg-type of chaparral and a slope-class of steep?" To answer this question you need to combine the membership function values from the two input coverages. In this case the combined membership function value is equal to the minimum of the veg-m and slope-m values for the polygon. Because the question is concerned with uncertainty in both input attributes (i.e., veg-type and slope-class), the result cannot be any more certain than the least certain of these attributes. In the example, veg-m = 0.8 and slope-m = 0.9 for the selected polygon, and therefore the combined membership function value is 0.8.

One might also ask, "How certain are you that the selected polygon actually has a veg-type of chaparral or a slope-class of steep?" This question is based on the logical operator "or" while the previous question is based on the logical operator "and." In the case of the or operator, the combined membership function value is defined as the maximum (rather than the minimum) of veg-m and slope-m. Because you are concerned with uncertainty in either input attribute (not both) the result is as certain as the most certain of these attributes. Thus the combined membership function value for the selected polygon is 0.9.

For each set of selection criteria below, list the set of polygons that meets the criteria. Then compute the combined membership function value for each polygon based on the specified logical operator.

<table>
<thead>
<tr>
<th>Selection criteria</th>
<th>Logical operator</th>
<th>Combined membership function value</th>
</tr>
</thead>
<tbody>
<tr>
<td>veg-type = &quot;barren&quot; and slope-class = &quot;gentle&quot;</td>
<td>and</td>
<td>0.8</td>
</tr>
<tr>
<td>veg-type = &quot;sage brush&quot; and slope-class = &quot;flat&quot;</td>
<td>and</td>
<td>0.9</td>
</tr>
<tr>
<td>veg-type = &quot;barren&quot; and slope-class = &quot;steep&quot;</td>
<td>or</td>
<td>0.9</td>
</tr>
<tr>
<td>veg-type = &quot;sage brush&quot; and slope-class = &quot;gentle&quot;</td>
<td>or</td>
<td>0.9</td>
</tr>
</tbody>
</table>

**Similarity Relations:** Fuzzy logic can also be used to examine how closely a set of selected polygons conform to a particular set of criteria. For example, imagine that you selected all polygons with veg-type = "sage brush" and slope-class =
"flat". Note that several other polygons have similar attributes but do not conform exactly to the criteria you have specified (e.g., polygons with veg-type = "sage brush" and slope-class = "gentle"). These polygons are quite similar to the set of polygons that you have selected, but would not actually be included in the selected set.

Such similarities can be handled using the concept of "similarity relations." A similarity relation is simply a table indicating the degree of similarity between different values of an attribute. Consider the following two similarity relations, the first for veg-type values and the second for slope-class values.

<table>
<thead>
<tr>
<th></th>
<th>chaparral</th>
<th>sage brush</th>
<th>grass</th>
<th>barren</th>
</tr>
</thead>
<tbody>
<tr>
<td>chaparral</td>
<td>1.0</td>
<td>0.8</td>
<td>0.4</td>
<td>0.0</td>
</tr>
<tr>
<td>sage brush</td>
<td>0.8</td>
<td>1.0</td>
<td>0.6</td>
<td>0.0</td>
</tr>
<tr>
<td>grass</td>
<td>0.4</td>
<td>0.6</td>
<td>1.0</td>
<td>0.0</td>
</tr>
<tr>
<td>barren</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>steep</th>
<th>gentle</th>
<th>flat</th>
</tr>
</thead>
<tbody>
<tr>
<td>steep</td>
<td>1.0</td>
<td>0.5</td>
<td>0.0</td>
</tr>
<tr>
<td>gentle</td>
<td>0.5</td>
<td>1.0</td>
<td>0.5</td>
</tr>
<tr>
<td>flat</td>
<td>0.0</td>
<td>0.5</td>
<td>1.0</td>
</tr>
</tbody>
</table>

The interpretation of these tables is relatively straightforward. For example, look at the second table, and follow the row labelled "steep" until you hit the column labelled "gentle." The value of 0.5 indicates the degree of similarity between "steep" and "gentle." There is more similarity between "steep" and "gentle" than between "steep" and "flat," for which the similarity value given in the table is 0.0. Note that the similarity between any value and itself is always 1.0, and that each table is symmetrical.

It is easier to conceptualize the similarities between slope classes than between vegetation types, but, as shown in the second table, there may be context-dependent dimensions along which the similarity between different vegetation types can also be measured.

With the aid of these similarity relations, it is possible to formulate "fuzzy" responses to questions about the attributes of any polygon. For example, you might ask how closely the polygon with merged-id = 1 conforms to the criteria of veg-type = "sage brush" and slope-class = "gentle". From MERGED.PAT you can easily determine that, for this polygon, veg-type = "chaparral" and slope-class = "steep". The above similarity relations show that the similarity between "sage brush" and "chaparral" is 0.8 and that the similarity between "gentle" and "steep" is 0.5. Since you asked a question involving an and (rather than an or), the minimum of the two similarity values is used. Thus the polygon has a similarity of 0.5 to the criteria you specified. By substituting the appropriate attribute values, the same approach can be used for any other polygon.

If the question involves an or, then the maximum of the two similarity values is used. For example, if you asked how closely the polygon with merged-id = 1 conforms to the criteria of veg-type = "sage brush" or slope-class = "gentle", the answer would be 0.8, the maximum of the two similarity values.

Compute how closely each polygon in MERGED conforms to the following criteria:

- veg-type = "grass" and slope-class = "steep"
- veg-type = "chaparral" or slope-class = "flat"

**Optional:** Implement the membership function concept in ARC/INFO. You can use the SITES and DEVELOP coverages from Assignment 3. Each item in the polygon attribute table for a coverage may have its own membership function. (You can add items to the table if you want.) Use the relational join capability of INFO to incorporate the membership function values into the polygon attribute tables of each coverage. Use the UNION command to overlay the two coverages and merge their attributes. Show how the membership function values from the input coverages would be combined when manipulating the attribute data in different ways.
Assignment 7
Answer Key

VEG.PAT

<table>
<thead>
<tr>
<th>veg-id</th>
<th>veg-type</th>
<th>veg-μ</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>chaparral</td>
<td>0.8</td>
</tr>
<tr>
<td>2</td>
<td>sage brush</td>
<td>0.7</td>
</tr>
<tr>
<td>3</td>
<td>grass</td>
<td>0.8</td>
</tr>
<tr>
<td>4</td>
<td>sage brush</td>
<td>0.7</td>
</tr>
<tr>
<td>5</td>
<td>barren</td>
<td>0.5</td>
</tr>
</tbody>
</table>

SLOPE.PAT

<table>
<thead>
<tr>
<th>slope-id</th>
<th>slope-class</th>
<th>slope-μ</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>steep</td>
<td>0.9</td>
</tr>
<tr>
<td>2</td>
<td>gentle</td>
<td>0.8</td>
</tr>
<tr>
<td>3</td>
<td>flat</td>
<td>0.6</td>
</tr>
<tr>
<td>4</td>
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MERGED.PAT

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<td>barren</td>
<td>steep</td>
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Selection criteria: veg-type = "barren" and slope-class = "gentle"
Logical operator: and
Selected polygons (merged-id): 12
Membership function values: 0.5

Selection criteria: veg-type = "sage brush" and slope-class = "flat"
Logical operator: and
Selected polygons (merged-id): 6 8
Membership function values: 0.6 0.6
Selection criteria: veg-type = "barren" and slope-class = "steep"
Logical operator: or
Selected polygons (merged-id): 13
Membership function values: 0.9

Selection criteria: veg-type = "sage brush" and slope-class = "gentle"
Logical operator: or
Selected polygons (merged-id): 3 5 9 11
Membership function values: 0.8 0.8 0.8 .8

veg-type = "grass" and slope-class = "steep"

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veg-type = "chaparral" or slope-class = "flat"

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